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2012 J. Phys. G: Nucl. Part. Phys. 39 085011

(http://iopscience.iop.org/0954-3899/39/8/085011)

J. Phys. G: Nucl. Part. Phys. **39** (2012) 085011 (10pp) doi:10.1088/0954-3899/39/8/085011

Absorptive one-loop corrections and the complex-mass prescription for the Δ resonance propagator

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Received 12 October 2011
Published 12 July 2012
Online at stacks.iop.org/JPhysG/39/085011

Abstract
It is shown that the absorptive one-loop self-energy corrections to the Δ^{++} propagator can reproduce the complex-mass prescription for its resonant form. The effects of absorptive corrections beyond the ones considered in the complex-mass (CM) prescription for the propagator turn out to be negligible in the cross section of $\pi^+ p$ elastic scattering. This result brings support to the CM approximation used in previous calculations to include the Δ baryon in processes involving the production of the πN system in the resonance region.

1. Introduction

The mass $M_{\rm res}$ and decay width $\Gamma_{\rm res}$ are intrinsic properties of a resonance for which there are at least two commonly used definitions [1]. One definition is obtained from the real and imaginary parts of the pole position $s_p \equiv M^2 - {\rm i}M\Gamma$ of the S-matrix amplitude[3]; this complex pole is located at the value $s = s_p$, where s is the square of the invariant mass of the decay products of the resonance. A different definition is provided by the mass parameter m which is obtained from the renormalized propagator that includes the resummation of 1PI self-energies computed from the field theory Lagrangian that describes the dynamics of the resonance; in this case, the corresponding energy-dependent decay width $\Gamma(s)$ is determined by the interactions of the resonance with other fields and the decay width is given by $\Gamma(m^2)$. Hereafter, we will refer to these definitions as the *pole* and *field-theoretic* (FT) parameters of the resonance, respectively. We note that some authors used to call the second case Breit–Wigner parameters [2]. While the pole position is a physical (model and process-independent) property of the S-matrix amplitude [3], the FT resonance parameters usually depend on the particular model used to compute the decay width $\Gamma(s)$ and on the choice of the model-dependent background contributions to the scattering amplitude [5]. Furthermore, some authors use generalized Breit–Wigner formulas

[3] An equivalent definition is provided by $s_p \equiv (M' - {\rm i}\Gamma'/2)^2$. The two sets of pole parameters are related by $M = M'\gamma'$, $\Gamma = \Gamma'/\gamma'$, where $\gamma' \equiv \sqrt{1 - (\Gamma'/2M')^2}$ is usually very close to unity.

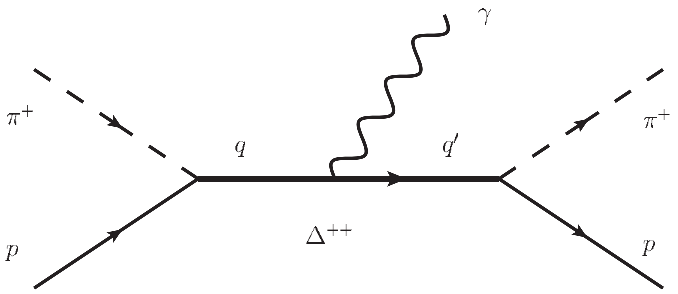

Figure 1. Photon emission off the Δ^{++} internal line in radiative $\pi^+ p$ scattering.

to describe hadronic resonances, which partially include the effects of background terms (see for example discussion in [2]).

The $\Delta(1232)$ baryon ($I = J = 3/2$), which manifests as a peak in the πN p-wave invariant-mass distribution, is one of the most familiar resonances whose properties have been measured in different reactions. Values extracted from data for the pole and FT parameters of the $\Delta(1232)$ are very different among themselves [1]: ($M_\Delta \approx 1210$, $\Gamma_\Delta \approx 100$) MeV and ($m_\Delta \approx 1232$, $\Gamma(m_\Delta) \approx 118$) MeV for the pole and FT values, respectively. The pole and FT resonance parameters can be related by imposing the pole condition $s_p - m^2 + \mathrm{i}\sqrt{s_p}\Gamma(s_p) = 0$ on the FT expression for the inverse propagator [4]. The choice of a particular definition depends on whether we are interested in fitting data to test a specific model of resonances or extracting its parameters in a model-independent way.

In the presence of gauge interactions, the transition amplitudes that describe the production and decay of resonances must satisfy gauge invariance. The naive introduction of the finite width effects in the resonance propagator can spoil the gauge invariance of the amplitude. As has been extensively discussed in the literature in the case of unstable gauge bosons [5, 6] and of hadronic resonances [7, 8], there are several procedures to render the amplitude gauge invariant in the presence of finite width effects of unstable particles. One of these prescriptions is the so-called complex-mass scheme (CMS), which consists in replacing $m^2 \to m^2 - \mathrm{i}m\Gamma$ (or equivalently, $m \to m - \mathrm{i}\Gamma/2$) in all the Feynman rules where the mass of the unstable particle appears. As a result, the transition amplitude has a pole located at the complex-mass position; note, however, that using the complex-mass prescription *only* in the pole of the propagator spoils the gauge invariance of the amplitude.

To make clear the point about gauge invariance, consider the diagram depicted in figure 1, which contributes to radiative $\pi^+ p$ scattering and is not gauge invariant by itself. Six additional diagrams involving photon emission either from external lines or from $\pi^+ p \Delta^{++}$ vertices must be included to obtain the total gauge-invariant amplitude [7]. If one introduces an energy-dependent width only in the denominator of the Δ^{++} resonance propagators, the gauge-invariance property of the total scattering amplitude can be spoiled unless these finite widths are introduced in such a way that the Ward identity between the Δ^{++} electromagnetic vertex and propagator is respected. This can be achieved by incorporating consistently the dressed Δ^{++} electromagnetic vertex at the corresponding order. A similar situation is encountered in the study of the $\gamma p \to \gamma p \pi^0$ reaction close to the resonance region, where a real photon can couple to an internal Δ^+ line [9]. In both cases, gauge invariance can be preserved in the simplest way if the bare mass of the resonance is replaced by its complex mass in all Feynman rules [6, 7]. Note that the resonance contribution to the amplitudes of processes like pion photoproduction off nucleons is self-gauge-invariant by construction of the $\pi\Delta\gamma$ vertex, independently of the choice of the Δ baryon propagator.

In previous studies of the elastic and radiative $\pi^+ p$ scattering [10], pion photoproduction off nucleons [11] and of the one-pion production in νN reactions [12] in the Δ resonance region, we have assumed the CMS prescription [7] to introduce the finite width of the Δ resonance in the tree-level expression of the amplitudes. It is worth noting that the finite width effects can be introduced also by considering the absorptive one-loop corrections in the propagator *and* electromagnetic vertex of the resonance (see for example [6, 8]). This gives rise to gauge-invariant amplitudes as these corrected Green functions satisfy the electromagnetic Ward identity. For example, in the case of the $\Delta^+(1232)$ resonance, this calculation was implemented in the study of the $\gamma p \to p\pi^0\gamma$ reaction in the framework of the chiral effective field theory [13].

By considering the absorptive part of the one-loop self-energy corrections to the Δ propagator, in this paper we show how the CMS prescription advocated in our previous papers can be derived in the limit of massless particles appearing in loop corrections (similar results were obtained in the case of spin-1 resonances in [6, 8]). The effects on the cross section of elastic $\pi^+ p$ scattering due to the absorptive corrections beyond the ones considered in the CMS expression of the propagator turn out to be negligibly small. This result provides further support to the use of the complex-mass prescription in processes involving the production and decay of resonances.

2. Resonant spin-3/2 propagator

The Lagrangian that describes the spin-3/2 Rarita–Schwinger field $\psi^\mu(x)$ and its interactions with other fields is invariant under contact interaction transformations $\psi^\mu \to \psi^\mu + \bar{a}\gamma^\mu\gamma_\alpha\psi^\alpha$ [14]. The corresponding Feynman rules involving the interacting spin-3/2 field depend upon an arbitrary parameter A, although the S-matrix amplitudes derived thereof are independent of A, as they should be [14]. This led us to consider a set of *reduced* Feynman rules which are explicitly independent of the parameter A and give rise to the same invariant amplitudes [7].

In this reduced set of Feynman rules, the bare propagator is given by

$$\begin{aligned} \mathrm{i}G_0^{\mu\nu}(p) = \frac{\not{p}+m}{p^2-m^2}\left\{-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3m}(\gamma^\mu p^\nu - \gamma^\nu p^\mu) + \frac{2}{3m^2}p^\mu p^\nu\right\} \\ - \frac{2}{3m^2}[\gamma^\mu p^\nu - \gamma^\nu p^\mu + (\not{p}+m)\gamma^\mu\gamma^\nu], \end{aligned} \tag{1}$$

where m is the bare mass of the spin-3/2 field and p its 4-momentum. If we focus on the Δ baryon, the corresponding interaction for the $\Delta \to N\pi$ vertex is given by $-\mathrm{i}gk^\mu$ [7], where g is the strong coupling with dimensions of mass^{-1} and k denotes the 4-momentum of the outgoing pion.

In terms of the basis of spin projection operators [15, 16],

$$\begin{aligned} (\mathcal{P}^{3/2})^{\mu\nu} &= g^{\mu\nu} - \frac{2}{3}\frac{p^\mu p^\nu}{p^2} - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3p^2}(\gamma^\mu p^\nu - \gamma^\nu p^\mu)\not{p}, \\ (\mathcal{P}_{11}^{1/2})^{\mu\nu} &= \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3}\frac{p^\mu p^\nu}{p^2} - \frac{1}{3p^2}(\gamma^\mu p^\nu - \gamma^\nu p^\mu)\not{p}, \\ (\mathcal{P}_{22}^{1/2})^{\mu\nu} &= \frac{p^\mu p^\nu}{p^2}, \\ (\mathcal{P}_{21}^{1/2})^{\mu\nu} &= \sqrt{\frac{3}{p^2}}\frac{1}{3p^2}(-\mathrm{i}\sigma^{\mu\alpha}p_\alpha)\not{p}p^\nu, \end{aligned} \tag{2}$$

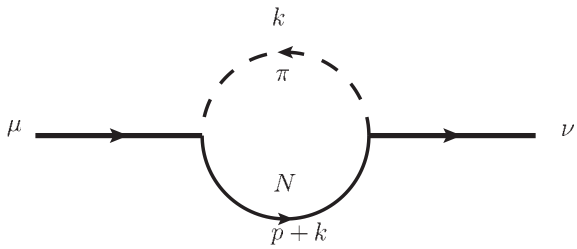

Figure 2. One-loop πN self-energy correction to the Δ propagator.

$$\left(\mathcal{P}_{12}^{1/2}\right)^{\mu\nu} = \sqrt{\frac{3}{p^2}}\frac{1}{3p^2}(-\mathrm{i}\sigma^{\nu\alpha}p_\alpha)\not{p}\,p^\mu,$$

with $\sigma^{\mu\nu} = \mathrm{i}[\gamma^\mu, \gamma^\nu]/2$, the bare propagator given in equation (1) can be re-written as

$$\mathrm{i}G_0^{\mu\nu}(p) = \frac{m+\not{p}}{m^2-p^2}(\mathcal{P}^{3/2})^{\mu\nu} - \frac{2}{m^2}(m+\not{p})\left(\mathcal{P}_{11}^{1/2}\right)^{\mu\nu} + \frac{\sqrt{3}}{m\sqrt{p^2}}\not{p}\left[\left(\mathcal{P}_{21}^{1/2}\right)^{\mu\nu} - \left(\mathcal{P}_{12}^{1/2}\right)^{\mu\nu}\right]. \quad (3)$$

Note that only a partial subset of projection operators given in equation (3) appear in the expression of the tree-level propagator. The pole of the propagator ($p^2 = m^2$), which gives rise to the pole of the physical amplitude, appears only in the pure spin-3/2 component as it should. The remaining pieces of $\mathrm{i}G_0^{\mu\nu}(p)$, which propagate the spin-1/2 components of the field, do not develop a pole.

An alternative and useful basis of projection operators is the following [17]:

$$\begin{aligned}
(\mathcal{P}_{1,2})^{\mu\nu} &= \Lambda^\pm(\mathcal{P}^{3/2})^{\mu\nu},\\
(\mathcal{P}_{3,4})^{\mu\nu} &= \Lambda^\pm\left(\mathcal{P}_{11}^{1/2}\right)^{\mu\nu},\\
(\mathcal{P}_{5,6})^{\mu\nu} &= \Lambda^\pm\left(\mathcal{P}_{22}^{1/2}\right)^{\mu\nu},\\
(\mathcal{P}_{7,8})^{\mu\nu} &= \Lambda^\pm\left(\mathcal{P}_{21}^{1/2}\right)^{\mu\nu},\\
(\mathcal{P}_{9,10})^{\mu\nu} &= \Lambda^\pm\left(\mathcal{P}_{12}^{1/2}\right)^{\mu\nu},
\end{aligned} \quad (4)$$

where $\Lambda^\pm = \dfrac{\sqrt{p^2}\pm\not{p}}{2\sqrt{p^2}}$.

The expression for the dressed propagator $\mathrm{i}G^{\mu\nu}(p)$ can be obtained by solving the Schwinger–Dyson equation that is satisfied by the inverse propagators:

$$(\mathrm{i}G^{-1})^{\mu\nu}(p) = \left(\mathrm{i}G_0^{-1}\right)^{\mu\nu}(p) - \Sigma^{\mu\nu}(p), \quad (5)$$

where $\Sigma^{\mu\nu}(p)$ denotes the self-energy correction of Δ as shown in figure 2. In the following we will consider only the absorptive (imaginary) parts of the self-energy correction, i.e. we will assume [6, 8] that the parameter m represents the 'renormalized' mass of Δ (we place quotation marks as a reminder that the Lagrangian is not renormalizable; only the absorptive corrections are finite in this case). If we denote by p the 4-momentum of Δ and compute the one-loop absorptive corrections by applying the cutting rules to figure 2, we obtain

$$\Sigma_{\mathrm{abs}}^{\mu\nu}(p) = \mathrm{i}\frac{g^2}{2(2\pi)^2}\int\frac{\mathrm{d}^3k}{2k_0}\frac{1}{2\sqrt{p^2}}\delta\left(k_0 + \frac{p^2+m_\pi^2-m_N^2}{2\sqrt{p^2}}\right)$$
$$\times\,\theta\left(p^2-(m_N+m_\pi)^2\right)(\not{p}+\not{k}+m_N)k^\mu k^\nu, \quad (6)$$

which, in terms of the basis of projection operators given in equation (4), we can write as

$$\Sigma_{\mathrm{abs}}^{\mu\nu}(p) = \sum_i \bar{J}_i(\mathcal{P}_i)^{\mu\nu}. \quad (7)$$

The coefficients $\bar{J}_i$ can be computed by projecting out equations (6) and (7) into the set of ten projection operators introduced in equation (4). Thus we obtain (we use $s = p^2$):

$$\begin{aligned}
\bar{J}_1 &= \bar{J}_3 = -\mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{1}{12s}\left[\frac{(\sqrt{s}+m_N)^2 - m_\pi^2}{4\sqrt{s}}\right]\lambda(s, m_N^2, m_\pi^2)\\
\bar{J}_2 &= \bar{J}_4 = \mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{1}{12s}\left[\frac{(\sqrt{s}-m_N)^2 - m_\pi^2}{4\sqrt{s}}\right]\lambda(s, m_N^2, m_\pi^2),\\
\bar{J}_5 &= \mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{1}{4s}(s - m_N^2 + m_\pi^2)^2\left[\frac{(\sqrt{s}+m_N)^2 - m_\pi^2}{4\sqrt{s}}\right],\\
\bar{J}_6 &= -\mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{1}{4s}(s - m_N^2 + m_\pi^2)^2\left[\frac{(\sqrt{s}-m_N)^2 - m_\pi^2}{4\sqrt{s}}\right],\\
\bar{J}_7 &= \bar{J}_8 = \bar{J}_9 = \bar{J}_{10} = \mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\sqrt{\frac{3}{s}}\frac{1}{48s}(s - m_N^2 + m_\pi^2)\lambda(s, m_N^2, m_\pi^2),
\end{aligned} \tag{8}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, and [17]:

$$I_0 = \theta(s - (m_N + m_\pi)^2)\left(\frac{\pi}{2}\right)\frac{\lambda^{1/2}(s, m_N^2, m_\pi^2)}{s}. \tag{9}$$

For the purposes of comparison with the results of [17], we obtain the closely related coefficients:

$$\begin{aligned}
J_1 &= J_3 = -\mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{m_N}{24s}\lambda(s, m_N^2, m_\pi^2),\\
J_2 &= J_4 = -\mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{1}{48s^2}(s + m_N^2 - m_\pi^2)\lambda(s, m_N^2, m_\pi^2),\\
J_5 &= \mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{m_N}{8s}(s - m_N^2 + m_\pi^2)^2,\\
J_6 &= \mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\frac{1}{16s^2}(s + m_N^2 - m_\pi^2)(s - m_N^2 + m_\pi^2)^2,\\
J_7 &= J_9 = \mathrm{i}\frac{g^2 I_0}{(2\pi)^2}\sqrt{\frac{3}{s}}\frac{1}{48s}(s - m_N^2 + m_\pi^2)\lambda(s, m_N^2, m_\pi^2),\\
J_8 &= J_{10} = 0,
\end{aligned} \tag{10}$$

which are defined from the relations: $\bar{J}_{2n-1} \equiv J_{2n-1} + \sqrt{s}J_{2n}$ and $\bar{J}_{2n} \equiv J_{2n-1} - \sqrt{s}J_{2n}$, for $n = 1, \ldots, 5$. The results displayed in equations (10) are identical to the results given in equation (56) of [17]. Note that our expressions are related to the so-called discontinuities ΔJ_i calculated in [17] as follows $J_i = \Delta J_i/2(2\pi)^2$ and that our reduced Feynman rules correspond to the choice $a = 0$ in their notation.

By inserting the results shown in equations (8)–(10) into equations (7) and (5), we obtain the following form of the dressed propagator:

$$\begin{aligned}
\mathrm{i}G^{\mu\nu}(p) = \frac{1}{1-J_2}\Bigg\{&\frac{\widetilde{m} + \not{p}}{\widetilde{m}^2 - p^2}(\mathcal{P}^{3/2})^{\mu\nu}\\
&+ \frac{1}{2}\left[\frac{2\widetilde{m} - 2\sqrt{p^2} + A_+}{-\widetilde{m}^2 + X_+} + \frac{2\widetilde{m} + 2\sqrt{p^2} + A_-}{-\widetilde{m}^2 + X_-}\right](\mathcal{P}_{11}^{1/2})^{\mu\nu}\\
&+ \frac{1}{2\sqrt{p^2}}\left[-\frac{2\widetilde{m} - 2\sqrt{p^2} + A_+}{-\widetilde{m}^2 + X_+} + \frac{2\widetilde{m} + 2\sqrt{p^2} + A_-}{-\widetilde{m}^2 + X_-}\right]\not{p}(\mathcal{P}_{11}^{1/2})^{\mu\nu}\\
&+ \frac{1}{2}\left[\frac{3\frac{J_3 - \sqrt{p^2}J_4}{1-J_2}}{-\widetilde{m}^2 + X_+} + \frac{3\frac{J_3 + \sqrt{p^2}J_4}{1-J_2}}{-\widetilde{m}^2 + X_-}\right](\mathcal{P}_{22}^{1/2})^{\mu\nu}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2\sqrt{p^2}}\left[\frac{3\frac{J_3-\sqrt{p^2}J_4}{1-J_2}}{-\widetilde{m}^2+X_+}-\frac{3\frac{J_3+\sqrt{p^2}J_4}{1-J_2}}{-\widetilde{m}^2+X_-}\right]\not{p}\big(\mathcal{P}^{1/2}_{22}\big)^{\mu\nu}\\
&+\frac{\sqrt{3}}{2}\left[\frac{\widetilde{m}-\left(\frac{J_1+\sqrt{3}J_7}{1-J_2}\right)}{-\widetilde{m}^2+X_+}-\frac{\widetilde{m}-\left(\frac{J_1-\sqrt{3}J_7}{1-J_2}\right)}{-\widetilde{m}^2+X_-}\right]\big[\big(\mathcal{P}^{1/2}_{21}\big)^{\mu\nu}+\big(\mathcal{P}^{1/2}_{12}\big)^{\mu\nu}\big]\\
&-\frac{\sqrt{3}}{2\sqrt{p^2}}\left[\frac{\widetilde{m}-\left(\frac{J_1+\sqrt{3}J_7}{1-J_2}\right)}{-\widetilde{m}^2+X_+}+\frac{\widetilde{m}-\left(\frac{J_1-\sqrt{3}J_7}{1-J_2}\right)}{-\widetilde{m}^2+X_-}\right]\not{p}\big[\big(\mathcal{P}^{1/2}_{21}\big)^{\mu\nu}-\big(\mathcal{P}^{1/2}_{12}\big)^{\mu\nu}\big]\Bigg\},
\end{aligned}
\tag{11}
$$

where we have defined

$$
\begin{aligned}
X_\pm &\equiv \frac{2m(J_1+J_3\pm\sqrt{3}J_7\mp pJ_4)+2p(\mp J_3+pJ_4)+J_1^2}{(1-J_2)^2}\\
A_\pm &\equiv \frac{3(J_5\pm pJ_6)-2(J_1\pm pJ_2)}{1-J_2}.
\end{aligned}
\tag{12}
$$

Note that $X_\pm$ and $A_\pm$ are functions that start at $O(g^2)$.

In equation (11) we have introduced the effective mass term:

$$
\begin{aligned}
\widetilde{m} &= \frac{m+J_1}{1-J_2}\\
&= m+(J_1+\sqrt{s}J_2)+(m-\sqrt{s})J_2+O(g^4)\\
&\approx m-\mathrm{i}\frac{\Gamma_\Delta(s)}{2},
\end{aligned}
\tag{13}
$$

where we have neglected terms of $O(g^4)$ and $O((m-\sqrt{s})g^2)$ in the last result, because these terms are expected to be very small in the resonance region ($\sqrt{s}\approx m$). In equation (13) we have introduced the $\Delta\to N\pi$ energy-dependent decay width which is defined as

$$
\Gamma_\Delta(s)=\frac{g^2}{4\pi}\left(\frac{(\sqrt{s}+m_N)^2-m_\pi^2}{48s^{5/2}}\right)\lambda^{3/2}\big(s,m_N^2,m_\pi^2\big).
\tag{14}
$$

Next, we define a renormalized propagator $\mathrm{i}G_R^{\mu\nu}$ as follows:

$$
\mathrm{i}G^{\mu\nu}(p)=(1-J_2)^{-1}\left[\mathrm{i}G_R^{\mu\nu}(p)\right],
\tag{15}
$$

where the factor $(1-J_2)^{-1/2}$ can be absorbed as a component of the Δ wavefunction renormalization constant. If we keep only the leading (in g^2) terms in the coefficients of the projection operators in equation (11), we obtain

$$
\mathrm{i}G_R^{\mu\nu}(p)=\frac{\widetilde{m}+\not{p}}{\widetilde{m}^2-p^2}(\mathcal{P}^{3/2})^{\mu\nu}-\frac{2}{\widetilde{m}^2}(\widetilde{m}+\not{p})\big(\mathcal{P}^{1/2}_{11}\big)^{\mu\nu}+\frac{\sqrt{3}}{\widetilde{m}\sqrt{p^2}}\not{p}\big[\big(\mathcal{P}^{1/2}_{21}\big)^{\mu\nu}-\big(\mathcal{P}^{1/2}_{12}\big)^{\mu\nu}\big].
\tag{16}
$$

A comparison of equations (16) and (3) shows that the renormalized propagator has identical form to the tree-level propagator under the replacement $m\to\widetilde{m}=m-\mathrm{i}\Gamma_\Delta(s)/2$. Note, however, that this expression involves a complex mass with an energy-dependent width and therefore it does not correspond exactly to the complex-mass prescription which involves a constant width.

In [6, 8] we have shown that, in the limit of massless particles appearing in absorptive one-loop corrections, the renormalized propagator reproduces exactly the one obtained by means of the CMS prescription. If we consider this massless limit[4] in equation (14), namely $m_N = m_\pi = 0$, we obtain the following expression for the decay width:

$$\Gamma_\Delta(s) = \frac{g^2}{192\pi} s^{3/2} = \left(\frac{\sqrt{s}}{m}\right)^3 \Gamma_\Delta, \qquad (17)$$

where $\Gamma_\Delta \equiv \Gamma_\Delta(s = m^2)$ in the limit of massless nucleon and pion.

The width given in equation (17) grows with energy as $s^{3/2}$ while, in the corresponding massless limit, the widths of the W boson and ρ meson grow as $s^{1/2}$ [6, 8]. This different behavior can be easily understood because in the massless limit the decay width behaves as $\Gamma(s) \sim g^2 s^X$; thus, for dimensionless couplings $X = 1/2$, while for the Δ resonance $X = 3/2$ because its coupling has $(\text{mass})^{-1}$ units. Therefore, if we assume that the $\Delta N\pi$ coupling behaves as $g(s) = \tilde{g}/\sqrt{s}$ when the Δ resonance is off its mass shell (now $\tilde{g}$ is dimensionless and constant), we get that $\Gamma_\Delta(s)$ grows with energy in a similar way as the W boson and ρ meson decay widths. In this case, we can remove the energy dependence in the complex mass introduced in equation (13) by using a proper redefinition of the mass and decay width (as done for example in [6, 8]). Thus, we can propose the following conjecture as a general rule: 'the massless limit of the absorptive self-energy corrections to the propagator of a resonance can reproduce its complex-mass prescription if the couplings involved in loop corrections are dimensionless.'

3. Effects of absorptive corrections

One may wonder about the size of terms that have to be neglected in equation (11) in order to obtain equation (16). We can expect these terms to be relatively small because they appear only in the non-resonant spin-1/2 pieces of the Δ propagator (see equation (11)). Their observable effects can be estimated by comparing the amplitudes for the production and decay of a Δ resonance when we use the propagators given in equations (11) and (16).

In figure 3 we compare the total cross sections for $\pi^+ p$ elastic scattering when we consider three different forms of the Δ^{++} propagator (background terms in the amplitude are not considered in these plots). The result corresponding to the exact propagator (terms within curly brackets in equation (11)) is represented with a solid line, and the one corresponding to the complex-mass approximation, equation (16) with a constant width, is shown by a dashed line. Just for comparison, we also display (dotted line) the result obtained using the pure spin-3/2 component of the propagator (first term in the r.h.s. of equation (16)). In all three cases, we have used $\widetilde{m} = m_\Delta - i\Gamma_\Delta/2$, with $m_\Delta = 1211.20$ MeV and $\Gamma_\Delta = 88.16$ MeV as was obtained from fits to data in [10]; the corresponding value of the strong coupling constant $g = 14.3\ \text{GeV}^{-1}$ [10] is used to quantify the effects of subleading terms in equation (11). Small differences are observed only for energies above the resonance peak. This means that the terms in the propagator that describe the propagation of virtual spin-1/2 components give very small contributions.

As a second example, we consider the differential cross section of elastic $\pi^+ p$ scattering off the peak of the resonance. The angular distribution of pions is expected to be more sensitive to the spin components of the resonance. Furthermore, the interference of the resonance and background contributions can make more visible the difference between the exact calculation given in equation (11) and the one based on the CMS prescription. In figure 4 we plot this

[4] This is, of course, a formal limit. We know that the nucleon mass does not disappear in the chiral limit, as was the case for particles involved in the examples considered in [6, 8].

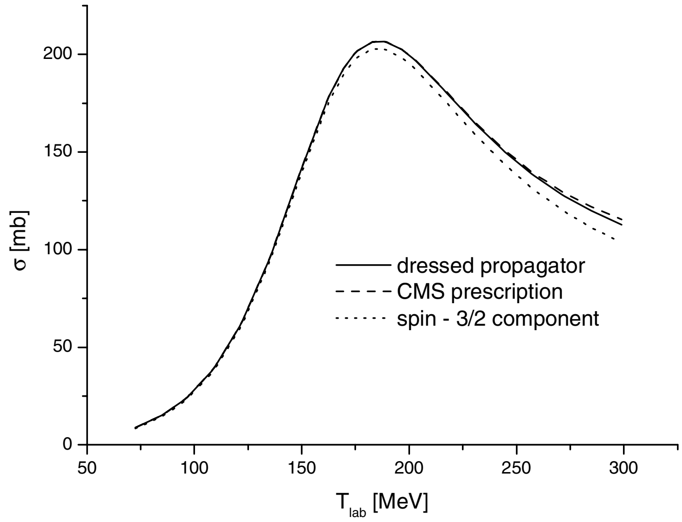

Figure 3. Cross section of $\pi^+ p$ elastic scattering in the $\Delta(1232)$ resonance region. The different plots are described in section 3. The horizontal axis represents the kinetic energy of pions in the lab frame.

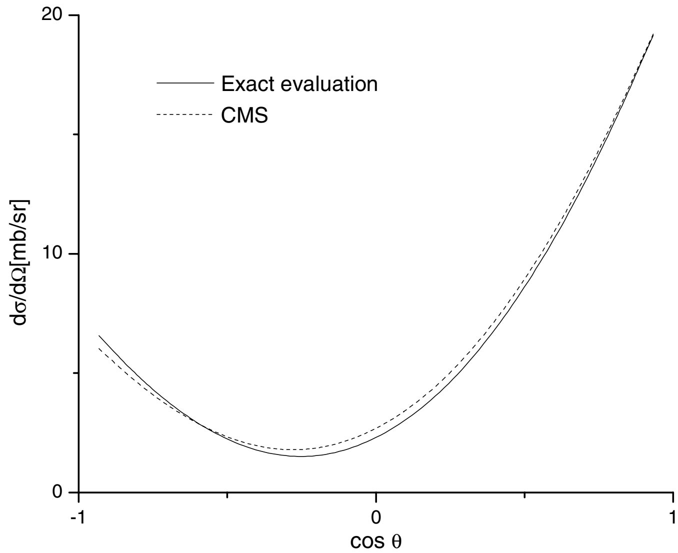

Figure 4. Differential cross section of $\pi^+ p$ elastic scattering at a pion kinetic energy T = 291.5 MeV in the lab frame. The solid line corresponds to the exact calculation, and the dashed line to the use of the CMS prescription.

observable as a function of $\cos\theta$, where θ is the scattering angle of pions in the lab frame. All the parameters have been fixed to the values of the previous example. In figure 4 the solid line corresponds to the results of the exact calculation, while the dashed line represents the use of the CMS prescription. Both calculations include the background contributions of the Δ^0

crossed channel, the nucleon pole and the exchange of ω, ρ^0 and σ mesons in the t channel as described in [10]. The most significant differences are observed close to $\theta = \pi/2$ and π, which however do not affect the good agreement with data as can be checked through a simple inspection of figure 6 in [10].

We can conclude that the effects of absorptive corrections beyond the ones included in the complex mass $\widetilde{m}$ are negligibly small. Furthermore, since the pure spin-3/2 component of the Δ^{++} propagator (which is obtained from the absorptive corrections without further approximations) largely dominates the cross section, we can understand why the CMS provides a good approximation to the propagator.

4. Conclusions

The complex-mass prescription for the propagator of a resonance, namely the replacement of the bare mass by the complex mass $\widetilde{m}^2 = m^2 - \mathrm{i}m\Gamma = (m' - \mathrm{i}\Gamma'/2)^2$, is a useful and easy rule to implement gauge invariance in the amplitudes for processes involving the production and decay of resonances [7]. On the other hand, the mass and width involved in the complex mass $\widetilde{m}$ allow a direct link between the field-theoretic and pole definitions of the resonance parameters described in the introduction.

In this paper we have shown that the complex-mass prescription for the $\Delta(1232)$ resonance can be obtained from the massless limit of the one-loop absorptive self-energy corrections to its propagator. We conjecture that this rule follows from any model where the coupling of resonances to their decay products is dimensionless. We observe that the absorptive corrections beyond the ones considered in the complex mass $\widetilde{m}$ have negligible observable effects in the cross section of π^+p scattering, which is largely dominated by the Δ^{++} in the resonance region. We conclude from this that the complex-mass prescription used in our previous calculations is well justified.

Acknowledgments

GLC acknowledges financial support from Conacyt (Mexico). CB and AM are fellows of CONICET, CCT La Plata (Argentina) and thank the Agencia Nacional de Promoción Científica y Técnica for supporting this work through PICT-2007-00861.

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