Physica A 391 (2012) 410–413

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effect upon universal order of Hubble expansion

B.R. Frieden [a,*], A. Plastino [b], A.R. Plastino [c,d]

[a] College of Optics, University of Arizona, Tucson, AZ 85721, United States
[b] National University La Plata, Physics Institute (IFLP-CCT-CONICET), C.C. 727, 1900 La Plata, Argentina
[c] CREG-National University La Plata-CONICET, C.C. 727, 1900 La Plata, Argentina
[d] Instituto Carlos I de Fisica Teorica y Computacional, Departamento de Fisica Atomica, Molecular y Nuclear, Universidad de Granada, Granada, Spain

ARTICLE INFO

Article history:
Received 31 March 2011
Received in revised form 2 August 2011
Available online 11 August 2011

Keywords:
Coarse graining
Fisher information
Order and complexity
Hubble expansion

ABSTRACT

The level of order R in a spherical system of radius r_0 with a probability amplitude function $\psi(\mathbf{x})$, $\mathbf{x} = r, \theta, \phi$ obeys $R = (1/2)r_0^2 I$, where $I = 4\int d\mathbf{x}|\nabla\psi|^2$ is its Fisher information level. We show that a flat space universe obeying the Robertson–Walker metric has an invariant value of the order as it undergoes either uniform Hubble expansion or contraction. This means that Hubble expansion per se does not cause a loss of universal order as time progresses. Instead, coarse graining processes characterizing decoherence and friction might cause a loss of order. Alternatively, looking backward in time, i.e. under Hubble contraction, as the big bang is approached and the Hubble radius r_0 approaches small values, the structure in the amplitude function $\psi(\mathbf{x})$ becomes ever more densely packed, increasing all local slopes $\nabla\psi$ and causing the Fisher information I to approach unboundedly large values. As a speculation, this ever-well locates the initial position of the universe in a larger, multiverse.

1. Background

The concept of disorder, as measured by an entropic quantifier has an illustrious past. In this paper, we consider a complementary concept, namely, the level of order, or complexity, in a system. What, quantitatively, is a system's "order"? Numerous authors have considered this question to be equivalent to the statement of the second law of thermodynamics, that disorder must increase. On this basis order must decrease, where order is in some sense the opposite of disorder. Important properties of order that arise in specific applications are often mentioned, such as its spontaneous, coherent, or statistical nature, but the term itself is never rarely quantified in mathematical fashion. By comparison, in a recent effort [1], Frieden and Hawkins derived a mathematical form for order from first principles, on the basis of adequate considerations regarding the Fisher information measure (FIM) I, on the one hand, and "coarse graining" on the other one.

Obviously, coarse-grained systems consist of fewer, larger components than fine-grained systems; a coarse-grained description of a system regards large subcomponents while a fine-grained description regards smaller components of which the larger ones are composed. A fine-grained description of a system is a detailed, low-level model of it, while a coarse-grained description refers to a model for which some of its fine details have been smoothed over or averaged out. The replacement of a fine-grained description with a lower-resolution, coarse-grained model is what one calls coarse graining. For instance, coarse-grained models have been developed for investigating the longer time- and length-scale dynamics that are critical to many biological processes, such as lipid membranes and proteins [2]. The property [3–5] that order R should decrease under coarse graining was used [1] as the single postulate for finding its form. For a shift-invariant system with a

* Corresponding author. Tel.: +1 520 621 4904.
E-mail addresses: roy.frieden@optics.arizona.edu (B.R. Frieden), angeloplastino@gmail.com (A. Plastino), arplastino@ugr.es (A.R. Plastino).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.08.005

general number K of coordinates $x_{1,\ldots,}x_K$ the order obeys

$$R = 8^{-1}L^2 I = 2^{-1}r_0^2 I. \tag{1}$$

On this basis, the total order R is linear in the Fisher information I, the latter defined in Eq. (2) below. The first expression (1) for R was derived in Ref. [1] for a one-dimensional system, but obeys the same expression for a K dimensional system. This follows a straightforward but lengthy derivation, which for brevity is not shown here. By hypothesis the system fills a closed surface that is convex outward, with L the maximum chord connecting two surface points. For example, a $K = 3$ dimensional spherical system of radius r_0 has a maximum chord length $L = 2r_0$. This yields the second equality in Eq. (1).

As is well-known, the Fisher information measure reads [6–10]

$$I = 4\int \mathrm{d}\mathbf{x}\nabla q\cdot\nabla q; \qquad p \equiv q^2(\mathbf{x}), \quad \mathrm{d}\mathbf{x} \equiv \mathrm{d}x_1\cdots\mathrm{d}x_K. \tag{2}$$

Thus, $q(\mathbf{x})$ is a (real) probability amplitude function. The differentials dq in gradient ∇q are small perturbations caused by the coarse graining. In (2) the real probability amplitudes $q(\mathbf{x})$ can be most generally identically replaced by the generally *complex* amplitudes $\psi(\mathbf{x})$ describing a quantum system, where $p \equiv \psi^*\cdot\psi$ and $\nabla q\cdot\nabla q$ is replaced identically by $\nabla\psi^*\cdot\nabla\psi$. However, we defer this quantum generalization to later discussion.

In this communication we investigate "order" in connection with an expanding universe. The simplest universe consistent with relativity is one which appears isotropic to a set of privileged observers, called co-moving observers. Each observer sees the others as moving along with the overall cosmic expansion, although reference to such observers is mostly a figure of speech. Emphasis is to be placed on the existence of sites from which the universe would appear isotropic if there were anyone there to observe. In such circumstances, the interval (space–time separation) between events ("points" in space–time) can be described by the Robertson–Walker metric [11]. By fixing the distances between all points, the metric also defines the geometry of space–time, and, because there is a meaningful cosmic time, the geometry of space at a given time.

2. Effect of uniform expansion on order

Suppose that a system of PDF $p(\mathbf{x})$ is uniformly expanded spatially, to a new system with coordinates $\mathbf{x}'$ obeying

$$\mathbf{x}' = \alpha\mathbf{x}; \quad \text{where } \alpha = const. \tag{3}$$

Thus, α is to be constant in variables $\mathbf{x}$. However, α *may be* a function of coordinates other than the $\mathbf{x}$ that are integrated over in definition (2). An example is the fourth coordinate t in the often used $(3+1)$ dimensioned analysis of cosmology. This is our case below, where generally $\alpha = \alpha(t)$. Space is here defined by rectangular coordinates and hence is assumed flat. More generally, the transformation (3) can be used even in scenarios where space has finite (positive or negative) curvature, provided the system's linear scales are much smaller than space's radius of curvature.

Taking into account the Jacobian [7] associated with the transformation (3), the PDF $p(\mathbf{x})$ is distorted to a new PDF $p_1(\mathbf{x}')$ according to

$$p(\mathbf{x}) \longrightarrow p_1(\mathbf{x}') = \alpha(t)^{-3}p(\alpha(t)^{-1}\mathbf{x}'). \tag{4}$$

(The power -3 traces from the three-dimensional nature of our coordinate space.) However, despite the distortion, one can readily show [1] that R defined by Eqs. (1) and (2) *is invariant* to this uniform change of scale, obeying

$$R' = R, \tag{5}$$

where order R' is the order in the stretched system. The key effect is that L^2 transforms to value α^2L^2 whereas I transforms to $\alpha^{-2}I$, so that their product L^2I in (1) keeps the original value. It is worth mentioning that invariance properties akin to the one embodied in Eq. (5) also play a central role in connection with statistical complexity measures (see [12–14] and references therein).

3. Robertson–Walker metric

Assume an isotropic, homogeneous, expanding universe characterized by $K = 3$ flat-space variables $\mathbf{x} = x, y, z$ plus a time variable t. Let it be viewed in large scale, temporarily ignoring quantum effects (and thus avoiding replacements $q(x) \to \psi(x)$ discussed above). Let the "perfect cosmological principle" hold, according to which the universe looks the same to any observer moving with the expansion. This is also conveniently described by a coordinate system co-moving with the observers, since it allows for a unique definition of the "same" point at different times. Thus, the distances between any pairs of points are well defined. In such a system, the space–time interval ds^2 between neighboring events (or "points") in space–time can be described by the Robertson–Walker metric [9]

$$\mathrm{d}s^2 = (c\,\mathrm{d}t)^2 - a^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2). \tag{6}$$

This metric defines a geometry of space–time, and, because there is a meaningful cosmological time, the geometry of space at a given time. The factor $\alpha(t) \to \frac{a(t)}{a(t_0)}$ manifestly defines a *uniform* (in coordinates $\mathbf{x}$, see (3) above) change of scale, at each instant of time. To be definite we have set it equal to unity at some standard time t_0 by simply dividing through the original scale change function by its value at time t_0.

Let the time-dependent scale change characterize the Hubble temporal expansion. Notice that since a operates equally on each term dx^2, dy^2, dz^2 in (4) the associated scale change is homogeneous, preserving the homogeneity required at the outset. Thus, the expansion (3) described by $\alpha(t)$ simply *uniformly* magnifies the 'system', here the universe, at each instant of time. Here we have considered the Robertson–Walker metric with *zero curvature (flat)* space (this last feature is a "global" property of the particular Robertson–Walker solution considered here that evidently holds for all times).

Therefore by Eq. (5) *the order R holds constant during Hubble expansion*.

4. Quantum generalization

As mentioned at the outset, the order R for a generally *complex* system $\psi(\mathbf{x})$ still has the form Eq. (1), with information form (2) replaced by (see Refs. [15,16] for interesting discussions concerning the Fisher information as applied to wave functions)

$$I = 4\int d\mathbf{x}\nabla\psi^* \cdot \nabla\psi; \quad p \equiv \psi^* \cdot \psi. \tag{7}$$

We can thereby regard $\psi(\mathbf{x})$ as a wave function associated with the quantum analysis of a process akin to the one considered here, including effects holding over distances extending over a range from below the Compton scale to the Hubble radius. Of course $\psi(\mathbf{x})$ must be significantly affected by gravitational effects, and there is currently no known theory for incorporating them into $\psi(\mathbf{x})$. Hence we must regard $\psi(\mathbf{x})$ as currently unknown. However, result (5) of invariance to scale change is a *mathematical* equality, valid [3] for *any* amplitude function $\psi(\mathbf{x})$. Hence (5) should hold at any scale for which space–time coordinates $\mathbf{x}, t$ remain well defined. At sufficiently tiny scales, say approaching the Planck scale, space–time coordinates are believed to become random and ill-defined, so that form (7) would not hold (although it might if, e.g., it were replaced by its expectation over the newly random space–time geometry).

We conclude that, at any scales for which space–time coordinates are well defined, including those possibly much smaller than (say) the Compton length, the order of the universe remains constant in time.

5. Discussion

We have found that if space is flat and well approximated by the Robertson–Walker metric (5), then under spatially uniform Hubble expansion the order R of the universe *remains constant*. This holds true for *any rate* of expansion $da(t)/dt$. Such expansion does not decrease the overall order, contrary to intuition. Alternatively, looking backward in time, as the big bang is approached the system extension $L \equiv 2r_0$ approaches small values so that, with R constant, by Eq. (1) it must be that information I approaches unboundedly large values. Mathematically this is because the structure in $q(\mathbf{x})$ or $\psi(\mathbf{x})$ is becoming ever more densely packed, increasing all local slopes ∇q in Eq. (2) or $\nabla\psi$ in Eq. (7). A toy example [5] is the case of $q(\mathbf{x}) \propto \sin(n\pi x/l)\sin(n\pi y/l)\sin(n\pi z/l)$ within a cube of $K = 3$ dimensions and side l. Here, *each dimension contain n oscillations or fringes*. The resulting information (2) is value $I = 12(n\pi/l)^2$, strongly increasing as extension l decreases.

On the other hand, during the expansion there are *other* processes transpiring that may decrease the order. These are so-called "coarse graining" events [3–5] such as decoherence and frictional losses. *By hypothesis* [1] such events cause order changes $\Delta R \le 0$, either reducing or not affecting the order level. This hypothesis was the sole premise of the derivation [1] of form (1) for R. We conclude that if the order of the universe decreases with time, it is not due to Hubble expansion but, rather, to coarse graining processes. Another important process that was not taken into account here, is the self-gravitational interaction between the different elements of the evolving density distribution, which leads to the evolution of structure within this mass distribution.

6. Speculation

The above result, that I increases beyond limit as time is run backward, has a strange ramification. The Cramer–Rao inequality [6,7] states that the accuracy with which an unknown parameter may be determined from data is limited by the level of the Fisher information in the data. Specifically, the mean-squared error e^2 of estimation of a parameter has a minimum value obeying $e^2_{\min} = 1/I$. Let the parameter be the initial spatial 'location' of the compressed density distribution. Thus, as the big bang is approached backward in time and, as we saw, I increases beyond bound, the true initial location of the compressed distribution becomes increasingly well defined. This suggests the existence of a larger space surrounding our own, i.e. the 'multiverse', within which universes are formed from pointlike regions that are extremely well localized. These could be, e.g., the initial 'seed' positions of black holes that have been hypothesized [17] to form universes provided their mass–energy values are large enough.

Acknowledgments

This work was partially supported by the programs FQM-2445 and FQM-207 of the Junta de Andalucia-Spain, and by CONICET (Argentine Agency).

References

[1] B.R. Frieden, R.J. Hawkins, Phys. Rev. E 82 (2010) 066117.
[2] J.J. de Pablo, Ann. Rev. Phys. Chem. 62 (2011) 555.
[3] J.J. Halliwell, Contemp. Phys. 46 (2005) 93.
[4] S.-L. Luo, Commun. Theor. Phys. 47 (2007) 597.
[5] P. Gibilisco, F. Hiai, D. Petz, IEEE Trans. Inf. Theory 55 (2009) 439.
[6] R.A. Fisher, Statistical Methods and Scientific Inference, second ed., Oliver and Boyd, London, 1959.
[7] B.R. Frieden, Probability, Statistical Optics and Data Testing, third ed., Springer-Verlag, NY, 2001.
[8] B.R. Frieden, Science from Fisher Information, Cambridge Univ. Press, Cambridge, UK, 2004.
[9] F. Olivares, F. Pennini, A. Plastino, Physica A 389 (2010) 2218.
[10] V. Kapsa, L. Skala, J. Chen, Physica E 42 (2010) 293.
[11] L. Bergstrom, A. Goobar, Cosmology and Particle Astrophysics, Springer, Berlin, 2004.
[12] R. Lopez-Ruiz, A. Nagy, E. Romera, J. Sanudo, J. Math. Phys. 50 (2009) 123528.
[13] R. Lopez-Ruiz, J. Sanudo, Open Syst. Inf. Dyn. 16 (2009) 423.
[14] C. Anteneodo, A.R. Plastino, Phys. Lett. A 223 (1996) 348.
[15] S. Luo, Int. J. Theor. Phys. 41 (2002) 1713.
[16] S. Luo, J. Phys. A 35 (2002) 5181.
[17] L. Smolin, The Life of the Cosmos, Oxford Univ. Press, Oxford, UK, 1997.