On big rip singularities

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Abstract

In this comment we discuss big rip singularities occurring in typical phantom models by violation of the weak energy condition. After that we compare them with future late-times singularities arising in models where the scale factor ends in a constant value and there is no violation of the strong energy condition. In phantom models the equation of state is well defined along the whole evolution, even at the big rip. However, both the pressure and the energy density of the phantom field diverge. In contrast, in the second kind of model the equation of state is not defined at the big rip because the pressure burst at a finite value of the energy density.

1 Introduction

Investigations on theories with matter fields that violate the weak energy condition were triggered by the influential paper of Caldwell [1] in which he showed that dark energy of that sort would fit very well the supernova-derived luminosity. These models were dubbed phantom cosmologies, and it has been show that in general relativity and some of its generalizations such matter might make the universe end up in a kind of singularity [2] characterized by divergences in the scale factor a, the Hubble factor H and its time-derivative $\dot{H}$. In other words, the scale factor expands so quickly that the scalar curvature R burst in the limit $a \to \infty$, which is reached in a finite amount of proper time [3]. These singularity is commonly called the "big rip".

In a recent paper [4], the author suggested that the violation of the weak energy condition would not be necessary for producing a singularity in an expanding universe at a finite late-time. In particular, he showed that such a singularity can be constructed in such a way that the strong weak energy condition is preserved. Although this singularity is different than the usual big rip, both singularities share the same physical attribute of producing a divergent scalar curvature. This is an interesting point because if we enlarge the definition of a big rip by saying it occurs when the scalar curvature diverges, then we will have enriched our understanding about the meaning of a singularity reached at finite time. Although, phantom cosmologies have been investigated from different perspectives, here we will be concerned with those issues related with analytical properties of the model [**?**]. Hence, it is interesting to clarify the nature of both types of singularities [**?**], and we devote this comment to that purpose.

2 Future late-times singularities

Let us discuss the two approaches to obtaining big rip singularities. An example of the first one can be obtained when phantom cosmologies are generated by a scalar field with negative kinetic term. There, the energy density and pressure of the field are

$$\rho_{ph} = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1}$$

$$p_{ph} = -\frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{2}$$

while the Einstein-Klein-Gordon equations read

$$3H^2 = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3}$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} = 0. \tag{4}$$

For the exponential potential

$$V = \frac{2(6+A^2)}{A^4}\, e^{-A\phi}, \tag{5}$$

with

$$\phi = \frac{2}{A}\ln t, \tag{6}$$

the power-law evolutions

$$a = (-t)^{-2/A^2}, \tag{7}$$

are solutions of the Eqs. (3)-(4). They represent a universe without initial singularity begining in the remote past with a null scale factor. Thereafter, the scale factor grows monotonically till a future big rip is reached at $t = 0$. In our models, $\rho > 0$ but the weak energy condition is violated because $\rho + p = -\dot{\phi}^2 < 0$. So, if we associate a perfect fluid with the phantom field, then the baryotropic index for the power law solutions (7)

$$\gamma_{ph} = -\frac{\dot{\phi}^2}{\rho_{ph}} = -\frac{A^2}{3}, \tag{8}$$

becomes negative while both the energy density ρ_{ph} and the pressure p_{ph} of the field

$$\rho_{ph} = \frac{12}{A^4 t^2} \tag{9}$$

$$p_{ph} = -\frac{4(3+A^2)}{A^4 t^2}, \tag{10}$$

burst at $t = 0$. Note as well that the pressure is negative, as it must, because at the big rip super-acceleration occurs. This kind of solutions were also found solving the semiclassical Einstein equations for conformally invariant free fields and a conformally coupled massive scalar field in spatially flat Friedmann-Robertson-Walker (FRW) spacetimes with no classical radiation or matter, see Ref. [5].

In Ref. [4] it was showed that a future finite-time singularity can arise in as FRW expanding universe even when the strong-energy condition $\rho+3p > 0$ holds [1]. For instance, inserting a selected scale factor with a future finite-time singularity in the the Einstein equations for a flat FRW space

[1] Recalling that for super-acceleration $p < 0$ is required it may be seen that a consequence of the preservation of the strong-energy condition is the simultaneous fulfillment of the weak-energy condition

$$3H^2 = \rho, \tag{11}$$

$$6\frac{\ddot{a}}{a} = -(\rho + 3p), \tag{12}$$

we can find the energy density and the pressure satisfying the strong energy condition. In Ref. [4] it was chosen a model given by

$$a(t) = \left(\frac{t}{t_s}\right)^q (a_s - 1) + 1 - \left(1 - \frac{t}{t_s}\right)^n, \tag{13}$$

where the scale factor evolves in the interval $0 < t < t_s$ with $a_s \equiv a(t_s)$, $1 < n < 2$ and $0 < q \leq 1$. This model represents a universe beginning at a singularity, where the energy density ρ and the pressure p are divergent while the scale factor behaves as $a(t) \approx (a_s - 1)(t/t_s)^q$, with $a_s > 1$. It ends in a big rip at $t = t_s$ where the expansion rate H_s and energy density ρ_s are finite positive quantities but the pressure p_s is a positive divergent quantity. This type of big rip singularity is different than that appearing in phantom cosmologies. In fact, the final behaviour of the scale factor in this model is driven by the peculiar asymptotic form of the equation of state near t_s. To see that, it will be intersting to find $p = p(\rho)$, in the limit $t \rightarrow t_s$, using that the scale factor is a given function of the cosmological time. To begin with, let us expand the scale factor (13) at late times in powers of $t_s - t$. The first correction to the constant a_s is given by

$$a \approx a_s + \frac{q(1 - a_s)}{t_s}(t_s - t). \tag{14}$$

Inserting this approximate solution in the Einstein equations (11) and (12) we obtain the approximate energy density, expressed in powers of $a_s - a$, and the corresponding equation of state

$$\rho \approx \frac{3q(a_s - 1)}{a_s^2 t_s^2}\left[q(a_s - 1) + 2n\left[\frac{a_s - a}{q(a_s - 1)}\right]^{n-1}\right], \tag{15}$$

$$p \approx \frac{6n(n-1)}{a_s t_s^2}\left[\frac{t_s^2}{2nq(a_s - 1)}\left[\frac{a_s^2}{3}\rho - \frac{q^2(a_s - 1)^2}{t_s^2}\right]\right]^{(n-2)/(n-1)}. \tag{16}$$

Hence, in the limit $a \rightarrow a_s$, we have a finite energy density

$$\rho(t) \to \rho_s = \frac{3q^2(a_s - 1)^2}{a_s^2 t_s^2}, \tag{17}$$

and a pressure singularity

$$p(t) \to \infty, \tag{18}$$

which is of a logarithmic or a pole-like singularity depending on whether $(n-2)/(n-1)$ is respectively a real or an integer number. Equivalently, (15) and (16) correspond to a finite expansion rate $H_s = (\rho_s/3)^{1/2}$ and a divergent $\dot{H} \to -\infty$, which means, an infinite acceleration $\ddot{a} \to -\infty$.

Let us interprete now the peculiar fluid given by (15)-(16) as a perfect fluid with equation of state $p = (\gamma - 1)\rho$. In the asymptotic limit $t \to t_s$ the barotropic index becomes

$$\gamma \approx \frac{2n(n-1)a_s}{3q^2(a_s-1)^2} \left[\frac{a_s - a}{q(a_s - 1)} \right]^{n-2}. \tag{19}$$

The exotic fluids that lead to future late-time singularities represented by the bariotropic indexes (8) and (19) do not share the usual properties of the physical fluids i.e., $1 \leq \gamma \leq 2$. In fact, the former is negative $\gamma_{ph} = -A^2/3$, and the latter diverges asymptotically in the limit $t \to t_s$.

Nevertheless, it must be stressed that the presence of a logarithmic or a pole-like singularity in the equation of state is not a sufficient condition for a big rip singularity to occur. This can be understood by considering a particular example of the the van der Waals fluid [7]. Its equation of state is

$$p = \frac{8w\rho}{3-\rho} - 3\rho^2, \tag{20}$$

with w a constant. Near some finite value ρ_0 such that $|\rho_0 - \rho_s|$ is sufficiently small we have $|p| \gg \rho$ so that we may set

$$p \approx \frac{8w\rho}{3-\rho} \tag{21}$$

and

$$2\dot{H} \approx -p\,; \tag{22}$$

that is, we have a pressure dominated regime. Now, let us assume that the model enter this regime when ρ takes a value ρ_0 slightly smaller (larger) than ρ_s. The singularity will be reached when $3H^2 = \rho_s$, and this will only be possible if H grows (decreases) with time during this pressure dominated epoch till ρ reaches the value ρ_s. Clearly, equation (22) tells us that ρ will solely transit between ρ_0 and ρ_s if $w < 0$ is negative (positive), and, in particular, for all phantom models ($w < -1$). We may view the situation as if we had two branches of solutions, depending on the sign of w, and the reachability of the big rip depends on which branch is being taken.

The latter example gives us hints for drawing more general conclusions. Let us consider a fluid with an equation of state of the form

$$p \approx \frac{f(\rho)}{|\rho - \rho_s|^\alpha} \tag{23}$$

where $\alpha > 0$ is a real or integer number and such that f is an arbitrary function with definite sign and which remains finite as long as ρ does too. The reason why we choose expression (23) is that not only it includes (21) in the $\rho \to \rho_s$ limit, but also (16).

Following an argumentation identical to the previous one, we conclude that the big rip singularity will only be reached if $\mathrm{sign}(\mathrm{f}(\rho)) = \mathrm{sign}(\rho_0 - \rho_\mathrm{s})$, where as before ρ_0 stands for the value of ρ when the pressure dominated regime begins.

3 Conclusions

Summarizing, we believe it is worth widening the definition of big rip singularities by considering cases in which the divergence only appears in the pressure as those models which preserve the strong or weak energy conditions. Nevertheless, the difference between this new way of viewing big rip singularities and the customary one must always be kept in mind.

We have also shown that in the examples discussed the pressure may diverge due to a logarithmic or a pole-like singularity in the asymptotic equation of state. Along this line we have also shown that the presence of such singularities in the equation of state does not ensure *per se* that the singularity is reachable, and we have discussed which condition must be met for that to occur.

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