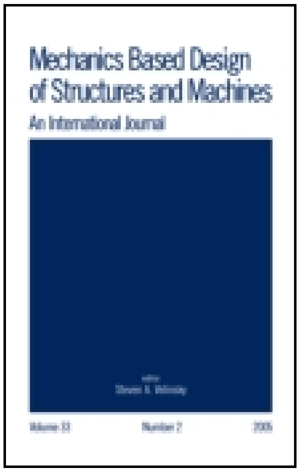

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TOPOLOGICAL SYNTHESIS OF PLANAR METAMORPHIC MECHANISMS FOR LOW-VOLTAGE CIRCUIT BREAKERS#

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This article presents a systematic graph theory-based method for the topological synthesis of planar metamorphic mechanisms including metamorphic transformations of links and changes in the degrees-of-freedom. The parts to move, with input and output motion defined, and the topological design space, are represented by graphs of simple-jointed mechanisms. The topological requirements involving link transformations are expressed in terms of subgraphs with a given degrees-of-freedom containing prescribed input and output parts. The algorithm executes two subgraph searches inside atlases of mechanisms with different degrees-of-freedom. An application to the design of a family of low-voltage circuit-breaker mechanisms is shown.

Keywords: Finite-state machine; Graph theory; Low-voltage circuit breakers; Metamorphic mechanisms; Topological synthesis.

1. INTRODUCTION

A metamorphic mechanism (MM)—also called reconfigurable mechanism (Zhang and Dai, 2009) or mechanism with variable topology (Yan and Kuo, 2006)—has the capacity of changing its topology and configuration under different operation conditions. The transformations of links and/or joints produce changes on the mobility of one or more members preserving or changing the degrees-of-freedom (DOFs) of the mechanism. The transformations of links can consist in changes of connectivity by collapsing or releasing bodies subject to contact (e.g., binary to ternary link) or changes of function (e.g., input to passive, movable to fixed), among other possibilities. The transformations of joints can consist of changes in type (e.g., cam to revolute) or changes in a property; for instance, the axis orientation of a joint can change from planar to outer-planar (e.g., prismatic to slider).

In the last 20 years, the mathematical modeling and computer-aided synthesis of MMs (Zhang and Dai, 2009) have attracted new attention of the multibody

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community including packaging (Dai and Rees Jones, 2005; Li et al., 2011), machine, mechanism (Kuo, 2004; Yan and Kuo, 2006), and robot designers (Martins and Simoni, 2009). However, the redesign of MMs is relatively new. From the topological point of view, several advances have been made on the representation of MMs (Dai and Rees Jones, 2005; Kuo, 2004; Lan and Du, 2008; Martins and Simoni, 2009; Slaboch and Voglewede, 2011); however, few of them are focused on the enumeration (Yan and Kang, 2009; Martins and Simoni, 2009). The size of the enumeration impacts on the number of required multibody simulations and optimizations at the detailed design stage (Fig. 1). For this reason, the rules for topological enumeration must contain all information related to desired kinetostatic and dynamic behaviors.

This work has the practical objective of finding a methodology for cataloguing and enumerating the existent and eventually new low-voltage circuit-breaker (LVCB) mechanisms from the point of view of mechanism topology. The final goal is to formalize the current design knowledge and improve designs. These electromechanical devices are used to protect human lives in electrical circuits. The mechanisms under study form a family among a wide variety of circuit breakers with different features and requirements. Existing LVCB mechanisms of this family have complex requirements involving multiple stable states, multiple operations, and several functions. Some of the most important operational requirements are: fast interruption of the electric circuit (to be performed in some milliseconds), low energy of actuation (of manual, electrical, thermal, or magnetic origin), and reduced variability of forces and moments required by the mechanical parts. Their performance is constantly improved in current designs using well-known

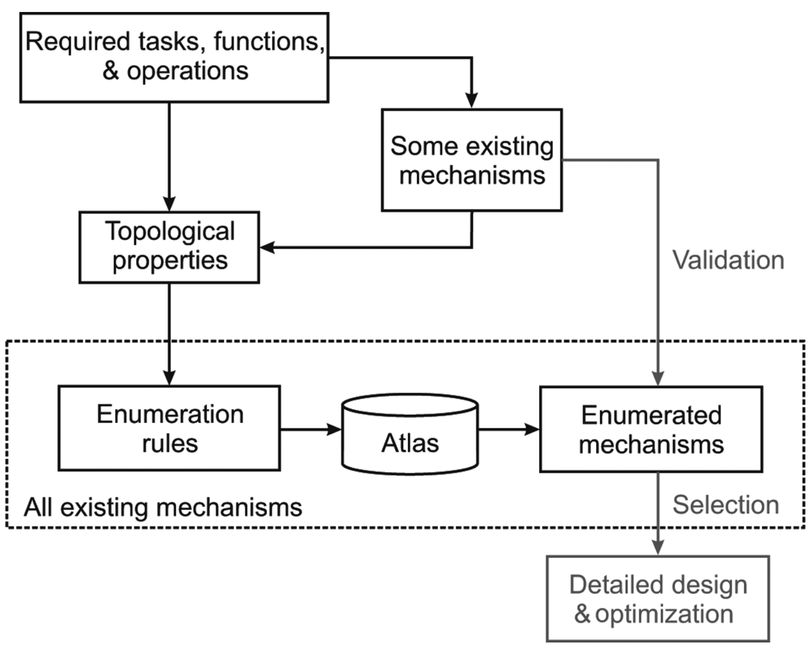


Figure 1 Automated conceptual design of mechanisms.

experimental and numerical optimization procedures. However, these procedures do not allow establishing whether a design with a different topology would provide a better performance.

The conceptual design of MMs is a difficult combinatorial problem: the number of solutions grows exponentially with the number of operations to be satisfied and with the number of changes allowed to be performed by the different links and joints. This article presents a systematic method for the topological synthesis of mechanisms, taking into account metamorphic transformations of links and DOFs required for the main operations. The method is based on Graph Theory concepts (Jobes et al., 1990; Pucheta and Cardona, 2007, 2008; Liu and McPhee, 2007; Pucheta, 2008) and can be applied to the design and re-design of mechanisms satisfying complex metamorphic requirements. An application to the design of circuit-breaker mechanisms is shown.

2. PROBLEM DESCRIPTION

A LVCB mechanism has two kinds of inputs: a manual handle, denoted as I_1 in Fig. 2, and one or more internal inputs (actuated by a bimetal, magnetic plunger, relay, etc.), which actuate over a delatching lever (DL), see I_2 and I_3 in Fig. 2. The main output O_1 of the mechanism is the contact carrier (CC); it contains and also isolates the metallic contacts, which close the electric circuit. The CC is closed only by closing the handle (manually or automatically through accessories that move the handle remotely). Under electrical failure conditions, the internal inputs must open the contacts even when the handle is intentionally locked; and hence, the configuration of the remaining parts of the mechanism may also be considered as an output O_2. The problem to solve consists in enumerating the mechanisms that contain these input and output parts, and fulfill a given set of operations or transitions as described next. Springs and members, able to store energy, are ignored at this initial stage.

2.1. Stable States, Energy Requirements, and Transitions

The operations of the mechanisms can be represented by a finite-state machine (FSM), either in tabular or graphic form (Kuo, 2004; Yan and Kuo, 2006; Rosen, 2007), see the digraph of an LVCB mechanism in Fig. 3. An FSM is an algebraic structure, denoted as $M = (\boldsymbol{S}, \boldsymbol{I}, f, s_0, \boldsymbol{O})$, and consists of a finite set of states $\boldsymbol{S}$, a finite input alphabet $\boldsymbol{I}$, a transition function f that assigns a next state to every pair of state and input ($f: \boldsymbol{S} \times \boldsymbol{I} \rightarrow \boldsymbol{S}$), an initial state s_0, and a subset $\boldsymbol{O}$ of $\boldsymbol{S}$ consisting of final or output states. In the MMs context, each state corresponds to configurations with different mobility, including partial structures and overconstrained mechanisms.

This useful discrete representation is not enough to describe the continuous transitions between states, which can be further composed of more discrete substates with topological changes between them.

In this work, the FSM representation is used to identify the number of stable states of the LVCB mechanisms (and thus the n-stability requirements) and is also useful for analyzing the requirements of the transitions. It is worth to

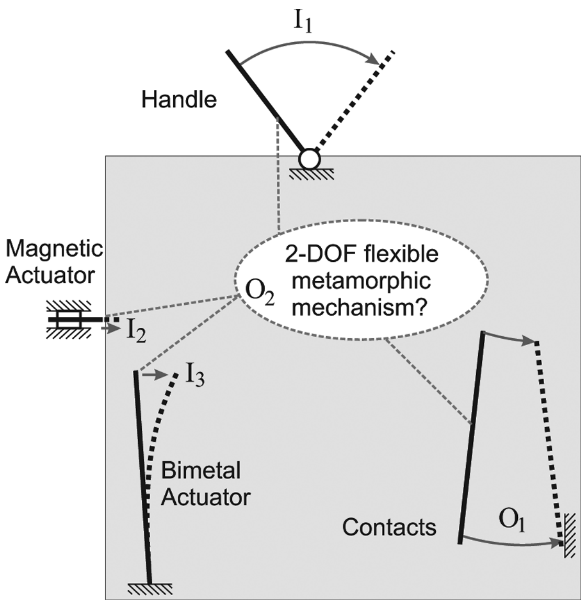


Figure 2 Required input and output parts to move.

mentioning that mechanisms designed for other applications can have the same FSM representation (motion homomorphic; Yan and Kuo, 2006), thus the number of existing designs can be increased. The basic maneuvers common to all circuit breakers can be represented by one FSM, see Fig. 3 and its tabular representation in Appendix A.

The meaning of the states in Fig. 3 is the following:

1. s_0: Contacts in off status, armed mechanism.
2. s_1: Contacts in on status, armed mechanism.

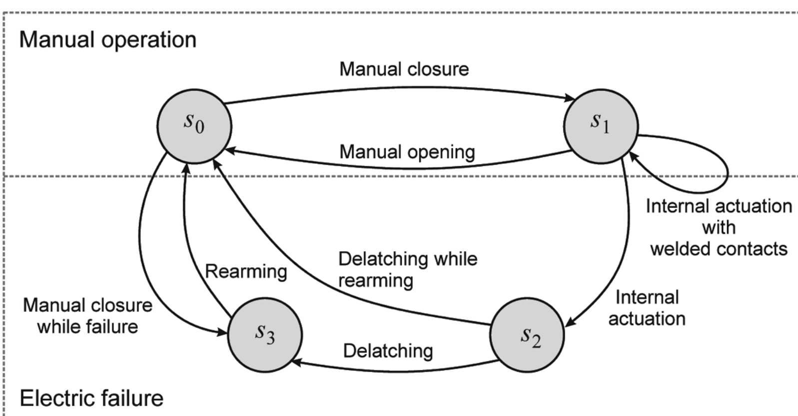


Figure 3 Required operations and stable states: s_0: open (armed), s_1: closed, s_2: open contacts after delatching, s_3: safe delatched position.

3. s_2: Contacts still in on status, disarmed mechanism produced by motion of the magnetic or bimetal actuator (electric failure).
4. s_3: Contacts in off status, disarmed mechanism in safe configuration.

The operations consist of a set of the following transitions:

1. Manual Closure s_0 to s_1 with delatching submechanism armed.
2. Manual Opening s_1 to s_0 with delatching submechanism armed.
3. Electromechanical Disarming s_1 to s_2 by means of internal actuation.
4. Mechanical Delatching and Rearming s_2 to s_0 by means of delatching process with unloaded handle.
5. Mechanical Delatching s_2 to s_3 by means of delatching process with locked handle.
6. Automatic mechanical Rearming s_3 to s_0 by means of handle spring.
7. Manual Closure s_0 to s_3 with disarmed delatching submechanism (some internal input is activated).
8. Failure with welded contacts; this undesired situation corresponds to a failure of the mechanism due to several reasons, e.g., a failure in the internal actuators.

Manual operation consists of opening and closing between two stable states. While the handle is in one of these two states or in a transition, a submechanism, independent of the handle position, hereafter called delatching submechanism, must be able to open the contacts under electric failure. Therefore, the mechanism topology must have at least 2 DOFs. Since this opening under failure must be performed in a prescribed short time, necessarily, a considerable amount of energy must be stored when the mechanism is closed. This energy is stored in the main springs, often connected directly to the CC and depends on human hand force, length of the handle, and demultiplication of the mechanism.

In terms of energy requirements, s_0, s_1, and s_3 are constrained stable states, whereas s_2 is a highly unstable state, see, for instance, the state G in Fig. 4. The mechanism has a bi- or tri-stable behavior, depending on whether the handle position is free or locked.

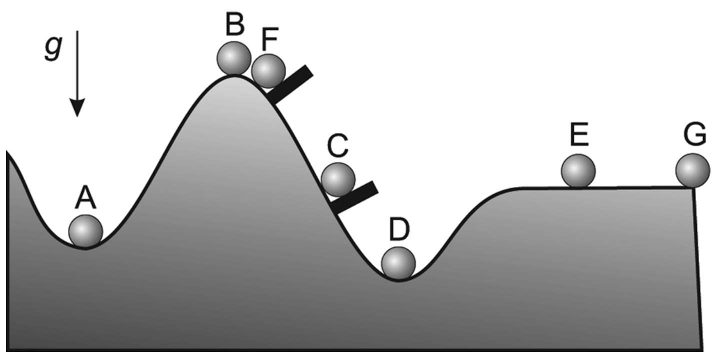


Figure 4 Possible energy states. (A) and (D) are stable; (B) is unstable; (C) and (F) are externally constrained stable; (E) is neutrally stable; and (G) is in a neutral state but near to a highly unstable transition.

3. GRAPH REPRESENTATIONS OF MECHANISMS WITH VARIABLE TOPOLOGY

Graph representations are adequate for modeling the initial topology and for imposing desired topological constraints (Pucheta and Cardona, 2007, 2008; Pucheta, 2008). This technique is well defined for mechanisms with simple joints; bodies are represented by vertices and joints by edges connecting a pair of vertices (Tsai, 2001).

Mechanisms with multiple joints (i.e., joints with more than two incident members) can be represented by larger mechanisms with simple joints; however, the conversion is not unique and several binary joints mechanisms equivalent to a mechanism with multiple joints can be found in Yan (1998, pp. 107–108).

A mechanism with higher pairs has its own graph representation but also admits—using several conversion rules—a graph representation in terms of lower pairs (allowing 1-DOF per joint) called associated linkage (Tsai, 2001) or generalized linkage (Yan, 1998). These simplified linkage mechanisms with only lower pairs permit the representation of any more complex planar mechanism and also allow the systematic enumeration of new mechanisms. Yan (1998, pp. 99–106) described the lower pair representations of most joints and links and called them generalized joint and members, respectively.

Figure 5 illustrates some joints commonly used in LVCB mechanisms and also shows their graph representation. The edges or lines connecting bodies will be, hereafter, drawn as “dotted,” “dashed,” “solid,” and “double,” in line with the reduction of 0, 1, 2, and 3 DOFs, respectively. Using this representation, several existing mechanisms of the market were analyzed and described by hand and

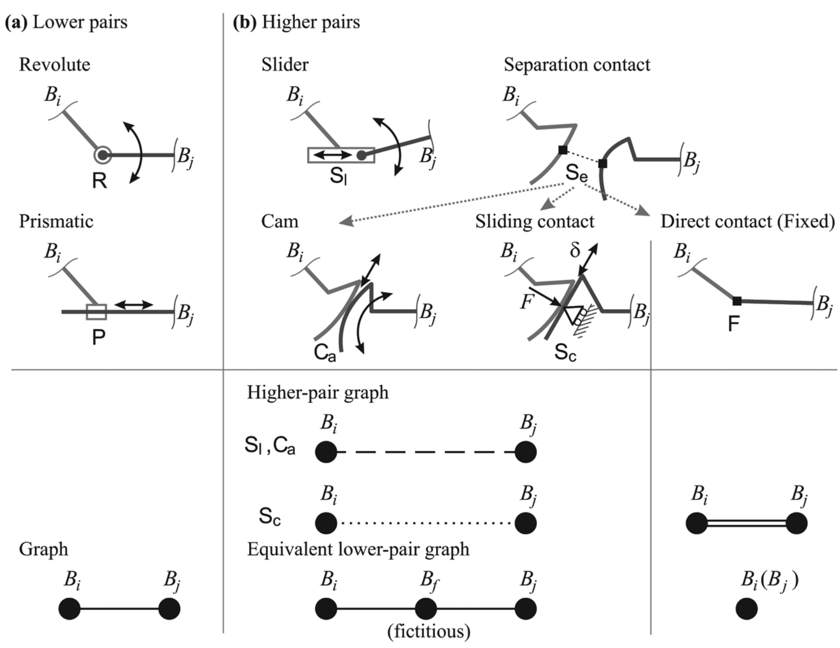


Figure 5 Types of joints and their lower-pair graph representation.

converted into a lower pair representation, useful to express requirements and to validate the results.

The design resources used in MMs can be represented as follows:

- Variable topology: The representation is the conventional kinematic chain with additional indicators of links and joints. Changes can be expressed as a sequence of graphs (one graph for each state or configuration) or, as a unique graph with a sequence of joints labels (e.g., see the unified graph by Kuo, 2004).
- Metamorphic bodies or links: Bodies joined together are represented by a main body followed by another one attached to it, parenthesized; for instance, 0(1,2) means that bodies B_1 and B_2 are considered as attached to the ground, denoted by default as B_0.
- Metamorphic joints: A metamorphic joint can basically change its:
  - Type: Type changes can be produced inside the same order or between different orders, i.e., from a lower kinematic pair to a higher kinematic pair and vice versa. A joint can also lose or gain mobility, e.g., a 3-DOF spherical joint can be converted into a 1-DOF hinge by locking two of its axes.
  - Characteristics: change of joint characteristics, for instance, the change of orientation of the joint axis, denoted as x, y, z, or as v, if its direction is made arbitrary; a change of motion orientation for a prismatic joint is illustrated by Yan and Kang (2009).

It should be noted that changes are produced either by different kinetostatic conditions while the motion takes place or by external actuation; magnitude and direction of the reaction forces in singular configurations are the key for obtaining the desired energy characteristics and behavior in dynamic transitions.

3.1. Example of Graph Representation

The mechanism shown in Fig. 6(a) is used as an example for the identification of the topology; see also the names of the bodies in Fig. 6(b) and Table 1. This mechanism has a metamorphic joint between the rod (R), the CC, and the DL; it is composed by two sliders formed by DL and CC. The rod is trapped by three points, two from DL and one from CC, and they form a revolute joint denoted as [R]. This metamorphic joint works as a revolute joint in normal operation, and it is converted into a slider in the delatching operation (the rod slides over the CC). While rearming, the rod slides in the opposite sense over CC and pushes DL until forming the revolute joint again.

In Fig. 6(c), the graph representation is shown, the bodies are shown as vertices with their labels inside them and the kinematic pairs are represented by edges, it also contains the contacts and springs represented by multiple edges. Thus, the complete representation of the mechanism is a multigraph. The separated points are denoted as S_e and as S_c when they effectively enter in contact.

In Fig. 6(d), the line types for edge representation defined in Fig. 5 were used and the joint types were ignored to further simplify the graph, a unique edge is used for the pair of bodies in contact. Only some joints with an important functional meaning were included (see labels a–e). Finally, the conventional lower pair graph is deduced using the DOF equivalence. Note that fictitious bodies B6 and B7 were

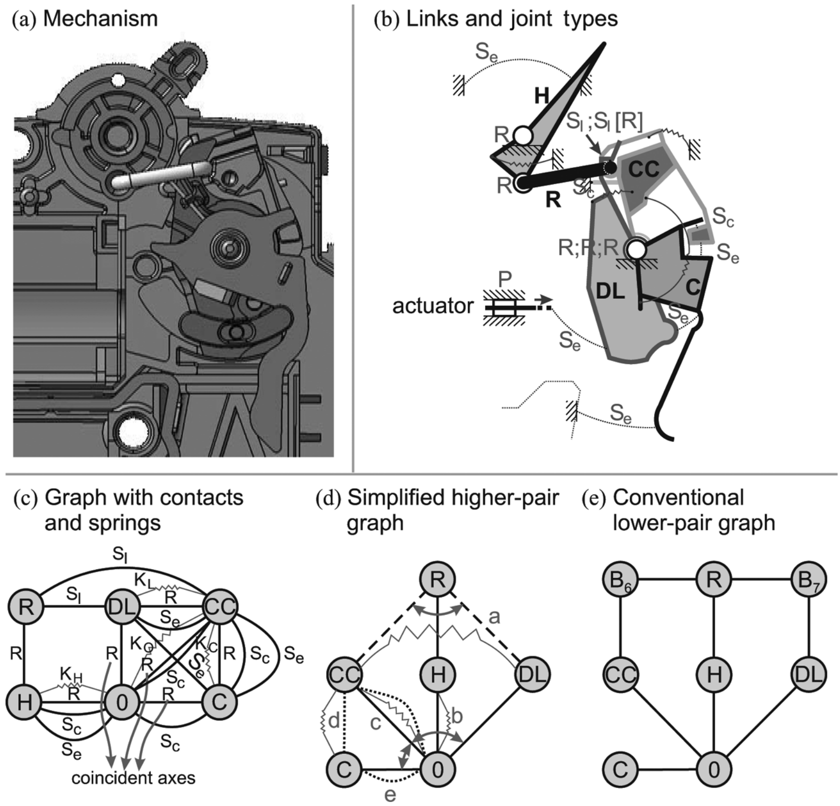


Figure 6 Example of graph representation of a circuit breaker mechanism.

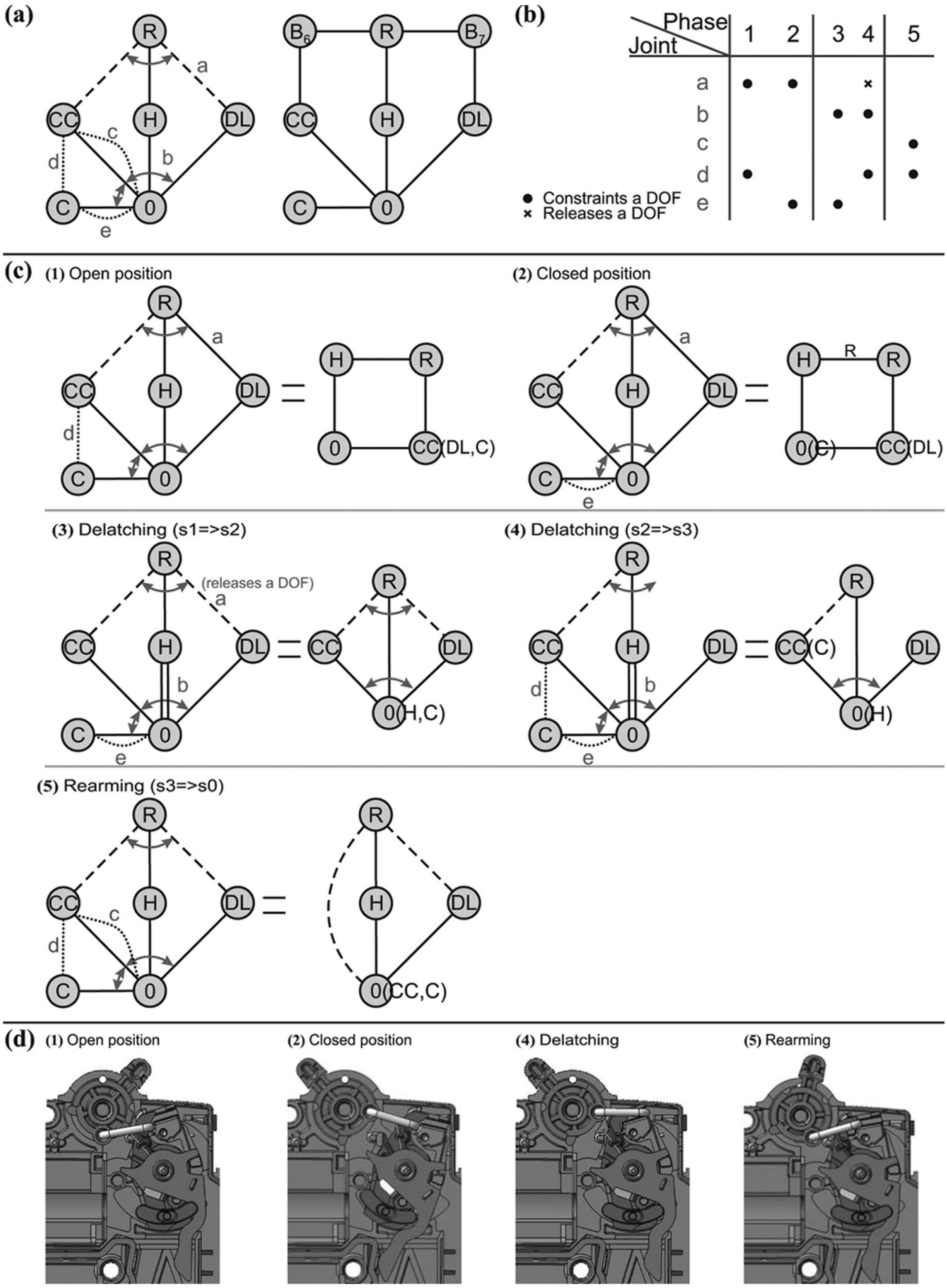
included to emulate the DOF of the slider joints. The two latter representations can be combined with a phase/joint table to express the metamorphic changes as shown in Fig. 7. This table considers the DOFs restrained by a joint for each state. A black bullet means that the joint restrains a DOF, and a cross represents that the DOF is released when the delatching takes place.

The topology with $2 + 1$ DOF shown in Fig. 7(a) is used as example to explain the table:

(1) In open position, see first column of the table in Fig. 7(b) and graphs in Fig. 7(c-1), the contacts C are kept in contact with CC by joint **d**, and another

Table 1 Names given to the parts of a circuit-breaker mechanism

Body	Name	Body	Name
Handle	H	Rod	R
Delatching lever	DL	Contact-carrier	CC
Mobile contact	C	Magnetic actuator	M_A
Latching spring	K_L	Contact spring	K_C
Opening spring	K_O	Handle spring	K_H



Joint \ Phase	1	2	3	4	5
a	•	•		×	
b			•	•	
c					•
d	•			•	•
e		•	•		

● Constraints a DOF
× Releases a DOF

Figure 7 (a) Chosen graph representation; (b) phase/joint table; (c) graphs states; and (d) virtual model.

DOF is reduced by joint **a** and it behaves as a revolute joint avoiding that the rod could slide through CC. The mechanism behaves as a four-bar linkage where CC, DL, and C are collapsed by the springs as a unique driven link.

(2) In closed position, the contact C is collapsed with the ground. Joint **e** enters in contact and joint **d** is in separation, thus, CC (collapsed with DL) can rotate as the driven link of a four-bar linkage.

(3) In the first part of the delatching, the actuator moves the lever DL and it releases 1-DOF, thus the rod R can slide over both DL and CC during a short stroke of the actuation.

(4) In the second part of the delatching, the rod R can slide only over CC until reaching the end of the slot. Even with locked handle (where joint b is fixed), the CC and the contacts C are collapsed and are able to rotate to a safe opening position where they enter in contact with the case.

(5) For rearming, the handle is rotated and moves the rod. The rod firstly slides over CC, then, over both CC and DL as it is illustrated in Fig. 7(c-5), and finally reaches the end of the slot forming again a fictitious revolute joint a where the rod is kept in contact with CC and DL but CC and DL are separated in joint c, see Fig. 7(c-1).

Using the presented representation, 26 existing designs were catalogued and stored for validation of the results. All states can also be catalogued and represented in a more compact form called unified graph by Yan (Yan and Kuo, 2006; Pucheta et al., 2011a). However, we found it more convenient to give one graph representation per state.

4. METHODOLOGY FOR ENUMERATION

The lower pair representation of linkages is used in this work: both the description of the parts to move and the atlas database are given in this form. An enumeration synthesis solver (Pucheta and Cardona, 2007, 2008) was modified to take into account new design constraints related to metamorphic changes (Pucheta et al., 2011a,b). This solver will be used to search the parts to move inside every mechanism taken from an atlas without repetitions and satisfying topological constraints.

An atlas data base of mechanisms determines the design space of the search. A set of 39 graphs of 2-DOF kinematic chains were taken from Tables D7–D14 of Tsai (2001; Appendix D) and codified by their adjacency matrices: 1 kinematic chain with 5 links–5 joints, 3 kinematic chains with 7 links–8 joints, and 35 kinematic chains with 9 links–11 joints. A procedure for the assignment of the fixed link (ground) to each kinematic chain and a code-based identifier for isomorphism testing (Pucheta and Cardona, 2007) was used to obtain 232 different mechanisms (in agreement with those obtained by Simoni et al., 2009) included as design space.

By representing existing mechanisms in terms of graphs, several common topological properties were identified for the different operational conditions and then used in the enumeration. The enumeration also serves to establish a topological classification of existing mechanisms.

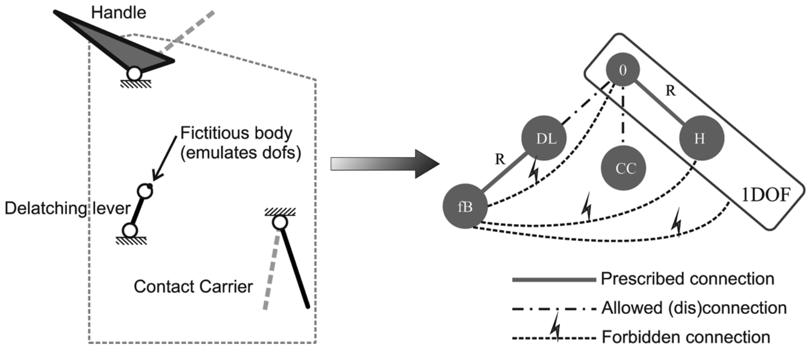


Figure 8 Topological configuration imposed on the initial parts.

4.1. Initial Graph and Topological Constraints

The parts to move were first represented in a graph as shown in Fig. 8.

Labels were assigned to each part. A series of constraints were defined, based on the functioning requirements, for the search of feasible mechanism topologies.

- Required bodies:

  [B1] There must exist at least five bodies: 0 (ground), H (handle), CC (contacts carrier), DL (delatching lever), and fB (fictitious body emulating the DOF of the contact).

- Connectivity constraints imposed on the initial graph:

  [C1] H is connected to 0.
  [C2] DL is connected to a fictitious body fB.
  [C3] fB is never hinged to 0.
  [C4] fB is never hinged to H
  [C5] fB is never hinged directly to a 1-DOF subgraph containing both 0 and H.
  [C6] DL can either be connected to 0 or be a floating link (hereafter, "floating" means not connected to ground).
  [C7] CC can either be connected to 0 or be a floating link.
  [C8] If CC is a floating link, it cannot be connected to H.

- Degree constraint (number of bodies connected) imposed on bodies of the desired solution:

  [D1] H: binary.
  [D2] DL: binary.
  [D3] fB: binary.

- Metamorphic constraints related to prohibited or allowed changes:

  [T1] There is not a 1-DOF subgraph containing: 0, H, CC.
  [T2] There is not a 1-DOF subgraph containing: 0, DL, fB, H.
  [T3] There is not a 1-DOF subgraph containing: 0, DL, fB, CC.

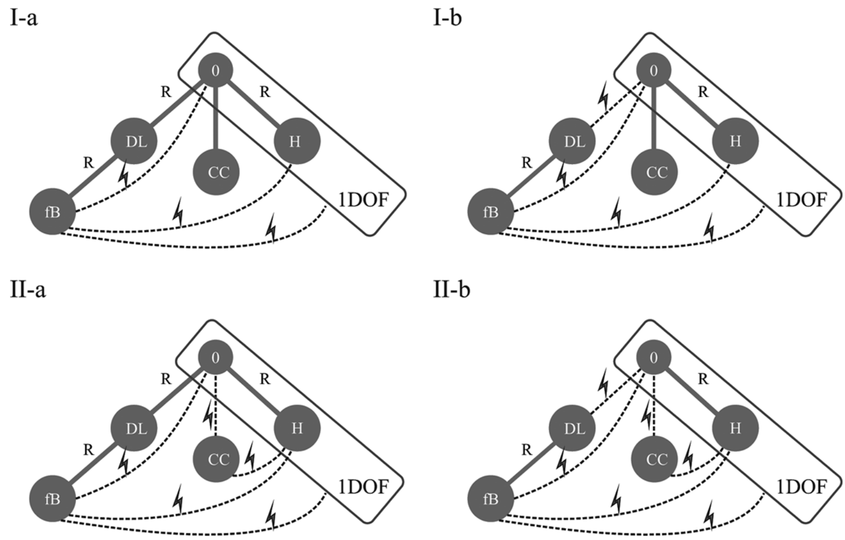


Figure 9 Subdivision of the problem into four mutually exclusive cases: (Ia) grounded DL and CC; (Ib) floating DL and grounded CC; (IIa) grounded DL and floating CC; and (IIb) floating DL and CC.

The metamorphic constraints are expressed in terms of subgraphs constraints. For instance, if the handle and the CC belong to a 1-DOF submechanism, locking of the handle will also lock the contacts; topologies with these undesired configurations are rejected by constraint T1. The constraints T2 and T3 ensure that the mechanism is able to delatch even when the handle is fixed. Note also that no degree constraint is imposed on the CC and on the rod.

To compute constraints C5, T1, T2, and T3, the method involves the simultaneous use of two atlases: a 2-DOF atlas as design space and a 1-DOF atlas to compute the constraints. Candidate solutions are taken from the design space and all those that contain any submechanism of those defined in the 1-DOF constraints atlas, are rejected.

In order to satisfy constraints like C6 and C7, the problem was split into four cases or subproblem, which are mutually exclusive. They are denoted as Ia, Ib, IIa, and IIb; see Fig. 9. In this way, the complexity is reduced and the information is easier to be handled.

4.2. Subgraph Search Subject to Metamorphic Constraints

For each subproblem, the algorithm executes two subgraph searches inside atlases of mechanisms with different DOFs. An outer loop of the algorithm performs a subgraph search of the initial graph G_{ini} inside every mechanism G_A taken from an atlas of 2-DOF mechanisms. For each occurrence ($G_{\text{ini}} \subseteq G_A$), a new mechanism G_A^P is created, its links labels are copied from their homologous links in G_{ini} and new links are labelled as $B_6, B_7, \ldots, B_n$., where n is the number of links of G_A^P. Then, the constraints are validated using G_A^P in two steps. First, the connectivity constraints C1–C4, and constraints C6 and C7 (in correspondence with the subproblem) are validated; then, degree constraints D1–D3 are validated; this procedure was explained in Pucheta and Cardona (2007). Second, to compute constraints C5 and metamorphic constraints T1–T3, an inner loop with a second subgraph search is performed. Given a potential 2-DOF solution G_A^P, every 1-DOF mechanism G_1, taken from a 1-DOF atlas, is sequentially searched as a subgraph inside the potential solution. For each sub-graph occurrence ($G_1 \subseteq G_A^P$), the constraints C5, and T1–T3 are validated. Every mechanism G_A^P that satisfies all constraints and is nonisomorphic to previous solutions is saved as a feasible solution.

5. RESULTS

The aforementioned methodology was implemented in the mentioned solver and written in C++ language. The searches inside the atlas of 2-DOF mechanisms,

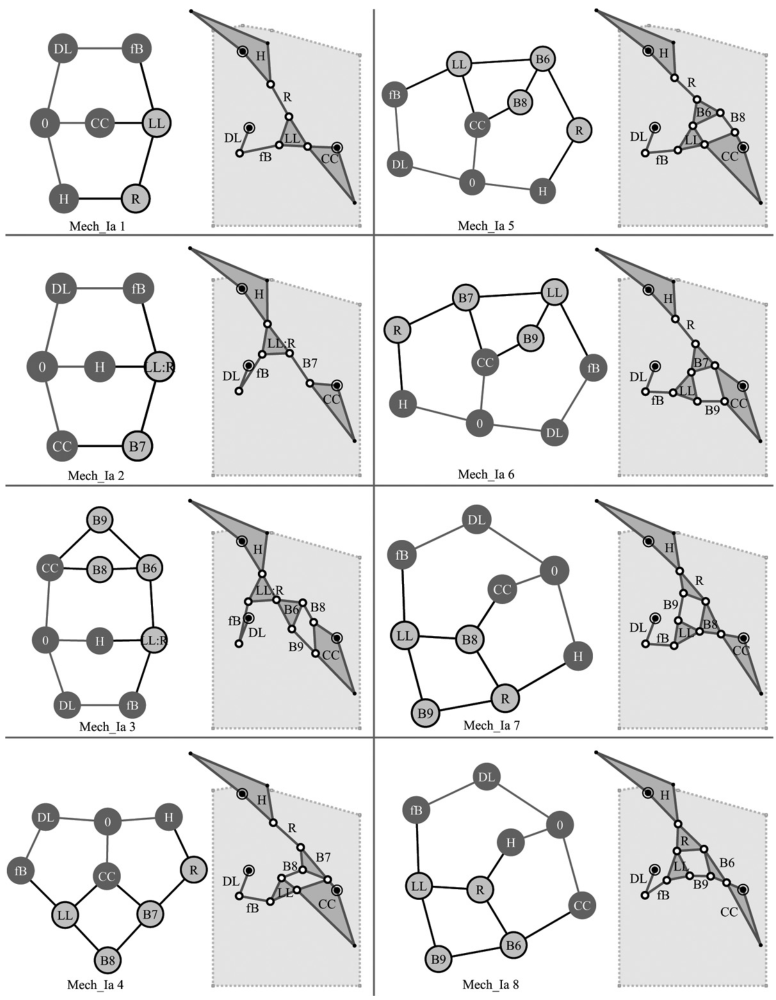


Figure 10 Some mechanisms satisfying the initial graph and constraints Ia.

with up to 9 links–11 joints, resulted in a set of 617 topologically different mechanisms distributed by case as follows: 30 in Ia, 218 in Ib, 60 in IIa, and 309 in IIb.

The results were represented by automatic sketching to help designers understand the proposals. The first results of the subproblem Ia are shown in Fig. 10. Most existing mechanisms for circuit breakers appeared within this set (e.g., the existing mechanism in Fig. 7(a) can be found as $\text{Mech}_{-\text{Ia}}$ 1 in Fig. 10), and several potential candidates for new mechanisms were also established.

5.1. Postprocessing of the Results and Further Research

All topological simplifications assumed in the enumerated topologies must be evaluated as possible forms of adding parts: e.g., to consider a contact mounted over the CC, the use of a slider for achieving a contact pressure DOF, and the use of 0-DOF chains for amplifying the mechanical advantage of the DL.

The presented enumeration can be used as a source of new designs. First, the designs can be dimensioned using synthesis methods for planar linkages with lower pairs until functionality is ensured. Then, using manual assignment, the designers can try the inverse transformation from lower pairs to multiple joints and higher pairs (such us contact pairs, sliders, and others), possibilities of making axes of joints coincident, addition of springs, separation contacts (stops), etc. All of these design considerations will increase the combinatorial explosion from each of the enumerated alternatives. The automation procedures of the mentioned transformation will be addressed in future research.

6. CONCLUSIONS

Mechanisms for fulfilling complex operations involving metamorphic changes were enumerated using Graph Theory concepts and combinatorial algorithms. The presented problem is difficult because one submechanism of the mechanism must have a mobility, independent from other submechanism, which can or cannot be locked.

The main contribution of this work is to give a way to express topological constraints for metamorphic changes in the form of subgraph constraints.

An application to the design of a family of LVCB mechanisms was shown. The many feasible concepts were represented in the form of physical sketches and are currently available as a source of potential new designs.

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APPENDIX: A TRANSITION AND OUTPUT FUNCTIONS OF THE LVCB MECHANISM

The transition function f is a mapping from state space to the state space for the feasible combinations of the inputs. It can be tabulated as shown in Table A1, which is also known as the "next state" table (Rosen, 2007; Yan and Kuo, 2006).

Table A1 Transition function of the circuit-breaker mechanism (normal operation in gray)

	Input			
$f(\mathbf{S}, \mathbf{I}) \to \mathbf{S}$	$I_1 = \mathbf{on}$ I_2 and $I_3 = \mathbf{off}$	$I_1 = \mathbf{off}$ I_2 and $I_3 = \mathbf{off}$	$I_1 = \mathbf{on}$ I_2 or $I_3 = \mathbf{on}$	$I_1 = \mathbf{free}$ I_2 and $I_3 = \mathbf{off}$
State	Next State			
s_0	s_1	–	s_3	–
s_1	–	s_0	s_2	–
s_2	s_3	–	–	s_0
s_3	–	–	–	s_0

For a given present state and the same set of input values of the previous table, the "outputs" table (Table A2) shows the values of the outputs for the associated "next state."

Table A2 Output function of the circuit-breaker mechanism (normal operation in gray)

	Input			
$g(\mathbf{S}, \mathbf{I}) \to \mathbf{O}$	$I_1 = \mathbf{on}$ I_2 and $I_3 = \mathbf{off}$	$I_1 = \mathbf{off}$ I_2 and $I_3 = \mathbf{off}$	$I_1 = \mathbf{on}$ I_2 or $I_3 = \mathbf{on}$	$I_1 = \mathbf{free}$ I_2 and $I_3 = \mathbf{off}$
State	Ouputs			
s_0	$O_1 = \mathbf{on}$; $O_2 = \mathbf{armed}$	–	$O_1 = \mathbf{off}$; $O_2 = \mathbf{disarmed}$	–
s_1	–	$O_1 = \mathbf{off}$; $O_2 = \mathbf{armed}$	$O_1 = \mathbf{on}$; $O_2 = \mathbf{disarmed}$	–
s_2	$O_1 = \mathbf{off}$; $O_2 = \mathbf{disarmed}$	–	–	$O_1 = \mathbf{off}$; $O_2 = \mathbf{armed}$
s_3	–	–	–	$O_1 = \mathbf{off}$; $O_2 = \mathbf{armed}$

In these tables, the cells with grey background in the first two columns of data correspond to the normal or manual operation. The other cells correspond to the electric failure.