INVERSE METHOD FOR THE ANALYSIS OF INSTRUMENTED IMPACT TESTS OF POLYMERS

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ABSTRACT

Impact testing has become an important technique to determine the parameters associated to dynamic fracture of polymer materials. These parameters are commonly calculated from the experimentally measured load versus time curves. However, this curves are not what theoretically should be used for this purpose, because the measured load is not equal to the load exerted on the tested specimen, load from which the mechanical performance of the material must be evaluated. The recorded load is corrupted by the other forces acting during the experimental run, which depend in part on the characteristics of the tester and in part on the properties and geometry of the tested material. In order to extract from the corrupted load the useful information, a simple model composed of springs, point masses, and viscoelastic elements is used. The model is employed to formulate an inverse problem from which the load on the specimen is obtained using the recorded load. The methodology is tested using simulated as well as experimental curves of different polymer materials such as: RT-PMMA, PP, and PE. The simulated curves demonstrate the validity of the inverse technique applied. The experimental curves confirm the methodology in a real situation.

KEY WORDS

Instrumented impact test, three-point bending, bending force, analogical model, inverse problem.

INTRODUCTION

Since the advent of high speed recording equipment, impact testing has become a more useful technique than it was before, to test the most severe load conditions to which a material can be subjected. From impact testing the fracture resistance of a material can be inferred if proper interpretation of the collected data is performed. In fact, fracture resistance parameters are directly related to the bending force exerted on the tested specimen. However, the force registered by the testing instrument is not actually the bending force but is the one applied on the striker, where the transducer is mounted. The relationship between the recorded force and the one needed for the determination of the fracture parameters is not straightforward. This is due, in large part, to the very complicated nature of the dynamic phenomena involved during the time in which the tested specimen interacts with the striker and finally breaks. This fact has imposed a limitation to the direct interpretation of load-time registers, and has led several authors to propose dynamic models with different degrees of complexity with the aim to extract the actual force applied on the tested material from the measured one.

One of the first models available is that proposed in a paper by Williams [1], from which several other studies have been initiated. This model has been challenged in a paper by Zanichelli et al. [2]. These authors have presented a detailed study of the first moments of the impact testing event and proposed a model that is based on experimental evidence that shows that: i) at the beginning the tested specimen does not interact with the support, ii) the mass initially involved is not the equivalent total mass of the specimen but only a part of it initially in contact with the striker, and iii) the stiffness that really plays a role at the beginning is a local one also related to the contact area. Later, Marur et al. [3], using auxiliary measurements, have validated experimentally a complete model similar to those proposed by the authors mentioned before. More recently Pavan and Draghi [4] have developed a more complete model than the ones already available and verified it for the case in which the specimen is tested without using supports.

The need for models has been envisioned from two angles: a) as a tool to improve the understanding of the dynamic phenomena involved in impact testing; and b) as a way to connect the remote measurement (force exerted on the striker) to the sought measurement (force exerted on the material), with the purpose of extracting the latter from the former. This last approach was taken in the past by Cain [5] who used frequency domain techniques to filter the recorded registers out of spurious oscillations associated to the dynamics of impact testing. This author used a model of the same characteristics as the ones mentioned before, but no analytical development was performed. The model was just used to numerically estimate the type of filters that could be used to clean the recorded signals of unwanted oscillations.

The work presented here will consider in detail the problem of recovering the bending force acting in a three point bend test under impact loading, carried out on a falling weight impact machine, using the recorded force exerted in the striker. The methodology will be developed based on the model described by Pavan and Draghi [4]. The recovery of the bending force acting in the specimen from impact force measurements will be reported for different polymer materials. The usual differential equations that describe the behaviour of the mass-spring-dashpot configuration were transformed into a discrete model. In this form, the problem of obtaining the bending force acting in the specimen from impact measurements becomes an algebraic inverse problem. It will be shown that the solution of this problem is not easy due to the small errors present in the measurements, which appear greatly amplified in the solution. In order to obtain useful results the problem will be regularized using the Philips-Tikonov technique [6]. The parameters of the model needed to apply this methodology will be independently calculated through single determinations. Simulated and experimental load-time registers will be processed for different polymer materials. The results will demonstrate that this methodology yields adequate recoveries of the bending force.

MATHEMATICAL MODEL

As mentioned in the introduction, different models have been proposed in the literature to describe impact testing. Some of them are accurate but rather complex, others are more simple but less accurate. Based on the needs of the application proposed here, the model developed by Pavan and Draghi [4] will be used in all what follows. Figure 1 illustrates the model. This model describes in a simple manner the main effects that take place when the striker of a falling weight impact machine hits a sample in three point bending mode. The model consists of a series of point masses, springs and viscoelastic elements, connected together in a form in which the first contact between the striker and the specimen, the evolution of the portion of the specimen first in contact with the striker, and the evolution of the remaining part of the specimen, are taken into account. This model has the advantage that, despite the inclusion of all the relevant dynamic effects, it keeps the simplicity of being linear and one-dimensional. The model equations are easily derived and are the following:

$$m_t \ddot{z}_{m_t} = k_t (Vt - z_{m_t}) - k_c (z_{m_t} - z_{m_{sc}}) - r_c (\dot{z}_{m_t} - \dot{z}_{m_{sc}}) \quad (1)$$

$$m_{sc} \ddot{z}_{m_{sc}} = k_c (z_{m_t} - z_{m_{sc}}) + r_c (\dot{z}_{m_t} - \dot{z}_{m_{sc}}) - k_b (z_{m_{sc}} - z_{m_{sw}}) - r_b (\dot{z}_{m_{sc}} - \dot{z}_{m_{sw}}) \quad (2)$$

$$m_{sw} \ddot{z}_{m_{sw}} = k_b (z_{m_{sc}} - z_{m_{sw}}) + r_b (\dot{z}_{m_{sc}} - \dot{z}_{m_{sw}}) - k_a z_{m_{sw}} \quad (3)$$

The meaning of the variables and parameters involved in the equations is given in Figure 1. The striker is assumed as a large mass that moves at constant speed V. Its tup or nose is represented by a separate unit with mass m_t and stiffness k_t. Evidence of the need of representing the tup in this form are the oscillations observed when the tup is out of contact with the specimen. The force that is experimentally measured is the one acting in the tup. Ideally the spring represented by k_t accounts for the stiffness of the gauging device. Thus the force sensed in the experiment is given by:

$$P_t = k_t (Vt - z_{m_t}) \quad (4)$$

The contact force exerted on the tup by the specimen dynamically balances this force. The tup/specimen contact is modelled using a Kelvin-Voigt element having stiffness k_c and damping coefficient r_c. Thus the contact force is given by:

$$P_c = k_c (z_{m_t} - z_{m_{sc}}) + r_c (\dot{z}_{m_t} - \dot{z}_{m_{sc}}) \quad (5)$$

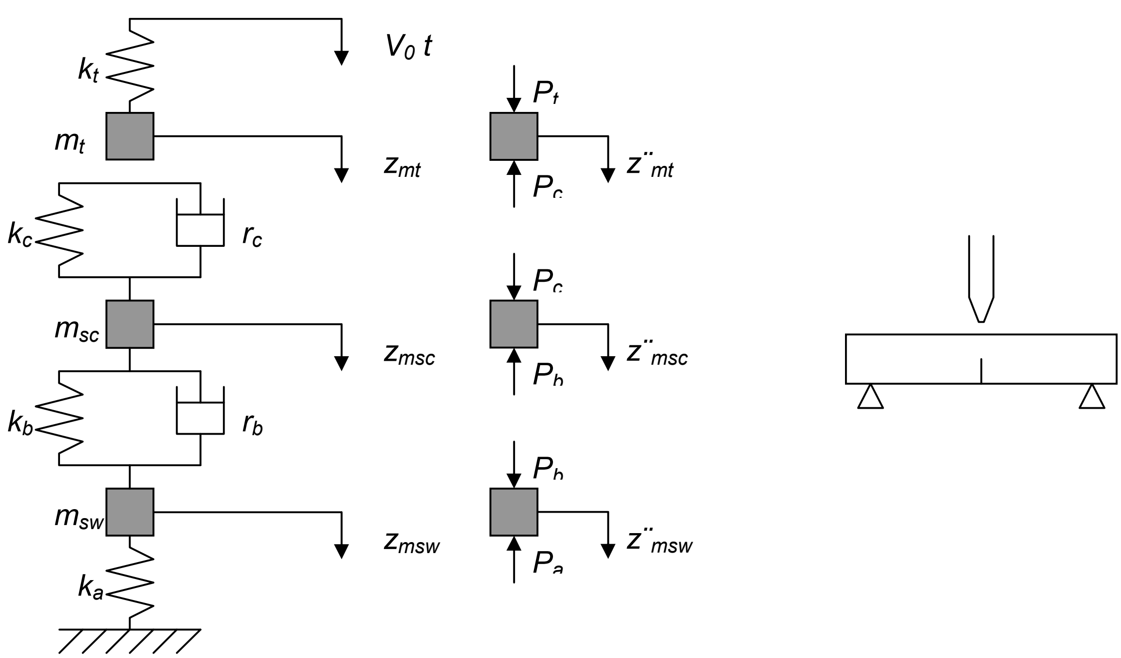

Fig. 1. Schematic representation of the model used to analyse the actual tests, and configuration of the test.

Experimentally it has been observed that the tup may loose contact with the specimen [4, 7]. This event can be included in the model equations when they are solved. In fact this can be done by stating that the contact force, P_c, is non-negative. Initially, contact between tup and specimen is assumed and the equations solved with proper boundary conditions. As soon as P_c becomes negative, integration is stopped and, the model and boundary conditions updated. This is done by removing the Kelvin-Voigt element and setting the new boundary conditions equal to the speeds and positions of the point masses of the different elements. In this form Eq (1) runs now independently of Eqs (2-3). This situation is kept until the two point masses become in contact again. The procedure is repeated as many times as P_c becomes negative.

To model the flexural dynamics of the test specimen, two masses and a Kelvin-Voigt element are used. The first mass, m_{sc}, represents the inertia of the central part of the specimen and it is also the mass first involved in the local interaction at the contact point. The second mass, m_{sw}, represents the inertia of the wings of the specimen.

It is important to notice that under the assumptions of this model, P_c does not represent the force responsible for the flexural deformation of the specimen. Under dynamical conditions P_c acts only locally producing mainly indentation. The actual force acting in the specimen and related to its bending is:

$$P_b = k_b(z_{m_{sc}} - z_{m_{sw}}) + r_b(\dot{z}_{m_{sc}} - \dot{z}_{m_{sw}}) \qquad (6)$$

The bending force, P_b, is the one that ideally should be determined from the impact test. What is proposed in this work is to treat P_b as an unknown function, and try to infer it from what is actually measured; i.e. P_t. In order to follow this approach not all the equations of the model are needed. With this in mind, the model to be used is reduced to Eqs (1,2) with the last two terms of Eq (2) replaced by Eq (6). In this form, from all the parameters originally involved in the model (nine), only k_t, m_t, k_c, r_c and m_{sc} need to be determined to complete the model. If these five parameters are determined independently, a model that relates P_b and P_t would be available. Thus P_t can be certainly calculated from P_b using Eqs (1,2). As stated before this is not the problem of interest, what is sought is to estimate P_b from P_t, something that in principle is not obvious.

Mathematical Model in Matrix Form

In order to extract P_b, from P_t and the model, a possible strategy is first to transform the differential model into an integral one, and then discretize it to obtain a set of linear algebraic equations. These equations in matrix form can be, in principle, easily inverted to estimate P_b. The first step is to obtain the transfer function between P_t and P_b. First Eqs (1-2) are expressed in terms of P_t and P_b as follows:

$$m_t\ddot{z}_{m_t} = P_t - k_c(z_{m_t} - z_{m_{sc}}) - r_c(\dot{z}_{m_t} - \dot{z}_{m_{sc}}) \qquad (7)$$

$$m_{sc}\ddot{z}_{m_{sc}} = k_c(z_{m_t} - z_{m_{sc}}) + r_c(\dot{z}_{m_t} - \dot{z}_{m_{sc}}) - P_b \qquad (8)$$

Applying Laplace transform to Eqs (7,8) the following transfer function is obtained:

$$\overline{P}_t(s) = H_1(s)\overline{\delta}(s) + H_2(s)\overline{P}_b(s) \qquad (9)$$

where the upper bar indicates Laplace transformed variable. $\overline{\delta}(s)$ is the transform of the Dirac $\delta(t)$ function and the transfer functions of Eq (9) are given by:

$$H_1(s) = -m_{sc}Vk_t(r_cs + k_c)/a(s) \qquad (10)$$

$$H_2(s) = -k_t(r_cs + k_c)/a(s) \qquad (11)$$

with

$$a(s) = m_tm_{sc}s^4 + r_c(m_t + m_{sc})s^3 + (k_c(m_t + m_{sc}) + k_tm_{sc})s^2 + k_tr_cs + k_tk_c \qquad (12)$$

In time domain Eq (9) can be written as

$$P_t(t) = \int_0^t \delta(\tau) h_1(t-\tau) d\tau + \int_0^t P_b(\tau) h_2(t-\tau) d\tau \qquad (13)$$

with

$$h_1(t) = L^{-1}\{H_1(s)\} \qquad (14)$$

$$h_2(t) = L^{-1}\{H_2(s)\} \qquad (15)$$

where L^{-1} indicates inverse Laplace transform.

Eq (13) can be discretized using any quadrature formula and written in algebraic form as follows:

$$\mathbf{p}_t = \mathbf{h} + \mathbf{A}\mathbf{p}_b \qquad (16)$$

Here $\mathbf{p}_t$ is a vector containing the values of $P_t(t)$ at the discretization times, **h** is a vector containing the values of the first integral in Eq (13) at the discretization times, **A** is a matrix result of the quadrature process used in the second integral in Eq (13), and $\mathbf{p}_b$ is a vector containing the unknown values of $P_b(t)$ at the discretization times. It is important to note that, as pointed out before, the value of m_{sc} must also be considered as unknown. Therefore, it must be kept in mind that in Eq (16) the elements of **h** and **A** are functions of m_{sc} and could be more precisely written as $\mathbf{h}(m_{sc})$ and $\mathbf{A}(m_{sc})$.

EXPERIMENTAL DETAILS

Instrumented Impact Tests

Experiments were conducted on different commercial polymeric materials. Polypropylene homopolymer (PP), mid-density polyethylene (MDPE), and rubber toughened polymethylmetacrylate (RT-PMMA), kindly supplied by Petroquimica Cuyo SAIC, Siderca, and Ineos Acrylics, respectively. Pellets of the materials were compression molded into 10 mm thick plaques. Rectangular bars for fracture experiments were cut and then machined to reach the final dimensions and improve edge surface finishing. Sharp notches were introduced by scalpel-sliding a razor blade having an on-edge tip radius of 13 µm. The specimen thickness, B, and the span to depth ratio, S/W, were always kept equal to $W/2$ and 4 respectively. The notch-depth to specimen-width ratio was varied from 0.1 to 0.9 in every case. Crack length, a_0, was determined posmorten from the fracture surface using a Profile Projector with a magnification of 20x.

Pre-cracked specimens were tested in three point bending (mode I) at room temperature and at V_0 =1 m/s using a falling weight type machine Fractovis 6789 by Ceast.

Determination of the Model Parameters

The striker stiffness (k_t). The stiffness of the striker may be obtained by making it to hit a highly rigid surface, such as steel. Under these conditions, the model of Fig. 1 can be precisely approximated during the first moments of the impact, by the simple configuration of Fig. 2. The condition imposed before, i.e. the constant speed of the large mass M of the striker is removed now and replaced by an initial speed, V_0 at the moment of impact. The solution of this model in terms of the force applied on the sensing device is:

$$P_t = V_0 k_t \frac{sen\sqrt{k_t/M}\, t}{\sqrt{k_t/M}} \qquad (17)$$

The derivative of P_t vs. t at t=0 gives V_0k_t. Thus the slope of the recorded load-time curve at t=0 (see Fig. 3) together with the known speed at impact, gives the value of k_t. This parameter, which depends only on the machine, was obtained at a speed of 0.5 m/s. With this value of k_t, M can also be calculated by fitting Eq (17) to the experimental register.

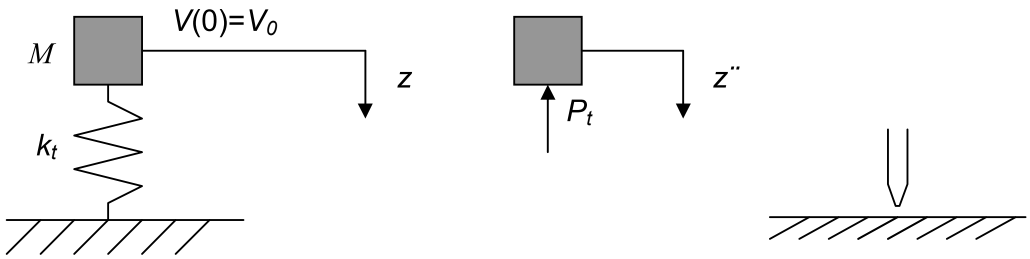

Fig. 2. Schematic representation of the model and test configuration used to estimate k_t and M.

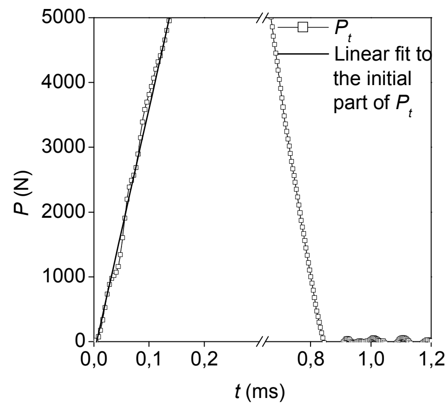

Fig. 3. Rebound test on steel placed on a flat rigid surface: recorded force P_t (‐□‐) and linear fit to the initial part of P_t (—).

The tup equivalent mass (m_t). The mass of the tup can be obtained when it freely oscillates after the completion of the impact test or when no sample is used. From the load-time curve recorded after fracture or without sample, the frequency of the free oscillations, $\omega = \sqrt{k_t / m_t}$, can be obtained. With the value of k_t already available, m_t can be determined.

The tup/specimen contact stiffness and damping coefficient (k_c and r_c). The tup/specimen contact stiffness, k_c, and damping coefficient, r_c, can be determined by performing an additional rebound test in which the specimen is tested laid on a flat rigid surface. In this case the proposed model is reduced to that shown in Fig. 4. Again, and because this is also a rebound test, the assumption of constant speed is replaced by an initial speed at t=0. In this particular set up only the two contact parameters are unknown. These will be estimated for different materials by fitting the model simplified as in Fig. 4, to the initial parts of the rebound test registers in which the speed of the striker remains fairly constant, more precisely up to a time when the speed reduced 10% of its initial value. This is a condition imposed by the fact that the damping coefficient is rate dependent. Contact parameters were estimated for three materials: polypropylene homopolymer (PP), mid-density polyethylene (MDPE), and rubber toughened polymethylmetacrylate (RT-PMMA). In Fig. 5 the model fit to the experimental points is shown for PP as an example.

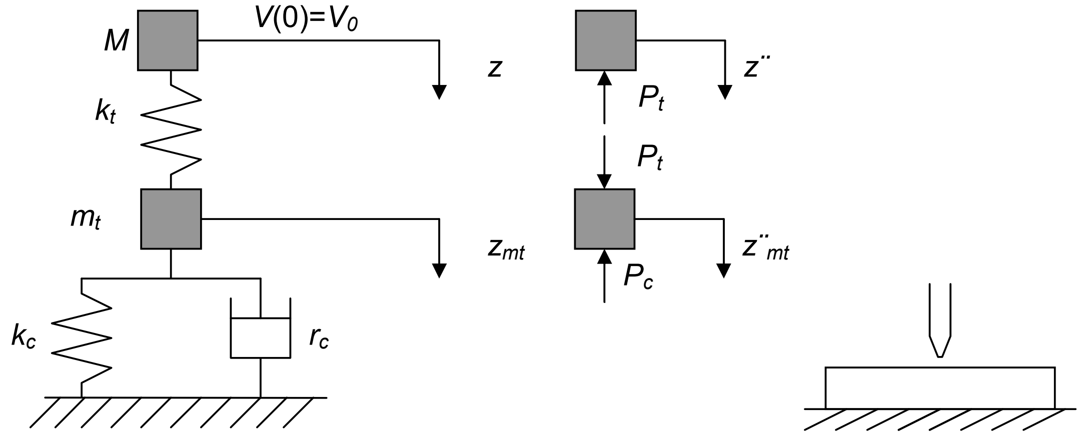

Fig. 4. Schematic representation of the model and configuration test used to estimate k_c and r_{sc}.

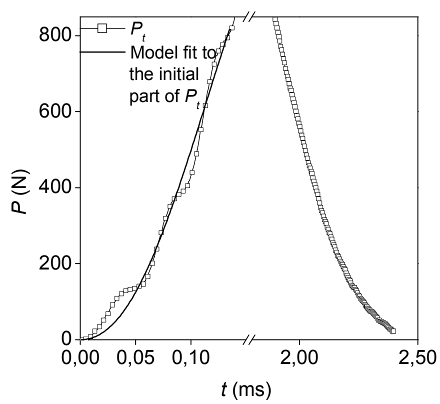

Fig. 5. Rebound test on PP placed on a flat rigid surface: recorded force P_t (-□-) and model fit to the initial part of P_t (—).

The value of the contact mass, m_{sc}, depends not only on the material but also on the geometry of the sample. For this reason, it is highly desirable to obtain its value from the register acquired when the actual sample is tested. In the next section it will be explained how the value of m_{sc} can be estimated at the same time as the main unknown, P_b.

ANALYSIS: THE INVERSE PROBLEM

As stated before, given the force exerted on the tup, P_t; P_b, the bending force, must be obtained from Eq (16). Assume for simplicity that the value of m_{sc} is known. The solution of this equation for the general case in which the number of experimental determinations of P_t, i.e. m, is larger than the number of elements of P_b, i.e. n, is given, in principle, by the least squares solution of an over specified system of linear equations

$$\mathbf{p}_b = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T(\mathbf{p}_t - \mathbf{h}) = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{p}'_t \qquad (18)$$

Although this solution appears to be straightforward, it is well documented in the literature [6,8-10] that small errors in $\mathbf{p}_t$ (i.e., quadrature and experimental errors) result in large errors in $\mathbf{p}_b$. The amplification of errors occurs independently of the fact that the inverse of $(A^T A)$ can be calculated exactly, and it is a direct consequence of the near singularity of the matrix A (if $m=n$), or more generally (if $m>n$) of its near incomplete rank.

However, by constraining the least-squares solution by means of a penalty function, approximate useful solutions can be obtained [11]. This implies to extend the original least squares problem to

$$\min_{\hat{\mathbf{p}}_b, m_{sc}} \left\{ \left| \mathbf{p}_t'(m_{sc}) - \mathbf{A}(m_{sc})\hat{\mathbf{p}}_b \right|^2 + \gamma q(\hat{\mathbf{p}}_b) \right\} \tag{19}$$

where $q(\hat{\mathbf{p}}_b)$ is a scalar function that measures the correlation or smoothness of $\hat{\mathbf{p}}_b$, and γ is a nonnegative parameter that can be varied to emphasize more or less one of the terms of the objective functional given by the previous equation. Note that we have included the value of m_{sc} as unknown that makes the inverse problem to be solved, non-linear. If γ is set to 0, the equation reduces to the previous case, a solution that generally exhibits large oscillations. On the other hand, when $\gamma \rightarrow \infty$ the minimization leads to a perfectly smooth solution judged by the measure of $q(p_b)$ but totally independent of p_t and, therefore, useless. Clearly, intermediate values of γ, that balance the amount of fitting to the data, p_t, against the amount of smoothness of p_b, are the ones expected to produce acceptable solutions to the original problem. Several functions can be chosen to establish the desired correlation level or the smoothness of p_b. An interesting class of functions can be formulated by using a quadratic form of the vector p_b with the advantage that they yield in part an analytical solution to the minimization problem. For example,

$$q(\hat{\mathbf{p}}_b) = \sum_{i=2}^{n-1} (2\hat{p}_{b_i} - \hat{p}_{b_{i-1}} - \hat{p}_{b_{i+1}})^2 = \hat{\mathbf{p}}_b^T \mathbf{H} \hat{\mathbf{p}}_b \tag{20}$$

Where matrix $\mathbf{H} = \mathbf{K}^T \mathbf{K}$ is given by

$$\mathbf{H} = \begin{bmatrix} 1 & -2 & 1 & 0 & . & . & 0 \\ -2 & 5 & -4 & 1 & 0 & . & 0 \\ 1 & -4 & 6 & -4 & 1 & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & 1 & -4 & 6 & -4 & 1 \\ 0 & . & 0 & 1 & -4 & 5 & -2 \\ 0 & . & . & 0 & 1 & -2 & 1 \end{bmatrix} \tag{21}$$

Thus, it can be shown that the solution to the minimization problem of Eq (19) is given by[12]:

$$\hat{\mathbf{p}}_b = [\mathbf{A}^T(m_{sc})\mathbf{A}(m_{sc}) + \gamma \mathbf{H}]^{-1} \mathbf{A}^T(m_{sc}) \mathbf{p}_t'(m_{sc}) \tag{22}$$

where the value of m_{sc} is the one that minimizes the single variable equation obtained after replacing Eq (22) in Eq (19), i.e.:

$$\phi(m_{sc}) = \left\{ \left| \mathbf{p}_t'(m_{sc}) - \mathbf{A}(m_{sc})\hat{\mathbf{p}}_b \right|^2 + \gamma q(\hat{\mathbf{p}}_b) \right\} \tag{23}$$

To compute γ several methods have been proposed. In this work, the Generalized Cross Validation (GCV) technique will be preferred. GCV is explained in detail elsewhere [13]. The value of γ computed by means of GCV is the one that minimizes the following function:

$$V(\gamma) = \frac{\left| (\mathbf{I} - \mathbf{Z}) \mathbf{p}_t \right|^2}{\left| Trace(\mathbf{I} - \mathbf{Z}) \right|^2} \tag{24}$$

where $\mathbf{Z} = \mathbf{A}(\mathbf{A}^T\mathbf{A}+\gamma\mathbf{H})\mathbf{A}^T$ and $\mathbf{I}$ is the identity matrix. It is clear that for each value of m_{sc} there is a different value of γ that minimizes Eq (24) and then Eq (23) must be minimized under this condition.

RESULTS

Verification of the Methodology: Application to a Simulated Load-Time Curve

In order to test the proposed methodology before using it with real data, a synthetic experiment is generated using the complete model of Fig. 1 with the parameters obtained by other authors for RT-PMMA [14] (a/W=0.5, k_t=110 MPa.m, k_c=3.94 MPa.m, k_b=0.459 MPa.m, k_a=11.58 MPa.m, m_t=29.5 g, m_{sc}=6.5 g, r_c=62 Ns/m, r_b=3.2 Ns/m, V=1 m/s).
The simulated register of P_t generated using all the parameters is plotted in Fig. 6. The "real" P_b is also plotted to compare the curves estimated at different values of γ. We follow the proposed methodology and then assume that only the values of k_t, m_t, k_c, and r_c are known as a result of preliminary tests. Equations (22) and (23) are then used to estimate P_b and m_{sc}. The results are also plotted in Fig. 6 for different values of γ. Note that for very small γ (10^{-25}) the recovered P_b is very oscillatory and it differs completely from the "real" one. This result validates the preventions taken at the moment of inverting Eq (17); such small γ is in practice equivalent to have inverted Eq (17) without any precaution; i.e. without regularization. When a value of $\gamma=10^{-15}$ was used the estimated P_b follows quite well the "real" curve. This γ makes the estimated value of m_{sc} be exactly the "real" one.
These results theoretically validate the proposed methodology and alert on the attention that must be put on the selection of the regularization parameter γ, because it affects not only the estimated curves with, sometimes, obvious spurious oscillations, but also the parameter m_{sc} with no simply observable effects.

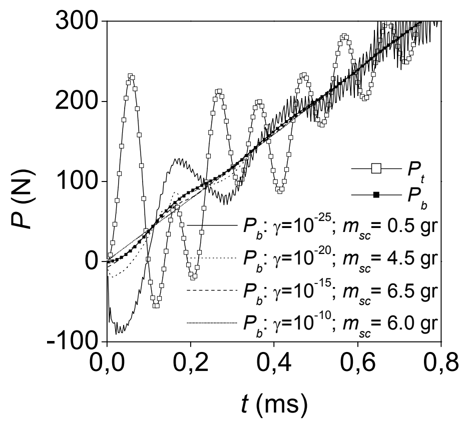

Fig. 6. Simulation example using a set of parameters taken from the literature [ref]: recorded force P_t (–□–), actual bending force P_b (–■–), estimated P_b with $\gamma=10^{-25}$ (—), estimated P_b with $\gamma=10^{-20}$ (······), estimated P_b with $\gamma=10^{-15}$ (-----), estimated P_b with $\gamma=10^{-10}$ (—).

Application to Real Load-Time Curves

The estimated tup parameters are: k_t=72.6 MPa.m, M=5.317 kg and was m_t=0.189 kg. The tup/specimen contact parameters estimated as stated in previous sections are listed in Table I.

Three typical load-time curves, with different notch-depth to specimen-width ratio, *a/W*, of PP, MDPE and RTPMMA are plotted in fig. 7, 8 and 9 respectively. Also, the recovered bending force is plotted together with the recorded force, for all cases. The estimated values of the mass of the central part of the specimen, m_{sc}, are listed in Table II. The values of P_b and m_{sc} obtained are in agreement with those expected. Being the former a smoothed version of the force registers and the latter in harmony with the total mass of the tested specimens.

Table I. Tup/specimen contact parameters

Material	PP	MDPE	RTPMMA
k_c (MPa.m)	4.447	0.237	0.5403
r_c (Ns/m)	209	15	469

Table II. Estimated masses of the central part of the specimen for all tested materials

Material	PP			MDPE			RTPMMA		
a/W	0.26	0.34	0.48	0.2	0.4	0.5	0.16	0.23	0.57
m_{sc} (gr)	2.5	2	1.5	0.5	0.5	0.5	9	7.5	9.5

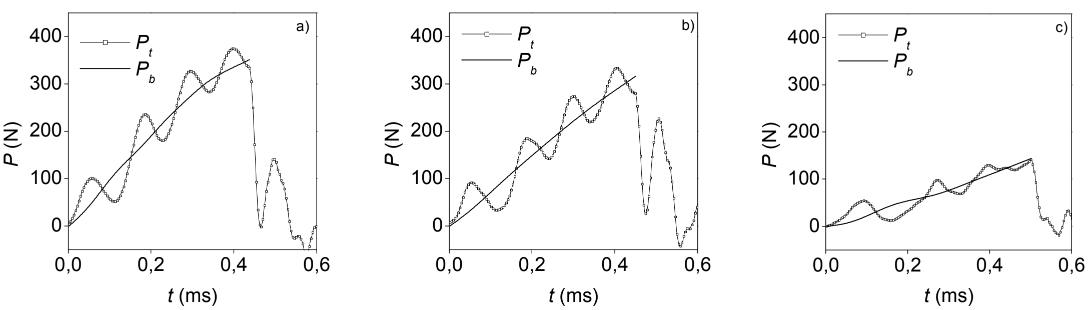

Fig. 7: Load-time signals for PP, measured P_t (-□-) and recovered P_b (—). a) a/W=0.26; b) a/W=0.34; and c) a/W=0.48.

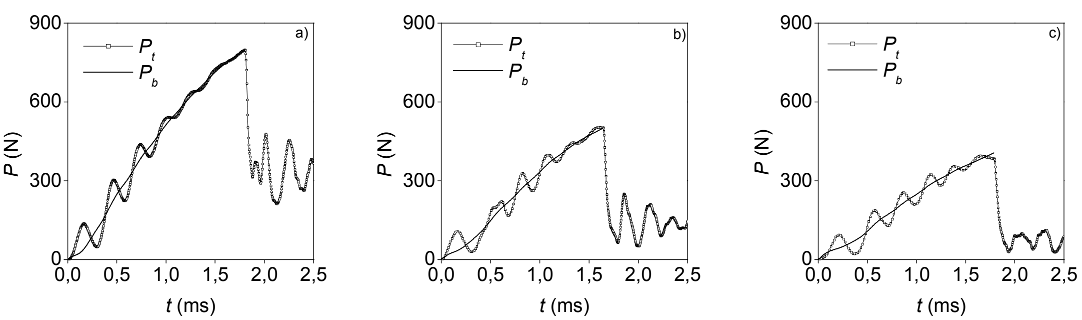

Fig. 8. Load-time signals for PE, measured P_t (-□-) and recovered P_b (—). a) a/W=0.2; b) a/W=0.4; and c) a/W=0.5.

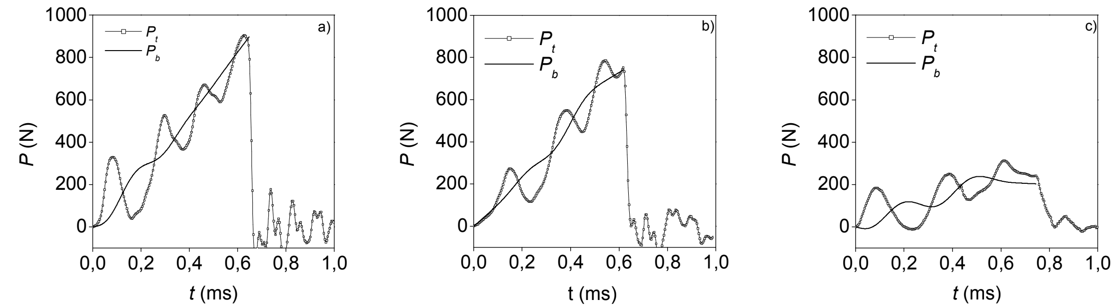

Fig. 9. Load-time signals for RT-PMMA, measured P_t (–□–) and recovered P_b (—). a) a/W=0.16; b) a/W=0.23; and c) a/W=0.57.

CONCLUSIONS

A comprehensive methodology to process load-time registers from impact tests in three point bending mode has been proposed. This methodology is based on a model available from the literature. A non-linear inverse problem in which the unknowns are the bending force and the contact mass must be solved. The solution of the resulting inverse problem requires the calculation of a regularization parameter. This parameter is automatically calculated for each run based only on the model an the experimental register. As far as the authors know this is the first work in which inversion techniques are used to solve this problem.

The proposed method has the following characteristics: a) Not all the parameters of the model used to develop the methodology need to be known. From the original nine parameters only five are required. b) Independent calibration is only needed for the two parameters related to the testing machine (k_t and m_t) and the two related to the material but not to the geometry and other characteristics of the tested specimen (k_c and r_c), the fifth parameter, m_{sc}, related to both material and geometry is estimated during the actual test. This fact reduces the number of independent calibration experiments to: 1) one to characterize the stiffness of the tup, 2) one to characterize the mass of the tup, and 3) one for each new material to be tested, regardless of geometry and other characteristics of the sample and the machine such as crack length, span, and rigidity of the supports. In other words no model for the tested specimen needs to be adopted. c) The tests designed to determine the parameters that must be known in advance (k_t, m_t, k_c and r_c), are strictly based on the original model and estimated under similar dynamic conditions as the actual tests.

Compared to other methods used to extract the bending force from impact tests, it can be noted that using the present approach several drawbacks existing with the other methodologies are removed: a) the increase of fracture time and addition of nonlinearities characteristic of mechanical damping; b) the lack of foundation of methods used to numerically smooth the experimental registers; c) the additional cost incurred when samples are instrumented.

Finally, the results obtained when real load-time curves are processed are in good agreement with the ISO protocole.

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REFERENCES

[1] Williams, J. G. and Adams, G. C. (1987) *Int. J. Fracture* **33**, 209

[2] Zanichelli, C., Rink, M., Pavan, A. and Ricco, T. (1990) *Polym. Eng. Sci.* **30**, 1117

[3] Marur, P. R., Simha, K. R. Y. and Nair, P. S. (1994) *Int. J. Fracture* **20**, 139
[4] Pavan, A. and Dragui, S. (2000) In: *Fracture of Polymers, Composites and Adhesives*, pp. 347-361, Williams, J. G. and Pavan, A. (Eds). Elsevier, The Netherlands
[5] Cain, P.J. (1987) In: *Instrumented Impact Testing of Plastics and Composite Materials*, pp. 81-102, S.L. Kessler, G.C. Adams, S.B. Driscoll, and D.R. Ireland (Eds). American Society for Testing and Materials, Philadelphia.
[6] Phillips, D. L. (1962) *J. Assoc. Comput. Mach.* **9**, 84
[7] Kalthoff, J. F. (1985) *Int. J. Fracture* **27**, 277
[8] Twomey, S. (1963) *J. Assoc. Comput. Mach.* **10**, 97
[9] Twomey, S. (1977) *Introduction to the Mathematics of Inversion in Remote Sensing and Indirect Measurements*. Elsevier, New York.
[10] Bertero, M., De Mol, C. and Viano, G. A. (1980) In: *Inverse Scattering Problems in Optics; Topics in Current Physics*, pp. 161, Baltes, H. P. (Ed). Springer Verlag, New York.
[11] Eliçabe, G. E. and Garcia Rubio, L. H. (1990) In: *Polymer Characterization. Physical Property, Spectroscopic, and Chromatographic Methods*, pp. 83-104, C. Craver and T. Provder (Eds). Advances in Chemistry Series, **227**.
[12] Frontini, G. L. and Eliçabe G. E. (2000) *J. Chemometrics* **14**, 51
[13] Golub, G. H., Heath, M. and Wahba, G. (1979) *Technometrics* **21**, 215
[14] Dragui, S. (1996) Thesis, Politecnico di Milano, Italy.