Plane-wave diffraction at the periodically corrugated boundary of vacuum and a negative-phase-velocity material

Ricardo A. Depine*
Grupo de Electromagnetismo Aplicado, Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina

Akhlesh Lakhtakia†
Computational and Theoretical Materials Science Group, Department of Engineering Science and Mechanics, The Pennsylvania State University, University Park, Pennsylvania 16802–6812, USA
(Received 13 November 2003; published 24 May 2004)

Considering the diffraction of a plane wave by a periodically corrugated half-space, we show that the transformation of the refracting medium from positive (negative) phase velocity to negative (positive) phase velocity type has an influence on the diffraction efficiencies. This effect increases with increasing corrugation depth, owing to the presence of evanescent waves in the troughs of the corrugated interface.

DOI: 10.1103/PhysRevE.69.057602 PACS number(s): 42.25.Fx, 78.20.Ci

I. INTRODUCTION

The rediscovery [1] of isotropic dielectric-magnetic materials exhibiting phase velocity vector opposed in direction to the time-averaged Poynting vector has prompted a flurry of publications during the last four years [2,3]. Many interesting effects have been predicted, with some experimental backing as well [4–6].

Though several names have been proposed for this class of materials, we think that the most descriptive is negative-phase-velocity (NPV) materials [7]. In contrast, the phase velocity and the time-averaged Poynting vectors are coparallel in positive-phase-velocity (PPV) materials. PPV materials are, of course, commonplace and require no introduction.

That the intrinsic difference between NPV and PPV materials has recognizable consequences is easily gauged from a simple problem: reflection and refraction of a plane wave due to a homogeneously filled half-space. Let vacuum be the medium of incidence, while ϵ_2 and μ_2 denote the relative permittivity and relative permeability of the medium of refraction. Let a linear plane wave be incident on the planar interface of the two mediums at an angle θ_0 ($|\theta_0| < \pi/2$) from the normal to the interface and $\rho(\theta_0)$ be the reflection coefficient. If the transformation $\{\epsilon_2 \to -\epsilon_2^*, \mu_2 \to -\mu_2^*\}$ is implemented, then $\rho(\theta_0) \to \rho^*(\theta_0)$, where the asterisk denotes the complex conjugate [8]. Thus, the replacement of a refracting NPV (PPV) half-space by an analogous refracting PPV (NPV) half-space changes the phase of the reflection coefficient but not its magnitude.

What would happen if the interface were to be corrugated [9]? Surface-relief gratings are periodically corrugated surfaces that are commonly used in electromagnetics, and many theoretical techniques are available to compute their diffraction efficiencies [10]. Therefore, we decided to compute and compare the diffraction efficiencies of PPV and NPV surface-relief gratings. In this Brief Report, we present our chief results. Section II contains a sketch of the theoretical method we chose, while Sec. III is a discussion of the numerical results obtained. An $\exp(-i\omega t)$ time dependence is implicit, with ω the angular frequency.

II. THEORY

In a rectangular coordinate system (x,y,z), we consider the periodically corrugated boundary $y=g(x)=g(x+d)$ between vacuum and a homogeneous, isotropic, linear material, with d being the corrugation period. The region $y>g(x)$ is vacuous, whereas the medium occupying the region $y<g(x)$ is characterized by complex-valued scalars $\epsilon_2=\epsilon_{2R}+i\epsilon_{2I}$ and $\mu_2=\mu_{2R}+i\mu_{2I}$. If this medium is of the NPV type, then [7,11]

$$\epsilon_{2R}|\mu_2| + \mu_{2R}|\epsilon_2| < 0; \tag{1}$$

otherwise

$$\epsilon_{2R}|\mu_2| + \mu_{2R}|\epsilon_2| > 0. \tag{2}$$

We note here that refracting material need only be *effectively* homogeneous at the angular frequency of interest [12]. A linearly polarized electromagnetic plane wave is incident on this boundary from the region $y>g(x)$ at an angle θ_0 ($|\theta_0|<\pi/2$) with respect to the y axis.

Let the function $f(x,y)$ represent the z-directed component of the total electric field for the s-polarization case and the z-directed component of the total magnetic field for the p-polarization case [13]. Outside the corrugations, $f(x,y)$ is rigorously represented by the following Rayleigh expansions [9]:

$$f(x,y) = \exp[i(\alpha_0 x - \beta_0^{(1)} y)] + \sum_{n=-\infty}^{+\infty} \rho_n \exp[i(\alpha_n x + \beta_n^{(1)} y)],$$
$$y > \max g(x), \tag{3}$$

*Electronic address: rdep@df.uba.ar
†Electronic address: AXL4@psu.edu

$$f(x,y) = \sum_{n=-\infty}^{+\infty} \tau_n \exp[i(\alpha_n x - \beta_n^{(2)} y)], \quad y < \min g(x). \quad (4)$$

Here, $\{\rho_n\}_{n=-\infty}^{+\infty}$ and $\{\tau_n\}_{n=-\infty}^{+\infty}$ are scalar coefficients to be determined and

$$\alpha_0 = \frac{\omega}{c}\sin\ \theta_0,$$

$$\alpha_n = \alpha_0 + 2n\pi/d,$$

$$\beta_n^{(1)} = \sqrt{\frac{\omega^2}{c^2} - \alpha_n^2},$$

$$\beta_n^{(2)} = \sqrt{\frac{\omega^2}{c^2}\epsilon_2\ \mu_2 - \alpha_n^2}, \quad (5)$$

where c is the speed of light in vacuum. Note that $\beta_n^{(1)}$ is either purely real or purely imaginary, and the conditions

$$\left.\begin{aligned} \mathrm{Re}(\beta_n^{(1)}) \geqslant 0 \\ \mathrm{Im}(\beta_n^{(1)}) \geqslant 0 \end{aligned}\right\} \forall\ n \quad (6)$$

are appropriate for plane waves in the vacuous half-space $y > g(x)$. The refracting half-space $y < g(x)$ being filled by a material medium, $\epsilon_{2I} > 0$ and $\mu_{2I} > 0$ by virtue of causality. The refracted plane waves must attenuate as $y \to -\infty$, which requirement leads to the condition

$$\mathrm{Im}(\beta_n^{(2)}) > 0. \quad (7)$$

Fulfillment of this condition automatically fixes the sign of $\mathrm{Re}(\beta_n^{(2)})$, regardless of the signs of ϵ_{2R} and μ_{2R}. We must note here that the transformation $\{\epsilon_{2R} \to -\epsilon_{2R}, \mu_{2R} \to -\mu_{2R}\}$ alters the signs of the real parts of all $\beta_n^{(2)}$.

Boundary conditions at $y = g(x)$ require the continuity of the tangential components of the total electric field and the total magnetic field. Hence,

$$f(x,g(x)+) = f(x,g(x)-),$$

$$\hat{n}\cdot\nabla f(x,g(x)+) = \sigma^{-1}\hat{n}\cdot\nabla f(x,g(x)-), \quad (8)$$

where $\sigma = \mu_2$ for the s-polarization case and $\sigma = \epsilon_2$ for the p-polarization case, while $\hat{n}$ is a unit vector normal to the boundary.

At this stage we invoke the Rayleigh hypothesis [9] — that is, we assume that expansions (3) and (4), which are strictly valid outside the corrugated region, can be used in the boundary conditions (8). Doing so and then projecting into the Rayleigh basis $\{\exp(i\ \alpha_m x)\}_{m=-\infty}^{+\infty}$, we obtain a system of linear equations for $\{\rho_n\}_{n=-\infty}^{+\infty}$ and $\{\tau_n\}_{n=-\infty}^{+\infty}$. [This step involves multiplying both sides of a boundary condition by $\exp(i\alpha_m x)$ and then integrating with respect to x over one period, for each m.] Following Maradudin ([14] p. 427), we write down the system in matrix form as

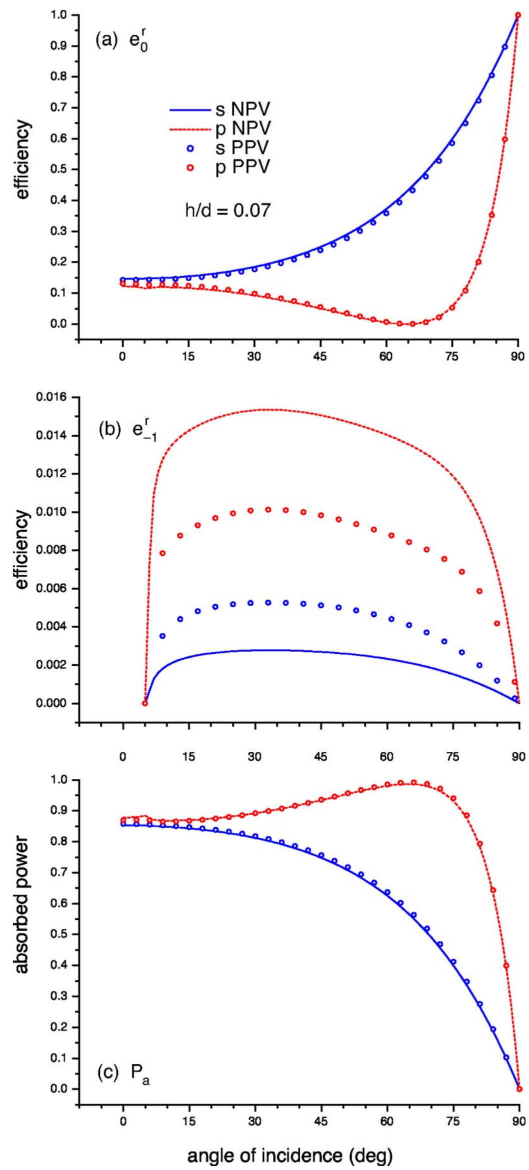

FIG. 1. Diffraction efficiencies e_0^r and e_{-1}^r as well as the normalized absorbed power P_a as functions of the incidence angle θ_0, for a sinusoidally corrugated interface between vacuum and a linear homogeneous medium. The interface function $g(x) = 0.5\ h\cos(2\pi x/d)$, where $h/d = 0.07$ and $\omega d/c = 2\pi/1.1$. The refracting medium is of either the PPV ($\epsilon_2 = 5 + i0.01, \mu_2 = 1 + i0.01$) or the NPV ($\epsilon_2 = -5 + i0.01, \mu_2 = -1 + i0.01$) type. Calculations were made for both the s- and the p-polarization cases.

$$\begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{R} \\ \mathcal{T} \end{bmatrix} = \begin{bmatrix} \mathcal{U} \\ \mathcal{V} \end{bmatrix}. \quad (9)$$

The (m,n)th elements of the four matrixes on the right-hand side of Eq. (9) are

$$\mathcal{M}_{11}|_{mn} = -D_{mn}(\beta_n^{(1)}),$$

$$\mathcal{M}_{12}|_{mn} = D_{mn}(-\beta_n^{(2)}),$$

$$\mathcal{M}_{21}|_{mn} = \beta_n^{(1)}\ D_{mn}(\beta_n^{(1)}) - \alpha_n\ E_{mn}(\beta_n^{(1)}),$$

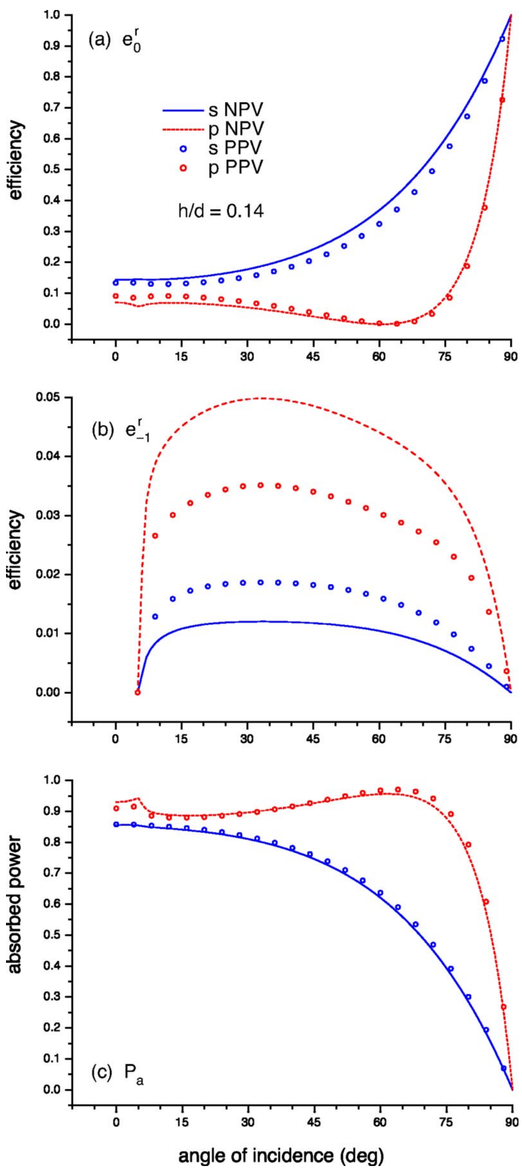

FIG. 2. Same as Fig. 1, but for $h/d = 0.14$.

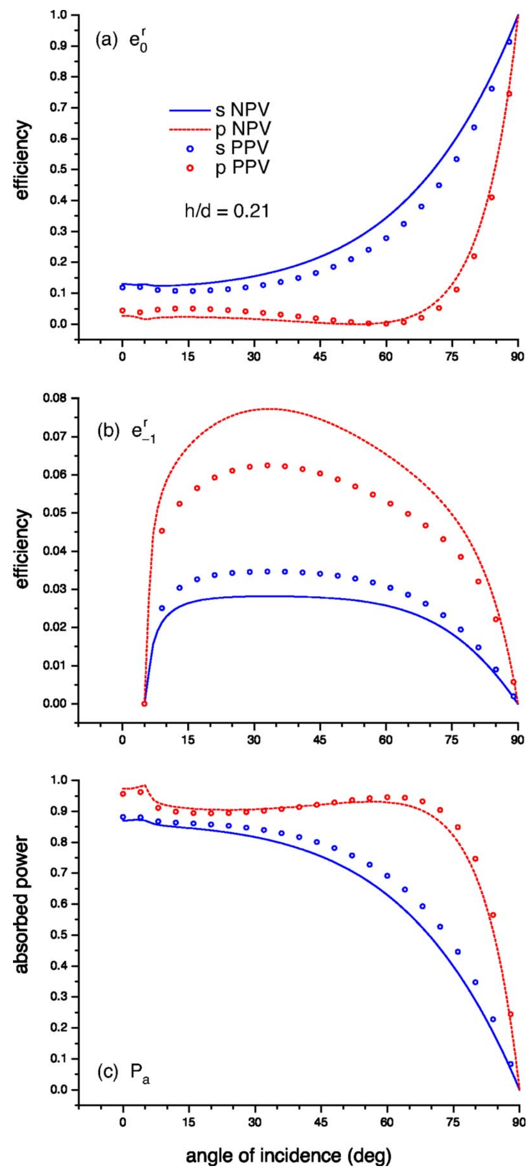

FIG. 3. Same as Fig. 1, but for $h/d = 0.21$.

$$\mathcal{M}_{22}|_{mn} = \frac{1}{\sigma}[\beta_n^{(2)}\, D_{mn}(-\beta_n^{(2)}) + \alpha_n\, E_{mn}(-\beta_n^{(2)})], \quad (10)$$

while the mth elements of the four column vectors in the same equation are

$$\mathcal{R}|_m = \rho_m$$

$$\mathcal{T}|_m = \tau_m,$$

$$\mathcal{U}|_m = D_{m0}(-\beta_0^{(1)}),$$

$$\mathcal{V}|_m = \beta_0^{(1)}\, D_{m0}(-\beta_0^{(1)}) + \alpha_0\, E_{m0}(-\beta_0^{(1)}). \quad (11)$$

The integrals appearing in the foregoing equations are defined as

$$D_{mn}(u) = \frac{1}{d}\int_0^d \exp\left(-i\frac{2\pi}{d}(m-n)x + iug(x)\right)dx \quad (12)$$

and

$$E_{mn}(u) = \frac{1}{d}\int_0^d g'(x)\exp\left(-i\frac{2\pi}{d}(m-n)x + iug(x)\right)dx, \quad (13)$$

with the prime denoting differentiation with respect to argument.

Equation (9) has to be appropriately truncated and solved to determine the reflection coefficients ρ_n and refraction coefficients τ_n. Diffraction efficiencies

$$e_n^r = \frac{\mathrm{Re}(\beta_n^{(1)})}{\beta_0^{(1)}}|\rho_n|^2 \quad (14)$$

are defined for the reflected orders. The normalized power absorbed across one period of the corrugated interface is given by

$$P_a = \mathrm{Re}\left[\frac{1}{\beta_0^{(1)}\sigma} \sum_{n,m} \{\alpha_n\, E_{mn}((\beta_m^{(2)})^* - \beta_n^{(2)}) + \beta_n^{(2)}\, D_{mn}((\beta_m^{(2)})^* - \beta_n^{(2)})\}\tau_n\, \tau_m^* \right], \quad (15)$$

The principle of conservation of energy requires that

$$\sum_n e_n^r + P_a = 1. \quad (16)$$

When we implemented the procedure presented, we checked that the condition (16) was satisfied to an error of 10 ppm. This was usually achieved by retaining 15 terms (i.e., $-7 \leqslant n \leqslant 7$) in the Rayleigh expansions (3) and (4) of the fields.

III. NUMERICAL RESULTS AND DISCUSSION

We chose the corrugations to be sinusoidal: $g(x) = 0.5\, h \cos(2\pi x/d)$. For this type of boundary between vacuum and a penetrable dielectric medium, good results have been obtained for $h/d < 0.3$ [15,16]. We calculated diffraction efficiencies for refracting mediums of both the PPV ($\epsilon_2 = 5 + i0.01, \mu_2 = 1 + i0.01$) and the NPV ($\epsilon_2 = -5 + i0.01, \mu_2 = -1 + i0.01$) types. Calculations were made for both the s-and p-polarization cases. Fixing the ratio $\omega d/c = 2\pi/1.1$, we plotted the diffraction efficiencies e_0^r and e_{-1}^r as well as the absorption P_a as functions of $\theta_0 \in [0, \pi/2)$ for $h/d = 0.07$ (Fig. 1), $h/d = 0.14$ (Fig. 2), and $h/d = 0.21$ (Fig. 3).

When $h/d = 0$—i.e., when the interface is planar—it has been shown [8] that the transformation $\{\epsilon_{2R} \to -\epsilon_{2R}, \mu_{2R} \to -\mu_{2R}\}$ does not change e_0^r. No wonder, the same transformation does not seem to be very effective in affecting e_0^r when $h/d = 0.07$. As the corrugations grow deeper (i.e., as h/d increases in value), the presented data show that the transformation of the refracting medium from NPV (PPV) to PPV (NPV) increasingly affects e_0^r and P_a.

Why should this be so? Now, for a planar interface, the transformation $\{\epsilon_{2R} \to -\epsilon_{2R}, \mu_{2R} \to -\mu_{2R}\}$ leaves the magnitude of the reflection coefficient *only* unchanged for nonevanescent incident plane waves, but that is not a true statement for incident evanescent plane waves [17]. In the troughs of the corrugated interface, the total field that exists has both specular ($n = 0$) and nonspecular ($n \neq 0$) components. Most of the nonspecular components are like evanescent plane waves because they are characterized by $\mathrm{Re}(\beta_n^{(1)}) = 0$. Their presence ensures that the diffraction efficiencies are affected by the transformation of the refracting medium from NPV (PPV) to PPV (NPV).

Before concluding, let us point out that the numerical results presented here for NPV surface-relief gratings agree with the results of a perturbational approach, thereby validating the limited use of the Rayleigh hypothesis for NPV gratings in the same way as for PPV gratings [18]. Also, the emergence of homogeneous NPV materials promises new types of gratings which could be significantly different from their PPV counterparts.

ACKNOWLEDGMENTS

R.A.D. acknowledges financial support from Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Agencia Nacional de Promoción Científica y Tecnológica (Grant No. ANPCYT-BID 802/OC-AR03-04457), and Universidad de Buenos Aires (UBA). A.L. acknowledges partial support from the Penn State Materials Research Science and Engineering Center.

[1] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[2] A. Lakhtakia, M. W. McCall, and W. S. Weiglhofer, Arch. Elektr. Uebertrag. **56**, 407 (2002).
[3] A. Lakhtakia, M. W. McCall, and W. S. Weiglhofer, in *Introduction to Complex Mediums for Optics and Electromagnetics*, edited by W. S. Weiglhofer and A. Lakhtakia (SPIE Press, Bellingham,WA, 2003).
[4] A. Grbic and G. V. Eleftheriadis, J. Appl. Phys. **92**, 5930 (2002).
[5] C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbah, and M. Tanielian, Phys. Rev. Lett. **90**, 107401 (2003).
[6] A. Houck, J. Brock, and I. Chuang, Phys. Rev. Lett. **90**, 137401 (2003).
[7] M. W. McCall, A. Lakhtakia, and W. S. Weiglhofer, Eur. J. Phys. **23**, 353 (2002).
[8] A. Lakhtakia, Electromagnetics **23**, 71 (2003).
[9] Lord Rayleigh, Proc. R. Soc. London, Ser. A **79**, 399 (1907).
[10] *Selected Papers on Diffraction Gratings*, edited by D. Maystre (SPIE Press, Bellingham, WA, 1993).
[11] R. A. Depine and A. Lakhtakia, Microwave Opt. Technol. Lett. **41**, 315 (2004).
[12] *Selected Papers on Linear Optical Composite Materials*, edited by A. Lakhtakia (SPIE Press, Bellingham, WA, 1996).
[13] M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon Press, Oxford, 1980).
[14] A. Maradudin, in *Surface Polaritons*, edited by V. Agranovich and D. L. Mills (North-Holland, Amsterdam, 1982).
[15] R. Petit and M. C. Cadilahc, C. R. Acad. Sci., Ser. A-B **262**, 468 (1966).
[16] N. R. Hill and V. Celli, Phys. Rev. B **17**, 2478 (1978).
[17] A. Lakhtakia, Microwave Opt. Technol. Lett. **40**, 160 (2004).
[18] R. A. Depine and A. Lakhtakia (unpublished).