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Propagation and excitation of eigenmodes at isotropic–gyroelectromagnetic index-matched interfaces

Miriam L. Gigli *, Marina E. Inchaussandague

Consejo Nacional de Investigaciones Científicas y Técnicas, Rivadavia 1917, Buenos Aires, Argentina
Grupo de Electromagnetismo Aplicado, Departamento de Física Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina

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Abstract

We study the relation between the excitation of eigenmodes at isotropic-gyroelectromagnetic flat interfaces under index-matching condition and the singular behaviour of the reflectivity at grazing incidences. We show that, as in the isotropic case, these eigenmodes can be excited by waves at other angles of incidence by introducing a periodic corrugation to the interface between the two media. The coupling produces strong peaks in the curves of reflectivity versus angle of incidence, whose intensity and shape depend on the height of the corrugation. Numerical examples for a particular orientation of the optic axis are presented.

Keywords: Diffraction; Grating; Index-matched interfaces

As it is well known, the reflectivity of a flat interface between permeable, isotropic dielectrics with equal refractive index (index-matched interface) is a constant, independent of the angle of incidence and polarization of the incident wave [1–4]. Moreover, it has been demonstrated that the reflectivity exhibits a singular behaviour at grazing incidence and that this feature is related to the excitation of surface modes [5]. Taking into account these results for isotropic interfaces, the purpose of the present paper is to investigate this singular behaviour at isotropic-gyroelectromagnetic interfaces under conditions of index matching.

In the anisotropic medium, there are two possible refractive indices, one associated with waves of the electric (n_e) and the other with waves of the magnetic (n_m) type. Therefore, if the refractive index of the isotropic medium is denoted by n_i, two different cases of index matching can be considered: $n_i = n_e$ or $n_i = n_m$.

* Corresponding author. Tel.: +541145763390; fax: +541145763357.
E-mail address: mlg@df.uba.ar (M.L. Gigli).

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Let us consider an electromagnetic plane wave (frequency ω) incident from the isotropic side onto a flat interface at $y = 0$. The dielectric zone (ϵ_1, μ_1) is the half space $y \geqslant 0$, while the anisotropic medium $(\tilde{\epsilon}, \tilde{\mu})$ occupies the half space $y \leqslant 0$. The anisotropic medium is described by the constitutive relations

$$\tilde{\epsilon} = \epsilon_{\perp}\tilde{I} + (\epsilon_{\parallel} - \epsilon_{\perp})\hat{c}\hat{c}, \tag{1}$$

$$\tilde{\mu} = \mu_{\perp}\tilde{I} + (\mu_{\parallel} - \mu_{\perp})\hat{c}\hat{c}. \tag{2}$$

In these expressions, $\tilde{I}$ is the unit dyadic and $\hat{c} = (c_x, c_y, c_z)$ (optic axis) is a unit eigenvector of both $\tilde{\epsilon}$ and $\tilde{\mu}$ corresponding to the nonrepeated eigenvalues $\epsilon_{\parallel}$ or $\mu_{\parallel}$, respectively. $\epsilon_{\perp}$ ($\mu_{\perp}$) is the other repeated eigenvalue of $\tilde{\epsilon}(\tilde{\mu})$.

Plane waves in the isotropic medium are expressed as

$$\vec{E} = [A_{\perp}\hat{z} + A_{\parallel}(-\alpha\hat{y} + \beta\hat{x})/k_{\mathrm{i}}]\exp(\mathrm{i}\mathbf{k}_{\mathrm{r}} \cdot \mathbf{r}) + [B_{\perp}\hat{z} + B_{\parallel}(-\alpha\hat{y} - \beta\hat{x})/k_{\mathrm{i}}]\exp(\mathrm{i}\mathbf{k}_{\mathrm{i}} \cdot \mathbf{r}), \tag{3}$$

$$\frac{\omega\mu_1}{c}\vec{H} = [(\beta\hat{x} - \alpha\hat{y})A_{\perp} - k_{\mathrm{i}}A_{\parallel}\hat{z}]\exp(\mathrm{i}\mathbf{k}_{r} \cdot \mathbf{r}) + [(-\beta\hat{x} - \alpha\hat{y})B_{\perp} - k_{\mathrm{i}}B_{\parallel}\hat{z}]\exp(\mathrm{i}\mathbf{k}_{\mathrm{i}} \cdot \mathbf{r}), \tag{4}$$

where c is the velocity of light in vacuum, $\mathbf{k}_{\mathrm{i}} = \alpha\hat{x} - \beta\hat{y}$, $\mathbf{k}_{r} = \alpha\hat{x} + \beta\hat{y}$, $k_{\mathrm{i}} = |\mathbf{k}_{\mathrm{i}}|$, $\alpha = (\omega/c)(\epsilon_1\mu_1)^{1/2}\sin\theta$, $\beta^2 = (\omega/c)^2\epsilon_1\mu_1 - \alpha^2$ and θ is the angle of incidence. In the anisotropic half space, plane waves are written as

$$\vec{E} = A_{\mathrm{e}}\mathbf{e}_{\mathrm{e}}\exp(\mathrm{i}\mathbf{k}_{\mathrm{e}} \cdot \mathbf{r}) + A_{\mathrm{m}}\mathbf{e}_{\mathrm{m}}\exp(\mathrm{i}\mathbf{k}_{\mathrm{m}} \cdot \mathbf{r}), \tag{5}$$

$$\vec{H} = A_{\mathrm{e}}\mathbf{h}_{\mathrm{e}}\exp(\mathrm{i}\mathbf{k}_{\mathrm{e}} \cdot \mathbf{r}) + A_{\mathrm{m}}\mathbf{h}_{\mathrm{m}}\exp(\mathrm{i}\mathbf{k}_{\mathrm{m}} \cdot \mathbf{r}), \tag{6}$$

where subscript e (m) denotes waves of the electric (magnetic) type. $\mathbf{k}_{\mathrm{e,m}} = \alpha\hat{x} + \gamma_{\mathrm{e,m}}\hat{y}$ are the wavenumbers associated with the electric or magnetic wave, $\gamma_{\mathrm{e,m}}$ are calculated from the dispersion relation for the anisotropic medium, $\mathbf{e}_{\mathrm{e}}$ ($\mathbf{e}_{\mathrm{m}}$) is an electric field vector that specifies the polarization of the electric (magnetic) wave and $\mathbf{h}_{\mathrm{e}}$ ($\mathbf{h}_{\mathrm{m}}$) is related to $\mathbf{e}_{\mathrm{e}}$ ($\mathbf{e}_{\mathrm{m}}$) according to Maxwell equations.

The solution of the reflection-transmission problem is obtained by imposing the boundary conditions at the interface, that is the continuity of the tangential components of the fields at $y = 0$. Doing so, we obtain

$$-A_{\perp} - B_{\perp} + A_{\mathrm{e}}(\mathbf{e}_{\mathrm{e}} \cdot \hat{z}) + A_{\mathrm{m}}(\mathbf{e}_{\mathrm{m}} \cdot \hat{z}) = 0, \tag{7}$$

$$\frac{\beta}{k_{\mathrm{i}}}(A_{\parallel} - B_{\parallel}) - A_{\mathrm{e}}(\mathbf{e}_{\mathrm{e}} \cdot \hat{x}) - A_{\mathrm{m}}(\mathbf{e}_{\mathrm{m}} \cdot \hat{x}) = 0, \tag{8}$$

$$\frac{c}{\omega\mu_1}k_{\mathrm{i}}(A_{\parallel} + B_{\parallel}) + A_{\mathrm{e}}(\mathbf{h}_{\mathrm{e}} \cdot \hat{z}) + A_{\mathrm{m}}(\mathbf{h}_{\mathrm{m}} \cdot \hat{z}) = 0, \tag{9}$$

$$\frac{c}{\omega\mu_1}\beta(A_{\perp} - B_{\perp}) - A_{\mathrm{e}}(\mathbf{h}_{\mathrm{e}} \cdot \hat{x}) - A_{\mathrm{m}}(\mathbf{h}_{\mathrm{m}} \cdot \hat{x}) = 0. \tag{10}$$

Equations above form a system of four linear equations with four unknowns ($A_{\perp}$, $A_{\parallel}$, A_{e} and A_{m}) from which the expressions of the Fresnel reflection and transmission coefficients are obtained [6,7]. Expressions for these coefficients are long and complicated. However, for particular orientations of the optic axis of the anisotropic medium, the expressions of Fresnel coefficients are much simpler. In this work, we will investigate the behaviour of the reflectivity in the anisotropic interface under conditions of index matching, in the three following cases: (a) $\hat{c} = \hat{z}$, (b) $\hat{c} = \hat{y}$ and (c) $\hat{c} = \hat{x}$. It is easy to demonstrate that for cases (a)–(c), there is no polarization conversion; that is $R_{\mathrm{sp}} = R_{\mathrm{ps}} = 0$. The expressions of the Fresnel reflection coefficients are:

- Case (a)

$$R_{\mathrm{ss}} = \frac{\mu_{\perp}\beta + \mu_1\gamma_{\mathrm{e}}}{\mu_{\perp}\beta - \mu_1\gamma_{\mathrm{e}}}, \quad R_{\mathrm{pp}} = \frac{\epsilon_{\perp}\beta + \epsilon_1\gamma_{\mathrm{m}}}{\epsilon_{\perp}\beta - \epsilon_1\gamma_{\mathrm{m}}}. \tag{11}$$

- Case (b)

$$R_{\mathrm{ss}} = \frac{\mu_{\perp}\beta + \mu_1\gamma_{\mathrm{m}}}{\mu_{\perp}\beta - \mu_1\gamma_{\mathrm{m}}}, \quad R_{\mathrm{pp}} = \frac{\epsilon_{\perp}\beta + \epsilon_1\gamma_{\mathrm{e}}}{\epsilon_{\perp}\beta - \epsilon_1\gamma_{\mathrm{e}}}. \tag{12}$$

- Case (c)

$$R_{\mathrm{ss}} = \frac{\mu_{\parallel}\beta + \mu_1\gamma_{\mathrm{m}}}{\mu_{\parallel}\beta - \mu_1\gamma_{\mathrm{m}}}, \quad R_{\mathrm{pp}} = \frac{\epsilon_{\parallel}\beta + \epsilon_1\gamma_{\mathrm{e}}}{\epsilon_{\parallel}\beta - \epsilon_1\gamma_{\mathrm{e}}}. \tag{13}$$

The transmitted waves of the electric and magnetic type, have refractive indices $n_{\mathrm{e}}^2 = \mathbf{k}_{\mathrm{e}}^2c^2/\omega^2$ and $n_{\mathrm{m}}^2 = \mathbf{k}_{\mathrm{m}}^2c^2/\omega^2$, respectively. For the orientations of the optic axis selected above, the refractive indices in the anisotropic medium, and the polarization of the electric fields for each mode, are given by

- Case (a)

$$n_e^2 = \mu_\perp \epsilon_\parallel, \quad n_m^2 = \mu_\parallel \epsilon_\perp, \tag{14}$$

$$\mathbf{e}_e = \hat{z}, \quad \mathbf{e}_m = \gamma_m \hat{x} - \alpha \hat{y}. \tag{15}$$

- Case (b)

$$\begin{aligned} n_e^2 &= \mu_\perp \epsilon_\perp - (\epsilon_\perp/\epsilon_\parallel - 1)\alpha^2 c^2/\omega^2, \\ n_m^2 &= \epsilon_\perp \mu_\perp - (\mu_\perp/\mu_\parallel - 1)\alpha^2 c^2/\omega^2, \end{aligned} \tag{16}$$

$$\mathbf{e}_e = -\gamma_e \hat{x} + \frac{\epsilon_\perp}{\epsilon_\parallel}\alpha \hat{y}, \quad \mathbf{e}_m = \hat{z}. \tag{17}$$

- Case (c)

$$\begin{aligned} n_e^2 &= \mu_\perp \epsilon_\parallel - (\epsilon_\parallel/\epsilon_\perp - 1)\alpha^2 c^2/\omega^2, \\ n_m^2 &= \epsilon_\perp \mu_\parallel - (\mu_\parallel/\mu_\perp - 1)\alpha^2 c^2/\omega^2, \end{aligned} \tag{18}$$

$$\mathbf{e}_e = -\gamma_e \hat{x} + \frac{\epsilon_\parallel}{\epsilon_\perp}\alpha \hat{y}, \quad \mathbf{e}_m = \hat{z}. \tag{19}$$

Note that in cases (b) and (c) the index-matching condition holds for a unique angle of incidence. To study the case of index matching in anisotropic interfaces, we have to consider the matching with either the electric or the magnetic refractive index. When the refractive index of the isotropic medium is equal to the refractive index of the electric type, we obtain

- Case (a)

$$\begin{aligned} R_{ss} &= \frac{\mu_\perp - \mu_1}{\mu_\perp + \mu_1}, \\ R_{pp} &= \frac{\epsilon_\perp \beta - \epsilon_1 \sqrt{\frac{\omega^2}{c^2}\mu_\parallel \epsilon_\perp - \frac{\omega^2}{c^2}\mu_\perp \epsilon_\parallel + \beta^2}}{\epsilon_\perp \beta + \epsilon_1 \sqrt{\frac{\omega^2}{c^2}\mu_\parallel \epsilon_\perp - \frac{\omega^2}{c^2}\mu_\perp \epsilon_\parallel + \beta^2}}, \end{aligned} \tag{20}$$

- Case (b)

$$\begin{aligned} R_{ss} &= \frac{\mu_\perp \beta - \mu_1 \sqrt{\frac{\omega^2}{c^2} n_m^2(\alpha) - \frac{\omega^2}{c^2} n_e^2(\alpha) + \beta^2}}{\mu_\perp \beta + \mu_1 \sqrt{\frac{\omega^2}{c^2} n_m^2(\alpha) - \frac{\omega^2}{c^2} n_e^2(\alpha) + \beta^2}}, \\ R_{pp} &= \frac{\epsilon_\perp - \epsilon_1}{\epsilon_\perp + \epsilon_1}, \end{aligned} \tag{21}$$

- Case (c)

$$\begin{aligned} R_{ss} &= \frac{\mu_\parallel \beta - \mu_1 \sqrt{\frac{\omega^2}{c^2} n_m^2(\alpha) - \frac{\omega^2}{c^2} n_e^2(\alpha) + \beta^2}}{\mu_\parallel \beta + \mu_1 \sqrt{\frac{\omega^2}{c^2} n_m^2(\alpha) - \frac{\omega^2}{c^2} n_e^2(\alpha) + \beta^2}}, \\ R_{pp} &= \frac{\epsilon_\parallel - \epsilon_1}{\epsilon_\parallel + \epsilon_1}, \end{aligned} \tag{22}$$

whereas

- Case (a)

$$\begin{aligned} R_{ss} &= \frac{\mu_\perp \beta - \mu_1 \sqrt{\frac{\omega^2}{c^2}\mu_\perp \epsilon_\parallel - \frac{\omega^2}{c^2}\mu_\parallel \epsilon_\perp + \beta^2}}{\mu_\perp \beta + \mu_1 \sqrt{\frac{\omega^2}{c^2}\mu_\perp \epsilon_\parallel - \frac{\omega^2}{c^2}\mu_\parallel \epsilon_\perp + \beta^2}}, \\ R_{pp} &= \frac{\epsilon_\perp - \epsilon_1}{\epsilon_\perp + \epsilon_1}, \end{aligned} \tag{23}$$

- Case (b)

$$\begin{aligned} R_{ss} &= \frac{\mu_\perp - \mu_1}{\mu_\perp + \mu_1}, \\ R_{pp} &= \frac{\epsilon_\perp \beta - \epsilon_1 \sqrt{\frac{\omega^2}{c^2} n_e^2(\alpha) - \frac{\omega^2}{c^2} n_m^2(\alpha) + \beta^2}}{\epsilon_\perp \beta + \epsilon_1 \sqrt{\frac{\omega^2}{c^2} n_e^2(\alpha) - \frac{\omega^2}{c^2} n_m^2(\alpha) + \beta^2}}, \end{aligned} \tag{24}$$

- Case (c)

$$\begin{aligned} R_{ss} &= \frac{\mu_\parallel - \mu_1}{\mu_\parallel + \mu_1}, \\ R_{pp} &= \frac{\epsilon_\parallel \beta - \epsilon_1 \sqrt{\frac{\omega^2}{c^2} n_e^2(\alpha) - \frac{\omega^2}{c^2} n_m^2(\alpha) + \beta^2}}{\epsilon_\parallel \beta + \epsilon_1 \sqrt{\frac{\omega^2}{c^2} n_e^2(\alpha) - \frac{\omega^2}{c^2} n_m^2(\alpha) + \beta^2}}, \end{aligned} \tag{25}$$

when the matching is with the refractive index of the magnetic type. From expressions above, we can see that, conversely to what happens in index-matched isotropic interfaces, coefficients R_{ss} and R_{pp} are different. Moreover, only one reflection coefficient R_{ss} or R_{pp} (depending on the case) results independent on the angle of incidence. This result is not surprising as we are dealing with anisotropic media for which two different index-matching are possible.

The expressions above are valid for $\theta \neq \pi/2$. When we approach grazing incidences, $\beta = \gamma_e = \gamma_m = 0$ and the coefficients in Eqs. (11)–(13) become singular. This is so because in this situation, the system (7)–(10) has more unknowns than equations. As incident and diffracted fields have both the same spatial dependence for $\theta = \pi/2$, this situation is completely equivalent to the propagation of an eigenmode along the surface. This result indicates that, as it happens in the isotropic case, the singular behaviour of the reflectivity at grazing incidence is intimately related with the existence of guided modes along the interface.

To investigate the possibility of exciting other surface modes in index-matched anisotropic interfaces, we will introduce a periodic sinusoidal corrugation $g(x) = \frac{h}{2}\sin(2\pi x/d)$ to the interface between the two media.

In the examples below, we will consider $\hat{c} = \hat{z}$. In Fig. 1, we plot the reflected power (RP) as function of the angle of incidence for $0 < \theta < 90°$ for both polarizations of the incident wave. Other parameters are $\epsilon_1 = 2.5$, $\mu_1 = 1$, $\epsilon_\perp = 2.5$, $\epsilon_\parallel = 2.08333$, $\mu_\perp = 1.2$, $\mu_\parallel = 1.05$, $h/d = 0.1$ and $\lambda/d = 1$. These values correspond to the matching between the refractive index of the isotropic side and the refractive index of the electric type. As it can be observed in this figure, when the polarization of the incident wave is TE, the curve exhibits a sharp peak (RP $\approx$ 0.97) at $\theta \approx 21.56°$. At this angle of incidence, the reflected 1st order becomes evanescent and the coupling condition for the electric type mode

$$\sqrt{\mu_1\epsilon_1}\sin(\theta) + n\frac{\lambda}{d} = \sqrt{\mu_\perp\epsilon_\parallel}, \tag{26}$$

is satisfied for the 1st order. In contrast, the curve corresponding to a TM polarized incident wave does not present any singularity. As we have explained before, this occurs because, if $n_i = n_e$, the coupling occurs when the electric field is perpendicular to the plane of incidence (TE polarization).

In Fig. 2, we plot the efficiency of the zeroth reflected order ($r(0)$) as a function of the angle of incidence for the same parameters as Fig. 1 for incident TE and TM polarizations. In these figures, we also appreciate a peak at $\theta \approx 21.56°$ for TE incidence, when the surface mode is excited.

In Fig. 3, we plot the efficiency of the 1st reflected order ($r(1)$) as a function of the angle of incidence, for the same parameters as the previous figures, for TE and TM incidence. The intensity of this efficiency varies from 0.0017 and 0.0015 for $\theta < 21.5°$, increases abruptly for $21.5° < \theta < 21.56°$-due to the coupling with the surface mode-, and vanishes for $\theta > 21.56°$.

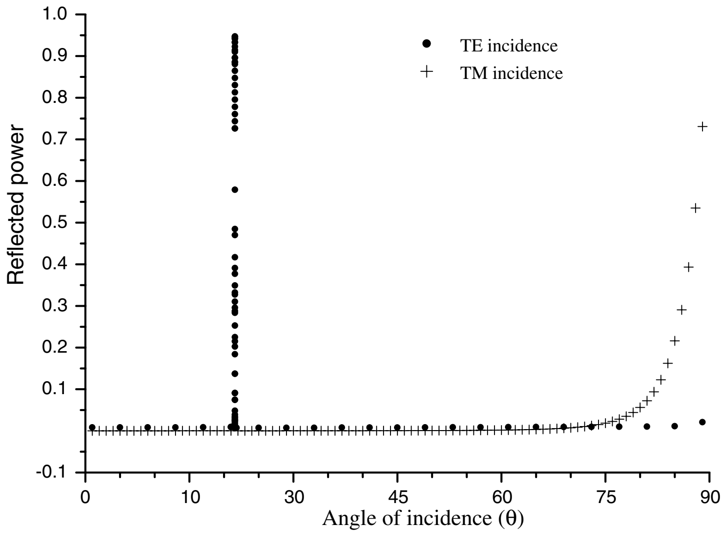

Fig. 1. Reflected power versus angle of incidence θ for a sinusoidal profile with $h/d = 0.1$. The grating parameters are $\epsilon_1 = 2.5$, $\mu_1 = 1$, $\epsilon_\perp = 2.5$, $\epsilon_\parallel = 2.08333$, $\mu_\perp = 1.2$, $\mu_\parallel = 1.05$ and $\lambda/d = 1$.

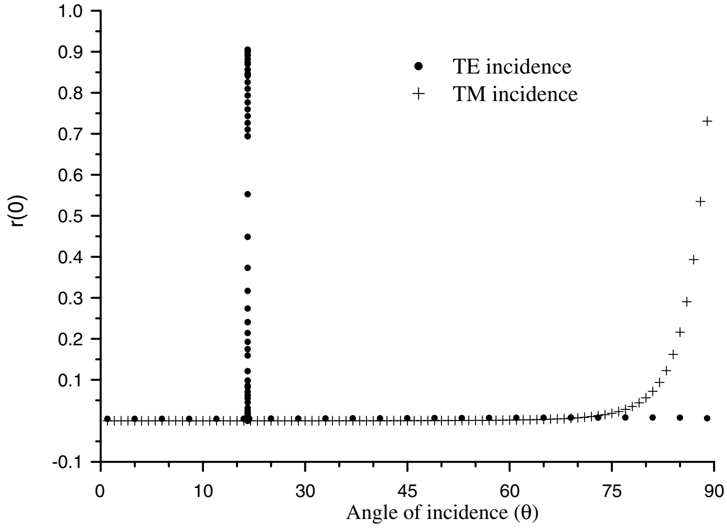

Fig. 2. Efficiency of the zeroth reflected order $r(0)$ versus angle of incidence for the same parameters as in Fig. 1.

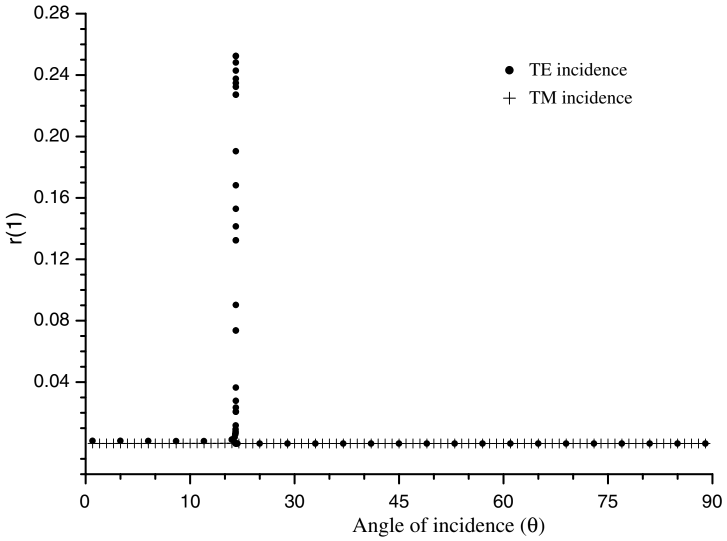

Fig. 3. Efficiency of the 1st reflected order $r(1)$ versus angle of incidence for the same parameters as in Fig. 1.

Next, we consider the matching with the refractive index of the magnetic type. To illustrate the behaviour of the curves in this case, we plot in Figs. 4–6 the reflected power, and the efficiencies of the 0th ($r(0)$) and 1st ($r(1)$) reflected orders, respectively, as functions of the angle of incidence for a grating with $\epsilon_1 = 2.5$, $\mu_1 = 1$, $\epsilon_\perp = 1.5$, $\epsilon_\parallel = 2$, $\mu_\perp = 1$, $\mu_\parallel = 1.66$, $h/d = 0.1$ and $\lambda/d = 1$. We consider both polarizations of the incident wave. In this case, as it was expected, the coupling takes place when the electric field vector is contained in the plane of incidence (TM polarization) and when the coupling condition

$$\sqrt{\mu_1 \epsilon_1}\sin(\theta) + n\frac{\lambda}{d} = \sqrt{\mu_\parallel \epsilon_\perp}, \tag{27}$$

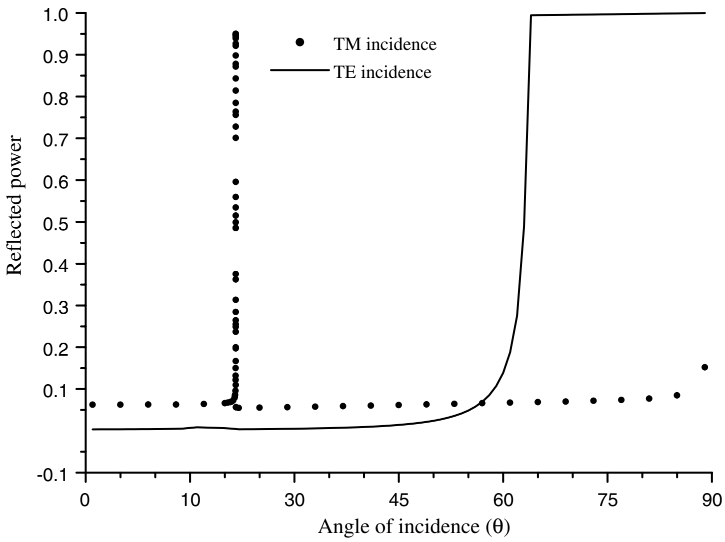

Fig. 4. Reflected power versus angle of incidence θ for a sinusoidal profile with $h/d = 0.1$. The grating parameters are $\epsilon_1 = 2.5$, $\mu_1 = 1$, $\epsilon_\perp = 1.5$, $\epsilon_\parallel = 2$, $\mu_\perp = 1$, $\mu_\parallel = 1.66$ and $\lambda/d = 1$.

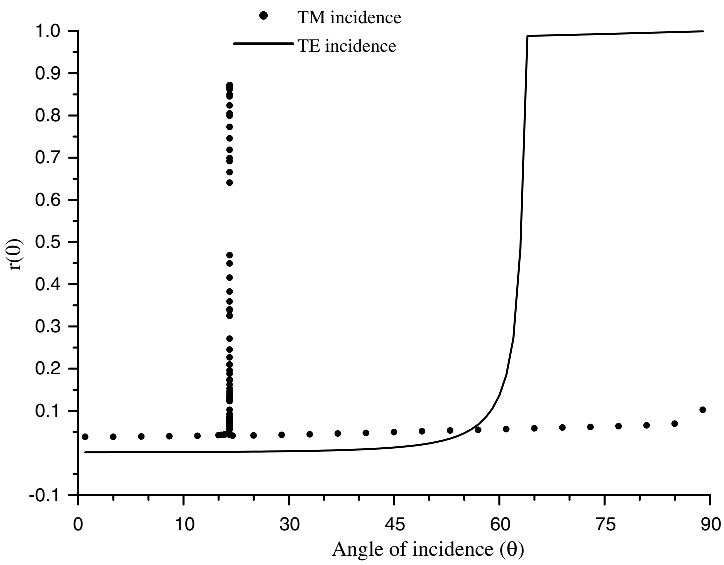

Fig. 5. Efficiency of the zeroth reflected order $r(0)$ versus angle of incidence for the same parameters as in Fig. 4.

for the magnetic type mode is satisfied.

In Fig. 7, we show curves of the reflected power as functions of the angle of incidence for different values of the groove height-to-period ratio and for the same parameters as in Fig. 1. We observe that the position of the peaks is approximately the same in all the curves. For low values of h/d the peaks are narrow; as h/d is increased they become broader.

In summary, we have shown that the excitation of eigenmodes at isotropic-gyroelectromagnetic surfaces is connected with the singular behaviour of the reflectivity at grazing incidences for flat inter-

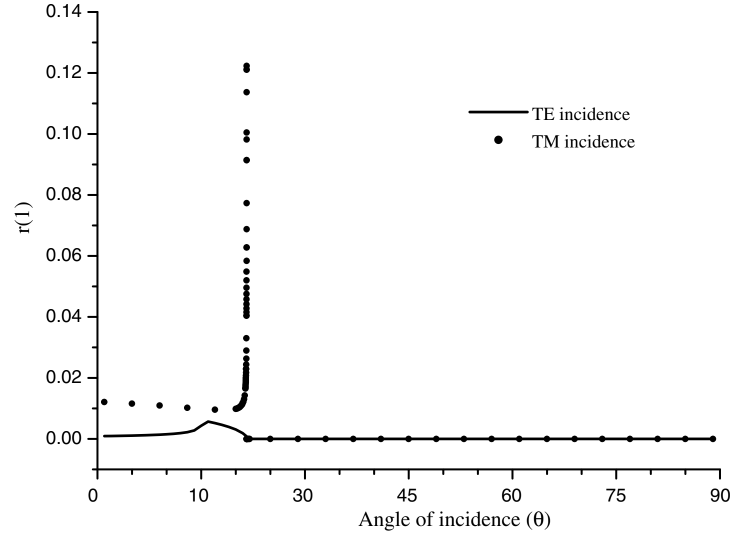

Fig. 6. Efficiency of the 1st reflected order $r(1)$ versus angle of incidence for the same parameters as in Fig. 4.

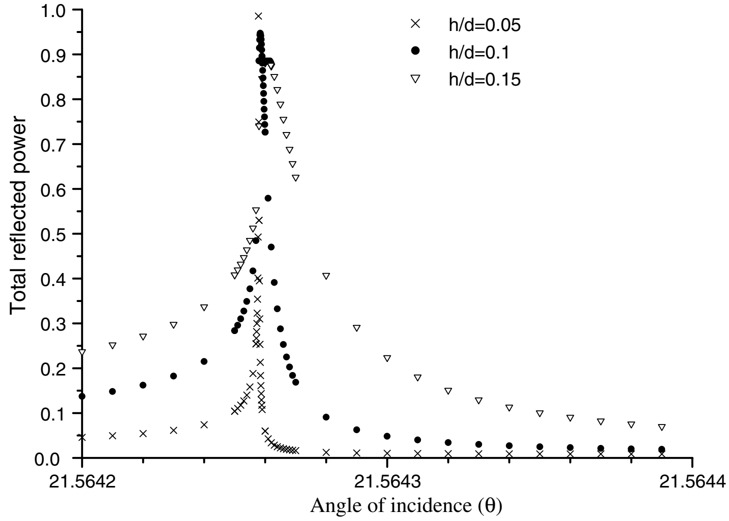

Fig. 7. Reflected power versus angle of incidence θ for a sinusoidal profile with different values of h/d. Other parameters are the same as Fig. 1.

faces, and that these modes can be coupled with incident waves at other angles of incidence through a corrugation of the interface between the two media.

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