Exact model of conformal quintessence

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A nonminimally coupled quintessence model is investigated and the conditions for a stationary solution to the coincidence problem are obtained. For a conformally coupled scalar field and dissipative matter, a general solution possessing late acceleration is found. It fits rather well the high redshift supernovae data and gives a good prediction of the age of the Universe. Likewise, the cold dark matter component dominates the cosmological perturbations at late times albeit they decrease with expansion.

I. INTRODUCTION

After some years of research, the accelerated expansion of the Universe appears to have gained further ground [1] but the nature of dark energy—the substratum behind this acceleration—remains as elusive as ever [2,3]. While one may expect that one or another of the dark energy candidates (a very small cosmological constant [4], quintessence [5], Chaplygin gas [6], tachyon field [7], interacting quintessence [8,9], nonminimally coupled quintessence [10,11], etc.) will finally emerge as the successful model, at the time being none of them is in position to claim such status.

In introducing dark energy as a novel component most of these candidates encounter the so-called "coincidence problem," namely, "why are the energy densities of both components (dark energy and dark matter) of the precisely same order today?" As shown by the authors, this problem has a dynamical solution provided that the dark matter component is assumed to be dissipative [12] or interacts with the dark energy [8]. In such a case, it can be demonstrated that the equations governing the cosmic evolution imply the stationary condition [Eq. (10) below] and that the system is attracted to a stationary and stable solution characterized by the constancy of both density parameters, i.e., Ω_m and Ω_ϕ tend to constant values at late times.

The aim of this paper is to provide an exact quintessence model, nonminimally coupled to the Ricci curvature. Nonminimal coupling naturally arises in generalizing the Klein-Gordon equation from Minkowski space to a curved space—for a recent review on this subject and further motivations, see Ref. [13]. We believe it is rather reasonable to explore whether a nonminimal coupling of the scalar field acting as dark energy to the Ricci curvature may be of help to understand the present stage of accelerated expansion and shed some light into the nature of the dark component. As a first step toward weighing the contribution of the coupling to the evolution of the Universe we shall consider the simplest case

of a nonminimally coupled quintessence with a constant potential. As it turns out, there is a stationary solution that ensures that for late times the ratio between both energy densities remains a constant while the Universe asymptotically approaches a de Sitter expansion. In this late regime, the density perturbations of large wavelength decrease and are dark matter dominated. Further, our results are consistent with the high redshift supernovae data and provide a very reasonable estimate of the age of the Universe.

Section II presents our model which, aside from the nonminimally coupled scalar field, introduces a dissipative pressure in the dark matter fluid; this pressure turns out to be key for the solution of the coincidence problem. Section III derives the statefinder parameters as well as the age of the Universe. Section IV studies the observational constraints on our model imposed by the high redshift supernova data. Section V investigates the long wavelength density perturbations. Finally, Sec. VI summarizes our findings. As usual, a subindex zero indicates that the corresponding quantity must be evaluated at the present time. We have chosen units so that $c = 8\pi G = 1$.

II. CONFORMALLY COUPLED SCALAR FIELD

Let us consider a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime filled with two components, namely, dissipative matter and a nonminimally coupled quintessence field. The equation of state of the first component is of baryotropic type $p_m = (\gamma_m - 1)\rho_m$, where the baryotropic index of matter is restricted to the range $1 \leq \gamma_m \leq 2$. In addition to this equilibrium pressure the matter component is assumed to have a nonequilibrium (dissipative pressure) π connected to entropy production. It should be noted that barring superfluids (as helium superfluid), this quantity is ever-present in every matter fluid found in Nature and is negative for expanding fluids [14]. In the case at hand, it may either come from interactions within the dark matter, or the decay of dark matter particles into dark particles [15], or from the nonlinear growth of cosmic structures [16], and it proves crucial to solve the coincidence problem. Likewise, the equation of state of the quintessence component can be written as $p_\phi = (\gamma_\phi - 1)\rho_\phi$ with $\gamma_\phi < 1$.

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The Friedmann equation and the conservation equations for the matter fluid and quintessence read

$$3H^2 + 3\frac{K}{a^2} = \rho_m + \rho_\phi \quad (K=1,0,-1), \quad (1)$$

$$\dot{\rho}_m + 3H(\gamma_m\rho_m + \pi) = 0, \quad (2)$$

$$\ddot{\phi} + 3H\dot{\phi} + \xi R\phi + \frac{dV(\phi)}{d\phi} = 0, \quad (3)$$

where ξ denotes the nonminimal coupling constant and the Ricci curvature scalar R is related to the quintessence scalar field by [10,11]

$$[1-\xi(1-6\xi)\phi^2]R = -(1-6\xi)\dot{\phi}^2 + 4V - 6\xi\phi\frac{dV(\phi)}{d\phi}$$
$$+(4-3\gamma_m)\rho_m - 3\pi. \quad (4)$$

Likewise, the energy density and pressure of the quintessence field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + 3\xi H\phi(H\phi + 2\dot{\phi}), \quad (5)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \xi[4H\phi\dot{\phi} + 2\dot{\phi}^2$$
$$+2\phi\ddot{\phi} + (2\dot{H}+3H^2)\phi^2]. \quad (6)$$

Obviously, these two reduce to their minimally coupled expressions for vanishing ξ. In terms of the density parameters $\Omega_m \equiv \rho_m/(3H^2)$, $\Omega_\phi \equiv \rho_\phi/(3H^2)$ and $\Omega_K = -K/(aH)^2$, the set of equations (1)–(3) become

$$\Omega_m + \Omega_\phi + \Omega_K = 1, \quad (7)$$

$$\dot{\Omega}_m + 3H\left(\frac{2\dot{H}}{3H^2} + \gamma_m + \frac{\pi}{\rho_m}\right)\Omega_m = 0, \quad (8)$$

$$\dot{\Omega}_\phi + 3H\left(\frac{2\dot{H}}{3H^2} + \gamma_\phi\right)\Omega_\phi = 0, \quad (9)$$

respectively.

The simplest solution to the system of equations (8), (9) that solves the coincidence problem is that $\Omega_m = \Omega_{m0}$ and $\Omega_\phi = \Omega_{\phi0}$ at late times, with Ω_{m0} and $\Omega_{\phi0}$ constants. This automatically implies the stationary condition [12]

$$\gamma_m + \frac{\pi}{\rho_m} = \gamma_\phi = -\frac{2\dot{H}}{3H^2}. \quad (10)$$

It is readily seen that on the stationary solution, Eq. (10), one has $K=0$ thereby we shall focus on spatially flat FLRW spacetimes hence forward.

We are interested in obtaining a simplified, analytically integrable model that still retains the essentials of nonmini-

mally coupled approaches and leads to an accelerated phase of expansion at late times. This may be accomplished by choosing for the coupling constant the conformal value $\xi = 1/6$ and a constant value for the potential, $V(\phi) = V_0$. As a consequence, Eqs. (1), (4) and (3) take a very simple form on the stationary solution

$$3H^2 = (1+r)\left[\frac{1}{2}(\dot{\phi}+H\phi)^2 + V_0\right], \quad (11)$$

$$R = 4(1+r)V_0, \quad (12)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\omega^2}{2}\phi = 0, \quad (13)$$

where $r \equiv \Omega_m/\Omega_\phi$ stands for the density ratio, and $\omega^2 \equiv (4/3)(1+r)V_0$. In deriving Eq. (12) we have made use of the fact that the Ricci scalar is $R = 6(\dot{H}+2H^2)$. It is interesting to note that, notwithstanding the potential being a constant, the gravitational interaction induces an effective mass given by $m_{\mathrm{eff}}^2 = \omega^2/2$ in the effective potential $V_{\mathrm{eff}} = (\omega^2\phi^2/4) + V_0$ of the generalized Klein-Gordon equation. Thereby, in a loose sense, one might associate a particle—the "conformalon"—with the field ϕ.

In order to integrate this system of equations it is expedient to introduce the conformal time η and define a new field ψ as

$$\eta = \int\frac{dt}{a}, \quad \psi = \phi a. \quad (14)$$

Then Eqs. (11)–(13) become

$$3(a')^2 = (1+r)\left[\frac{1}{2}(\psi')^2 + a^4 V_0\right], \quad a'' = \frac{\omega^2}{2}a^3, \quad \psi'' = 0, \quad (15)$$

where a prime indicates derivation with respect to η.

The general solution of this system of equations is given by

$$a = \sqrt{c\,\sinh\omega t}, \quad (16)$$

$$\phi = \frac{\psi}{a} = \frac{\sqrt{2c^2 V_0}\,\eta + b}{\sqrt{c\,\sinh\omega t}}, \quad (17)$$

where c and b are arbitrary integration constants, and the initial singularity has been fixed at $t=0$. Combining Eq. (16) with the Friedmann equation on the stationary solution, $3H^2 = (1+r)\rho_\phi$, we get the following expression for the conformal quintessence energy density

$$\rho_\phi(t) = V_0\coth^2\omega t. \quad (18)$$

In the late time accelerated regime, its equation of state

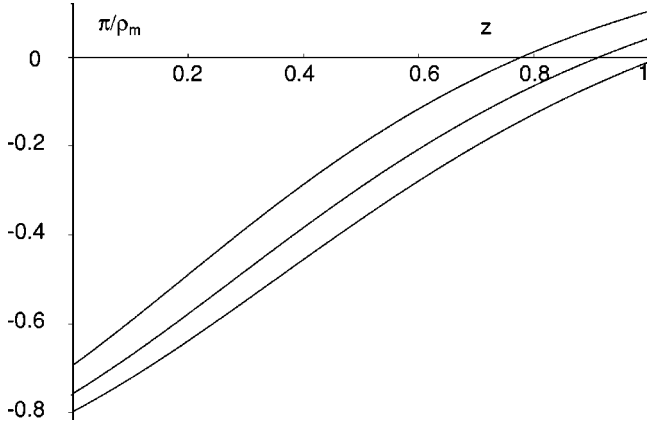$$p_\phi(t) = \left(\frac{4}{3\cosh^2\omega t} - 1\right)\rho_\phi \quad (19)$$

FIG. 1. Selected curves of the ratio dissipative pressure vs energy density of matter π/ρ_m vs the redshift z between the present $z=0$ and $z=1$. From top to bottom, the curves correspond to parameter $\sigma=0.3$, 0.224, and 0.18 defined by $\sigma=(\sinh \omega t_0)^{-2}$.

becomes that of a cosmological constant, and both density parameters $\Omega_\phi(t)$ and $\Omega_m(t)=r\Omega_\phi(t)$ asymptotically approach constant values.

Inserting Eq. (16) into Eq. (14) it follows that

$$\eta(t)=\frac{2}{\omega\sqrt{c}}F\left[(1-\exp(-\omega t))^{1/2},\frac{1}{\sqrt{2}}\right], \qquad (20)$$

where F is the elliptic integral of the first kind. The conformal time is a growing monotonic function of t that behaves like $\sqrt{t}$ close to the singularity and has a finite upper bound

$$\eta(\infty)=\frac{2K(1/\sqrt{2})}{\omega\sqrt{c}}, \qquad (21)$$

where K is the complete elliptic integral and $2K(1/\sqrt{2}) \simeq 3.7$.

From Eq. (17) it is apparent that depending on the choice of the constants several cases arise. If $b\neq 0$, then $\phi\propto 1/\sqrt{t}$ for $t\rightarrow 0$ while if $b=0$, the field has an extremum at the initial singularity. On the other hand, at late times the quintessence field evolves toward the minimum of its effective potential.

The dissipative pressure

$$\pi=\rho_m\left[-\gamma_m+\frac{4}{3\cosh^2 \omega t}\right] \qquad (22)$$

follows from Eqs. (10) and (16); and the evolution of the ratio π/ρ_m, depicted in Fig. 1, shows that the relative relevance of the dissipative pressure grows with expansion. From the last equation, when $\gamma_m<4/3$, it is seen that there is a critical time t_c given by

$$\cosh^2 \omega t_c=\frac{4}{3\gamma_m} \qquad (23)$$

that separates the epoch with $\pi>0$ from the one with $\pi <0$. From Eqs. (23) and (16) we see that it corresponds to the redshift

$$z_c=\left[\sigma\left(\frac{4}{3\gamma_m}-1\right)\right]^{-1/4}-1, \qquad (24)$$

where $\sigma\equiv(\sinh \omega t_0)^{-2}$ is a characteristic parameter of the model with t_0 denoting the "age" of the Universe. As is well known, the second law of thermodynamics requires π to be nonpositive for expanding fluids [14]. This means that our model is valid for $t>t_c$ only; when $t<t_c$ it describes a fictitious positive pressure contribution implying that any realistic evolution of the Universe would yield a corrected value for the critical time $\tilde{t}_c>t_c$. Hence we would obtain a corrected age of the Universe $\tilde{t}_0=t_0-t_c+\tilde{t}_c>t_0$. This model may describe the growth of dissipative effects within dark matter as a consequence of the development of density inhomogeneities [16]. Its characteristic growth time is given by the inverse of the conformalon mass, i.e., by V_0 and r. In the late time accelerated regime, the transport equation for the dissipative pressure becomes $\pi\simeq -3\zeta_b H$. So, we obtain for the dissipative coefficient $\zeta_b\simeq \gamma_m r[V_0/(3(1+r))]^{1/2}$ which satisfies $\zeta_b\geqslant 0$ as demanded by the second law.

III. COSMOLOGICAL PARAMETERS

The acceleration of the Universe is usually evaluated by the dimensionless deceleration parameter $q=-\ddot{a}/(aH^2)$, where $q<0$, $q=0$, $q>0$ describes an accelerating, a linearly expanding (or contracting), and a decelerating universe, respectively. The present value q_0 does not uniquely characterize the current accelerating phase thereby different dark energy models can lead to the same value. Useful additional information is encoded in the statefinder parameters $\bar{r}$ and $\bar{s}$, defined as [17,18]

$$\bar{r}=\frac{\dddot{a}}{aH^3}, \quad \bar{s}=\frac{\bar{r}-1}{3\left(q-\frac{1}{2}\right)}. \qquad (25)$$

It is to be hoped that the pairs $(\bar{r},q)$ and $(\bar{s},q)$ will provide an accurate description of the present dynamics of the Universe and give us some insight into the nature of dark energy. This is only natural because $\bar{r}$ and $\bar{s}$ are directly connected to the third order term in Taylor's expansion of the scale factor around its present value [19]. In the case of an expansion given by Eq. (16) these parameters are found to be

$$q=1-\frac{\alpha^2}{2}, \quad \bar{r}=3-\frac{\alpha^2}{2}, \quad \bar{s}=\frac{4-\alpha^2}{3(1-\alpha^2)}, \qquad (26)$$

in terms of the adimensional ratio

$$\alpha\equiv\frac{\omega}{H}=\frac{2}{[1+\sigma(1+z)^4]^{1/2}}. \qquad (27)$$

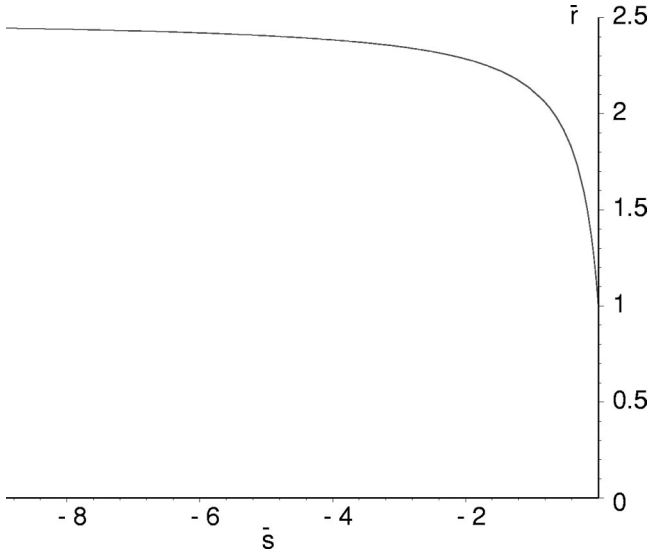Hence there is a single functionally independent cosmological parameter, and the relationships

FIG. 2. Statefinder parameter $\bar{r}$ in terms of $\bar{s}$, the other member of the pair. The evolution goes from the point $(\bar{s}_c, \bar{r}_c) = (-\infty, 5/2)$, corresponding to the critical time to the point $(\bar{s}_\infty, \bar{r}_\infty) = (0,1)$, corresponding to the asymptotically exponential expansion at large times. The present state of the universe corresponds to $(\bar{s}_0, \bar{r}_0) = (-0.109 \pm 0.030, 1.191 \pm 0.043)$.

$$\bar{r} = 2 + q, \quad \bar{s} = \frac{2}{3} \frac{\bar{r} - 1}{2\bar{r} - 5}, \tag{28}$$

between them hold [the dependence $\bar{r}(\bar{s})$ is depicted in Fig. 2], so that the history of the deceleration parameter completely describes the evolution of this universe. Since $q \to 1$ when $t \to 0$ and $q \to -1$ when $t \to \infty$, this model describes a transition from a non-accelerated era to an accelerated era in the present Universe (see Fig. 3). As the accelerated phase begins at a time t_{ac}, where $\sinh(\omega t_{ac}) = 1$, we have that $a_{ac} = \sqrt{c}$,

$$t_{ac} = \frac{1}{\omega} \cosh^{-1} \sqrt{2} = 4.31 \frac{\sqrt{1+\sigma}}{h} \text{Gyr}, \tag{29}$$
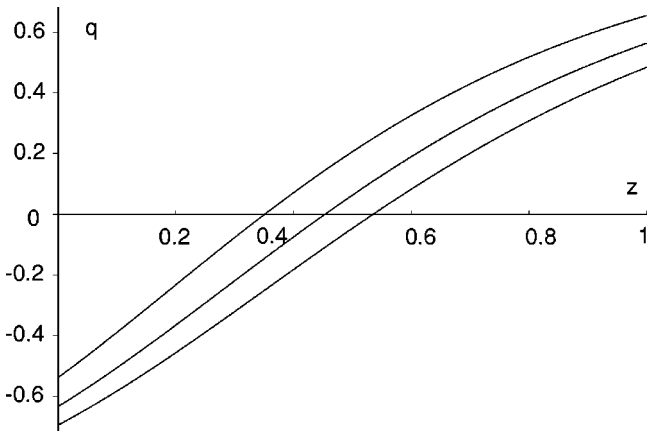

FIG. 3. Selected curves of the deceleration parameter q vs the redshift z between the present $z = 0$ and $z = 1$. From top to bottom, the curves correspond to $\sigma = 0.3$, 0.224, and 0.18.

where h indicates the current value of Hubble's constant in units of 100 km/s/Mpc and the corresponding redshift is $z_{ac} = \sigma^{-1/4} - 1$. For cold dark matter ($\gamma_m = 1$), the ratio $\cosh(\omega t_{ac})/\cosh(\omega t_c) = \sqrt{3/2}$ shows that at the commencement of this phase the dissipative pressure was already negative. On the other hand, in virtue of Eq. (27), the age of the Universe can be expressed by

$$t_0 = \frac{1}{\omega} \sinh^{-1} \frac{1}{\sqrt{\sigma}} = 4.31 \frac{\sqrt{1+\sigma}}{h} \sinh^{-1} \frac{1}{\sqrt{\sigma}} \text{Gyr}, \tag{30}$$

implying that σ must be lower than unity if the Universe is to be accelerated at present. The time span since the critical time to the present is given by

$$t_0 - t_c = \frac{1}{\omega} \left(\sinh^{-1} \frac{1}{\sqrt{\sigma}} - \sinh^{-1} \frac{1}{\sqrt{3}} \right). \tag{31}$$

The stationary condition (10) shows that $\gamma_\phi = \gamma_m$ at the critical time. Provided that by then the matter is cold, i.e., $\gamma_m = 1$ (in the next section we will verify the consistency of this premise) we can assume a smooth extension of our model towards earlier times, as a cold dark matter dominated era for the purpose of obtaining a corrected age of the Universe. Indeed, using Eqs. (24), (26) and (27) we find $q(z_c) = 1/2$ so that the deceleration parameter of this two-stage Universe is continuous. Hence, imposing the continuity of the matter energy density at the critical time, we have $H_c = \omega$ and $\tilde{t}_c = 2/(3\omega)$. This yields the corrected age

$$\tilde{t}_0 = \frac{1}{\omega} \left(\sinh^{-1} \frac{1}{\sqrt{\sigma}} - \sinh^{-1} \frac{1}{\sqrt{3}} + \frac{2}{3} \right). \tag{32}$$

As shown in the next section, this simple estimate produces a rather satisfactory result.

IV. OBSERVATIONAL CONSTRAINTS

It appears that supernovae of type Ia (SNeIa) may be used as standard candles. Properly corrected, the difference in their apparent magnitudes is related to the cosmological parameters. Confrontation of cosmological models to recent observations of high redshift supernovae ($z \lesssim 1$) have shown a good fit in regions of the parameter space compatible with an accelerated expansion [20–25]. We note, however, that models like ΛCDM and QCDM usually require fine tuning to account for the observed ratio between dark energy and clustered matter, while our conformalon model, as well as QDDM-QIM models [12,8], simultaneously provides a late accelerated expansion and solves the coincidence problem.

Ignoring gravitational lensing effects, the predicted magnitude for an object at redshift z in a spatially flat homogeneous and isotropic universe is given by [26]

$$m(z) = \mathcal{M} + 5 \log \mathcal{D}_L(z), \tag{33}$$

where $\mathcal{M}$ is its Hubble radius free absolute magnitude and $\mathcal{D}_L$ is the luminosity distance in units of the Hubble radius,

$$\mathcal{D}_L = (1+z)\int_0^z dz' \frac{H_0}{H(z')}. \tag{34}$$

In virtue of Eq. (27) we obtain a representation in terms of the elliptic integral of the first kind

$$\mathcal{D}_L = \frac{\sqrt{2}(1+z)(1+\sigma)^{1/2}}{(1+i)\sigma^{1/4}}\left[F\left(\frac{(1+i)}{\sqrt{2}}\sigma^{1/4}(1+z),i\right) \right.$$
$$\left. - F\left(\frac{(1+i)}{\sqrt{2}}\sigma^{1/4},i\right)\right]. \tag{35}$$

We have used the sample of 38 high redshift ($0.18\leqslant z$ $\leqslant 0.83$) supernovae of Ref. [21], supplemented with 16 low redshift ($z<0.1$) supernovae from the Calán/Tololo Supernova Survey [27]. This is described as the "primary fit" or fit C in Ref. [21], where, for each supernova, its redshift z_i, the corrected magnitude m_i and its dispersion σ_i were computed. We have determined the optimum fit of the conformalon model by minimizing a χ^2 function

$$\chi^2 = \sum_{i=1}^N \frac{[m_i - m(z_i;\sigma,\mathcal{M})]^2}{\sigma_i^2}, \tag{36}$$

where $N=54$ for this data set. The most likely values of these parameters are found to be $(\sigma,\mathcal{M})=(0.2041,23.93)$, yielding $\chi^2_{min}/N_{DF}=1.104$ ($N_{DF}=52$), and a goodness-of-fit $P(\chi^2 \geqslant \chi^2_{min})=0.282$. These figures show that the fit of conformalon cosmology to this data set is even better than the fit of the ΛCDM or QDDM-QIM models in spite of the fact that we have at our disposal just one free parameter, namely, σ.

We estimate the probability density distribution of the parameters by evaluating the normalized likelihood

$$p(\sigma,\mathcal{M}) = \frac{\exp(-\chi^2/2)}{\int d\sigma \int d\mathcal{M} \exp(-\chi^2/2)}. \tag{37}$$

Then we obtain the probability density distribution for σ marginalizing $p(\sigma,\mathcal{M})$ over $\mathcal{M}$. This probability density distribution $p(\sigma)$ is shown in Fig. 4 and it yields $\sigma=0.224$ ± 0.054. We next use Eq. (30) to obtain $H_0t_0=0.833$ ± 0.045. Likewise, taking $h=0.7\pm0.07$ (cf. [28]) we get from Eq. (31) a period since the critical time $t_0-t_c=7.46$ ± 1.03 Gyr, hence a corrected age of the Universe from Eq. (32) of $\tilde{t}_0=12.7\pm1.4$ Gyr. This one standard deviation range for the corrected age of the Universe falls within the 95% confidence age range 11.2–20 Gyr derived from the age of the oldest globular clusters and it is fully consistent with the recent estimation of 13.4 ± 0.3 Gyr reported by the WMAP team [29] though the latter was reached on the basis of the standard ΛCDM model.

By resorting to Eq. (26) we obtain $q_0=-0.809\pm0.043$, $\bar{r}_0=1.191\pm0.043$ and $\bar{s}_0=-0.109\pm0.030$. Assuming $\gamma_m=1$ we get from Eq. (24) the critical redshift $z_c=0.931$ ±0.123. This figure is consistent with the assumption that
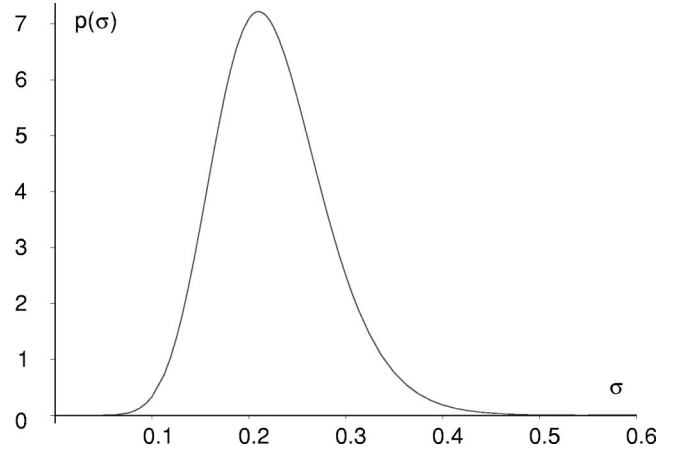

FIG. 4. The estimated probability density distribution (normalized likelihood) for the parameter σ.

matter is already cold at critical time and with the range of redshifts of the supernovae used in the fit. On the other hand, the accelerated expansion era begins at $z_{ac}=0.467\pm0.093$. This value matches the estimation from ΛCDM and the two epoch model of Ref. [30], where the deceleration parameter is constant within each stage.

From Eq. (27) we obtain the frequency parameter ω $=0.129\pm0.013$ Gyr^{-1}, corresponding to a conformalon effective mass $m_{eff}=1.91\pm0.20\times10^{-33}$ eV; and combined with the current density ratio $r_0\simeq0.56\pm0.07$ [31–33] yields $V_0=8.16\pm1.69\times10^{-3}$ Gyr^{-2}. Finally, from Eq. (22) we get for the current ratio

$$\left.\frac{\pi}{\rho_m}\right|_0 = \frac{\sigma-3}{3(1+\sigma)} = -0.758\pm0.051, \tag{38}$$

implying that nowadays the dissipative pressure plays a rather prominent role.

V. COSMOLOGICAL PERTURBATIONS

This section considers the evolution of long-wavelength scalar perturbations of this model. We shall follow the method employed by Perrotta and Baccigalupi in [34,35] based on the formalism developed by Hwang [36] to describe the evolution of perturbations in the synchronous gauge. In this gauge the perturbed metric takes the form

$$ds^2 = a^2[-d\eta^2 + (\delta_{ij}+h_{ij})dx^i dx^j], \tag{39}$$

where the tensor h_{ij} represents the metric perturbations and its Fourier transform can be written as

$$h_{ij}(\mathbf{x},\eta) = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}\left[\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j h(\mathbf{k},\eta)\right.$$
$$\left. + \left(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j - \frac{1}{3}\delta_{ij}\right)6\zeta(\mathbf{k},\eta)\right]. \tag{40}$$

Here h denotes the trace of the tensor h_{ij} and ζ represents its traceless component -for the sake of brevity we will omit the arguments $(\mathbf{k}, \eta)$ henceforth.

The perturbed Einstein equations read

$$k^2\zeta - \frac{1}{2}\mathcal{H}h' = -\frac{a^2\delta\rho}{2}, \tag{41}$$

$$k^2\zeta' = \frac{a^2(p+\rho)\theta}{2}, \tag{42}$$

$$h'' + 2\mathcal{H}h' - 2k^2\zeta = -3a^2\delta p, \tag{43}$$

$$h'' + 6\zeta'' + 2\mathcal{H}(h'+6\zeta') - 2k^2\zeta = -3a^2(p+\rho)\Sigma, \tag{44}$$

where $\mathcal{H} = a'/a$, and the perturbed density $\delta\rho$, pressure δp, velocity divergence θ and shear Σ take the form (for a detailed definition of these terms see [34,37])

$$\delta\rho = \frac{1}{F}\left[\delta\rho_m + \frac{\phi'\delta\phi'}{a^2} - \frac{1}{2}F_{,\phi}R\delta\phi - 3\frac{\mathcal{H}\delta F'}{a^2}\right.$$
$$\left. - \left(\frac{\rho+3p}{2} + \frac{k^2}{a^2}\right)\delta F + \frac{F'h'}{6a^2}\right], \tag{45}$$

$$\delta p = \frac{1}{F}\left[\delta p_m + \frac{\phi'\delta\phi'}{a^2} + \frac{1}{2}F_{,\phi}R\delta\phi + \frac{\delta F''}{a^2} + \frac{\mathcal{H}\delta F'}{a^2}\right.$$
$$\left. + \left(\frac{p-\rho}{2} + \frac{2k^2}{3a^2}\right)\delta F - \frac{F'h'}{9a^2}\right], \tag{46}$$

$$(p+\rho)\theta = \frac{(p_m+\rho_m)\theta_m}{F} - \frac{k^2}{a^2F}(-\phi'\delta\phi - \delta F' + \mathcal{H}\delta F), \tag{47}$$

$$(p+\rho)\Sigma = \frac{(p_m+\rho_m)\Sigma_m}{F} + \frac{2k^2}{3a^2F}\left[\delta F + 3\frac{F'}{k^2}\left(\zeta' + \frac{h'}{6}\right)\right], \tag{48}$$

where $F = 1 - (\phi^2/6)$. There remains the perturbed Klein-Gordon equation:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' + \left(k^2 + \frac{1}{6}a^2R\right)\delta\phi = \frac{\phi'h'}{6} + \frac{a^2}{2}F_{,\phi}\delta R, \tag{49}$$

where the perturbed Ricci scalar reads

$$\delta R = \frac{1}{3a^2}(h'' - 3\mathcal{H}h' + 2k^2\zeta). \tag{50}$$

It has been shown in Ref. [38] that the density contrast $\delta \equiv \delta\rho/\rho$ at large scales grows as $\eta^2 \sim a$ during the matter dominated era previous to the critical time $\tilde{t}_c$. Here we in-

vestigate the behavior of the large scale density perturbations in the asymptotically de Sitter era. In this regime we have $a \simeq (c/2)^{1/2}\exp(\omega t/2)$ and

$$\eta - \eta(\infty) \equiv \Delta\eta \simeq -\frac{2}{\omega a}, \tag{51}$$

hence $\mathcal{H} \simeq -1/\Delta\eta$ and $\phi \simeq -(b\omega/2)\Delta\eta$ for $\Delta\eta \to 0^-$, where we have made use of Eq. (17) to obtain ϕ. The solution of the system of equations (41)–(50) at the lowest order in $\Delta\eta$ and k^2 is readily found to be

$$h \simeq D\Delta\eta, \quad \zeta \simeq -\frac{D}{6}\Delta\eta,$$

$$\delta\rho \simeq \delta\rho_m \simeq -\delta R \simeq -\frac{\omega^2 D}{4}\Delta\eta,$$

$$\delta p \simeq \delta p_m \simeq \frac{\omega^2 D}{6}\Delta\eta \tag{52}$$

$$\delta\phi \simeq A_1\Delta\eta \tag{53}$$

where the integration constants D and A_1 are functions of the wave number k. Thus we find that matter perturbations are dominant at large times and it holds for the perturbation of the energy density ratio $\delta r \simeq \delta_m \equiv \delta\rho_m/\rho_m$. The density contrast decreases in the late time regime as $\delta \propto 1/a$, so that it has a peak during the period when viscous pressure grows.

VI. CONCLUDING REMARKS

We have presented a model of late acceleration that fits extraordinarily well the high redshift supernovae data, yields a good prediction for the age of the Universe and solves the coincidence problem. The only free parameter in the supernovae fit takes a natural value, $\sigma = 0.224 \pm 0.054$.

The dissipative pressure π in dark matter is a key ingredient of our model, and it can attain comparatively large values. Such pressure may arise from the interaction of cold dark matter with itself or the annihilation and/or decay of this component. Different models of cold dark matter that may show these features have been proposed recently—see, e.g., the pedagogical short review of Ref. [39]. No doubt the dissipative pressure inherent to these models may have a profound cosmological impact [40].

The quintessence scalar field has a constant potential and it is nonminimally conformally coupled to the Ricci curvature. In our view, the model possesses two appealing features, namely: (i) an exact and simple solution [Eqs. (16)–(20)], and (ii) notwithstanding the potential is a constant (and consequently plays the role of a cosmological constant), the nonstandard kinetic energy term and the dissipation in the matter allows a stationary regime where the ratio of the energy densities remains constant. Likewise we have calculated the statefinder parameters and have shown that on this regime they are functionally dependent so that in this case the deceleration parameter is enough to describe these solutions.

The present model may describe the growth of dissipative effects within dark matter with some kind of selfinteraction as a consequence of the development of density inhomogeneities after a critical time when these inhomogeneities become large enough. The large scale cosmological density

perturbations are seen to decrease in the asymptotic de Sitter phase, with the matter perturbations dominating over the quintessence perturbations.

We believe that because of its simplicity, our conformal quintessence model may serve as a starting point for more complete models that include a nontrivial selfinteraction quintessence potential, consider the approach of the energy density ratio towards this stationary regime, and extend to-

wards earlier times, when dissipative effects within dark matter become negligible.

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