International Studies in the Philosophy of Science

ISSN: 0269-8595 (Print) 1469-9281 (Online) Journal homepage: http://www.tandfonline.com/loi/cisp20

Adjusting Venus: The Use of Maximum Elongations in the Almagest and Ptolemy's Theory of Knowledge

Gonzalo L. Recio

To cite this article: Gonzalo L. Recio (2017) Adjusting Venus: The Use of Maximum Elongations in the Almagest and Ptolemy's Theory of Knowledge, International Studies in the Philosophy of Science, 31:2, 113-131

To link to this article: https://doi.org/10.1080/02698595.2018.1424762

Published online: 05 Mar 2018.

INTERNATIONAL STUDIES IN THE PHILOSOPHY OF SCIENCE, 2017
VOL. 31, NO. 2, 113–131
https://doi.org/10.1080/02698595.2018.1424762

Adjusting Venus: The Use of Maximum Elongations in the *Almagest* and Ptolemy's Theory of Knowledge

Gonzalo L. Recio

Departamento de Ciencias Sociales, Universidad Nacional de Quilmes

ABSTRACT
There has been much debate surrounding the way in which Ptolemy handled the observations that are behind his planetary models. This article aims at presenting an interesting case of manipulation of observational data within the epistemological frame of what we may call Ptolemy's philosophy of knowledge. It deals with an interesting case from the *Almagest*, that of the determination of the longitudes of the apogee and perigee of Venus's deferent, and shows that the selections and adjustments Ptolemy carried out in order to obtain the needed results are nothing but an analogous application of the epistemological doctrines expressed in his *On the Kriterion* and in his *Harmonics*.

1. Introduction

There has been much debate surrounding the way in which Ptolemy handled the observations that are behind his planetary models. Two centuries ago, the great astronomer Jean Baptiste Delambre asked, 'Did Ptolemy really observe? Are the observations which he claims to have made not just calculations by means of tables, and examples intended to create a better for a better understanding of his theories?' (Delambre [1817] 1965, xxv). This aspect of Ptolemaic astronomy has been the subject of renewed interest in recent years. The publication of *The Crime of Claudius Ptolemy* (Newton 1977), a book whose title correctly summarises the spirit of its contents, was perhaps one of the reasons why many scholars have devoted their time to studying the way in which Ptolemy dealt with the problems of the relation between theory and observation. Many publications have dealt with this subject: Swerdlow (1979, 1989, 2004), Long (1989), Grasshof (1990), Britton (1992), Goldstein and Bowen (1999), Jones (2006), and Carman (2009) are among them most relevant.

The way in which this matter is approached is usually a very technical one, where the main focus lies on the observational basis for Ptolemy's theories and the procedures used to adjust it. The intention is, most times, to try to show the logic behind the adjustment of empirical data in order to make a case for Ptolemy. Though this approach is certainly fruitful and necessary, in general, it lacks the contextualisation of those procedures within a more comprehensive frame that explains how Ptolemy conceived the relation

CONTACT Gonzalo L. Recio gonzalorecio@hotmail.com Departamento de Ciencias Sociales, Universidad Nacional de Quilmes, Roque Saenz Peña 352, B1876BXD Bernal, Provincia de Buenos Aires, Argentina

between theory and observation in scientific research. It is an omission that is, in some ways, surprising, since Ptolemy himself left at least two works in which he explicitly stated his opinions on the matter: *Harmonics* (Ptolemy 1989a) and *On the Kriterion and Hegemonikon* (Ptolemy 1989b). These works have also been the subject of several studies: Long (1989), Feke (2012), and Schiefsky (2014) are some examples.

Not much attention has been paid, however, to the concrete ways in which Ptolemy may have applied his epistemological doctrines to his real scientific practice, which in the case of astronomy is most importantly displayed in his *Almagest*. I am aware of only two works where this has been done: in Feke and Jones (2010), Ptolemy's character is outlined, and some procedures of the arguments of the *Almagest* are related to his epistemological points of view. There is, however, a more extensive treatment of the subject in Jones (forthcoming).[1] Following these approaches, this article aims at presenting an interesting case of manipulation of observational data within the epistemological frame of what we may call Ptolemy's philosophy of knowledge. In order to do that, I will focus on three of Ptolemy's works, the *Almagest* (Ptolemy 1984), *Harmonics*, and *On the Kriterion and Hegemonikon*.

This article has six parts. Firstly, I will briefly explain the origin of some aspects of Ptolemy's planetary models. Secondly, I will show a possible way through which Ptolemy may have determined the value of some parameters for his model of Venus. In this part, I will mainly follow the ideas in Noel M. Swerdlow's 'Ptolemy's Theory of the Inferior Planets' (Swerdlow 1989). Thirdly, I will show Ptolemy's epistemological ideas as they are extant in book I of his *Harmonics*. The fourth part is devoted to his *On the Kriterion*, where he dealt with his philosophy of knowledge in more detail. I will show how the ideas advanced there have a close similarity to the doctrines he expressedin his work on musical harmonies. In the fifth part, I will show how the procedure he applied in his theory of Venus to find its parameters is nothing but an analogous application of the doctrines expressed in his *On the Kriterion* and in his *Harmonics*, and therefore, that the accusation of Ptolemy as a fraudulent astronomer is, at least, a simplistic approach to his work. Lastly, I will briefly show two cases where Ptolemy follows quite different procedures for manipulating observational data.

2. Ptolemaic Planetary Models

The apparent movements of the planets were amongst the most important and difficult problems faced by ancient astronomers. As has been known since times immemorial, while most of the stars maintained fixed relative positions in the sky, there were five that wandered through them following somewhat regular paths along a circular band of limited width, the Zodiac.[2] These were the five planets.[3]

Unlike the fixed stars, the planets showed three ways in which their apparent trajectories departed from simple circular and uniform movements, thus manifesting the presence of an anomaly.[4] For reasons that exceed the purpose of this article, all astronomy before Johannes Kepler's *Astronomia Nova* maintained that celestial objects performed uniform circular motions, and that their apparent trajectories were the result of one of such movements, or of the combinations of two or more of them. The presence of an anomaly was the sign that the system of that celestial object—in our case, a planet—had some complexity that had to be accounted for in the model that the astronomer designed. This was done by the incorporation of a new element in the model: an eccentricity for the orbit, the addition of an extra circle, etc.

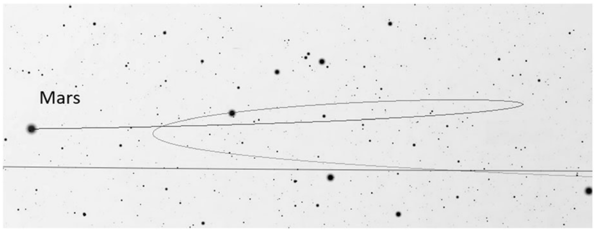

Figure 1. Complete Martian retrogradation. Mars is to the left with the ecliptic line crossing below the retrograde loop. The direction of the signs is from right to left.

The first and most spectacular way in which the planets showed the complexity of their systems was the phenomenon called retrogradation. In their movement across the Zodiac, the planets showed a cycle in their varying angular velocities, alternating between a maximum and a minimum. In all cases, the minimum angular velocity was negative, that is, the planet moved 'backwards' with respect to the signs (Figure 1).[5] These 'backward' movements were the so-called retrogradations.

Greek astronomers devised several models to account for retrogradations.[6] The most successful model they came up with was that of the deferent and epicycle, the existence of which can be traced back to at least the times of Apollonius of Perga (3rd century BCE; Evans 1998, 340). In that system (Figure 2), the planet P moves on a circle called *epicycle*, whose centre E in turn moves on another circle, called the *deferent*. The combination of the two movements, as seen from Earth T, causes the planet to retrograde with respect to the sphere of the fixed stars (the outer circle in the diagram).[7]

But this was not the only way in which the planets diverged from a simple, circular, and uniform model. Observations showed at least one other problem: the epicycle seemed to

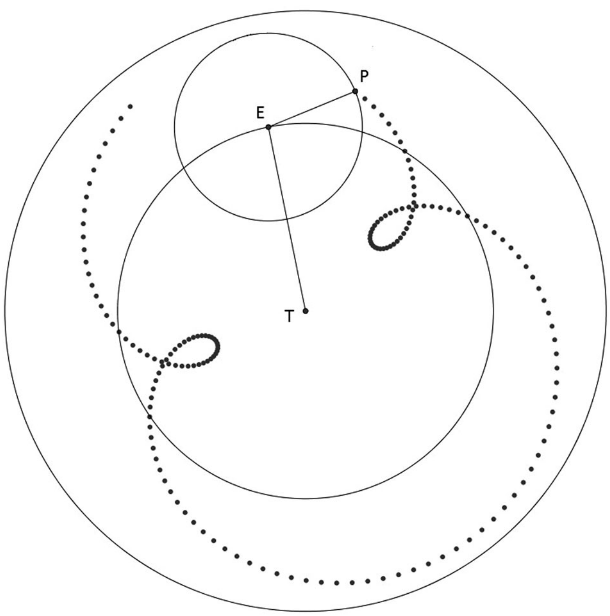

Figure 2. Simple epicyclic model, viewed from the south ecliptic pole.

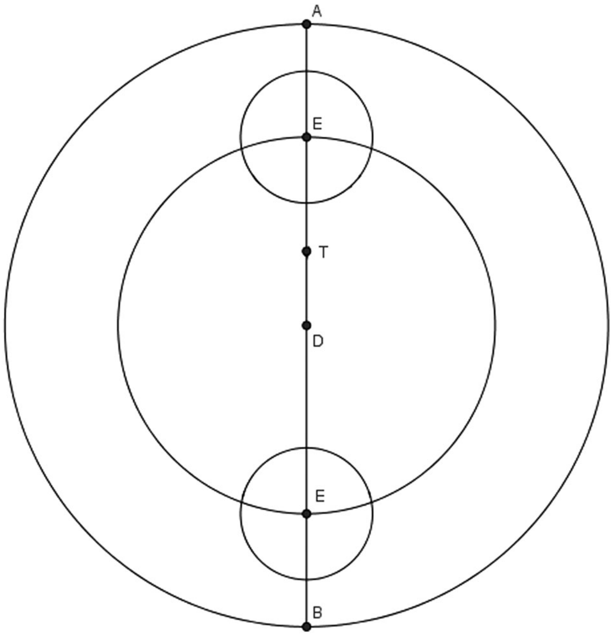

Figure 3. Eccentric solution for the problems with the variation of the epicycle's size and angular velocity.

vary its angular velocity, depending on the longitude its centre had. While there was a longitude *A* in which the epicycle seemed the fastest, when located in its opposite longitude *B* on the Zodiac, it seemed the slowest. The solution was very clever: Greek astronomers supposed that the centre of the deferent did not coincide with Earth *T* (Figure 3), but that it was instead located on another point *D* on the line that joined the two longitudes in question, *A* and *B*.

While the epicycle's angular velocity is always the same with respect to the centre of the deferent, the fact that the Earth is closest to the point of the deferent with longitude *A* ensures that when the epicycle is located there, it is seen as having a greater angular velocity. And the opposite happens at longitude *B*. Later investigations led to more modifications, mainly the existence of a centre of uniform angular velocity that is different from point *D*, a point later named *equant point*. Those aspects, nevertheless, are not relevant to this article and will not be mentioned.

3. The Derivation of the Longitudes for the Apogee and Perigee of Venus's Deferent

One of the problems any astronomer had regarding such a planetary model was the exact determination of the longitudes *A* and *B* or, in other words, the longitudes of the perigee—the point where the deferent is closest to Earth—and apogee—the point where it is farthest—of the deferent. In Swerdlow (1989, 36–43) there is a detailed discussion of the way through which Ptolemy could have arrived at these values in the case of the planet Venus. As Swerdlow says, Ptolemy gives a minute description of the construction of the model. The argument the astronomer presents, though, allows the reader also to peek at the research behind the compact reasoning given in the *Almagest*.

Swerdlow's thesis is that there were at least three stages in Ptolemy's research on Venus: a preliminary investigation where he assumed certain hypotheses—the existence of an epicycle, of an eccentricity of the deferent, etc.—and obtained, from observations, a rough estimate of the parameters of the model; a selection and adjustment of observations which was guided by the results of the previous step; and the derivation of accurate parameters from said selected and adjusted observations. The first stage is completely absent in the account given in the *Almagest*, and Ptolemy—according to Swerdlow's reconstruction—only partially presents the second stage, giving the selected and adjusted observations without explaining how he got them. Ptolemy's minute description corresponds to Swerdlow's third stage. Next, I will explain how these three steps might have developed in Ptolemy's research on Venus.

Ptolemy tells us in his *Almagest* that he calculated the position of the apogee and perigee of the deferent by a very simple method (Ptolemy 1984, 469–470), which uses observations of maximum elongations. The elongation of the planet depends on two factors: first the equation of anomaly, that is, the effect of the position of the planet on the epicycle. In Figure 4, when Venus is located at P_1, it shows a greater elongation than when located at P_2.

Second, a planet's elongation depends on the distance between the observer and the epicycle. And this distance depends in turn on the equation of centre, that is, the effect of the position of the centre of the epicycle on the deferent. In Figure 5, although in both cases the planet is located at the exact same point on the epicycle, when the centre of the epicycle is at E_1, the elongation is smaller than when it is at E_2.

For his method, Ptolemy looked for two equal and opposite maximum elongations (Figure 6) with the same value.

Given the fact that both the value of the maximum elongation and the influence of the equation of anomaly is the same in both cases, the influence of the equation of centre must also be the same. This is possible only if, in both cases, the centre of the epicycle is equally distanced from apogee and perigee. So, the middle points between the positions of the

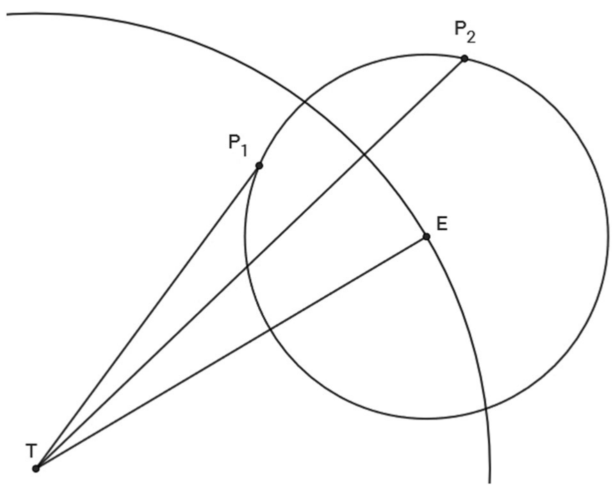

Figure 4. Influence of the equation of anomaly on the elongation of the planet.

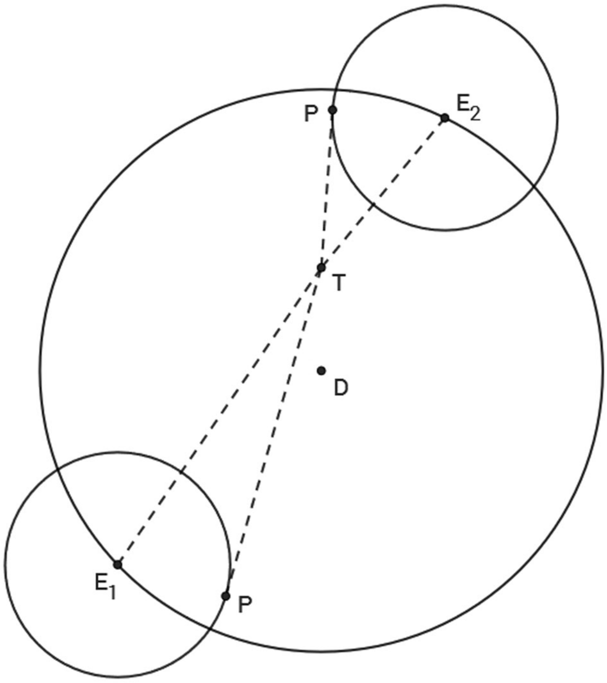

Figure 5. Influence of the equation of centre on the elongation of the planet.

centre of the epicycle are the apogee and perigee of the deferent: according to Ptolemy, the longitudes are 55° for the apogee and 235° for the perigee.[8]

Although in theory the method does not present any problem, in practice it is very difficult to determine the exact moment of maximum elongation for Venus. When the planet

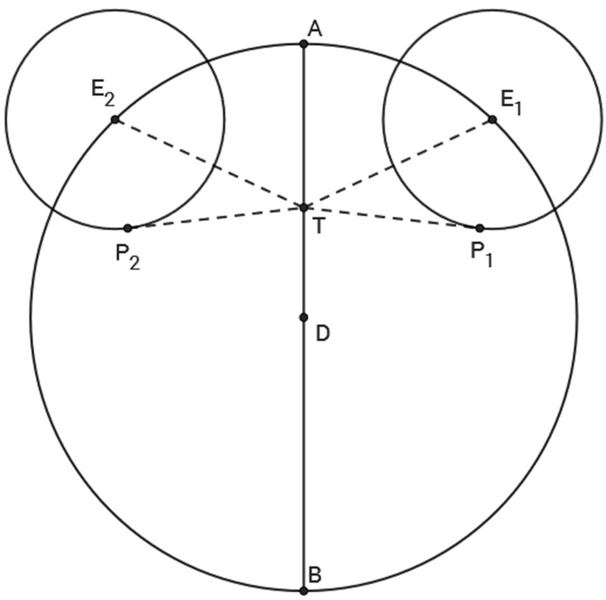

Figure 6. Demonstration of the position of apogee and perigee of Venus's deferent.

approaches the tangent point on the epicycle, its elongation varies very slowly in time, about 0.5° in the six previous and following days (Swerdlow 1989, 35). In this time, the centre of the epicycle moves about 12° on the deferent. The best thing Ptolemy must have had was a detailed record of the longitudes of both the planet and the centre of the epicycle in the time surrounding the moment of exact maximum elongation. Considering the rate of movement we mentioned earlier, this interval might have been probably a month before and after that moment. Thus, at this point, there are two options: on the one hand, the selection of any given moment as the moment of maximum elongation could be seen as an arbitrary one. On the other hand, it could be the result of a previous analysis, omitted from the *Almagest*, perhaps for didactic reasons. It is obvious that Ptolemy would not have chosen arbitrary values, which would have probably ended in grossly incorrect results. He must have had some criterion to choose a specific set of observations as the data on which to base his calculations. Let us see how he could have done it.

After laying out in detail the above argument, Ptolemy says that he has corroborated his results via another method: by the addition of opposite maximum elongations, with the centre of the deferent in the same parts of the ecliptic. He says: 'nowhere on the ecliptic do we find the sum of the greatest elongations from the mean on both sides to be less than the sum of both in Taurus, or greater than the sum of both in Scorpio' (Ptolemy 1984, 471). On the one part, this method does not need precise observations. If the astronomer knows that the planet is near its point of maximum elongation, he can choose two observations with opposite 'maximum' elongations and simply make the addition. On the other part, it does not give accurate longitudes, but merely aproximate locations. For this, Ptolemy could use any of the pairs at his disposal in the period between the years 127 and 141, when he conducted his research: he had two pairs in Taurus, two in Cancer, two in Libra, two in Sagittarius, and two in Aquarius.

In Figure 7, we can see the locations of the pairs of maximum elongations on the ecliptic for Ptolemy's period. Although Ptolemy did not know the exact location of the centre of the epicycle for the moment of maximum elongation simply because he did not know when it ocurred, he at least knew that its approximate longitude was in these areas. Under the name of the sign, the number indicates where the sum of maximum elongations were bigger, and where they were smaller, 1 being the smallest and 5 the biggest. From this information, it becomes obvious that the apogee must have a longitude between points W and X, and the perigee between points Y and Z.

Once Ptolemy had located the approximate longitudes of the apogee and perigee, he could use these same records in another way. He now had the selection and adjustment criteria he needed. If the apogee of the deferent was between W and X, then he knew that the equal and opposite 'maximum' elongations he was looking for in the first method (Figure 6) had to have a symmetry line—from now on, we shall call it *apsidal line*—that crossed the ecliptic between W and X on that side and between Y and Z on the opposite side. This posed a serious restriction on the undetermination that resulted from the fact that it is difficult to determine exact moment of maximum elongation. Given the 20 maximum elongations which Ptolemy could have observed, there are very few points where he could locate the centre of the epicycle while still having equal and opposite maximum elongations—whether they were exact or just approximate—and at the same time have an apsidal line that passed through the above-mentioned longitude ranges. In fact, there are only two pairs of this kind, one on the side of the apogee and

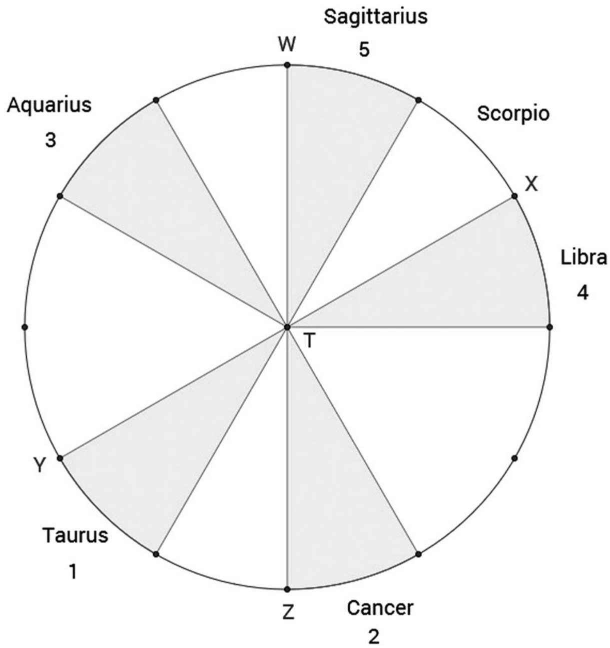

Figure 7. Locations of the pairs of maximum elongations in the Zodiac and their relative values.

one on the side of the perigee (Swerdlow 1989, 39). The results of this last step were the above-mentioned values of 55° and 235°. These observations are presented in the *Almagest* as actual observations of maximum elongations, and it is on top of these 'observational' data that Ptolemy builds his demonstration of the position of Venus's apsidal line.

It should be noted that Swerdlow (1989, 43) himself points out that the actual calculation of the position might have followed another path, at least in this third step. Swerdlow says that having obtained the approximate position of the line through the first step, Ptolemy then adjusted the other relevant 'maximum elongations' in order to allow for an apsidal line 90° of the observations in Cancer. In the *Almagest*, these observations are used not to determine the longitude of apogee and perigee—which at that point have already been located—but to find the correct value of the eccentricity of the centre of uniform motion, that is, the equant point (Ptolemy 1984, 472ff.). Swerdlow's claim is that, in fact, Ptolemy used those observations as the starting point from which to finely adjust the crude position of the apsidal line he had found from the 'sum of greatest elongations'.

This description of the order in which Ptolemy could have carried out his research has the advantage of explaining the way in which he used the astronomical records that he had at his disposal in a most ingenious way, avoiding, as much as he could, the imprecisions inherent to his observations. This kind of procedure was not an isolated fact in Ptolemy's scientific career. Throughout the *Almagest*, one can suspect similar applications of the same principle: to use rough observations to make preliminary determinations of the hypotheses and parameters and then to apply those values to select and adjust the observations—sometimes the same ones used in the first place—needed to conduct more precise and exact calculations. But Ptolemy was not naive about what he was doing. The way he intertwined theory and observation in the construction of his astronomical models has its parallels in other areas he worked on, but also in his theories about human perception, science in general, and, ultimately, how philosophers could attain accurate knowledge about the world.

4. Ptolemy's *Harmonics*

Claudius Ptolemy is one of the most famous scientists in history. His well-deserved fame is due to his astronomical works, which lasted for about one and a half millennia. His scientific interests had, nevertheless, a far wider range. Astrology, geography, optics, musical theory, all of these were among his areas of research. As Olaf Pedersen, one of the contemporary experts in his work, has said, 'It is almost as if he had intended to compile a huge "Encyclopedia of Applied Mathematics" as a counterpart to already existing comprehensive expositions of pure mathematics' (Pedersen 2010, 13).

Ptolemy's investigation on musical theory, his *Harmonics*, is one of his more detailed works. There, he deals mainly with harmonical combinations and the mathematical relations that rule them. His approach to such a subject is of great interest to our problems with observational adjustment in planetary observations. Due to its richness, I will quote most of the first paragraph:

> Harmonic knowledge is the power that grasps the distinctions related to high and low pitch in sounds: sound is a modification of air that has been struck (this is the first and most fundamental of things heard): and the criteria of *harmonia* are hearing and reason, not however in the same way. Rather, hearing is concerned with the matter and the modification, reason with the form and the cause, since it is in general characteristic of the senses to adopt from elsewhere what is accurate, and of reason to adopt from elsewhere what is approximate, and to discover what is accurate. For since matter is determined and bounded only by form, and modifications only by the causes of movements, and since of these the former [i.e. matter and modifications] belong to sense perception, the latter to reason, it follows naturally that the apprehensions of the senses are determined and bounded by those of reason, first submitting to them the distinctions that they have grasped in rough outline—at least in the case of the things that can be detected through sensation—and being guided by them towards distinctions that are accurate and accepted. … Thus just a circle constructed by eye alone often appears to be accurate, until the circle formed by means of reason brings the eye to a recognition of the one that is really accurate, so if some specified difference between sounds is constructed by hearing alone, it will commonly seem at first to be neither less nor more than what is proper: but when there is tuned against it the one that is constructed according to its proper ratio, it will often be proved not to be so. (Ptolemy 1989a, 276–277)

Ptolemy's ideas on the way musical harmonies could and should be achieved are part of a larger debate that can be traced back to several centuries before the life of the Alexandrian astronomer. Aristoxenus (4th century BCE) is the paradigmatic example of one of the two schools into which, broadly speaking, that discussion can be divided. His approach to the study of musical harmonies has a somewhat empiricist flavour, though not void of nuances. For him, musical data 'are described in autonomously musical terms (conceived as representing the phenomena in the way that perception grasps them … they are neither described nor explained by reference to concepts drawn from mathematics and physics' (Barker 1989, 5). The other group can be named after its founder, about whom we know very little: Pythagoras. The Pythagorean school conceived the study of harmonics as part of a much greater entreprise, the study of the ways in which number is at the foundation of all order in the world. For them, 'The order found in music is a mathematical order; the principles of the coherence of a coordinated harmonic system are mathematical principles' (Barker 1989, 6).

Ptolemy's approach can be identified more with the latter, though his regard for real musical practice and the importance he gives to the properly guided construction of instruments shows us that his doctrine is more complex. Far from the more extreme positions, where hearing had no value in the determination of musical harmonies (Barker 1989, 240), Ptolemy expressely states that perception plays an essential role in this science. The process of attaining scientific knowledge about musical harmonies is one in which reason and perception each play a determined role. The example he gives is, curiously, not a musical but a geometrical one. There is a first stage in which a person sees a circle, though not one that accords perfectly with the definition of circle. This perception, nevertheless, gives him a rough outline of *what it is to be a circle*. It is here that reason comes into play. By applying his rational faculty to that which his sense has perceived, the person acquires a more precise and exact knowledge of *what it is to be a circle* and compares this to the circle he has first seen, and through which he originally came in contact with the concept of circularity. By using that which his reason has obtained, which is more precise, as an adjusting criterion, the person can correct the original circle. It is important to stress out that it is not only the sensible circle that is rough and imprecise, but also the sensible perception our sight has of it. Matter and the perceptual powers that put us in contact with it share this common feature. The latter is, nevertheless, the one that is important here. This inherent imprecision of the senses is the reason why the hypothetical geometer cannot tell when a circle is imperfect by sight alone. It is only after the action of reason that he is equipped to judge about what he sees. At this stage, though, one can hardly say that it is sight that which is doing the judging.

It is not hard to see how this process could take place within musical practice and theory. A musician would tune, every day, two strings of the instrument in such a way that they would sound harmonic. Then, one day, using his reason, he would devise a way of measuring the proportions between the length of the strings accurately, and he would find that the proportion is always close to 2:1. I say *close* not only because, at this point, the tuning of the strings could have only been made by someone who did not know the precise proportion which would give a proper tuning, but also—and this is central to the analogy with astronomy—because the act of measuring cannot deliver an exact reflection of the true proportions of the real strings. Even if the strings were perfectly tuned, no measuring procedure would detect this perfect tuning with absolute certainty.

The aesthetic aspect of our musician's intelligence would then suggest to him that this is in fact the precise proportion that causes that combination to be harmonic, and he would thus be able to tune, using instruments specially devised for that task, every musical instrument according to the true harmonic principle.

5. Finding a Criterion of Truth

While the *Almagest* is, by far, Ptolemy's most famous work, among his manuscripts, there is a little treatise or essay, *On the Kriterion and Hegemonikon*, which has mostly fallen into oblivion. Its first translation into a modern language appeared only a little more than three decades ago (Manuli 1981), and little has been done, when compared to the amount of scholarship that has been devoted to its astronomical companion, on this philosophical work. It is divided into two parts of unequal size, the second of which deals with the way in which the body and the soul are related to each other. It is, nevertheless, the first and lengthier part that is of interest to us. There, Ptolemy declares his doctrine about the way the sense and reason interact in order to obtain accurate knowledge about the world.

Ptolemy's philosophical background was eclectic, and there has been much debate about where his sources lie: some see the influence of late Peripatetic doctrines, some argue in favour of the Middle Stoa, while others detect the influence of Middle Platonism, Epicurus and Academic sceptics. His greatest influence is probably Aristotle (Long 1989, 163). One of the reasons for the difficulty of identifying the philosophical background of this Ptolemaic work is that the author gives no clue about which school of thought he identifies himself with. He declares no intellectual genealogy, nor does he discuss any contrary doctrine. Unlike his astronomical works, where many times he cites his sources or explicitly declares his discontent with some predecessor, here Ptolemy limits himself to stating the way in which he thinks human knowing capabilities work.

To explain his doctrine, Ptolemy makes use of an analogy between the application of a criterion of truth and a lawcourt. The ideas he puts forward in this work, though presented in this particular manner, are concordant with what we saw in his studies about musical harmonies:

> thought possesses the fullest criterion of judgement covering objects, sense data, and even perception itself. Thought may have needed sense perception the first time it saw things of any given type. … We must not dismiss sense perception as contributing little or nothing to the appreciation of reality. Nor on the other hand must we prefer its conclusion to that of thought. Instead, we must allow sense perception and thought each to play its proper function, and employ each for the purposes which it is naturally able to fulfil infallibly. Sense perception we must employ for information about the affections which undergoes. It reports the truth about these and gives an honest answer if we confine ourselves to the question of how it has been affected, whereas it does sometimes make a false report about the nature of the object that has given rise to the affection. Thought, however, is to be employed for making judgements both about the affections themselves, and about the objects that have given rise to them. It links what is transmitted by the senses with its memories of the properties of each object, and in the ensuing process of assessment distinguishes things which have produced their appropriate affection in the senses from things which have given rise to an inappropriate affection. (Ptolemy 1989b, 199–201)

Knowledge of reality is gathered with the necessary help of the senses. The classical dictum, *nihil est in intellectu, quod prius non fuerit in sensu*, has in Ptolemy one of its defenders. Nevertheless, it is not with senses alone that man can attain accurate knowledge. Sensorial perceptions tell the truth only about the way in which external objects affect them. This effect, however, does not always conform to the object that causes it in the same way. Sensorial effects are neither precise nor constant. They are subjected to the variability natural to matter. Because of this, there is the need of reason to judge the material delivered by the senses in order to get accurate knowledge. In its proper ways, with its own tools, reason can distinguish what is truthful and what is not in sensorial perception. Thus, it can deliver its accurate results to the senses, which 'adopt from elsewhere what is accurate'. In this way, our sensorial perception of reality can, if it is scrutinised by our rational faculty—as it is the case with science—become more accurate and give, in the end, an account of the world that is trustworthy.

6. Ptolemy's Epistemology and Data Adjustment

At this point, we are ready to evaluate the procedure that Ptolemy followed, if Swerdlow's hypothesis is correct, in the case of the apogee and perigee of Venus's deferent. We assumed that, at the start, Ptolemy simply had a table of astronomical observations and calculations. In it, there were the longitudes of the planet in the days surrounding maximum elongation, and the longitude of the mean sun, whose position in the ecliptic was close with the centre of the epicycle and could be derived from his solar theory. These observations constitute the empirical basis of the entire work and are the analogue of the effects the senses suffer under the influence of external objects. As such, they are not precise, accurate: while Venus is mostly visible before, during and after the moment of maximum elongation, there is no way to sensibly distinguish that precise moment from those close to it. Astronomical observations had a margin of error, which prevented observers from deciding whether the planet was exactly at maximum elongation or very close to it.

I will show how the procedure Ptolemy could have followed to find the longitude of the apsidal line, which was explained above, is an analogical application of the procedures Ptolemy thought regarding musical theory and the finding of harmonical proportions, and the epistemological doctrines as are extant in his *On the Kriterion*.

With respect to the case of musical harmonies, we can distinguish five phases in the interplay between senses and reason: (a) a sensible perception of a set of notes which sound beautiful when played together; (b) a measurement of the lengths of the strings that produce those notes; (c) the arriving at an approximate harmonic proportion; (d) the determination of the exact harmonic proportion; and (e) the correction of the lengths of the strings according to that exact proportion.

Can we find analogue steps in the case of the longitude of Venus's apsidal line? Given the complexity of the subject, it is expected that the interplay between theory and observation in the case of astronomy would be more complex than in the case of musical harmonies. Nevertheless, both disciplines are parts of mathematics and thus prone to, at least to some extent, analogous research procedures. Both vision and hearing, which are their respective sensorial basis, share the common feature of being able to come in contact with the mathematical aspects of their objects. They are, of all the senses, the 'most closely tied to the ruling principle, and … are the only senses that asses their objects not only by the standard of pleasure but also, much more importantly, by that of beauty' (Ptolemy 1989a, 372). Astronomy and harmonics 'are as it were cousins, born of the sisters, sight and hearing, and brought up by arithmetic and geometry as children most closely related in their stock' (Ptolemy 1989a, 373).

In music, the entire effort to find an underlying mathematical proportion begins with the fact that there is an aesthetic experience of beauty. In the Pythagorean perspective, sensible beauty must be the effect of a deeper mathematical structure, which causes and explains such beauty. In that sense, phase (a) of harmonic research constitutes not only the starting point of the research, but also its psychological motivation: it is because I experience beauty that I assume there is some mathematical proportion to be found. In the case of planetary movements, though, it is doubtful that such aesthetic experience could have a similar role.

The cyclic character of celestial phenomena has had, of course, a fundamental influence in the very birth of astronomy as a science. Science can be done only about things that are stable, permanent, and the sky, and the events that happen in it, are perhaps the greatest and most prominent examples of this stability in all Nature.

At the time Ptolemy worked, the motivation to look for mathematical structures that explained planetary movements had already assumed a technical aspect: physics, metaphysics and the cosmology which they founded—and was in turn reshaped by them—all declared that the movements proper to celestial bodies had to be circular in shape and uniform in angular velocity. Ptolemy, or any other astronomer, knew that behind the apparent movements of the planets, there was a mathematical structure not because he experienced beauty when he looked at the way they moved, but because all his cosmology told him it had to be so.

The second phase described above, (b), can be compared to the measurement of successive elongations of the planet, and the determination of the period of time in which it seemed to be at maximum elongation. These observations were, by their own nature, and as Ptolemy knew very well, merely approximate: he had a list of observations where Venus seemed to be, in all of them, at maximum elongation, while he knew that this could not be true for all of them. To carry out his method of equal and opposite maximum elongations, Ptolemy needed to determine the precise moment of maximum elongation. This was, as we saw, beyond his observational capabilities. Observations alone could not do the job. Like the musician who has the approximate lengths of the strings and wants to know the true harmonic proportion, he needed some way to go from approximate maximum elongations to the true position of the apsidal line.

Phase (c) involves, in the case of music, a simple comparison of the measured lengths. Supposing the musical instrument had only two strings, the procedure to determine the approximate harmonical proportions involved only the simple mathematical action of constructing a ratio between them. If string A had a length of 4.2 and string B one of 1.9, then the harmonic proportion would be, approximately, 2.1/0.95.

In the case of Venus, as I said, the path was more complex. The method of equal and opposite maximum elongations did not allow one to determine even an approximate location of the apsidal line. As Swerdlow argues, Ptolemy must have used another method, probably the one with two elongations, one on each side of the epicycle, and with the centre of the epicycle at the same point—or at least, in the same region—of the ecliptic. This allowed him to obtain not only an approximate location for the apsidal line, but also the distinction between apogee and perigee of the deferent. So, sight alone, in spite of having the aid of specifically designed instruments of observation, and within a theoretical framework such as the one briefly described in section 2, is not sufficient to reach even an approximate position of the apsidal line. Because of the fact that the apparent movement produced by the epicyclic anomaly near maximum elongations is really slow and given the approximate character of the data delivered by the senses—even by the one that was brought up by geometry itself!—reason cannot draw any valuable accurate conclusions from them. To do this, theory must come into play: any basic analysis of the geometrical properties of an eccentric model shows that the angular size of the epicycle—the 'sum of maximum elongations' of which Ptolemy speaks—must be greatest and smallest at perigee and apogee, respectively. So Ptolemy could have found that, given the observed elongations at Taurus and Sagittarius, the position of the apsidal line had to be near those ecliptic locations.

There is here, of course, an important difference with the case of harmonics. While in that discipline the approximate harmonic proportion is obtained through simple observations—the measurement of the length of the strings—and a simple mathematical operations such as division, in the case of astronomy things are much more complicated. On the one hand, observations themselves are made within a heavy theoretical framework—epicyclic astronomy—which itself is responsible for the characterisation of the observation itself as the maximum elongation with respect to the centre of the epicycle. On the other, and much more importantly, there is no way to achieve even an approximate value of the parameter under consideration without the aid of theoretical considerations, and the help of a different method founded on them. The result Ptolemy could have obtained through this complex interplay between theory and observation is not, however, a final result.

The fourth phase, (d), is the one in which the musician was able to go from an approximate harmonic proportion to the exact one. In the case of music, one could consider that it involves an aesthetic aspect of reason, that, when it sees that all the approximate harmonic proportions derived from the—also approximate—measurements of the strings, are around 2:1, then it concludes that that must be the true proportion proper to that particular harmonic relation. In the case of Venus, this step is made by the application of the method of equal and opposite 'maximum' elongations, only that now the astronomer has the restriction of having to discard any pair of observations that do not allow the apsidal line to be within the limits that the previous step had set. This, as we saw, could have allowed Ptolemy to retain only two pairs of 'maximum' elongations and give him the final value for the position of the apsidal line. It is important to note that while in the case of harmonics, the correction or determination of the value obtained by the senses is done via a fairly straightforward application of some mathematical abstractions, in the case of astronomy the procedure is not so simple. In fact, complex geometrical models are the basis of the corrections, adjustments and determinations that observational data go through. Also, as we saw, these data are not so 'raw' as in the case of music: they are data that have already gone through an also complex process of selection that also was founded on these models.

Once the musician knows which the proper harmonic proportion is, he is able to adjust the strings of his instrument so that they are properly tuned: phase (e). Does anything similar happen in the case of Venus? As I said, in the *Almagest* Ptolemy presents a very clear path towards the longitudes of the apogee and perigee of the deferent, that is, the position of the apsidal line. In it, he presents a pair of equal and opposite maximum elongations and calculates the middle points between them: those are the apogee and perigee. The problem was, of course, that the determination of the moment or longitude of maximum opposition was beyond his observational capabilities. Can the procedure that Swerdlow describes, with one method giving an approximate value of the position of the apsidal line, and the second giving the precise one, allow him to select the correct moment of maximum elongation among his observational records? No, it cannot. But it can tell him which observations surrounding the moment of maximum elongation can be selected to calculate the position of the apsidal line and, in that way, be presented as if they were true maximum elongations. The influence of the theoretical analysis over observational data is, in this case, of the utmost importance: not only does it tell the astronomer which observations among many of the same type are useful to calculate the parameter he was looking for, serving as a selection criterion, but it also tells the teacher of astronomy, who is composing an astronomical text, which observations he can present as if they were true maximum elongations in order to make it more clear and straightforward.

7. The Many Faces of Ptolemy's Handling of Observations

The previous analysis has shown that the investigation of the position of Venus's apsidal line could be understood as a case in which Ptolemy's intertwining of theory and observation can be closely related to his doctrine of the relationship between senses and reason, as it is extant in some passages of his *Harmonics* and *On the Kriterion*. There is, nevertheless, an important question that can be raised when presented with this analysis: can Ptolemy's handling of observational data *always* be reduced to this procedure? I think that the text of the *Almagest* gives us sufficient material to give a definitive answer to that question: no.

To support this, I will briefly refer to two more cases where Ptolemy obviously adjusted observational data in order to obtain some precise result. In neither of those cases, can we find an approach equal to the one I described above. Even further, I will argue that, at least in one case, the guiding principle cannot even be said to be founded on theory.

In *Almagest*, book V, chapter 13 (Ptolemy 1984, 247ff.), the astronomer sets out to find the distance from the Earth to the Moon in Earth radii and from that value he calculates the distance to the Sun in the same unit. He obtains a final distance for the Sun of 1210^r. His geometrical argument is very complex and is ultimately founded on very precise observations of two lunar eclipses. It has been shown by Carman (2009) that is it indeed a calculation that is extremely sensitive to the smallest variations in the values given to the central instant of the eclipse, that is, the moment of maximum occultation, and to the magnitude of the eclipse, that is, the proportion of the obscured part of the Moon to the whole Moon at maximum occultation. A few minutes of difference or a little change in magnitude would imply a very different ultimate value for the distance of the Sun: a difference of 8 minutes in one eclipse would give a distance of 1024.03^r, almost 200 radii of difference. If, in the same eclipse, we assume that the magnitude was 2.8 instead of 3, then the distance would be 1938.5^r.

In the *Planetary Hypotheses* (Ptolemy 1967), a little simplified astronomical treatise that Ptolemy wrote after the *Almagest* was finished, Ptolemy arrived at the same value for the distance of the Sun: 1210^r. He did it using a different method from the one of the *Almagest*, one that was founded on the radii of the deferents and epicycles of his models, the distance to the Moon in Earth radii, and a philosophical principle that said that there could be no void space in the cosmos. He starts with the maximum distance to the Moon in Earth radii, and then proposes, because of that *horror vacui* I mentioned, that it coincided with the minimum distance of the next planet, Mercury. Given that minimum distance in Earth radii, and the proportional sizes of Mercury's deferent and epicycle, he can calculate the planet's maximum distance, also in Earth radii. Through the continuation of that argument, he arrives at a mean distance of the Sun of 1210^r.

The fact that so different methods should render identical results is striking. This is even more so, if we consider that the value is so far from the true one. Carman's argument is that, *before* the *Almagest* was written, Ptolemy had already calculated the distance to the Sun following the method explained in the *Planetary Hypotheses*. He already had all the material he needed: the distance to the Moon could be calculated by parallax, and the proportions between deferents and epicycles are extant in an earlier Ptolemaic work, the *Canobic Inscription* (Hamilton, Swerdlow, and Toomer 1987). Knowing that the method of the lunar eclipses was extremely sensitive to minute changes in the input values, he chose the ones he needed to obtain the result he already had.

Thus, the geometrical argument he presents in the *Almagest* is impracticable, but geometrically sensible. In that sense, it is a case similar to the one I presented in the previous section. This one, nevertheless, shows a very different way in which theory is used to adjust observations: in this case the observations used to carry out a certain method—the one with the two lunar eclipses—are corrected using a very different theory—one with a certain metaphysical flavour to it: the one of the coincidence of maximum and minimum distances.

Another example of Ptolemy's adjustment or correction of observational data where the correcting procedure is, I think, very different from the one described in section 5, is the case of the calculation of the length of the tropical year in *Almagest*, book III, chapter 1. As Ptolemy (1984, 139) himself says, Hipparchus had already calculated a value for the duration of the year, which was $365 + 1/4 - \sim 1/300$ days. Ptolemy's method for calculating the duration of the year is extremely simple: he chooses two equinoxes and a solstice which he says where carefully registered in the past, and then calculates the number of days that passed until two equinoxes and a solstice of his own times. He then divides the total sum by the number of years between the two respective observations, and he obtains a very precise value.

However, if we look at the times Ptolemy states for the three ephemerides he claims to have observed, we find that they could not have been observed in the way Ptolemy says they were: in every one of them we find errors that range from 21 to 36 hours.[9] The result he obtains is, nevertheless, exactly the same as the one given by Hipparchus. It is clear that, in this case, the only criterion Ptolemy used to correct his observations of solar phenomena was the authority of Hipparchus, and his value for the tropical year.

From these examples, we can conclude not only that Ptolemy was very aware of the difficulties of handling observational data—especially old and crude ones—but also that he had a very wide range of tools for dealing with these difficulties, tools that were used in different cases as he saw fit. No doubt, many other procedures for adjusting, selecting or correcting observational data can be found in the study of the Ptolemaic *corpus*. It is my hope that this article is a contribution to that enterprise.

8. Conclusion

In this article I have shown, following Swerdlow's article, a probable path that Ptolemy could have followed to calculate the position of the apsidal line of Venus. Then, I described Ptolemy's epistemological doctrine as it is extant in his *Harmonics* and *On the Kriterion*, describing the five main phases that, according to Ptolemy, constitute the relation between senses and reason, observation and theory. Then, I showed how the procedure described for the case of Venus is an analogical application of that doctrine, by explaining how each step in Swerdlow's path can be compared—though in an approximate way—to the five phases of Ptolemy's epistemology. Finally, I showed two cases where other ways in which Ptolemy handled observational data can be clearly seen.

The case of the investigation to find the longitudes of the apogee and perigee of Venus's deferent is one in which Ptolemy acted, with the particularities proper to the subject at hand, according to his doctrine on human knowledge and the interplay between reason and sensorial perception. I think, though, that to say that he was guided by his epistemology would give a false image of what really happened. It is likely that his scientific practice was not, in this respect, simply a derivation of a philosophical doctrine he held before his work. Rather, it was his practical experience with raw observational data and the demands the Greek astronomical tradition posed on him that were, at least in part, the source of his epistemological elucidations. In time, this relation must have been an iterative one, in which his scientific practice helped him understand the ways in which theory and observation, reason and perception, could work together in the constitution of a solid scientific discipline. Furthermore, his philosophical doctrine, placed within a richer and more ancient philosophical tradition, must have served as the frame in which to carry out his work. In any case, the understanding of these processes in the making of his *Almagest*—processes that are an intricate part not only of Ptolemy's astronomy, but of every scientific work that has some pretence of being original—is a vital part of any attempt to comprehend and correctly assess Ptolemy's contribution to the history and philosophy of science.

Notes

1. Jones presented some of the main contents of that chapter at the IV International Workshop on Epistemology and History of Astronomy, Rio de Janeiro, July 2017.
2. The Zodiac can be precisely determined by reference to the ecliptic line. This line is the one on which the Sun is always located, and through which it performs its apparent yearly motion around the Earth. The band of the Zodiac is the area which is located 10° to the north and south of the ecliptic. All the planets visible to the naked eye, the Moon, and obviously the Sun, move within these limits.
3. From the ancient Greek πλανήτης, *planétes*, wanderer. Although they usually marked a distinction between them, Greek astronomers frequently listed the Sun and Moon among the planets, because they shared this characteristic of not having a sidereal fixed movement.
4. In fact, the fixed stars also show a departure from a simple circular and uniform motion. As it was already known to Hipparchus (2nd century BCE), the celestial sphere performs a second motion, different from its daily rotation and much slower than it, in the direction of the signs. This second movement is a result of what we now call the precession of the equinoxes. Cf. Ptolemy (1984), 327ff.
5. The belt of the Zodiac was defined as the one whose limits were parallel to the ecliptic and located 10° to its north and south. It was divided into 12 parts called signs, of 30° each, which were associated with particular constellations. The longitudinal origin point of the ecliptic and the Zodiac was the vernal equinoctial point, which marked 0° and the beginning of Aries. The direction was determined by the tropical movement of the sun.
6. An example of a previous solution to the same problem is the system of homocentric spheres, famously explained in Aristotle's *Metaphysics* (cf. 1073a15–1074b15). Cf. Mendell (1998), Schiaparelli (1998).
7. In order to produce a retrogradation, a certain relation between the sizes of the circles and their angular velocities has to be met, that is, $\frac{r}{R-r} > \frac{vE}{vP}$, where r is the epicycle's radius, R the deferent's radius, vE the angular velocity of the epicycle on the deferent and vP the

angular velocity of the planet on the epicycle. In his *Almagest*, Ptolemy made a detailed study of this relation (Ptolemy 1984, 555ff.).

8. To know which point is which, Ptolemy goes through a separate discussion, looking at where the epicycle looked bigger and where it looked smaller (Ptolemy 1984, 470–471).
9. Calculated with Alcyone Ephemeris v4.3 (Meeus and Meshier algorithms).

Acknowledgements

The author would like to thank Dr Christián Carman for his many, many suggestions during the preparation of this paper, and Prof. Magdalena Battaglia for her help with the English. The author would also like to thank Dr James McAllister and the two anonymous referees of this journal for their comments and criticisms to previous versions of this paper.

References

Barker, A., ed. 1989. *Greek Musical Writings. Vol. 2, Harmonic and Acoustic Theory*. Cambridge: Cambridge University Press.

Britton, J. P. 1992. *Models and Precision: The Quality of Ptolemy's Observations and Parameters*. New York: Garland.

Carman, C. C. 2009. "Rounding Numbers: Ptolemy's Calculation of the Earth–Sun Distance." *Archive for History of Exact Sciences* 63: 205–242.

Delambre, J. B. [1817] 1965. *Histoire de l'astronomie ancienne*. New York: Johnson Reprint Corporation.

Evans, J. 1998. *The History and Practice of Ancient Astronomy*. New York: Oxford University Press.

Feke, J. 2012. "Mathematizing the Soul: The Development of Ptolemy's Psychological Theory from *On the Kritêrion and Hêgemonikon* to the *Harmonics*." *Studies in History and Philosophy of Science* 43: 585–594.

Feke, J., and A. Jones. 2010. "Ptolemy." In *The Cambridge History of Philosophy in Late Antiquity*, edited by L. P. Gerson, 197–209. Cambridge: Cambridge University Press.

Goldstein, B. R., and A. C. Bowen. 1999. "The Role of Observation in Ptolemy's Lunar Theories." In *Ancient Astronomy and Celestial Divination*, edited by N. M. Swerdlow, 141–159. Cambridge, MA: MIT Press.

Grasshof, G. 1990. *The History of Ptolemy's Star Catalogue*. Berlin: Springer.

Hamilton, N. T., N. M. Swerdlow, and G. J. Toomer. 1987. "The Canobic Inscription: Ptolemy's Earliest Work." In *From Ancient Omens to Statistical Mechanics: Essays on the Exact Sciences Presented to Asger Aaboe*, edited by J. L. Berggren and B. R. Goldstein, 55–73. Copenhagen: University Library.

Jones, A. 2006. "Ptolemy's Ancient Planetary Observations." *Annals of Science* 63: 255–290.

Jones, A. Forthcoming. "Limits of Observation and Pseudoempirical Arguments in Ptolemy's Harmonics and Almagest." In *Instruments—Observations—Theories: Studies in the History of Early Astronomy in Honor of James Evans*, edited by C. C. Carman and A. Jones.

Long, A. A. 1989. "Ptolemy on the Criterion: An Epistemology for the Practising Scientist." In *The Criterion of Truth: Essays Written in Honour of George Kerferd Together with a Text and Translation (with Annotations) of Ptolemy's "On the Kriterion and Hegemonikon"*, edited by P. M. Huby and G. C. Neal, 151–178. Liverpool: Liverpool University Press.

Manuli, P. 1981. "Claudio Tolomeo: Il criterio e il principio." *Rivista critica di storia della filosofia* 36: 64–88.

Mendell, H. 1998. "Reflections on Eudoxus, Callippus and Their Curves: Hippopedes and Callipopedes." *Centaurus* 40: 177–275.

Newton, R. R. 1977. *The Crime of Claudius Ptolemy*. Baltimore, MD: Johns Hopkins University Press.

Pedersen, O. 2010. *A Survey of the Almagest*, edited by A. Jones. New York: Springer.

Ptolemy, C. 1967. "Planetary Hypotheses." Translated by B. R. Goldstein, 5–9. In B. R. Goldstein, "The Arabic Version of Ptolemy's Planetary Hypotheses." *Transactions of the American Philosophical Society* 57: 3–55.

Ptolemy, C. 1984. *Ptolemy's Almagest*, edited by G. J. Toomer. Princeton, NJ: Princeton University Press.

Ptolemy, C. 1989a. "Harmonics." In *Greek Musical Writings. Vol. 2, Harmonic and Acoustic Theory*, edited by A. Barker, 270–391. Cambridge: Cambridge University Press.

Ptolemy, C. 1989b. "On the Kriterion and Hegemonikon." Translated by G. Kerferd and A. A. Long. In *The Criterion of Truth: Essays Written in Honour of George Kerferd Together with a Text and Translation (with Annotations) of Ptolemy's "On the Kriterion and Hegemonikon"*, edited by P. M. Huby and G. C. Neal, 178–230. Liverpool: Liverpool University Press.

Schiaparelli, G. V. 1998. *Scritti sulla storia della astronomia antica. Parte prima: Scritti editi*. 2 vols. Milan: Mimesis.

Schiefsky, M. J. 2014. "The Epistemology of Ptolemy's *On the Criterion*." In *Strategies of Argument: Essays in Ancient Ethics, Epistemology, and Logic*, edited by M.-K. Lee, 301–331. Oxford: Oxford University Press.

Swerdlow, N. M. 1979. "Ptolemy on Trial." *American Scholar* 48: 523–531.

Swerdlow, N. M. 1989. "Ptolemy's Theory of the Inferior Planets." *Journal for the History of Astronomy* 20: 29–60.

Swerdlow, N. M. 2004. "The Empirical Foundations of Ptolemy's Planetary Theory." *Journal for the History of Astronomy* 35: 249–271.