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TECHNICAL NOTE

Modeling the Fiber Addition Influence on the Small Strain Shear Modulus of Sand

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Abstract The mechanical behavior of fiber reinforced soils has been extensively studied in the last decades. Previous studies have shown that inclusion of fibers increases the shear strength of the reinforced soil. However, the presence of fibers can reduce, in some cases, the stiffness of the composite material. In this paper, we study the change on the initial stiffness in alluvial sand reinforced with polypropylene fibers. A model based on Hertz elastic contact theory is developed in order to explain the trends of shear wave velocity and maximum shear modulus in the fiber reinforced sand as the fiber content varies. The model assumes that the shear wave is transmitted through elastic distortions at the contacts, so the stiffness of the contacts governs the initial shear modulus, which in turn is affected by fiber additions. Furthermore, the ratio between the amount of grain to fiber contacts and the total of contacts on the shear wave path influence the maximum shear modulus. An experimental testing program involving confined compression tests with shear wave velocity measurements of unreinforced and fiber-reinforced sand specimens was undertaken to validate the proposed model trends. The model predictions were found to be in good agreement with the experimental results.

Keywords Polypropylene fibers · Reinforced soil · Shear modulus · Shear wave velocity

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List of symbols

α	Ratio between fiber-to-grain contact stiffness and grain-to-grain contact stiffness
β	Ratio of the number of fiber-to-grain contacts to the total of contacts
d_g	Soil grain diameter
d_f	Fiber diameter
FC%	Fiber content percentage
G_g	Shear modulus of the soil grains
G_f	Shear modulus of the fiber
G_{max}	Maximum shear modulus of soil without reinforcement
G'_{max}	Maximum shear modulus of fiber reinforced soil
G''_{max}	Maximum shear modulus when all contacts are fiber-to-grain type
G^*	Effective shear modulus
γ_s	Specific gravity of soil
γ_f	Specific gravity of fiber
L_f	Fiber length
μ_{gg}	Grain-to-grain contact shear stiffness
μ_{gf}	Fiber-to-grain contact shear stiffness
N	Contact force
N_c	Number of contacts
N_s	Number of spheres
N_{gg}	Number of grain-to-grain contacts
N_{gf}	Number of fiber-to-grain contacts
ν_g	Poisson ratio of the soil grains
ν_f	Poisson ratio of the fiber
r_c	Contact radius between two spheres
r'_c	Contact radius between a sphere and a cylinder
ρ	Soil bulk density
V_s	Shear wave velocity
W_s	Soil sample weight

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Introduction

Soil reinforcement by means of fiber addition has been reported in the last few decades by several investigators (e.g. [1–6], among others).

In granular soils, previous researchers agree that fiber reinforcement can increase the peak shear strength and significantly reduce the post-peak shear strength loss of the soil mass. However, for high fiber contents, the strength increment of the reinforced soil seems to approach an asymptotic upper limit [7–10]. In addition, the strain level required to attain the peak shear strength increases with fiber content [11].

On the other hand, only a few studies dealing with the effect of fiber inclusion on the low strain stiffness of the reinforced soil have been published. Among these, Heineck et al. [12] observed that the inclusion of fibers does not change the maximum stiffness at low strain levels (10^{-5}) of the reinforced soil, provided that the fiber content is up to 0.5% by weight of dry soil. Furthermore, Diambra et al. [13] observed that shear modulus at medium strain levels (10^{-3}) is not affected by fiber incorporation. Conversely, other investigators suggest that the stiffness of the reinforced soil at low strains is reduced when fiber content is higher than 0.5% by volume [11].

The dispersion and dissimilarities of the experimental conclusions achieved by different authors about the effect of fiber reinforcement on the stiffness of the soil may be due to the varied methodologies adopted by the investigators to prepare the soil specimens and to the way that void ratio was addressed. On analyzing the literature, two main different approaches were found. The first one considers that fibers fill the soil voids, with no modification of the soil fabric. The soil properties (G_s and Unit Weight) are considered constants that do not change with fiber incorporation. In this case, the fiber inclusion tends to densify the material [9]. On the other hand, the second approach considers the fiber reinforced soil as a composite material. In this case, properties such as void ratio are calculated for the soil-fiber compound, disregarding the original soil structure. The objective of the second approach is to quantify the mechanical properties of the composite in order to judge their influence on the efficiency of the reinforcement method. Since fiber reinforced specimens are not prepared in the same way along the literature, results are compared based on different variables which may cause the dispersion on experimental published data.

The main goal of the present paper is to explain, justify and understand the behavior and trends of the shear wave velocity and maximum shear modulus (G_{max}) in fiber reinforced sand as fiber content varies. The understanding of the soil mass behavior is based on the physics and mechanics of particulate material interactions. With this in mind, a model based on the Hertz theory (elastic contact theory) is developed. The model predictions are validated by means of data from an experimental program consisting of shear wave velocity measurements on a modified oedometer with bender elements. The model assumes that the inclusion of fibers into the soil mass does not change the number of contacts per particle (coordination number) if the preparation method and the relative density of the samples are kept constant.

Theoretical Model of Contacts

Introduction to the Model

At low strain levels (10^{-5} or lower), it can be assumed that soils behave elastically, so there is a unique and direct relation between shear wave velocity (V_s) and maximum shear modulus (G_{max}) given by:

$$G_{max} = \rho \cdot V_s^2 \tag{1}$$

where ρ is the soil bulk density.

The shear wave velocity is obtained by measuring the time that a mechanical shear wave needs to travel a certain distance along a soil specimen. This shear wave is transmitted through elastic distortions at the contacts of soil grains and the contacts of fibers to soil grains (Fig. 1).

Thus, the initial shear modulus of the fiber reinforced soil (G'_{max}) is a function of the grain-to-grain contact stiffness (μ_{gg}), the fiber-to-grain contact stiffness (μ_{gf}), the number of grain-to-grain contacts (N_{gg}) and the number of fiber-to-grain contacts (N_{gf}).

The contact shear stiffness (μ) between two elastic bodies is defined by the Hertz Theory [14] as the ratio between a tangential force applied at the contact and the resulting relative elastic displacement (Fig. 2):

$$\mu = \frac{T}{2\delta} \tag{2}$$

Parameter α is defined as the ratio between the fiber-to-grain contact stiffness and the grain-to-grain contact stiffness (Eq. 3), and parameter β as the ratio of the number of fiber-to-grain contacts to the total of contacts through which the shear wave is transmitted (Eq. 4).

$$\alpha = \frac{\mu_{gf}}{\mu_{gg}} \tag{3}$$

$$\beta = \frac{N_{gf}}{N_{gf} + N_{gg}} \tag{4}$$

When fiber content is zero, the maximum shear modulus is the shear modulus of the soil without

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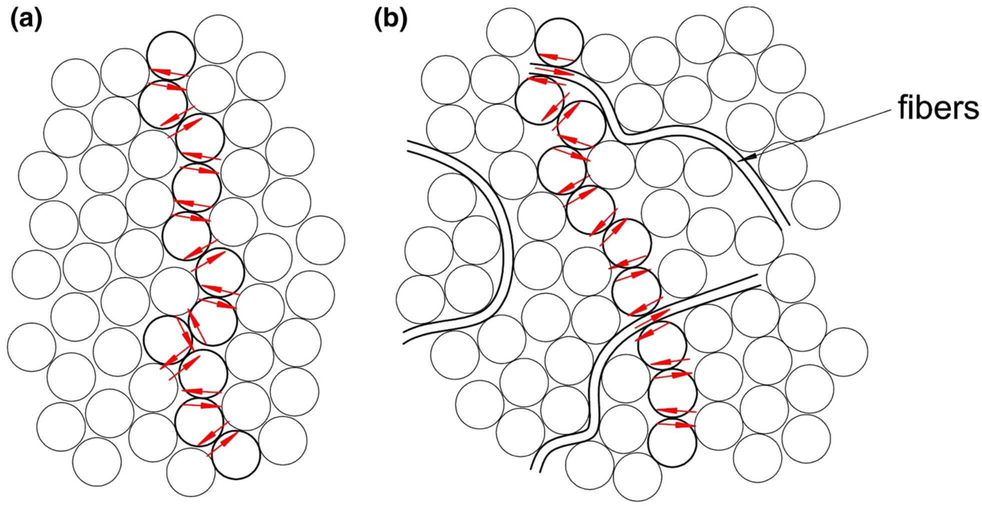

Fig. 1 **a** The shear wave is transmitted through elastic distortions at the grain-to-grain contacts; **b** the shear wave is transmitted through elastic distortions at the grain-to-grain and fiber-to-grain contacts

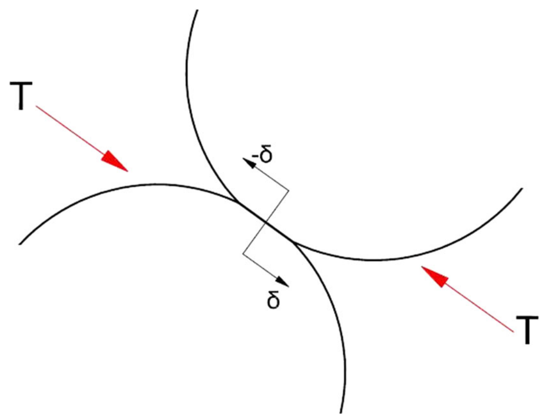

Fig. 2 Relative elastic displacement between two bodies under a tangential force at the contact

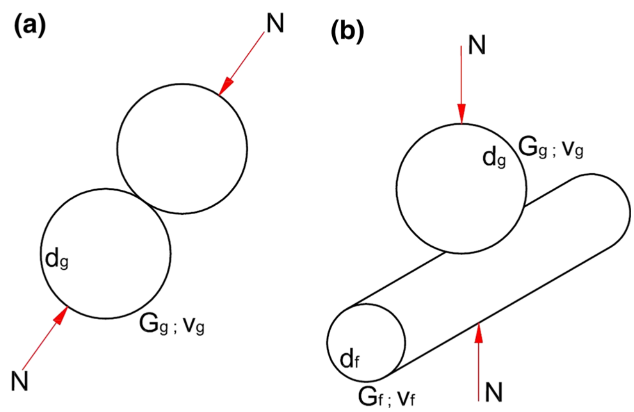

Fig. 3 Hertz theory of contact mechanics, **a** particles of soils are idealized as spheres and, **b** fibers are considered cylinders

reinforcement (G_{max}) and β is equal to zero. But if all contacts are of the fiber-to-grain type, the shear modulus will take a value G''_{max} directly related to the grain-to-fiber stiffness and β will be equal to one. Finally, the shear modulus G'_{max} of the fiber reinforced soil is given as a combination of the two limit conditions:

$$G'_{max} = \beta \cdot G''_{max} + (1 - \beta) \cdot G_{max} \quad (5)$$

As the grain stiffness is at least two orders of magnitude higher than the contact stiffness, it is reasonable to assume the relationship showed in Eq. 6:

$$\frac{G''_{max}}{G_{max}} = \frac{\mu_{gf}}{\mu_{gg}} = \alpha \quad (6)$$

Then, the initial shear modulus of the fiber reinforced soil is obtained as:

$$G'_{max} = G_{max} \cdot [1 - \beta \cdot (1 - \alpha)] \quad (7)$$

Determination of α

Parameter α relates the stiffness between fiber-to-grain contacts and grain-to-grain contacts. In order to quantify this parameter, the Hertz Theory of Contacts Mechanics is applied. Particles of soils are idealized as spheres, while fibers are assumed to be cylinders (Fig. 3). Both materials are considered elastic.

According to the Hertz Theory, the shear stiffness of a contact (μ) between two elastic bodies is [14]:

$$\mu = 2 \cdot G^* \cdot r_c \quad (8)$$

where r_c is the contact radius between the two bodies, and G* is the effective shear modulus given by a combination of the elastic properties of the two bodies under consideration (Eq. 9).

$$G^* = \left(\frac{2 - \nu_1}{4 \cdot G_1} + \frac{2 - \nu_2}{4 \cdot G_2}\right)^{-1} \quad (9)$$

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being v_1 and v_2 the Poisson ratio and G_1 and G_2 the shear modulus of the materials.

The contact radius depends on the shape of the bodies in contact. For two soil particles idealized as spheres, the contact radius is given by:

$$r_c = \sqrt[3]{\frac{3}{16} \cdot \frac{d_g \cdot (1 - v_g)}{G_g} \cdot N} \tag{10}$$

being N the contact force, d_g the grain diameter, Gg the shear modulus and v_g the Poisson ratio of the material of the soil grains. Introducing Eqs. 9 and 10 into Eq. 8 we obtain:

$$\mu_{gg} = \frac{4 \cdot G_g}{2 - v_g} \cdot \sqrt[3]{\frac{3}{16} \cdot \frac{d_g \cdot (1 - v_g)}{G_g} \cdot N} \tag{11}$$

On the other hand, since the fiber has a cylindrical shape, the fiber-to-grain stiffness is calculated also by Eq. 8, but using a contact radius given by the following equation:

$$r'_c = \sqrt[3]{\frac{3}{8} \cdot \left(\frac{1 - v_g}{2G_g} + \frac{1 - v_f}{2G_f}\right) \cdot \left[\frac{1}{d_g} \cdot \left(\frac{1}{d_g} + \frac{1}{d_f}\right)\right]^{-1/2} \cdot N} \tag{12}$$

where G_f is the shear modulus, v_f is the Poisson ratio of the fiber material and d_f is the fiber diameter. The value of r'_c is an average contact radius between a cylinder and a sphere, since the contact surface is oval. Combining Eqs. 8, 9 and 12, the fiber-to-grain contact stiffness is obtained as:

$$\mu_{gf} = 2 \cdot \frac{1}{\frac{2-v_g}{4 \cdot G_g} + \frac{2-v_f}{4 \cdot G_f}} \cdot \sqrt[3]{\frac{3}{8} \cdot \left(\frac{1 - v_g}{2G_g} + \frac{1 - v_f}{2G_f}\right) \cdot \left[\frac{1}{d_g} \cdot \left(\frac{1}{d_g} + \frac{1}{d_f}\right)\right]^{-1/2} \cdot N} \tag{13}$$

Finally, combining Eqs. 6, 11 and 13 we obtain parameter α:

$$\alpha = \frac{\mu_{gf}}{\mu_{gg}} = \frac{2}{1 + \frac{2-v_f}{2-v_g} \cdot \frac{G_g}{G_f}} \cdot \left[\left(1 + \frac{1 - v_f}{1 - v_g} \cdot \frac{G_g}{G_f}\right) \cdot \left(\frac{1}{1 + \frac{d_g}{d_f}}\right)^{1/2}\right]^{1/3} \tag{14}$$

Determination of β

In order to evaluate β, it is assumed that fibers are uniformly distributed and randomly oriented in the soil mass. To quantify the number of contacts inside a soil cube, only contacts between soil particles and contacts of soil particles with fibers are considered. No contacts at the boundaries of the soil mass are taken into account. In addition, in order to minimize the influence of the boundaries, a large enough volume of soil is considered to calculate β.

Now, we analyze the influence of the packing on β parameter. To do so, we study two cases: (1) a simple cubic packing representing a granular soil in a loose state with a relative density of zero per cent, and (2) a face-centered cubic packing for a soil in a dense state with a relative density of a hundred per cent (Fig. 4).

Simple Cubic Packing

In the simple cubic packing case, particles of soils are idealized as spheres of equal diameter, each of them in contact with other 6 spheres, so the coordination number (CN) is 6. Considering a cubic region of n^3 particles, the number of contacts will be:

$$N_c = 3 \cdot \left(n^3 - n^2\right) \tag{15}$$

Figure 5 shows the variation of the ratio "number of contacts" (N_c) to the "number of spheres" (N_s) with the sample mass, considering particles with specific gravity (γ_s) equal to 2.67. It can be seen in Fig. 5 that for a soil mass of over 0.25 kg the ratio N_c/N_s is almost constant and approximately equal to 3.

Thus, considering that the number of contacts (N_c) is equal to three times the number of spheres (N_s), and that the number of spheres is equal to the weight of a soil specimen divided by the weight of one sphere, the number of grain-to-grain contacts can be described as a function of the soil sample weight (W_s), particle diameter (d_g) and specific gravity of soil (γ_s) as:

$$N_{gg} = \frac{18}{\pi} \cdot \frac{W_s}{d_g^3 \cdot \gamma_s} \tag{16}$$

In order to evaluate the number of fiber-to-grain contacts, the number of fibers for a given fiber content (FC) is calculated by means of Eq. 17. Then, the number of

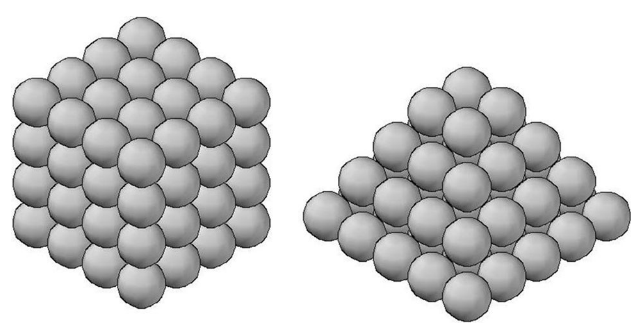

Fig. 4 Simple cubic packing (*left*) and face-centered cubic packing (*right*)

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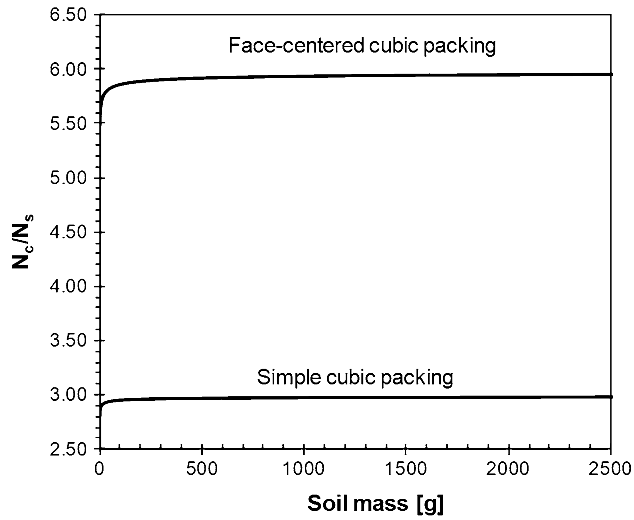

Fig. 5 Number of contacts to number of spheres ratio versus mass of soil specimen in a simple cubic packing and in a face-centered cubic packing

contacts between one single fiber and the surrounding grains of soil is approximated by Eq. 18.

$$n^{\underline{o}}fibers = \frac{4 \cdot W_s \cdot FC(\%)}{\pi \cdot d_f^2 \cdot L_f \cdot \gamma_f \cdot 100} \qquad (17)$$

$$n_{cf} = 2 \cdot \frac{L_f}{d_g} \qquad (18)$$

In Eq. 17, γ_f is the specific gravity of the fiber, and in Eq. 18, L_f represents the fiber length. The number of fiber-to-grain contacts is given by the combination of Eqs. 17 and 18:

$$N_{gf} = \frac{2}{25\pi} \cdot \frac{W_s \cdot FC(\%)}{\gamma_f \cdot d_g \cdot d_f^2} \qquad (19)$$

Therefore, parameter β will be:

$$\beta = \frac{N_{gf}}{N_{gg} + N_{gf}} = \left(1 + \frac{225}{FC(\%)} \cdot \frac{\gamma_f}{\gamma_s} \cdot \frac{d_f^2}{d_g^2} \cdot \right)^{-1} \qquad (20)$$

Face-Centered Cubic Packing

The coordination number for the face-centered cubic packing is 12. Taking a cubic region of n spheres by each side, the total number of spheres is:

$$N_s = n^3 - n^2 + \frac{n}{2} \qquad (21)$$

and the number of contacts is given by Eq. 22:

$$N_c = 6 \cdot n^3 - 16 \cdot n^2 + 14 \cdot n - 4 \qquad (22)$$

Figure 5 shows the variation of the number of contacts to the number of spheres ratio with the soil mass. As it was highlighted for the simple cubic packing, for a soil mass of over 0.25 kg the ratio N_c/N_s is nearly constant and, in this case, approximately close to 6.

Now we calculate the number of grain-to-grain contacts as a function of soil weight:

$$N_{gg} = \frac{36}{\pi} \cdot \frac{W_s}{d_g^3 \cdot \gamma_s} \qquad (23)$$

The number of fiber-to grain contacts is computed in the same way as for the simple cubic packing, but considering that each fiber has twice as many contacts as the same fiber in a simple cubic packing. The following result is obtained:

$$N_{gf} = \frac{4}{25\pi} \cdot \frac{W_s \cdot CF(\%)}{\gamma_f \cdot d_g \cdot d_f^2} \qquad (24)$$

Parameter β is the result of the combination of Eqs. 23 and 24:

$$\beta = \frac{N_{gf}}{N_{gg} + N_{gf}} = \left(1 + \frac{225}{CF(\%)} \cdot \frac{\gamma_f}{\gamma_s} \cdot \frac{d_f^2}{d_g^2} \cdot \right)^{-1} \qquad (25)$$

It can be seen that β does not depend on the packing, or on the void ratio of the soil mass, but it is a function of fiber content, of the specific gravity of each material and of the fiber diameter to grain diameter ratio.

Validation of the Model

Experimental Program

Materials

An alluvial siliceous well-graded sand was used in the present study. The main geotechnical properties of the sand are listed in Table 1. Since particle shape is a crucial characteristic in the theoretical model assumption, sphericity (S) and roundness (R) of sand particles were evaluated according to Santamarina et al. [15], obtaining mean values of R = 0.5 and S = 0.7. From these results, we conclude that the representation of soil grains as spheres is acceptable. Figure 6 presents microscopic images of the tested sand.

Polypropylene fibers of 0.16 mm in diameter were used throughout this work. A fiber length of 10 mm was chosen according to previous works on the shear strength of the reinforced soil [16].

Experimental Tests

Confined compression with shear wave velocity measurement tests were performed in order to quantify the maximum shear modulus of the reinforced soil at low strain levels ($\varepsilon \approx 10^{-5}$). These tests were carried out in a

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Table 1 Geotechnical properties of the soil used in this work

U.S.C.S.	C_U	C_g	%PT#200	γ_s	d_{50}
SW	7.9	1.4	4.9	2.67	0.6 mm

U.S.C.S., Unified Soil Classification System; $C_{U,}$ coefficient of uniformity; $C_{g,}$ coefficient of gradation; %PT#200, percentage of passing weight through the sieve IRAM Nº 200 (75 um); $\gamma_{s,}$ specific gravity; $d_{50,}$ diameter of 50% passing weight

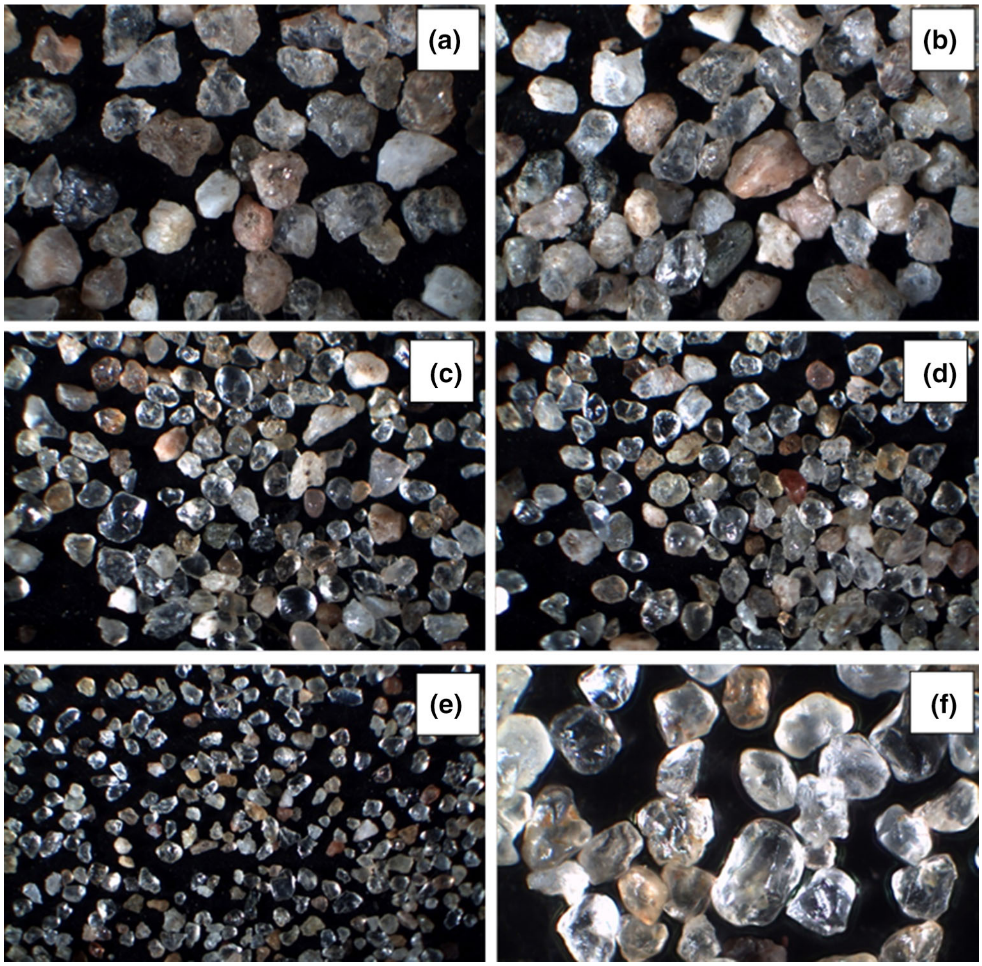

Fig. 6 Microscope images of the tested sand; **a**, **b** fraction of soil between sieves IRAM 10 (2 mm) and IRAM 40 (0.425 mm); **c**, **d** fraction of soil between sieves IRAM 40 (0.425 mm) and IRAM 60 (0.25 mm); **e**, **f** fraction of soil between sieves IRAM 60 (0.25 mm) and IRAM 100 (0.15 mm)

modified oedometer with the incorporation of bender elements in its upper and bottom caps. A detailed description of this equipment is given in [17]. A 10 V (peak to peak) in amplitude and 10 kHz frequency sine pulse was used. The signal arrival was selected following the criteria and recommendations of Patel et al. [18].

Fiber reinforced sand specimens were prepared with two different compaction states: a loose state with relative density of 0% and a dense state with relative density of 100%.

Shear wave velocity of fiber reinforced sand specimens with identical relative density but with different fiber content was measured. Then, the maximum shear modulus was obtained from shear wave velocity (Eq. 1).

Since the relative density was the comparison parameter used in this study, the void ratio of the reinforced sand specimens varied according to fiber content. The reason for choosing the relative density rather than the void ratio as the control variable was to compare the behavior of the composites with different fiber contents obtained by identical preparation methods and applying the same compaction effort. All the samples were tested in a dry state to avoid the influence of water.

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Table 2 Soil and fibers properties used in the calculation of the model's parameters

G_g	ν_g	d_g	γ_g	G_f	ν_f	d_f	γ_f
20,000 MPa	0.25	0.6 mm	2.67	400 MPa	0.50	0.16 mm	0.90

Quantification of the Model Parameters

Table 2 shows the data used to calculate parameters α and β. The properties of fibers were obtained from the manufacturer. Elastic parameters of the soil grains were assumed to be equal to the granite properties because of the nature and genesis of the sand.

Based on [19], quantification of parameter β was made considering a mono-sized distribution of spheres. Thus, the well-graded distribution of particles was replaced by a uniform one with grains of constant diameter assumed equal to d_{50} (diameter of 50% passing weight) of the sand. This is possible since the average coordination number does not change with the type and expansion of size distribution [19].

Results

Figures 7 and 8 show the effect of fiber content on the maximum shear modulus, for vertical stresses ranging from 28 to 440 kPa. Results displayed in Fig. 7 correspond to the sand in a loose state (0% of relative density), and results of Fig. 8 to the sand in a dense state (100% of relative density).

From Fig. 7 it can be seen that the predictive analytical model fits well for the lower vertical stresses, but not for the higher vertical stresses. The reason may be that at higher stresses, fibers undergo larger strains and soil particles are almost in contact with one another. Therefore, as the stress level increases, the shear wave velocity of loose sand tends to travel along the particle to particle contacts.

In Fig. 8, good agreement is observed between the trends of the experimental data and the model predictions, even at the highest vertical stresses. If the sand is in a dense state, soil particles suffer a lower contact stress for the same confining pressure than the sand in a loose state, because the coordination number is larger. As a consequence, at the highest vertical stresses applied in the experimental program, the shear wave seems to be transmitted according to the model hypothesis.

Finally, from the experimental results and analytical model it can be concluded that for both loose and dense sand, the inclusion of fibers tends to reduce the initial stiffness (and shear wave velocity) at low strain levels. This conclusion is valid if the compaction method and energy is not changed as fibers are added to the soil mass.

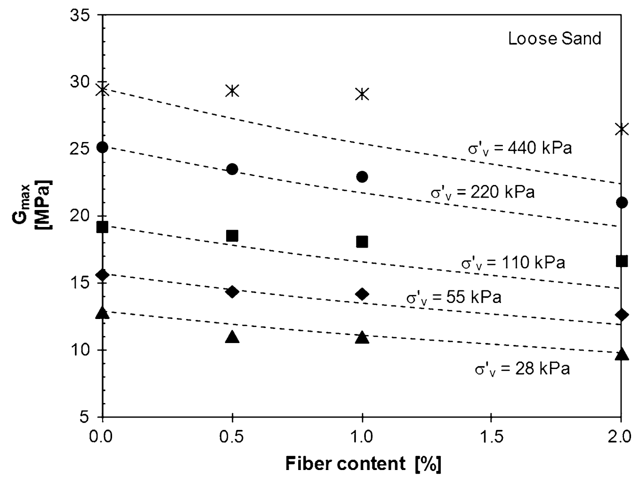

Fig. 7 Maximum shear modulus versus fiber content, for different vertical pressures. Comparison between test results (*bullets*) and predictive model (*dashed lines*) for the sand in a loose state (0% relative density)

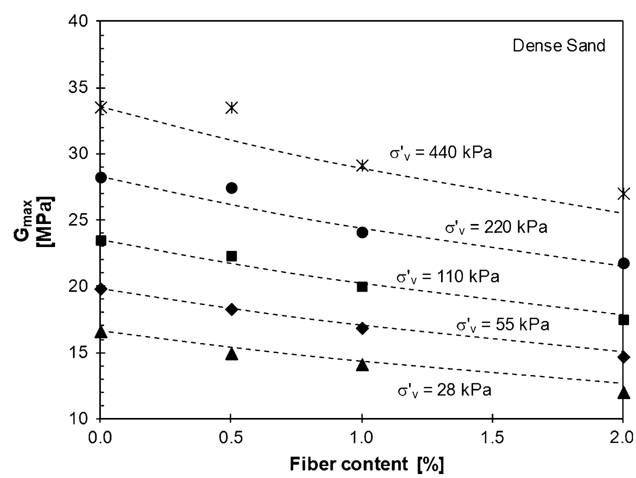

Fig. 8 Maximum shear modulus versus fiber content, for different vertical pressures. Comparison between test results (*bullets*) and predictive model (*dashed lines*) for the sand in a dense state (100% relative density)

Discussion

From the experimental program and the proposed model, we can conclude that the maximum shear modulus (G'_{max}) of the fiber reinforced sand tends to decrease with the increase of fiber content. This conclusion is valid only if the relative density of the compared specimens is the same,

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that is, if the compaction method and energy is kept constant along the preparation of the compared samples (with and without fibers).

If the preparation method and compaction energy applied to the samples with and without fibers is the same, the void ratio of the samples will turn out to be different because of fiber incorporation. The higher the fiber content, the higher the void ratio. Nevertheless, the coordination number will not change, and it will be the same for both specimens (with and without fiber reinforcement). Although the void ratio affects the shear modulus of a granular soil, it is the coordination number, in fact, the parameter that defines the shear wave path, and consequently the shear wave velocity and maximum shear modulus for a constant confining pressure.

The preparation method of the specimens with and without fiber is a key aspect at the time of comparing the behavior of the sand with and without reinforcement. If we compare the shear wave velocity of samples with different fiber content but with the same void ratio, the behavior and trends will be different from those observed in the present paper. This is the reason why other researchers have reported diverse trends and behaviors not matching the ones observed in this work.

In order to understand the influence of the relative density and preparation method on the coordination number, we can think about what happens to specimens of a material ideally constituted by spheres of identical diameter prepared with the maximum and minimum void ratio, that is, relative densities of 0 and 100%, respectively. The maximum void ratio (0% relative density) of a granular material formed by equal spheres can be obtained by pluviation and it corresponds to a simple cubic packing with a coordination number of 6. If fibers are added to the granular soil mass and the specimen is prepared by pluviation so as to achieve its maximum void ratio, once again the packing will be of a simple cubic type so the coordination number will keep on being 6, exactly like the soil without fibers. On the other hand, if the specimen is prepared with a very high compaction energy reaching a relative density of 100%, the spheres will accommodate in a face centered cubic packing, no matter whether fibers are or are not included into the soil mass. Consequently, the coordination number of the specimen of spheres on their own, will be the same as that of the spheres mixed with fibers.

According to the analytical model, the stiffness (G'_{max}) of the fiber reinforced soil depends on the quantification of parameters α and β.

Parameter α is a function of elastic properties and dimensions of the soil particles and fibers. For a given soil and fibers, α is a constant. But if we change the fiber rigidity, the higher the ratio of the fiber-to-grain contact stiffness to the grain-to-grain contact stiffness, the higher the drop of G'_{max} will be when fibers are added to the soil mass.

Parameter β depends on fiber content, showing that the larger the amount of fibers added to the granular skeleton, the lower the maximum shear modulus of the composite material will result, if the relative density is kept constant.

Conclusions

An analytical theoretical model based on the Hertz theory is developed in order to understand the maximum shear modulus drop as synthetic fibers are added to fiber reinforced sand. This behavior is valid when granular soils with the same coordination number are compared, which is possible if the specimens with and without fibers are prepared with the same relative density, that is, using the same energy and compaction method.

The analytical model assumes that shear wave velocity and maximum shear modulus of the reinforced sand depend mainly on the stiffness of the grain-to-grain and fiber-to-grain contacts.

The model predictions are compared with experimental results obtained by means of bender element measurements in an alluvial clean silica sand, reinforced with polypropylene fibers tested in a confined compression state. The model predictions fit very well with the laboratory measurements.

The proposed model leads us to conclude that the maximum shear modulus of the reinforced sand decreases as the fiber content increases because of the drop of stiffness at particle contact level when fibers are added to the soil mass.

The ratio between the amount of grain-to-fiber contacts and the total of contacts on the shear wave path controls the maximum shear modulus value.

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