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Hoffmann–Infeld black-hole solutions in Lovelock gravity

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Abstract

Five-dimensional black holes are studied in Lovelock gravity coupled to Hoffmann–Infeld nonlinear electrodynamics. It is shown that some of these solutions present a double peak behaviour of the temperature as a function of the horizon radius. This feature suggests that the evaporation process, though drastic for a period, leads to an eternal black-hole remnant. In fact, the form of the caloric curve corresponds to the existence of a *plateau* in the evaporation rate, which implies that black holes of intermediate scales turn out to be unstable. The geometrical aspects, such as the absence of conical singularity, the structure of horizons, etc are also discussed. In particular, solutions that are asymptotically AdS arise for special choices of the parameters, corresponding to charged solutions of five-dimensional Chern–Simons gravity.

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1. Introduction

In this paper we discuss the possibility of finding a class of black hole for which the behaviour of the temperature as a function of the horizon radius presents a *double peak* form. Typically, this particular behaviour leads to the presence of a *plateau* in the evaporation rate, implying a drastic evaporation for those black holes having sizes which are bounded between the two scales where the peaks are located. We show that these black holes actually appear as solutions of the Lovelock theory of gravity coupled to a particular nonlinear electrodynamics. The existence of such phase behaviour is due to the fact that the two models considered here (namely Lovelock theory and Hoffman–Infeld theory) represent short distance corrections to both general relativity and Maxwell electrodynamics respectively and, consequently, two peaks arise if the scales induced by both corrections do not coincide (scale splitting).

Specifically, we study five-dimensional solutions representing charged black holes in Hoffmann–Infeld electrodynamics within the framework of the Lovelock theory of gravity. Currently, in a particular context, the study of the combined problem of considering certain models of nonlinear (Born-Infeld like) electrodynamics and higher order gravitational theories acquires importance due to the role that these theories play in low energy string inspired models. Originally, the Hoffmann–Infeld model was proposed to avoid certain *pathological* features that Born–Infeld field theory presents when spherically symmetric static solutions are considered, such as the conical singularities that BIons present at the origin. Actually, the modification of Born–Infeld theory presented in [1] has been shown to lead to spherically symmetric particle-like objects whose associated metric is regular everywhere, so avoiding the conical singularity of the Einstein–Born–Infeld case previously studied in [2]. Nevertheless, the black-hole solutions in Einstein–Hoffmann–Infeld field theory are still singular since the curvature diverges at the origin.

The explicit form of Hoffmann–Infeld action is presented in [1]. This can be written as follows:

$$S_{\mathrm{HI}} = -\frac{b^2}{4}\int \mathrm{d}^5x\sqrt{-g}(1-\eta_{(F)}-\log\eta_{(F)})$$

with

$$\eta_{(F)} = \frac{b^{-2}F_{\mu\nu}F^{\mu\nu}}{\sqrt{1+2b^{-2}F_{\mu\nu}F^{\mu\nu}}-1} \tag{1}$$

and where b represents a characteristic field, analogue to that appearing in Born–Infeld theory. Actually, the Hoffmann–Infeld model corresponds to logarithmic modifications to a nonlinear Born–Infeld-like Lagrangian, and was originally designed in such a way that certain *regularity conditions* hold for both gravitational and electric fields when particle-like solutions are considered. In the case of the gravitational field, the regularity condition comes from the choice of an integration constant that amounts to stating the identity between gravitational and electromagnetic masses.

On the other hand, the short distance corrections carried by higher order theories, such as the Lovelock theory of gravity, automatically held the divergences associated with the Newtonian term [3]. Hence, the finiteness of the gravitational field at the origin is guaranteed *ab initio*. This means that the identification between electromagnetic and gravitational masses in Lovelock gravity does not come from imposing the requirement for the metric of the spacetime to be finite but by adding the requirement that no conical singularity should exist there [4].

In the following section, we describe the charged black-hole solutions in five-dimensional Lovelock gravity coupled to Hoffmann–Infeld nonlinear electrodynamics. In section 3, we study the thermodynamics of this solution, with particular interest focused on the evaporation phenomenon.

2. Charged black-hole solutions

The most general five-dimensional gravity action that depends on the metric and its derivatives up to the second order and, besides, leads to conserved field equations is the Lovelock gravitational action. This is given by supplementing Einstein–Hilbert action with Gauss–Bonnet terms. In lower dimensional models ($D < 5$) these terms represent topological invariants and, hence, Lovelock gravity turns out to coincide with general relativity. In five dimensions the action is

$$S = \frac{1}{16\pi}\int \mathrm{d}^5x\sqrt{-g}(R-2\Lambda+\alpha(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+R^2-4R_{\mu\nu}R^{\mu\nu}))+S_{\mathrm{HI}},$$

where α is the Gauss–Bonnet coupling constant, which defines a length scale. Actually, this theory introduces short distance corrections to general relativity, implying the existence of a scale $l_\alpha = \sqrt{4\alpha}$ where such corrections turn out to be relevant.

The gravitational equations of motion resulting from $\delta S = 0$ are

$$8\pi T_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} - \alpha\big(\tfrac{1}{2}g_{\mu\nu}(R_{\rho\delta\gamma\lambda}R^{\rho\delta\gamma\lambda} - 4R_{\rho\delta}R^{\rho\delta} + R^2) - 2RR_{\mu\nu} + 4R_{\mu\rho}R^{\rho}_{\nu} + 4R_{\rho\delta}R^{\rho\delta}{}_{\mu\nu} - 2R_{\mu\rho\delta\gamma}R_{\nu}{}^{\rho\delta\gamma}\big),$$

where $T_{\mu\nu}$ is the stress-tensor representing the *matter-field* distribution coming from the variation $\delta S_{\mathrm{HI}}/\delta g^{\mu\nu}$.

Here, we are interested in static spherically symmetric solutions satisfying the following ansatz:

$$\mathrm{d}s^2 = -g_\alpha(r)\,\mathrm{d}t^2 + g_\alpha^{-1}(r)\,\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\chi^2 + r^2\,\sin^2\theta\,\sin^2\chi\,\mathrm{d}\varphi^2$$

which, once replaced in the field equations, yields

$$-\frac{3}{r^3}\frac{\mathrm{d}}{\mathrm{d}r}(r^2(1-g_\alpha(r)) + 2\alpha(1-g_\alpha(r))^2) + 2\Lambda = 16\pi T^0{}_0. \tag{2}$$

Then, it is straightforward to prove that the following functional relation holds:

$$g_\alpha(r) - g_0(r) = \frac{2\alpha}{r^2}(1-g_\alpha(r))^2 \tag{3}$$

$g_0(r)$ being a spherically symmetric solution of the field equations with $\alpha = 0$, which is simply determined by solving Einstein equations in five dimensions. The meaning of equation (3) can be intuitively understood by the following heuristic argument: let us consider the gravitational potential $\phi_\alpha(r)$ defined as $g_\alpha(r) = 1 - 2\phi_\alpha(r)$; then, according to the above relation, the potential can be written as $\phi_\alpha(r) = \phi_0(r) + \frac{m_g(r)}{r^2}$ where the *mass* m_g is due to the gravitational potential itself and given by $m_g = -(l_\alpha\phi_\alpha(r))^2$.

Besides, when the total energy of the source is finite, g_0 can be written as

$$g_0(r) = 1 + \frac{16\pi}{3r^2}\int_0^r \mathrm{d}s\, s^3 T^0{}_0(s) - \frac{\Lambda}{6}r^2. \tag{4}$$

This solution amounts to the choice of a null integration constant in (2) (a possible term k/r^2 has been removed in (4)), and implies the identification of gravitational and electromagnetic masses. The finiteness of the total energy guarantees the Newtonian behaviour at infinity.

The nonlinear electrodynamics with finite total energy is characterized by a field scale b defining the typical length scale $l_b = (e/b)^{1/3}$, where e is the charge of the object. We will consider the electromagnetic stress-tensor for a particle-like source as having the generic form

$$T_0^0(r) = -\frac{b^2}{4\pi r^3}h_b(r). \tag{5}$$

In the case of the Hoffmann–Infeld model, the function $h_b(r)$ is given by

$$h_b(r) = \tfrac{1}{2}r^3\log\left(1 + l_b^6 r^{-6}\right), \tag{6}$$

and, as was already mentioned, corresponds to a charged particle-like source with electric field $E(r) = e\big/\big(r^3 + l_b^6 r^{-3}\big)$. On the other hand, in the case of the Born–Infeld model the function is $h_b(r) = \sqrt{r^6 + l_b^6} - r^3$. Moreover, in the generic case[4] we will demand the following *finiteness conditions*:

$$\lim_{r\to 0}\frac{1}{r^2}\int_0^r \mathrm{d}s\,h_b(s) = \delta < \infty, \qquad \int_0^\infty \mathrm{d}s\,h_b(s) = \gamma < \infty. \tag{7}$$

[4] Which are not satisfied by the particle-like source yielded from Maxwell theory.

Note that, in the case of the Hoffmann–Infeld model one finds $\gamma = \frac{\pi l_b^4}{4\sqrt{3}}$. The first condition in (7) is the requirement for the metric to be finite when a particle-like solution is considered as the source of Einstein gravity theory. On the other hand, the second condition means that the total energy of the particle turns out to be finite. The Born–Infeld charge fulfils both requirements (Hoffmann studied a Born–Infeld charged black hole in four dimensions, in the context of Einstein gravity [2]). However the Born–Infeld charge yields $\delta \neq 0$, which means that a conical singularity remains in the metric. Hoffmann and Infeld removed the conical singularity by modifying the Born–Infeld electrodynamics in order to obtain $\delta = 0$ [1]. However, Lovelock gravity also removes the conical singularity, as can be seen in relation (3). In fact, it results in

$$g_\alpha(r) = 1 + \frac{r^2}{l_\alpha^2} + \epsilon \frac{r^2}{l_\alpha^2}\sqrt{1 + \frac{8b^2 l_\alpha^2}{3r^4}\int_0^r \mathrm{d}s\, h_b(s) + \frac{l_\alpha^2}{l_\Lambda^2}}, \tag{8}$$

where $\epsilon = \pm 1$ and $l_\Lambda^2 = 3/\Lambda$. Thus, we find that $\lim_{r\to 0} g_\alpha(r) = 1$ for any finite δ. Note that this is a charged Deser–Boulware [3] black hole. Moreover, note that, in the large r/l_α limit, the following asymptotic behaviour is obtained:

$$g_\alpha(r) = 1 + \lambda_\epsilon r^2 + \frac{4\epsilon e^2}{3 l_b^6 r^2}\int_0^r \mathrm{d}s\, h_b(s) + \cdots, \tag{9}$$

where $\lambda_\epsilon = (1+\epsilon)\big/l_\alpha^2 + \epsilon\big/2l_\Lambda^2$ and the dots refer to subleading orders in l_α/r (and subleading orders in l_α/l_Λ as well). Furthermore, if the large r/l_b limit of the metric is performed, we find

$$g_\alpha(r) = 1 + \lambda_\epsilon r^2 - \frac{2m_\epsilon}{\pi r^2} + \cdots \tag{10}$$

once the mass is accordingly identified as $m_\epsilon = -\epsilon \frac{2\pi^2 e^2 \gamma}{3 l_b^6}$. Conversely, if we first take the limit $b \to \infty$ (i.e. $l_b/r \to 0$) and then explore the asymptotic behaviour, the geometry becomes

$$g_\alpha(r) = 1 + \lambda_\epsilon r^2 - \epsilon \frac{e^2}{3r^4} + \cdots \tag{11}$$

which, for instance, in the case $\epsilon = +1$ corresponds to a black hole with a dominant cosmological term $\lambda \sim l_\alpha^{-2}$ and a *wrong sign* Reissner–Nordström term $\sim -e^2/r^4$. This mimics a charged massless black hole with *imaginary electric charge*; though it has to be emphasized that the mechanism leading to such a metric is substantially different to the one leading to black holes with an analogous *tidal charge* term, cf [5]. It is relatively simple to verify that, considering subleading effects in powers of l_α/l_Λ, the mass of (8) is given by

$$m = \frac{2\pi b^2 \gamma}{3}\left(1 + \sum_{n=2}^{\infty} \frac{(2n-3)!!}{2^{n-1}(n-1)!}\left(-l_\alpha^2/l_\Lambda^2\right)^{n-1}\right), \tag{12}$$

where $l_\alpha < l_\Lambda$ and where the *dressing* of the Newtonian term manifestly appears due to the presence of the cosmological constant Λ (see [4]). This dressing effect is characterized by the expansion in powers of the dimensionless parameter $l_\alpha^2\big/l_\Lambda^2$. Moreover, the specific value of the first term in such an expansion is that required for the metric to be regular at $r = 0$.

On the other hand, for the specific value $l_\alpha^2 = -l_\Lambda^2$ ($\alpha > 0$, $\Lambda < 0$), the solution takes a rather different form; namely

$$g_\alpha(r) = 1 + \frac{r^2}{l_\alpha^2} + c(r) \tag{13}$$

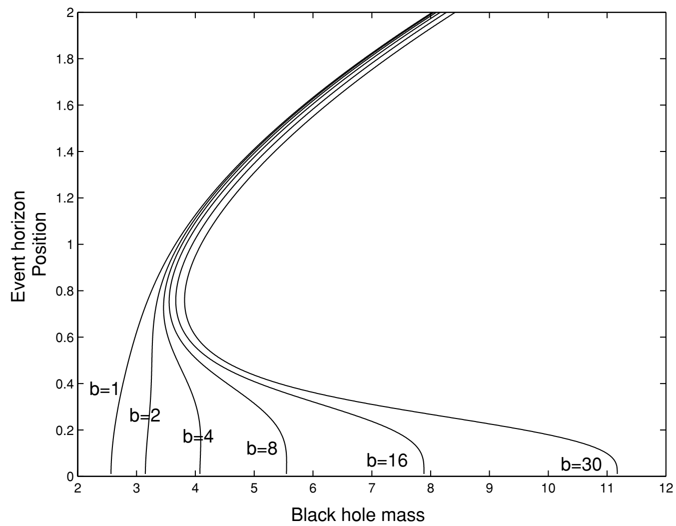

Figure 1. The position of the event horizon as a function of the black-hole mass for different values of b ($\alpha = 0.5$, $e = 1$, $\Lambda = 0$).

with the function

$$c^2(r) = \frac{8b^2}{3l_\alpha^2}\int_0^r \mathrm{d}s\, h_b(s). \tag{14}$$

This metric corresponds to an asymptotically anti-de Sitter space in five dimensions. This is due to the finite value $c^2(\infty) = \frac{8b^2\gamma}{3l_\alpha^2}$ at infinity. Geometries (14) present event horizons and are closely related to the black-hole solutions of Chern–Simons gravitational theory.

In particular, for the Lovelock–Hoffmann–Infeld black hole, the function $h_b(r)$ clearly satisfies the finiteness conditions described above. In this case, the charged black-hole solution (8) shares several properties with the one built for the Born–Infeld model, e.g. the existence of charged black holes with a unique horizon. However, the horizon structure of both theories is certainly different (cf figure 1 and the analogue presented in [6]), e.g. the fact that the internal radius r_- decreases when the mass m increases (for a fixed charge e) is strongly more evident for the case of the Hoffmann–Infeld black hole. Moreover, by comparing the solution (8) and the one corresponding to the Lovelock–Born–Infeld black holes [4–9], it is feasible to verify that in both theories $g_\alpha(r)$ can be set to 1 at the origin $r = 0$. Nevertheless, in the case of Lovelock–Hoffmann–Infeld solutions, we find that $\frac{\mathrm{d}g_\alpha(r)}{\mathrm{d}r}$ vanishes in the limit $r \to 0$ whereas it goes to $-\infty$ for Lovelock–Born–Infeld black holes (see figure 2 which manifests the difference between both solutions at the origin). This is because the short distance corrections to Maxwell theory involved in the Hoffmann–Infeld are, in some sense, stronger than the ones corresponding to the Born–Infeld model.

3. Thermodynamics

Now, the question arises as to what are the thermodynamical properties of this charged black-hole solution. Certainly, it is known that black holes in Lovelock gravity present special features which are not shared with their analogues in Einstein gravity theory; namely, these black holes typically have an infinite lifetime, present a positive specific heat for small radii,

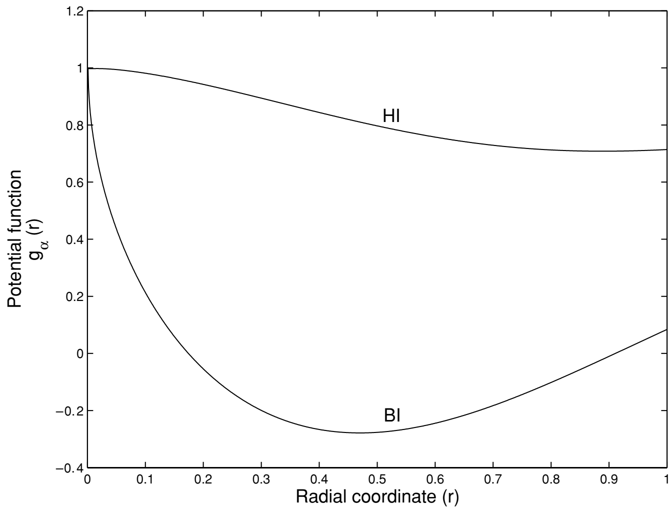

Figure 2. The (potential) function $g_\alpha(r)$ for both Hoffmann–Infeld (HI) and Born–Infeld (BI) solutions ($\alpha = 0.1$, $e = 1$, $\Lambda = 0$).

their isothermal graphs for the charged cases turn out to be rather different [6], violate the Bekenstein's area formula [7, 8] and the temperature formula in the general case presents an additional term which identically vanishes for $D = 5$. Here, we show that, besides these remarkable aspects, the Lovelock–Hoffmann–Infeld black holes present a *plateau* in the evaporation rate as a function of the horizon radius r_+. This aspect implies that the solutions charged under the Hoffmann–Infeld electrodynamics evaporate drastically when they have middle sizes within the range of scales where the specific heat is negative. Eventually, these black holes end up in a stable phase (region of positive specific heat) and their lifetimes result infinite. This can be intuitively inferred from the fact that the caloric curve presents a double peak form for certain tuning of the parameters.

Let us begin by writing the temperature for the case of vanishing cosmological constant, namely

$$T = \frac{1}{4\pi} \frac{\mathrm{d}g_\alpha(r)}{\mathrm{d}r}_{|r=r_+} = \frac{r_+ - \frac{2b^2}{3} h_b(r_+)}{2\pi\left(l_\alpha^2 + r_+^2\right)} \tag{15}$$

whose typical form is described in figures 3 and 4. The above expression corresponds to the temperature of solution (8) with $\epsilon = -1$ and $\Lambda = 0$; this solution is asymptotically flat, as can be verified by means of the expansion (10). Actually, the regime of general relativity is recovered in the limit $l_\alpha / r_+ \to 0$, where the expression for the temperature takes the form $T \sim 1/r_+$.

We also note that the specific heat changes its sign due to the short distance corrections imposed by both Lovelock and Hoffmann–Infeld models. The sign of the specific heat enables one to infer which are the regions of thermodynamical stability (where the black hole can be in thermal equilibrium with the environment). Consequently, the evaporation rate of these charged solutions is obtained from (15) by integrating over the energy flux. This is done by making use of the Stefan–Boltzmann law in five dimensions; namely $\frac{\mathrm{d}M_s}{\mathrm{d}\tau} \sim T^5$, M_s being the surface energy density $\left(M_s \sim m / r_+^3\right)$. Then, by using that the (three-dimensional) surface of

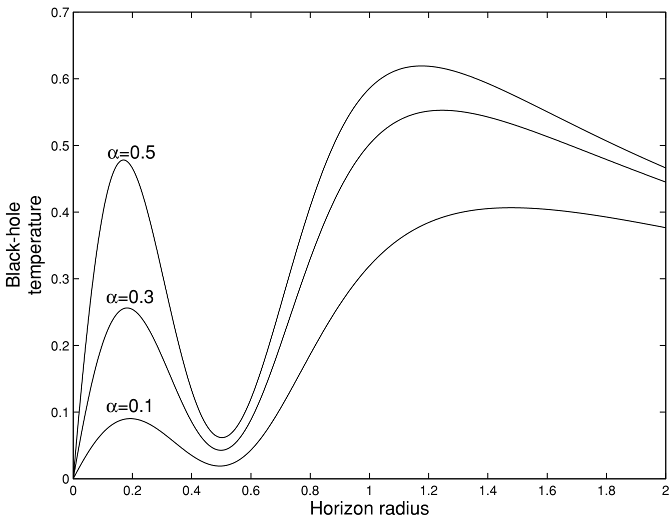

Figure 3. Black-hole temperature as a function of the horizon radius ($b = 2, e = 1$); representing the caloric curve.

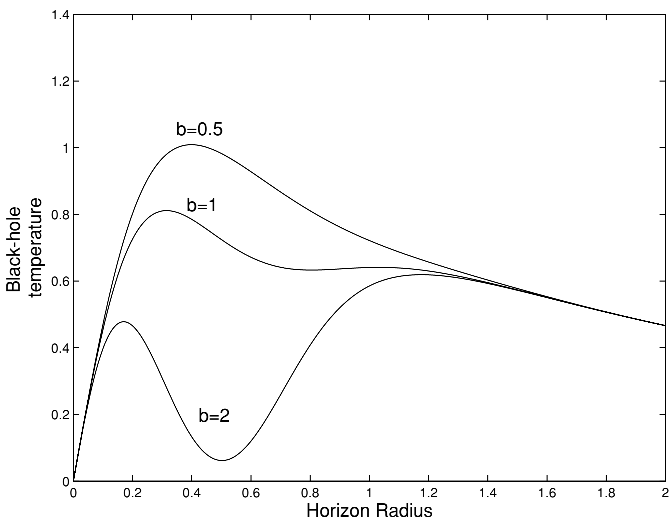

Figure 4. Black-hole temperature as a function of the horizon radius ($\alpha = 0.05, e = 1$); representing the caloric curve.

the horizon is given by $\frac{\pi}{2}r_+^3$ and the expression of the temperature is given by (15) we can integrate over the black-hole size in order to obtain the evaporation time

$$\tau \sim \int_{r_+}^{r_0} \mathrm{d}s \frac{\left(l_\alpha^2 + s^2\right)^5}{s^3\left(s - \frac{2}{3}b^2 h_b(s)\right)^4}. \tag{16}$$

This corresponds to the time required for a black hole to evaporate, starting with the initial size r_0 and ending with a size r_+. Note that r_+ is a monotonic function of the mass (energy) m. The sign $\sim$ stands in the above formula because of the presence of a positive multiplicative

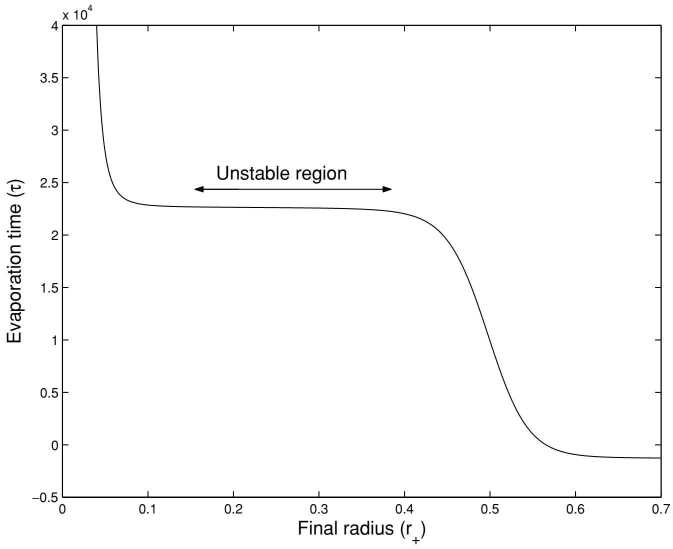

Figure 5. Evaporation time τ as a function of the final radius r_+ ($\alpha = 0.05, e = 1, b = 2$). A *plateau* manifestly appears in the evaporation rate. The graph has been normalized by means of an appropriate redefinition of the Stefan–Boltzmann constant.

constant which is given in terms of the (inverse of the) Stefan–Boltzmann constant in five dimensions.

This result leads us to observe the presence of the *plateau* which is studied in figure 5, showing a rapid transition between the two scales where the first maximum and the local minimum of figure 3 are located[5]. The evaporation rate is usually displayed by analysing the quantity $\mathrm{d}m/\mathrm{d}\tau$; let us note that figure 4 (showing the time τ required to reach a size r_+) basically gives the same information: this is because the *plateau* of the graph precisely corresponds to those scales for which the transition (evaporation) is abrupt and, hence, the quantity $\mathrm{d}m/\mathrm{d}\tau$ would present a peak precisely located in that region. Moreover, since we are interested in studying the scales where such an abrupt transition occurs, we find figure 4 convenient because it manifestly shows those scales within which such a drastic effect takes place.

An interesting analysis of the black-hole thermodynamics in Einstein–Gauss–Bonnet gravity and Chern–Simons gravity was recently performed in [10–16]. To make contact with Chern–Simons gravity, let us consider again the case $l_\alpha^2 = -l_\Lambda^2$, for which the formula of the temperature as a function of the horizon radius r_+ acquires a dominant linear term; namely

$$T = \frac{r_+}{2\pi l_\alpha^2} - \frac{b^2 h_b(r_+)}{3\pi\left(l_\alpha^2 + r_+^2\right)}. \tag{17}$$

This diverges in the limit $r_+/l_\alpha \to 0$.

Summarizing, the Lovelock theory of gravity in higher dimensions introduces short distance corrections to general relativity due to Gauss–Bonnet terms which, in addition to

[5] Let us remark the qualitative analogy existing between the caloric curve of the Lovelock–Hoffmann–Infeld black holes and those corresponding to the models with long-range interactions in condensed matter. In fact, it is well known that certain quasi-stationary states of those models exhibit a similar (double) change of sign in the specific heat and actually are qualitatively similar to the profile displayed in figure 4 (for instance, cf [17] and references therein). The caloric curve obtained here is reminiscent of that.

the Einstein–Hilbert action, have to be taken into account in the most general theory of gravity. These terms, corresponding to Lanczos quadratic gravity for $D = 5$, are such that the mentioned short distance effects imply substantial differences with respect to the black-hole thermodynamics of general relativity; these were listed above. In addition, we discussed here how the charged black holes in Lovelock five-dimensional gravity coupled to nonlinear Hoffmann–Infeld electrodynamics present other interesting features like the existence of the double peak profile in the caloric curve, leading to a particular evaporation effect for which two different thermodynamically stable regions exist. The black holes evaporate drastically for certain sizes that are bounded by two critical radii; this region corresponds to the one where the specific heat is negative. Eventually, a final phase is reached and the black holes become eternal; the explicit computation of the evaporation rate leads to an infinite lifetime as result. Because of the particular profile of the caloric curve, the evaporation phenomenon described here is qualitatively different to the one corresponding to the five-dimensional Lovelock–Born–Infeld solutions, cf [4–9].

Furthermore, the analysis of the static spherically symmetric solution presented in this paper is general enough as to be suitable for adaptation to the case of Lovelock black holes charged under a quite generic model of nonlinear electrodynamics. In particular, it would be relatively easy to extend it to those models of electrodynamics leading to regular black holes[6] in Einstein gravity [18–22]. The analysis of these geometries within the framework of Lovelock gravity could be an interesting subject for further study.

Before concluding, let us make a brief remark on the higher dimensional case. Certainly, the five-dimensional case presents a special feature: the fact that the expression for the temperature (15) acquires an additional term in D dimensions, which is proportional to $(D-5)$. This is precisely why previous papers on the subject (see for instance [6]) considered the case $D = 5$ as a special one. However, it is also true that, besides that, several qualitative aspects of the thermodynamics of the 5D Lovelock black holes are shared with their higher dimensional analogues: For instance, this is the case of the change of the sign of the specific heat at short distances and the existence of infinite lifetime remnants. Then, similar features to those analysed in this paper are expected to be valid in the D-dimensional Lovelock–Hoffmann–Infeld black holes.

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