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Use of an anisotropic absorber for simulating electromagnetic scattering by a perfectly conducting wire

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Abstract: A perfectly matched anisotropic absorber (PMAA) is used as a new type of medium in a formalism previously developed for investigating diffraction from an inhomogeneous isotropic aperture in a thick metallic screen. In this paper we consider that the aperture consists of five homogeneous regions: PMAA, transparent dielectric, perfect conductor, transparent dielectric and PMAA. This configuration could allow us to simulate a conducting cylinder of rectangular section in open space and it sets the basis for further extensions of the method to other geometries. We present near and far field simulations for different geometrical and constitutive parameters of the PMAA regions. The results seem to indicate that this new medium could be used to represent a lateral open space, thus enhancing the modal method and making it suitable to simulate scattering problems of finite objects.

Key words: Scattering – absorbing boundary condition – PML

1. Introduction

The perfectly matched layer (PML) is a fictitious material which does not reflect incident propagating waves regardless of the incident angle, frequency and polarization. It was firstly introduced by Berenger [1, 2] as a useful absorbing boundary condition to truncate the computational domain in finite-difference time-domain applications. A different formulation of the PML, given by Sacks et al. [3], is based on exploiting constitutive characteristics of anisotropic materials to provide a reflectionless interface. Compared with Berenger's formulation, the perfectly matched anisotropic absorber (PMAA) formulation offers the special advantage that it does not require modification of Maxwell equations [3]. Both formulations of the PML are very popular among the electrical engineering community and their use in the optics community has been growing in the last few years [4–7].

PMLs look attractive, mainly because of their potential capacity to simulate open-structure problems. We start from a multilayer modal formalism previously developed for investigating diffraction from an inhomogeneous aperture in a thick metallic screen [8]. This formalism allows us to deal with structures such as the one schematized in fig. 1a. The region $x_{\min} < x < x_{\max}$, $0 < y < h$ is the inhomogeneous aperture of a perfectly conducting thick screen. Although rather general inhomogeneities can be simulated, we wish to restrict ourselves to the case shown in fig. 1b: a perfectly conducting sample located at the otherwise homogeneous and transparent aperture in a metallic screen. The purpose of this paper is to explore the possibility of using PMLs, as shown in fig. 1c, in order to minimize the interaction between the metallic sample and the edges of the screen. If a PML actually represents a reflectionless and totally absorbing space, therefore the scattering of an isolated object could be modelled with the simplicity and versatility provided by the multilayer modal method, previously used in [8] for a very differ-

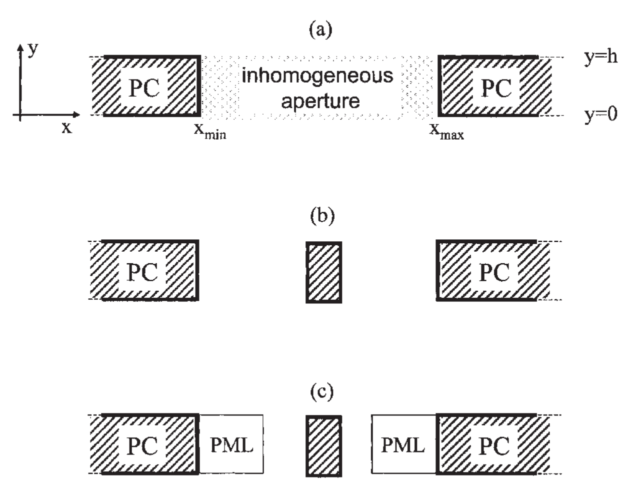

Fig. 1. a) A transparent inhomogeneous aperture in a perfectly conducting screen; b) aperture with a perfectly conducting sample; c) aperture with PMAA regions to simulate lateral open spaces.

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ent problem: that of the scattering of the same object inside the aperture of a conducting screen. The simple configuration shown in fig. 1c could be useful to minimize the interaction between the sample and the screen and establishes the basis for further extensions of the method to non-rectangular geometries. We present near and far field simulations for different geometrical and constitutive parameters of the PMAA regions.

2. Formulation of the problem

We consider the problem of a single rectangular wire (of width d and depth h) immersed in a dielectric, as schematized in fig. 2. The structure is invariant under translations in the $\hat{z}$ direction. A monochromatic Gaussian beam is incident on the wire forming an angle θ_0 with the y axis. We consider the case of s polarization (electric field perpendicular to the plane of incidence). To apply the modal method we divide the space into three regions: the superstrate (region 1) and the substrate (region 3) are homogeneous, isotropic and semi-infinite, with refractive indices ν_1 and ν_3, respectively; the intermediate region (region 2) contains the perfectly conducting scatterer, and is subdivided in zones of different materials and widths.

In regions 1 and 3 the electric field can be expressed as a continuous superposition of plane waves:

$$E_z^{(1)}(x,y) = \int_{-k}^{k} \mathcal{A}(\alpha)\, \mathrm{e}^{\mathrm{i}(\alpha x - \beta_1 y)}\, \mathrm{d}\alpha + \int_{-\infty}^{\infty} \mathcal{R}(\alpha)\, \mathrm{e}^{i(\alpha x + \beta_1 y)}\, \mathrm{d}\alpha\,, \tag{1}$$

$$E_z^{(3)}(x,y) = \int_{-\infty}^{\infty} \mathcal{T}(\alpha)\, \mathrm{e}^{i(\alpha x - \beta_3 y)}\, \mathrm{d}\alpha\,, \tag{2}$$

where

$$\beta_l = \begin{cases} \sqrt{k_l^2 - \alpha^2} & \text{if} \quad k_l^2 > \alpha^2 \\ \mathrm{i}\,\sqrt{\alpha^2 - k_l^2} & \text{if} \quad k_l^2 < \alpha^2 \end{cases}, \tag{3}$$

$k_l = 2\pi\nu_l/\lambda$ is the absolute value of the wave vector, the subscript $l = 1, 3$ denotes the region and i is the

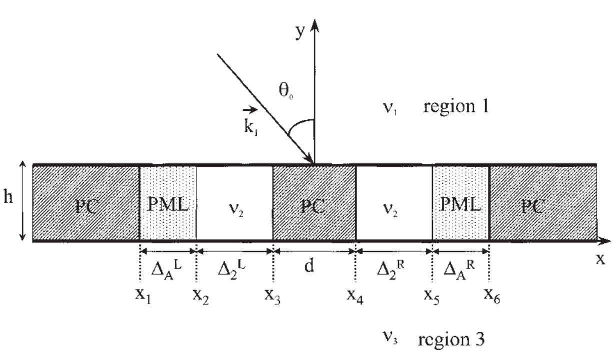

Fig. 2. Configuration of the problem.

imaginary unit. The first term in (1) represents the incident field, where $\mathcal{A}(\alpha)$ is a Gaussian distribution function

$$\mathcal{A}(\alpha) = \frac{w}{2\sqrt{\pi}} \exp\left\{-(\alpha - \alpha_0)^2 \left(\frac{w}{2}\right)^2\right\}, \tag{4}$$

$\alpha_0 = k_1 \sin\theta_0$, $\beta_0 = k_1 \cos\theta_0$ and $\mathcal{R}(\alpha)$ and $\mathcal{T}(\alpha)$ are unknown complex functions.

To simulate a lateral open space in the intermediate region, we put a perfectly matched anisotropic absorber (PMAA) at both sides of the inhomogeneous aperture (see fig. 2). This configuration permits us to avoid the non desired reflections that usually come from perfectly conducting boundaries. The PMAA is an anisotropic material characterized by electric permittivity $\varepsilon_\mathrm{A} = \varepsilon_2 \tilde{\Lambda}$ and magnetic permeability $\mu_\mathrm{A} = \mu_2 \tilde{\Lambda}$ tensors, where ε_2 and μ_2 are the permittivity and permeability of the isotropic zone within the intermediate region, respectively. For an interface in the $\hat{y}$ direction, the matrix $\tilde{\Lambda}$ is defined as:

$$\tilde{\Lambda} = \begin{bmatrix} 1/b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}, \tag{5}$$

which is especially built to avoid reflection for any angle of incidence [3]. The parameter b is a complex number: its real part is related to the wavelength inside the PMAA, and the imaginary part accounts for the losses in the material, i.e., the attenuation of the propagating waves.

Taking into account the characteristics of each medium within the inhomogeneous aperture, we can separate the differential equation and then express the electric field as a modal expansion, as the product of a function of x which satisfies the boundary conditions at the vertical interfaces, by a function of y which is a linear combination of trigonometric functions with unknown amplitudes:

$$E_z^{(2)}(x,y) = \begin{cases} \sum_m X_m^L(x)\, [a_m^L \cos v_m^L y + b_m^L \sin v_m^L y] & \text{for} \;\; x_1 < x < x_3 \\ \sum_m X_m^R(x)\, [a_m^R \cos v_m^R y + b_m^R \sin v_m^R y] & \text{for} \;\; x_4 < x < x_6 \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where the superscripts L and R account for left and right in fig. 2, a_m^q and b_m^q $(q = L, R)$ are unknown complex amplitudes and the functions $X_m^q(x)$ are given by:

$$X_m^L(x) = \begin{cases} \sin[u_{mA}^L(x - x_1)] & x_1 < x < x_2 \\ \sin[u_{mA}^L \Delta_A^L] \cos[u_{m2}^L(x - x_2)] + \cos[u_{mA}^L \Delta_A^L] \sin[u_{m2}^L(x - x_2)] & x_2 < x < x_3 \end{cases}, \tag{7}$$

$$X_m^R(x) = \begin{cases} \sin\left[u_{m2}^R(x - x_4)\right] & x_4 < x < x_5 \\ \sin\left[u_{m2}^R \Delta_2^R\right] \cos\left[u_{mA}^R(x - x_5)\right] & \\ + \cos\left[u_{m2}^R \Delta_2^R\right] \sin\left[u_{mA}^R(x - x_5)\right] & x_5 < x < x_6 \end{cases}, \quad (8)$$

where $(u_{m2}^q)^2 = k_2^2 - (v_m^q)^2$, $u_{mA}^q = u_{m2}^q b$, and Δ_A^q and Δ_2^q are the widths of the anisotropic and the isotropic zone, respectively (see fig. 2).

By requiring the continuity of the tangential electric and magnetic fields at the vertical interfaces we get two eigenvalues equations for u_{m2}^q whose complex roots are

$$u_{m2}^q = \frac{m\pi}{b\Delta_A^q + \Delta_2^q}, \qquad q = L, R, \quad (9)$$

where Δ_A^q and Δ_2^q are the widths of the zones occupied by the PMAA and the dielectric with refractive index ν_2, respectively. The complex coefficients $\mathcal{R}(\alpha)$ and $\mathcal{T}(\alpha)$, and consequently the reflected and transmitted intensities, are finally obtained by matching the fields at the interfaces $y = 0$ and $y = h$.

3. Numerical examples

In the following examples we illustrate the use of a PMAA as an artificial termination for the scattering problem of a rectangular cylinder. We consider the symmetric case in which $\Delta_A^L = \Delta_A^R = \Delta_A$, $\Delta_2^L = \Delta_2^R = \Delta_2$. In fig. 3 we plot the transmitted intensity vs. $\sin\theta$ for different widths Δ_A and b parameters of the PMAA. The incident beam of width $w/d = 3$ and wavelength $\lambda/d = 0.8$ incides normally on the aperture; the other parameters of the structure are: $h/d = 0.1$, $\Delta_2/d = 4$, $\nu_1 = \nu_2 = \nu_3 = 1$. The dark circles in figs. 3a and 3b correspond to the limit case in which there is no PMAA (schematized in fig. 1b), i.e., the aperture is terminated laterally by perfectly conducting walls, as in [8]. The other three curves correspond to varying values of b, with $\Delta_A/d = 1$. It can be observed in fig. 3a that the four curves coincide acceptably well and in this scale they seem to be good approximations of the diffraction pattern of a rectangular object. However, some discrepancies can be observed at a larger scale, as shown in fig. 3b for the sidelobes of the diffraction pattern. Due to interference, multiple reflections occuring at the perfectly conducting walls produce oscillations, mostly present in the case with no PMAA (dark circles curve). As soon as a little imaginary part is added to the b parameter of the PMAA, the curve of transmitted intensity begins to smooth. The dark rectangles correspond to $b = 1 + i0.3$, the thick solid line corresponds to $b = 2 + i0.3$, and the white circles to $b = 3 + i0.3$. Even though the structure is still limited by perfectly conducting walls, the waves are attenuated significantly inside the anisotropic layer, thus giving a much smaller reflection. The reflection from the sides is diminished and the oscillations in the transmitted pattern tend to disappear. However, as the real part of b is increased while keeping the imaginary part fixed, the curve improves up to a certain value of the ratio $\mathrm{Re}(b)/\mathrm{Im}(b)$. Beyond that value the fluctuations become significant again. In fig. 4 we show the electric field amplitude as a function of the coordinates $|E_z(x, y)|$ in the left isotropic zone within region 2. Fig. 4a corresponds to the case with no PMAA (represented by the dark circles in fig. 3), whereas the plot in fig. 4b corresponds to that represented by the thick solid line in fig. 3. When no absorber is introduced in the structure, the electric field exhibits a fringe pattern, which is related to the reflections produced at the perfectly conducting boundary on the left of the structure (see fig. 4a). On the other hand, for $b = 2 + i0.3$, the waves penetrate into the PMAA, where they are attenuated. In the contour plot of fig. 4b we observe that the spatial distribution of the field is significantly changed with respect to fig. 4a. A fringe pattern near the perfect conductor can still be appreciated, but with a weaker contrast. Therefore the introduction of a PMAA seems to produce the desired effect in the far field response of the structure, even though in the near field this effect is not as successfully achieved.

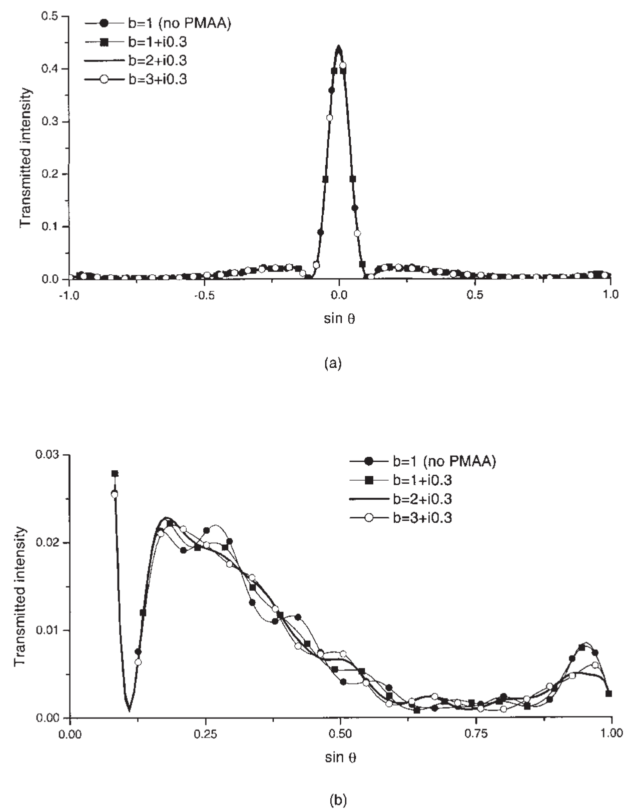

Fig. 3. Transmitted intensity vs. $\sin\theta$ for different widths and b parameters of the PMAA. The incidence parameters are $\theta_0 = 0°$, $w/d = 3$, and $\lambda/d = 0.8$, and the parameters of the structure are: $h/d = 0.1$, $\Delta_A/d = 1$, $\Delta_2/d = 4$, $\nu_1 = \nu_2 = \nu_3 = 1$.

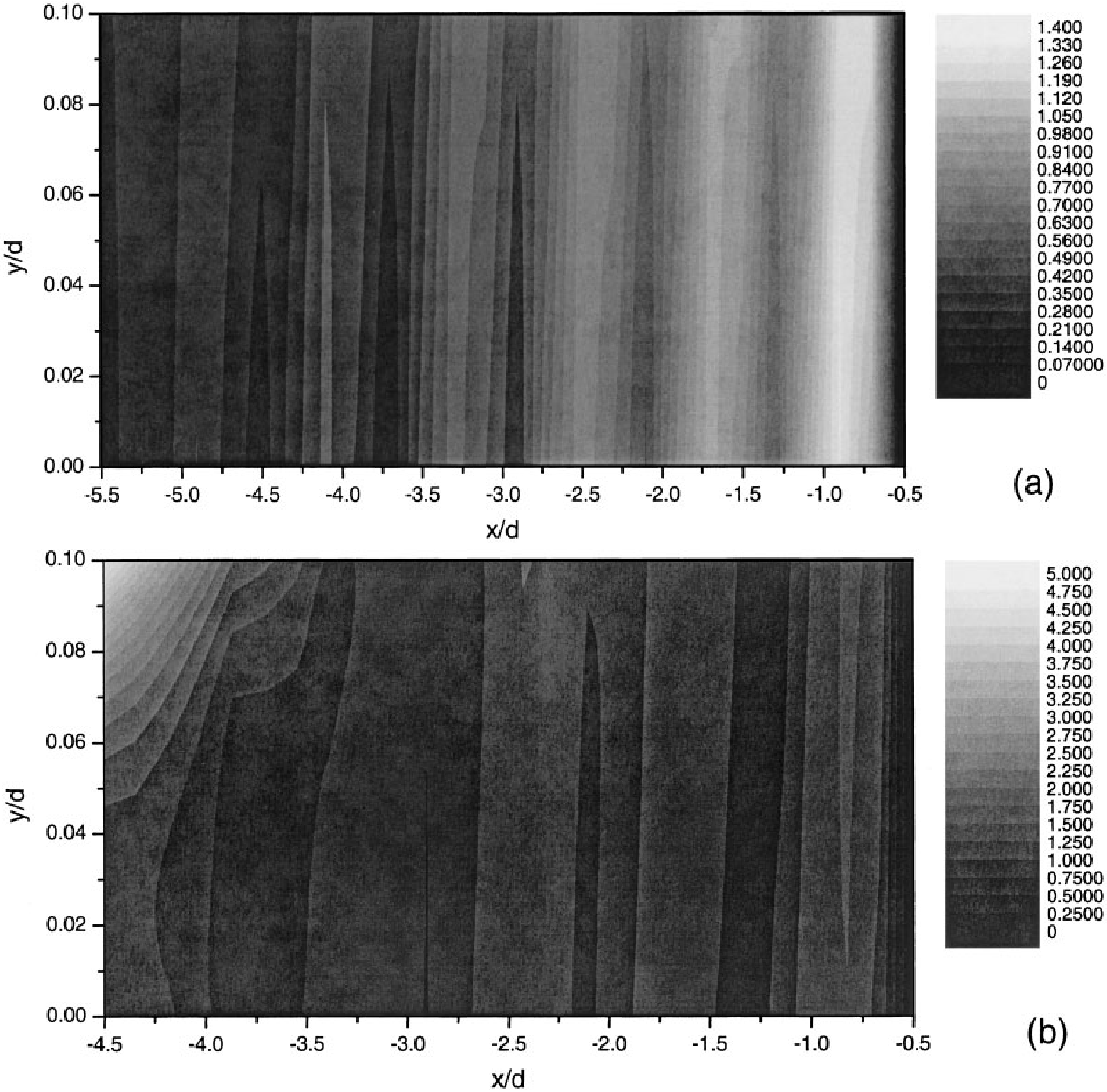

Fig. 4. Contour plot of $|E_z(x, y)|$ in the left isotropic zone within for $0 < y < h$. a) case with no PMAA (represented by the dark circles in fig. 3); b) case represented by the thick solid line in fig. 3.

4. Conclusion

The multilayer modal formalism, previously developed for investigating diffraction from an inhomogeneous isotropic aperture in a thick metallic screen, has been extended to include a perfectly matched anisotropic absorber (PMAA) as a new type of medium. From this extension we can get approximate solutions to the scattering problem of an isolated conducting cylinder of rectangular section in open space. Other non-rectangular geometries of the sample could be treated using a multilayer scheme, as in ref. [8].

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