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UNIFIED PHANTOM COSMOLOGIES

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We present a general algorithm based on the concept of form-invariance which can be used for generating phantom cosmologies. It involves linear transformations between the kinetic energy and the potential of the scalar field, and transforms solutions of the Einstein–Klein–Gordon equations which preserve the weak energy condition into others which violate it, while keeping the energy density of the field positive. Several known solutions representing phantom cosmologies are unified by this procedure. Using the general algorithm we obtain those solutions and show the relations between them. In addition, the scale factors of the product and seed solutions are related by a generalization of the well-known $a \to a^{-1}$ duality.

Keywords: Phantom fields; symmetry transformations; duality.

1. Introduction

Recently obtained astrophysical information ranging from high redshift surveys of supernovae to Wilkinson Microwave Anisotropy Probe (WMAP) observations, indicates that a considerable amount of the total energy of our universe might correspond to some dark energy which would be currently producing inflation thanks to a large and negative pressure.

Such exotic fluids may be framed in theories with matter fields that violate the weak energy condition.[1] These models were dubbed phantom cosmologies, and their study represents a currently active area of research in theoretical cosmology. Although phantom cosmologies have been investigated from different perspectives, here we will only be concerned with particular issues related with analytical properties.[2]

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At present, the theoretical understanding of the subject is limited, but at least one can rely on the motivation for phantom matter which is provided by string theory.[3] Precisely, the idea that the origin of dark energy should be searched within string theory or another fundamental theory has been recently reinforced by the discovery that the holographic principle cannot be used to tell whether dark energy is present or not.[4]

One interesting aspect of phantom matter is that it might make the universe end up in a kind of singularity[5] characterized by divergences in the scale factor a, the Hubble factor H and its time-derivative $\dot{H}$. This possibility has not only been shown to exist in general relativity,[6] but also in some of its generalizations like braneworld cosmology[7] or scalar-tensor theories of gravity.[8]

Another attractive idea put forward recently in connection with phantom fields is that they can be used for constructing bouncing universes.[9]

In this paper we present an algorithm for generating phantom cosmologies with Friedmann–Robertson–Walker (FRW) geometries. Interestingly, the method unifies others given in Refs. 11 and 14. In fact, here we will prove that the triality transformation implemented in Ref. 14 which chains inflationary, cyclic and phantom cosmologies is a form-invariance transformation; moreover, such transformation is itself a composition of three simpler form-invariance symmetry operations. Thus, that procedure is an application of general form-invariance transformations, which have been used successfully in Refs. 11 and 12 for generating new exact solutions.

Once we discuss the main general features of those transformations we concentrate on the linear ones. After that, we show there exist four transformations which preserve the positivity of the energy density, and we demonstrate they can be used for generating phantom cosmologies starting from standard scalar field universes. We will mainly be concerned with universes represented by scalar field solutions to the Einstein equations, and as such the energy density will be the sum of a kinetic and a potential term. Now, in the usual definition of the energy density of a scalar field the kinetic term is taken to be positive, but we will relax this hypothesis and will also consider the possibility that it is negative. The transformations we consider map between them solutions constructed by taking the two possible signs of the kinetic term. This will result in two possible ways of transforming those scalar field cosmologies into phantom ones. We will discuss this in detail and we will put some accent on comparison with related results which appeared earlier in the literature. Finally, we will draw our main conclusions.

2. The Form Invariance Symmetry

Our starting points are perfect fluid FRW spacetimes with flat sections given by the line element

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)\,. \tag{1}$$

Note that, even in such a simple setting as this, many questions remain open yet. The Einstein equations are

$$3H^2 = \rho \,, \tag{2}$$

$$\dot{\rho} + 3H(\rho + p) = 0 \,, \tag{3}$$

where ρ and p are respectively the energy density and the pressure of the fluid, and $H \equiv \dot{a}/a$. Now, the Einstein equations for a different perfect fluid with energy density $\bar{\rho}$ and pressure $\bar{p}$ will be the equations one would obtain after replacing all the quantities in (2) and (3) by their barred counterparts. The two equations obtained under this rule will be our transformed Einstein equations.

The system of transformed Einstein equations is known to admit form-invariance transformations,[10] which means there exist symmetry transformations which relate it to the system (2) and (3). Those transformations are given by

$$\bar{\rho} = \bar{\rho}(\rho) \,, \tag{4}$$

$$\bar{H} = \left(\frac{\bar{\rho}}{\rho}\right)^{1/2} H \,, \tag{5}$$

$$\bar{p} = -\bar{\rho} + \left(\frac{\rho}{\bar{\rho}}\right)^{1/2} (\rho + p)\frac{d\bar{\rho}}{d\rho} \,, \tag{6}$$

where $\bar{\rho} = \bar{\rho}(\rho)$ is an invertible function. The symmetry transformations (4)–(6) define a Lie group for any function $\bar{\rho}(\rho)$, which can connect scenarios with different evolutions. This will be discussed in the following sections.

Nevertheless, it is worth making a pair of clarifications before we move on. First, form-invariance transformations are defined without imposing any restriction on the fluid. Second, they relate solutions to different equations, unlike Lie point symmetry transformations, which relate different solutions to the same equation. Therefore, form-invariance may be viewed as an uncommon yet useful equivalence concept.

The implications of the symmetry transformations will be better understood if one is able to say how they change relevant physical parameters. We will look into this matter from a general perspective and also from that of well-known families of solutions. Under the symmetry transformations (4)–(6), the deceleration factor q, which by definition is

$$q = -H^{-2}\frac{\ddot{a}}{a} \,, \tag{7}$$

changes according to

$$\bar{q} + 1 = \left(\frac{\rho}{\bar{\rho}}\right)^{3/2} \frac{d\bar{\rho}}{d\rho} \, (q + 1) \,. \tag{8}$$

If we consider perfect fluids with equations of state

$$p = (\gamma - 1)\rho \,, \qquad \bar{p} = (\bar{\gamma} - 1)\bar{\rho} \,, \tag{9}$$

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respectively, then the barotropic index γ will become $\bar{\gamma}$ and the link between them will be

$$\bar{\gamma} = \left(\frac{\rho}{\bar{\rho}}\right)^{3/2} \frac{d\bar{\rho}}{d\rho}\, \gamma \,. \tag{10}$$

We now consider the linear transformation between H, ρ and p, given by

$$\begin{pmatrix} \bar{\rho} \\ \bar{H} \\ \bar{p} \end{pmatrix} = \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n & 0 \\ n - n^2 & 0 & n \end{pmatrix} \begin{pmatrix} \rho \\ H \\ p \end{pmatrix} \,, \tag{11}$$

where n is a real number. Even though the latter is a simple choice, it has got much in store. The transformed barotropic index and deceleration factor will read

$$\bar{\gamma} = \frac{\gamma}{n} \tag{12}$$

and

$$\bar{q} = -1 + \frac{q+1}{n} \,. \tag{13}$$

There will be inflation ($\bar{q} < 0$) for any $n > (q+1)$ if $n > 0$ or for any $n < (q+1)$ if $n < 0$. The first case was studied in full detail,[10] whereas the second one was partially investigated.[11] In this case we are going to devote ourselves to the second case.

On the other hand, note that the second row in Eq. (11) gives

$$\bar{H} = nH \,, \tag{14}$$

which leads to

$$\bar{a} = a^n \,. \tag{15}$$

The insight gained in this section will be used in the remainder with the purpose of generating phantom cosmologies.

3. Phantom Cosmologies and Duality

When solving the Einstein equations of flat FRW models (2) and (3), one looks for solutions of a system of equations in which a does not appear explicitly; H, ρ and p are the true variables of the problem, and mathematically speaking the most natural choice is making assumptions on them, instead of the scale factor.

Here we will consider models such that the Hubble factor is linear in $1/t$. They correspond to power-law evolutions and have a constant barotropic index. Thus, because of the equations of state (9) chosen earlier, we will have

$$H = \frac{2}{3\gamma t} \,, \tag{16}$$

together with its barred version to be obtained by replacing H with $\bar{H}$ and γ with $\bar{\gamma}$. Here and throughout γ and $\bar{\gamma}$ stand for the barotropic indexes of the seed and

transformed fluid respectively. The scale factor of the seed solution associated with (16) reads

$$a(t) = (\pm t)^{2/3\gamma} . \tag{17}$$

From Eq. (16), we have expanding solutions for $\gamma t > 0$ and contracting ones for $\gamma t < 0$. In order to be able to carry out the following discussion for a positive γ, we will take the plus sign for the $t > 0$ branch of the solution and the minus sign for the $t < 0$ one. The connection between these two time regions is made throughout a time reversal symmetry. Since this transformation does not change the Einstein equations (2) and (3), it is a form-invariance transformation. Under that sign choice the two branches will be mirror images of each other. Now, replacing Eq. (16) and its barred version in (14) and integrating one arrives at

$$|t|^{2/3\bar{\gamma}} = |t|^{2n/3\gamma} . \tag{18}$$

Since in the $n < 0$ case one has $a \to a^{-|n|}$, the transformation may be viewed as an extension of the well-known $a \to a^{-1}$ duality given in our paper,[11] and which was profusely studied in the pre-big bang scenario.[15] Now a solution of the form of (17) with $\gamma > 0$ describes for $t > 0$ a standard universe that expands ever after a big-bang at $t = 0$, but the $t < 0$ branch describes a contracting solution for which $t = 0$ represents a big crunch (see the continuous line in Fig. 1). Now, when we apply the transformation we obtain a new solution which contracts in its $t > 0$ branch, but expands in the $t < 0$ one (see the dashed line in Fig. 1). Precisely, the latter satisfies the definition of a phantom universe given in the literature because it is a expanding solution that violates the weak energy condition. Moreover, it reaches a big rip at finite time, which is a feature inherent to many phantom universes but
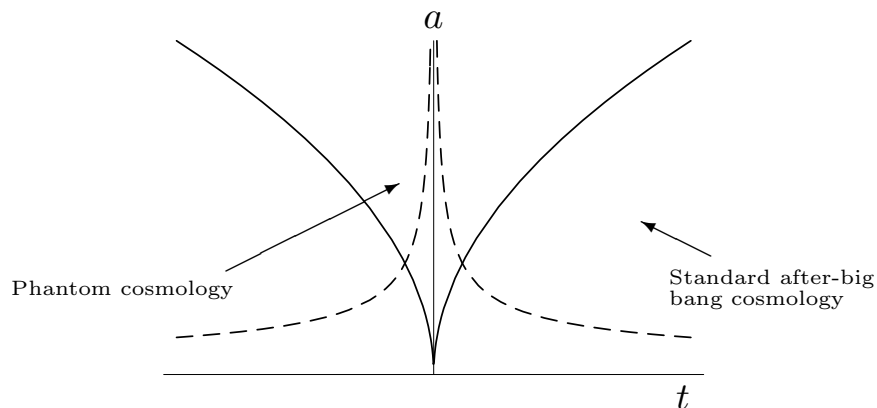

Fig. 1.   The continuous line is a plot of the scale factor $a = |t|^n$, with $0 < n < 1$ for a standard after-big bang cosmology $(t > 0)$ and its pre-big bang continuation. The dashed line represents a new solution obtained by applying to the latter a generalized duality transformation $a \to a^{-|n|}$. The $t < 0$ region of the new solution represents a phantom cosmology reaching a big-rip at finite time.

not to all of them. Thus, the phantom universe is dual to the universe described by the $t < 0$ branch of the seed solution.

4. A Unified Generation Algorithm

In this section we turn our attention to the scalar field picture; in particular, we study how scalar fields transform under the form invariance transformations considered above. To this end, we identify the corresponding energy–momentum tensor with that of a perfect fluid and write the energy densities and pressures in the invariant form

$$\rho(\phi) = s\frac{\dot{\phi}^2}{2} + V(\phi)\,, \qquad p(\phi) = s\frac{\dot{\phi}^2}{2} - V(\phi)\,, \tag{19}$$

and

$$\bar{\rho}(\bar{\phi}) = \bar{s}\frac{\dot{\bar{\phi}}^2}{2} + \bar{V}(\bar{\phi})\,, \qquad \bar{p}(\bar{\phi}) = \bar{s}\frac{\dot{\bar{\phi}}^2}{2} - \bar{V}(\bar{\phi})\,, \tag{20}$$

where s and $\bar{s}$ are two parameters of the scalar field models which can be chosen freely.

Since the energy density and pressure are written as a sum of two terms (kinetic and potential energy), there is in principle the possibility of combining them linearly so that the sums ρ and p (or $\bar{\rho}$ and $\bar{p}$) remain constant. This generates an additional group structure other than the one introduced by the transformation (11). However, since we are only interested in changes of sign in the kinetic energy we will restrict ourselves to the discrete symmetries obtained in that case. Therefore, redefining the scalar fields ϕ and $\bar{\phi}$, we can select the constant s and $\bar{s}$ as $s = \pm 1$ and $\bar{s} = \pm 1$. Thus, they determine the sign of the kinetic terms in the theories described by Eqs. (19) and (20) respectively.

If we insert (19) and (20) in (3) and its transformed counterpart respectively, the original and form-invariance transformed Einstein–Klein–Gordon equations will be obtained:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{sd\phi} = 0\,, \tag{21}$$

$$\ddot{\bar{\phi}} + 3\bar{H}\dot{\bar{\phi}} + \frac{d\bar{V}(\bar{\phi})}{\bar{s}d\bar{\phi}} = 0\,. \tag{22}$$

Applying the form-invariance transformations (4)–(6) and particularizing to our choice (11), we find that the kinetic and potential terms transform linearly as

$$\dot{\bar{\phi}}^2 = \frac{s}{\bar{s}}n\dot{\phi}^2\,, \tag{23}$$

and

$$\bar{V} = ns(n-1)\frac{\dot{\phi}^2}{2} + n^2 V\,. \tag{24}$$

Let us see now in more detail how the transformation operates. Under the definitions made above we have

$$-2\dot{H} = \rho + p = s\dot{\phi}^2\,. \tag{25}$$

and, in consequence, our transformation (14) gives

$$\bar{\rho} + \bar{p} = n(\rho + p)\,. \tag{26}$$

If the seed solution preserves the weak energy condition and considering we are in the $t < 0$ region with $n < 0$, then the transformed solution will violate it and will terminate in a big rip[5] (alternatively see Ref. 16 for big rip singularities in non-phantom settings). We concentrate in the remainder on the $n = -1$ case, which corresponds to $\rho \to \rho$, $H \to -H$ and $\rho + p \to -(\rho + p)$. Moreover, the barotropic index of the associate fluid is

$$\gamma = (\rho + p)/\rho = -2\dot{H}/3H^2 = s\frac{\dot{\phi}^2}{\rho} \tag{27}$$

and its transformation rule (10) reads $\gamma \to -\gamma$.

Due to the transformation law of the scalar field, the swap between cosmologies that preserve the energy condition and others that violate it can occur in two different ways, depending on whether it is the sign of s or $\dot{\phi}^2$ that flips in the transformation.[a] Combining this double possibility with the also double choice for the character of the seed solution ($s = 1$ or $s = -1$) we obtain, from Eqs. (23) and (24) four possible transformations. We list them in Table 1 for the particular case $n = -1$.

Table 1.   List of $n = -1$ transformations.

s	$\bar{s}$	$\bar{\phi}$	$\bar{V}$	$\bar{\gamma}$
1	1	$\mp i\phi$	$\dot{\phi}^2 + V$	$-\dot{\phi}^2/\rho$
1	-1	$\pm\phi$	$\dot{\phi}^2 + V$	$-\dot{\phi}^2/\rho$
-1	1	$\pm\phi$	$-\dot{\phi}^2 + V$	$\dot{\phi}^2/\rho$
-1	-1	$\mp i\phi$	$-\dot{\phi}^2 + V$	$\dot{\phi}^2/\rho$

Now, further insight on how the transformation works can be gained from the field equations. We will distinguish between conventional and non-conventional scalar fields by writing the Einstein equations using respectively positive or negative kinetic terms. For a conventional scalar field (CSF), the set of Einstein equations is

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{28}$$

$$\dot{\rho} + 3H\dot{\phi}^2 = 0\,, \tag{29}$$

[a]If the sign of $\dot{\phi}^2$ is preserved by the transformation the energy density we will say that the kinetic energy behaves as a scalar, whereas if that sign changes we will say that the kinetic energy behaves as a pseudo-scalar.

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whereas for an non-conventional scalar field (NCSF) the set of Einstein equations is

$$3H^2 = -\frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{30}$$

$$\dot{\rho} - 3H\dot{\phi}^2 = 0\,. \tag{31}$$

The first transformation in Table 1 makes $\dot{\phi}^2 \to -\dot{\phi}^2$, and $V \to \dot{\phi}^2 + V$, and leaves unaltered the CSF equation set. The second transformation makes $\dot{\phi}^2 \to \dot{\phi}^2$ and $V \to \dot{\phi}^2 + V$, so that it turns the set of equations governing the NCSF into the set CSF. The third transformation makes $\dot{\phi}^2 \to \dot{\phi}^2$ and $V \to -\dot{\phi}^2 + V$, and it turns the set of equations governing the CSF into the NCSF set. Finally, the fourth transformation makes $\dot{\phi}^2 \to -\dot{\phi}^2$, and $V \to -\dot{\phi}^2 + V$, and leaves unaltered the NCSF equation set.

It is also interesting to look at the implications of the transformations from the point of view of the barotropic index γ. A solution to the CSF set with a real scalar field will have a positive barotropic index, and its $t > 0$ and $t < 0$ branches will represent respectively expanding and contracting universes with a null scale factor at $t = 0$. Taking a seed of that sort one can construct phantom solutions to the CSF set with an imaginary scalar field (using the $s = \bar{s} = 1$ transformation) or with a real scalar field (using the $s = -\bar{s} = 1$ transformation). Specifically, these phantom universes will occupy the $t < 0$ region. In contrast, a solution to the NCSF set having a negative barotropic index with a real scalar field and its $t > 0$ and $t < 0$ branches will represent respectively contracting and expanding universes with an infinite scale factor at $t = 0$, so its $t < 0$ region branch is a phantom universe. Application of the duality transformation gives us a new solution such that its $t > 0$ and $t < 0$ branches will represent respectively expanding and contracting universes with a null scale factor at $t = 0$. We stress here again that if $n \neq -1$ the absolute value of the barotropic index will change in the transformation according to $\bar{\gamma} = \gamma/n$.

For illustration we consider now the same families of exact solutions considered earlier, that is, the models in which the Hubble factor is proportional to $1/t$. Choosing all the terms in each Eqs. (2) and (21) with the same degree of homogeneity, the existence of solutions is guaranteed. Hence, the corresponding scalar field has a logarithmic dependence on time t and the potential depends on the inverse square of t, so that

$$\bar{\phi} = \frac{2}{\bar{A}}\ln|t|\,, \qquad \phi = \frac{2}{A}\ln|t|\,, \tag{32}$$

and

$$\bar{V} = \frac{\bar{V}_0}{t^2}\,, \qquad V = \frac{V_0}{t^2}\,, \tag{33}$$

where $\bar{A}$, A, $\bar{V}_0$ and V_0 are parameters related by the transformation rules for the field and the potential (23) and (24). In addition, they are restricted by the

respective Einstein–Klein–Gordon equations. Then, inserting Eqs. (32) and (33) into the Eqs. (23) and (24), we get

$$\bar{A}^2 = \frac{\bar{s}}{ns} A^2 \,, \tag{34}$$

$$\bar{V}_0 = n^2 \left(\frac{2s}{A^2} + V_0 \right) - \frac{2ns}{A^2} \,, \tag{35}$$

while the composition of (32) with (33) gives the potentials as functions of the fields

$$\bar{V} = \bar{V}_0 e^{-\bar{A}\bar{\phi}} \,, \qquad V = V_0 e^{-A\phi} \,, \tag{36}$$

Requiring that the Hubble factor H, the field ϕ and the potential V given by (16), (32) and (33) respectively satisfy the unbarred Einstein–Klein–Gordon equations (2) and (21) we find that

$$\gamma = \frac{A^2}{3s} \,, \qquad V_0 = \frac{2s}{A^2} \left(\frac{6s}{A^2} - 1 \right) . \tag{37}$$

Finally, replacing the latter in (35) we arrive at the complete transformation law for the parameter $\bar{V}_0$:

$$\bar{V}_0 = \frac{2ns}{A^4} (6ns - A^2) \,. \tag{38}$$

For the seed power law solution $a(t) = |t|^{2/3\gamma}$ or $a(t) = |t|^{2s/A^2}$ (by using Eq. (37)), the symmetry operations of Table 1 give rise to four different kinds of solutions. We list them in Table 2.

Table 2. Transformations of the power-law solutions in the $n = -1$ case.

s	$\bar{s}$	$\bar{\phi}$	$\bar{a}$	$\bar{A}$	$\bar{V}_0$
1	1	$\mp\dfrac{2i}{A}\ln|t|$	$|t|^{-2/A^2}$	$\pm iA$	$\dfrac{2}{A^4}(6 + A^2)$
1	-1	$\pm\dfrac{2}{A}\ln|t|$	$|t|^{-2/A^2}$	$\pm A$	$\dfrac{2}{A^4}(6 + A^2)$
-1	1	$\pm\dfrac{2}{A}\ln|t|$	$|t|^{2/A^2}$	$\pm A$	$\dfrac{2}{A^4}(6 - A^2)$
-1	-1	$\mp\dfrac{2i}{A}\ln|t|$	$|t|^{2/A^2}$	$\pm iA$	$\dfrac{2}{A^4}(6 - A^2)$

The solution generation procedure we discussed in Ref. 11 corresponds to the first transformation. More specifically, if we take the usual power-law cosmologies with a constant barotropic index (the ones used above) and apply the transformation $\bar{s} = -1 = -s$ we get a family of power-law solutions that are, in fact, the late time limit of some generalized phantom cosmologies found in Ref. 13 in the context of k-essence cosmologies (see expressions (48) and (50) in that reference).

If we further set $A = \lambda$ and $n = -\lambda^4/4$ in expressions (32)–(36), then we can write those solutions in the form in which they were presented in Ref. 14, that is,

$$\bar{A} = \pm\frac{2}{\lambda}, \qquad \bar{\phi} = \pm\lambda\ln|t| = \pm\frac{\lambda^2}{2}\phi, \tag{39}$$

from where it is deduced that

$$\bar{A}\bar{\phi} = \pm\frac{2\bar{\phi}}{\lambda} = \lambda\phi, \tag{40}$$

and finally

$$\bar{V}_0 = \frac{\lambda^2}{4}(3\lambda^2 + 2), \tag{41}$$

$$\bar{V} = \bar{V}_0 e^{-\bar{A}\bar{\phi}} = \bar{V}_0 e^{\mp 2\bar{\phi}/\lambda} = \bar{V}_0 e^{-\lambda\phi}. \tag{42}$$

Then, the solution presented in Ref. 14 corresponds to a particular application of the second transformation type. Actually, in his paper,[14] the author only presented models with potentials that decrease as ϕ increases, which follow from taking the lower sign in (39) and (42).

Interestingly, using the concept form-invariance, and with the help of the parameters s y $\bar{s}$ we have unified the procedures already existing in the literature, and as a byproduct we have obtained the alternative procedures associated with the third and fourth transformations.

5. Conclusions

Our main goal here has been giving a general prescription for generating phantom cosmologies which unifies those obtained from the Einstein–Klein–Gordon with both sign of the kinetic term. The algorithm exploits form-invariance transformations, which have been discussed earlier in relation to cosmology as part of a long-term project. After some revision of the most important properties of the transformations, we have concentrated on linear transformations that multiply the Hubble factor by a negative constant factor n, so that contracting solutions will be transformed into expanding ones and vice versa. The transformation in the scale factor will be a generalized version of the $a \to a^{-1}$ duality investigated in Ref. 11 and appearing in the pre-big bang scenario, specifically our transformation will consist in $a \to a^{-|n|}$.

Let us assume $n = -1$ and that the seed solution is the typical contracting (expanding) solution in the $t < 0$ ($t > 0$) region ending (beginning) in a singularity at $t = 0$ (see Fig. 1). Given that $a \to a^{-1}$, the $t < 0$ branch of the transformed solution will have a expanding character and it will reach a big rip at $t = 0$, thus representing a phantom cosmology. The $t > 0$ branches of the transformed and seed solutions are the standard duals of each other.

For a constant n, the form-invariance transformations we consider can be viewed as group of uniparametric linear transformations acting on the sets of variables

(ρ, H, p) or $(\dot{\phi}^2, H, V)$, where $\dot{\phi}^2$ and V are the kinetic term and the potential of a conventional scalar field. Although we fix the value of the parameter of the group n, some discrete symmetry remains associated with the possibility of choosing a positive or a negative sign in kinetic term in the effective energy density of the scalar field. In addition, two different symmetry transformations can be constructed between the CSF equation set and the NCSF equation set. Combining those possibilities, we obtain four transformations that revert the sign of the barotropic index, and two of them generate phantom cosmologies.

In conclusion, the procedures to transform conventional scalar field cosmologies into phantom ones which existed up to now in the literature can be viewed as an application of form-invariance transformations.

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