

The first Copernican was Copernicus: the difference between Pre-Copernican and Copernican heliocentrism

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Abstract It is well known that heliocentrism was proposed in ancient times, at least by Aristarchus of Samos. Given that ancient astronomers were perfectly capable of understanding the great advantages of heliocentrism over geocentrism—i.e., to offer a non-*ad hoc* explanation of the retrograde motion of the planets and to order unequivocally all the planets while even allowing one to know their relative distances—it seems difficult to explain why heliocentrism did not triumph over geocentrism or even compete significantly with it before Copernicus. Usually, scholars refer to explanations of sociological character. In this paper, I offer a different explanation: that the pre-Copernican heliocentrism was essentially different from the Copernican heliocentrism, in such a way that the adduced advantages of heliocentrism can only be attributed to Copernican heliocentrism, but not to pre-Copernican heliocentrism proposals.

1 Introduction

It is universally accepted that heliocentrism was intermittently asserted in ancient and medieval times, starting at least during the third century B.C. with Aristarchus of Samos. Nevertheless, we must wait until a century after Copernicus to see heliocentrism accepted by the great majority of the scientific community. One needs to ask,

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therefore, why heliocentrism took so long to be accepted, or, better, why the heliocentric system was accepted in only one century in the early modern period, but was not during the previous eighteen centuries.

This question is not easy to answer because the strictly physical–astronomical reasons for accepting and rejecting heliocentrism were the same in Aristarchus’s and Copernicus’s times. Among the main reasons for accepting heliocentrism, the two that Copernicus most insisted on are: (a) that the system allowed one to calculate in a non-arbitrary manner the distances and order of the planets, and (b) that it explained in a better way the retrograde motion of the planets and other synodic phenomena (such as the limited elongation of the inner planets), without recurring to a *post hoc* explanation, as in the case of the epicycles introduced in the geocentric system. Among the main physical–astronomical objections that heliocentrism faced, the two most important were: (c) the non-observed physical apparent consequences that the movement of the Earth would imply, and (d) the non-observed parallax in the stars and other related phenomena followed by the change of distance from the center of the sphere of the fixed stars that the annual revolution of the Earth would produce.

But ancient astronomers such as Aristarchus, Hipparchus or Ptolemy would have understood advantages (a) and (b).¹ Moreover, the physical objections (c) to heliocentrism did not come to light for the first time in the early modern epoch, but existed since ancient times. It is enough to refer to book I.7 of the *Almagest* (Toomer 1998: 41–43) where Ptolemy enumerates and discusses the arguments against the rotation of the Earth based on physical grounds. There is also suggestive textual evidence that Aristarchus himself had in mind objection (d) relating to annual parallax. Archimedes mentioned Aristarchus’s heliocentrism in his *Sand Reckoner* (Heath 1897: 222), in what constitutes the oldest and most trustworthy testimony of Aristarchus’s system. From this passage, we can safely infer that Aristarchus had in mind the annual parallax objection and answered to it in exactly the same way Copernicus did in the fourth postulate of his *Commentariolus* (Swerdlow 1973: 436): if we accept that the proportion between the Sun’s orbit and the sphere of the fixed stars is the same as the proportion between the Earth and the Sun’s orbit, then we should not be worried by annual parallax.

Given, then, that the main internal reasons for accepting and rejecting heliocentrism were the same at Aristarchus’s and Copernicus’s times, most explanations of the different receptions of heliocentrism must appeal to external reasons, such as the status of crisis that the geocentric paradigm had in Copernicus’s, but not in Aristarchus’s times (Kuhn 1962, 75–76); the great authority that geocentric astronomers like Hipparchus (Heath 1913: 308) or geocentric philosophers like Plato and Aristotle had (Erhardt and Erhardt-Siebold 1942, 595, n. 45; North 2008: 86); or even the good luck that Copernicus had “to be born not only at a time when science was beginning to reach, so to say, a critical mass, but also at a time when scientific works were beginning to be printed” (Gingerich 1985: 41).²

¹ This is true for Aristarchus even if he lived before the development of the epicycle and deferent model, for these advantages arise even if we compare heliocentrism with Eudoxus’s system.

² There are, nevertheless, some scholars who appeal to other internal reasons. Stahl (Stahl 1945: 328), for example, says that in ancient times Aristarchus’s heliocentrism was not accepted precisely for the

I do not intend to discuss here each of these explanations. Rather, I hope to offer a very different answer: I will suggest that the main reason for the failure of Aristarchus's heliocentrism, or any other pre-Copernican heliocentrism, is not an external reason, but, rather, an internal one. I will argue that pre-Copernican heliocentrism has disadvantages (c) and (d), but does not imply advantages (a) and (b). This is the main reason that explains at the same time the failure of Aristarchus and the success of Copernicus.

Copernican heliocentrism is a complex system that is defined by many hypotheses. It seems natural to characterize pre-Copernican heliocentrism by the same set of hypotheses. I will show, however, that there are no reasons to do so. There is one particular hypothesis that Copernicus affirmed that is the main element responsible for his success, and, I think, cannot be attributed to pre-Copernican heliocentrism. In the following pages, I will first describe the main hypotheses of Copernican heliocentrism and show the role that each one of these plays in the reasons for accepting and rejecting heliocentrism. Then, I will argue that there are no reasons to attribute one of these hypotheses to pre-Copernican heliocentrism. I will revisit some historical testimonies of pre-Copernican heliocentrism to demonstrate this suggestion. Finally, I will draw some consequences related to the originality of Copernicus's heliocentrism.

2 The five hypotheses of Copernican heliocentrism

For the purpose of this analysis, I will characterize Copernican heliocentrism by five hypotheses: (1) The Earth rotates around its own axis once every (sidereal) day; (2) the Earth revolves around the Sun, which is fixed close to the center of the Universe, one turn per year; (3) the five planets (Mercury, Venus, Mars, Jupiter and Saturn) revolve around the Sun; (4) the Moon revolves around the Earth; (5) the sphere of fixed stars, with the Sun at its center, remains at rest. For reasons that I will make explicit at a later stage of my argument, hypothesis 3 could be subdivided into: (3a) the inner planets (Mercury and Venus) revolve around the Sun, and (3b) the outer planets (Mars, Jupiter and Saturn) revolve around the Sun.

Geocentrism shares hypothesis 4, so it could not play any role in the advantages or disadvantages of heliocentrism in relation to geocentrism.³ Hypothesis 1 explains the (now apparent) westward diurnal revolution of the stars, Sun, Moon and planets. Hypothesis 1 usually⁴ implies hypothesis 5: if one explains the (apparent) daily motion

Footnote 2 continued

same reason that made Tycho Brahe to offer his geo-heliocentrism: there was no observation of the annual parallax. Dreyer (1953: 148) adds that "the principal reason why the heliocentric idea fell perfectly flat, was the rapid rise of practical astronomy." Heliocentrism would be unable to account for the inequalities without complicating the original simplicity of the model. Of course, one must say against Dreyer that exactly the same situation happened with the geocentric model: it had to be complicated with eccentrics and equant points in order to be able to account for the anomalies.

³ This does not imply that a particular lunar theory proposed in the context of a heliocentric model could not play an important role. Actually, Copernicus's most important astronomical contribution was his lunar model, which superseded that of Ptolemy's. Cfr. Swerdlow and Neugebauer (1984, I: 193–283).

⁴ "Usually" because it is possible to assume at the same time the rotation of the Earth and the sphere of fixed stars. Ptolemy considered this possibility in *Almagest*, I,7 (Toomer 1998: 44–45). Technically,

of the stars by the Earth's rotation, the sphere of fixed stars does not need to rotate. Hypothesis 2 alone explains the (apparent) motion of the Sun through the zodiac.

On the one hand, hypotheses 1 and 2 are responsible for disadvantage (c), while hypothesis 2 alone is responsible for disadvantage (d). On the other, hypothesis 3 is responsible for both advantages (a) and (b), but for no disadvantage. Actually, Copernican heliocentrism is able to calculate in a non-arbitrary manner the distances and order of the planets—advantage (a)—because all of them share their center. Ptolemaic geocentrism was able to calculate the proportion between the radius of the epicycle and that of the deferent for each planet, but it could not calculate the distances without assuming a previously accepted planetary order together with other physical and metaphysical assumptions.⁵ Once Copernicus realized that the deferent of the inner planets and the epicycle of the outer planets represented the Earth's annual motion around the Sun, he found a common unit of measure for linking all the proportions of the epicycles and deferents of the planets, and, hence, he was able to calculate the distances. When Copernicus made all the planets, including the Earth, to revolve around the Sun at their correct heliocentric period, he was able to deduce that, observed from the Earth, the inner planets would retrograde at the inner conjunction, when the planet, being faster than the Earth, would overtake it, and that the outer planets would retrograde at opposition to the Sun, when the Earth, being faster than the planet, would overtake it. So, advantage (b) is also dependent on hypothesis 3.⁶

Hypothesis 3, then, is, at least, a necessary condition for having both advantages (a) and (b) and is not responsible for any of the disadvantages. This explains why, for example, the Tyconic system, which defended hypothesis 3 (but not 1 and 2), was so successful and immune to refutations. It follows from this analysis, then that hypotheses 1 and 2, i.e., the daily and annual motions of the Earth, did not imply any advantage for the Copernican system: the key reason for the advantages of the Copernican system was what it shared with Tycho's system, i.e., hypothesis 3.

Copernicus made the Earth and planets revolve around the Sun, but he left the Moon revolving around the Earth, even if the Earth revolved around the Sun. What Copernicus did with the Moon, pre-Copernican astronomers could have perfectly well done with all the planets. They could have put the Earth revolving around the Sun, while keeping the moving Earth as the center of the other *six* planets. The obvious question that follows is: what would an ancient system gain by allowing the planets to revolve around the Earth, rather than making them revolve, like the Earth, around the Sun? The revolving Earth (hypothesis 2) would explain the (apparent) annual motion of the Sun through the ecliptic, exactly in the same way that the daily rotation of the

Footnote 4 continued

hypothesis 1 implies that the sphere of the fixed stars remain at rest, but it doesn't imply that its center is the Sun. Probably, one should refer also to hypothesis 2 in order to imply that the Sun is the center of the sphere.

⁵ The situation is actually somewhat more complex. See Carman (2010).

⁶ To express the exact sense in which heliocentrism explained the retrograde motion of the planets better than geocentrism is not an easy task. It is not as simple as to assert, like Lakatos (1978: 185), that "the assumption that inferior planets have a shorter period while superior planets have a longer period than that of the Earth" is enough to arrive at the inference that "[p]lanets have stations and retrogressions." See Swerdlow (1984).

Earth (hypothesis 1) would explain the (apparent) daily motion of the stars. There are no advantages in the rotating Earth system: it would save the phenomena as well as a fixed Earth and rotating stars would. To save the phenomena as well as the other system was enough in ancient times for competing with the fixed Earth system. Equally, a revolving Earth would explain the annual motion of the Sun in the same way as a revolving Sun would do. Both save the phenomena equally well. As I will show later, this should have been enough evidence in ancient times for postulating the system.

I do not want to say that pre-Copernican heliocentrism never asserted hypothesis 3, since it does not follow from my argument. What does follow is that pre-Copernican heliocentrism *could* have not asserted hypothesis 3. Hypotheses 1 and 2 do not necessarily imply hypothesis 3, in the same way that hypothesis 3 does not imply hypotheses 1 and 2 (the Tychonic system asserted hypothesis 3 and not 1 and 2). Therefore, making the Earth revolve around the Sun leaves the question about the planets open: one can I) leave the planets revolving around the Earth; II) make the planets revolve around the Sun; or III) leave the planetary problem unanswered. Which of these three possibilities pre-Copernican heliocentrism followed is an empirical question that must be answered by examining historical records. I will do so in the next section.

3 The Pre-Copernican sources of heliocentrism

All scholars agree that Heraclides of Pontus (387–312 BCE) affirmed the daily rotation of the Earth on its own axis (hypothesis 1).⁷ Almost all scholars also agree that he did not affirm the revolution of the Earth around the Sun (hypothesis 2) (*Aëtius*. iii, 13, 3; Diels 1879: 378; trans. taken from Heath 1913: 251).⁸ But Heraclides of Pontus might have not been the first one to affirm the daily rotation of the Earth. There are testimonies of a certain Ecphantus (*Aëtius* iii, 13, 3; Diels 1879: 378, and also Hippolytus, *Refut.* i. 15; Diels 1879: 556) and of Hicetas of Syracuse (Cicero, *Acad. Prior*, Lib II, 39 § 123; Rackham 1933: 627) also associated with this hypothesis. And there is also an obscure text in the *Timaeus* (40c; Waterfield 2008: 29) that some scholars have used to attribute the daily rotation of the Earth to Plato (428–424 BCE). Ancient, medieval and

⁷ Simplicius, on *De Caelo*, ii.7 (289b 1); Heiberg, 1984, pp. 441.31–445.5; ii. 14 (297 a 2), Heiberg (1894), pp. 541.27–542.2; c.13, 293b, Heiberg (1894), pp. 519.9–519.11; *Schol. In Arist.* (Brandis 1836: 505 b, 46–47); Proclus, *In Tim.* 281 E (Festugière 1968). All the testimonies are translated by Heath (1913: 254–255).

⁸ The interpretation of a Simplicius's text from the *Commentary on Aristotle's Physics* (Evans and Berggren 2006: 250–252; Todd and Bowen 2009: 158–164) produced a significant discussion in the second half of nineteenth century and the beginning of the twentieth (Böckh 1852: 135–141; Bergk 1883: 148–152; Martin 1883; Tannery 1899; Heath 1913: 249–256; Schiaparelli 1926: 176–195). Nevertheless, following Tannery (1899, pp. 305–311), almost all scholars today agree that a later copyist interpolated the name of Heraclides in the text and, therefore, it has no value as a testimony (Evans and Berggren 2006, note 18: 254). Some other scholars, such as Duhem (1915, iii, 44–162), Dicks (1970, 136–137, 218–219) and Gottschalk (1980, 69–82), based mainly on Calcidius's commentary to Plato's *Timaeus* 38D (translated in Eastwood 1992: 239–240), considered whether one could attribute to Heraclides some sort of semi-Tychonic hypothesis according to which Venus and Mercury revolved around the Sun, while the Sun, the Moon and the outer planets orbited the Earth (hypothesis 3a, without hypothesis 2). The semi-Tychonic hypothesis, i.e., hypothesis 3a, was certainly known in ancient times, but as Neugebauer (1975, p. 694), Eastwood (1992) and Keyser (2009) show, the attribution to Heraclides was based on a misinterpretation of Calcidius's text.

modern scholars have discussed this passage of *Timaeus*, and the discussion continues today (Dreyer 1953: 71–79; Heath 1913: 174–180; McDonald Comford 1997:120–134; Evans 1998: 454, note 40). Current consensus is against attributing hypothesis 1 to Plato. There is no doubt, however, that Plato did not affirm hypotheses 2 or 3. Heraclides, on the other side, affirmed hypothesis 1, and he probably did not affirm hypothesis 2 and almost certainly did not affirm hypothesis 3.

These are the main authors that affirmed (or could have affirmed) the daily rotation of the Earth, but not its annual revolution around the Sun.⁹ According to the extant testimonies, the two ancient astronomers who held the revolution of the Earth around the Sun are Aristarchus of Samos and Seleucus of Seleucia.

In his *Arenarius* (Heath 1897: 221–252), Archimedes said that Aristarchus of Samos (310–230 BCE) “brought out a book consisting of certain hypotheses... His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circumference of a circle, [the sun] lying in the middle of the orbit” (*Arenarius* I, 4–7; Heiberg 1913, 2: 244; Heath 1897: 221–222).¹⁰ Of course, if the stars remain unmoved, the apparent daily rotation of the celestial bodies must be attributed to the rotation of the Earth, even if Archimedes did not mention hypothesis 1. Archimedes is too close in age to Aristarchus (one generation alone) to make it plausible to doubt about the authenticity of the attribution.¹¹ Plutarch explicitly attributes both hypotheses to Aristarchus. According to him (*De facie in orbe lunae*, c. 6, pp. 922f–923a; Plutarch 1957: 54–55), Cleanthes “thought that it was a duty of Greeks to indict Aristarchus of Samos on the charge of impiety for putting in motion the Hearth of the Universe, this being the effect of his attempt to save the phenomena by supporting the heaven to remain at rest and the earth to revolve in an oblique circle, while it rotates, at the same time, about its own axis” (translated by Heath 1913: 304). Actually, Cleanthes wrote a book titled *Against Aristarchus* (Diogenes Laertius, VII.174; 1935: 280–281). So, there is no doubt that Aristarchus held hypotheses 1 and 2. The same, however, cannot be said with regards to hypothesis 3. There is *not one single testimony* that refers to a theory of the planets by Aristarchus. There is no information regarding it at all and, as Neugebauer (1975: 692) says, “it seems unlikely that Aristarchus developed any specific planetary theory because our sources are silent on this topic.”

Plutarch (*Plat. Quaest.* viii. I, 1006 c) mentioned Seleucus of Seleucia (fl. c. 150 BCE) alongside Aristarchus in relation to the two motions of the Earth. According to Plutarch, both astronomers asserted both kinds of motion, but, while Aristarchus

⁹ It is impossible to enumerate and discuss all the pre-Copernican authors who mentioned hypothesis 1. Most of them are presented in McColley (1937). None of these authors ever held hypothesis 2 or 3.

¹⁰ The text has been discussed because of a supposed problem in one of its clauses, namely that the clause “lying in the middle of the orbit” should be grammatically attributed to the circle and not to the Sun, as Erhardt and Erhardt (1942: 579) propose because it is closer to *circle* than to *sun*. Neugebauer (1942, 6) proposes to add τῶν πλανητικῶν (“of the planets”) to reconcile syntax and meaning: “the earth moves around the sun through the circumference of a circle which lies in the midst of the course [of the planets].” Neugebauer’s addition, of course, assumes rather than proves that Aristarchus affirmed hypothesis 3. I do not see any problem with the attribution of the clause to the Sun. I follow, therefore, Heath’s translation. For another problem with the text, see Boter (2007).

¹¹ Wall (1975) alone has doubts regarding the attribution of heliocentrism to Aristarchus. His main argument is putting into question the authenticity of Archimedes in the *Arenarius*, based on the analysis of Erhardt and Erhardt-Siebold (1942).

proposed it only as a hypothesis (ὑποτιθέμενος μόνον), Seleucus did it as a definite opinion (καὶ ἀποφαινόμενος). The last verb has a wide range of meanings, ranging from “to give an opinion” up to “to prove.” The exact meaning is impossible to infer from such an isolated sentence. What seems clear is that Plutarch is contrasting Aristarchus’s and Seleucus’s epistemological compromise with regard to heliocentrism. It seems that, while Aristarchus proposed heliocentrism only as a hypothesis, Seleucus went further by offering probably some sort of assertion of the motion of the Earth. Van der Waerden (1970, 7, 51) inferred from this verb that Seleucus actually computed heliocentric planetary tables. Nevertheless, as Neugebauer (1975: 697) affirmed, this is too farfetched. According to von Erhardt and von Erhardt (1942: 598), Seleucus could have measured some annual parallax (obviously by mistake). An explanation of the tides is also attributed to Seleucus (*Aëtius* Iii. 17.9; Diels 1879: 383): the revolution of the Moon around the Earth would resist the rotation of the Earth and the agitation of the air produced by this resistance would move the water of the ocean producing the tides. Thus, it might be that this explanation of the tides was the reason Seleucus advanced. Nevertheless, Neugebauer (1975: 697–698) shows that Seleucus’ proofs that have reached to us are much more metaphysical than mathematical or physical. In summary, one can reasonably affirm that Seleucus also held hypotheses 1 and 2, probably adding new reasons for them, or at least having a more committed approach to the truth of the proposal. We do not possess any source referring to Seleucus’s planetary theory. Thus, we have no reasons to presume that he also affirmed hypotheses 3.

As I have already said, hypothesis 3a was known in Antiquity. The first certain reference belongs to Theon of Smyrna (fl. c. 100 C.E.) who, in his *Mathematics Useful for Understanding Plato* (Dupuis 1892), offered two different systems for the inner planets. The first one was similar to the one adopted by Ptolemy: the center of the epicycles of Mercury, Venus and the Sun formed a straight line with the center of the Earth:¹² thus, the centers of the three epicycles revolved around the Earth at the same speed. The smaller epicycle was for the Sun and served for explaining its anomaly; the epicycle of Venus was bigger than Mercury’s, because its maximum elongation with respect to the Sun was greater than Mercury’s. Theon affirms, however, that it could also be possible for Venus’s and Mercury’s epicycles to revolve around the Sun. He added: “it is understood that this position and this order is true (ἀληθεστέρων)” (Dupuis 1892: 301–302). In his justification, Theon invoked an analogy already used by Aristotle (*De Caelo* II. 13, 293b, Barnes 1995: 1059–1060) distinguishing two kinds of center that, according to Aristotle, Pythagoreans mixed up. In a living being, the life center is the heart, which is always warm and always moving, but the center of the body is different, toward the navel. The Sun, being always warm and always moving, is like the heart of the cosmos, but the geometrical center is, in contrast, the cold and immobile Earth. It is thus clear that Theon asserted hypothesis 3a, but not hypotheses 1 and 2: the Earth, as the navel of the universe, was immobile at the center of the universe.

¹² According to Ptolemy, the solar anomaly was not explained by an epicycle but by an eccentric (*Almagest* III.4; Toomer 1998: 153), but he showed that both models are equivalent (*Almagest* III.3; Toomer 1998: 141–153).

The other author who clearly held hypothesis 3a is Martianus Capella (fl. c. 410 CE), not only as a possible model, but also as a fact. He affirmed that “the heavens, swirling in a ceaseless and rotary motion, set the earth apart in a stationary position in the middle and at the bottom” (*De Nuptiis*, L. VIII, 814; Stahl and Johnson 1977: 318), “three of these [planets, i.e., Mars, Jupiter and Saturn] together with the sun and the moon, have their orbits about the earth, but Venus and Mercury do not go about the earth (...) Venus and Mercury, although they have daily risings and settings, do not travel about the earth at all; rather they encircle the sun in wider revolutions. The center of their orbits is set in the sun” (*De Nuptiis*, L. VIII, 856–857; Stahl and Johnson 1977: 332–333).¹³ Capella, thus, held, like Theon, hypothesis 3a, but not hypotheses 1 and 2.

Some scholars have also identified Macrobius (fl. c. 420 CE) as describing hypothesis 3a (Dreyer 1953: 129–130; Heath 1913: 258–259; Neugebauer 1975: 695; Keyser 2009: 219–220). Even if it is hard to be sure about this, it is clear that only hypothesis 3a is at stake in this argument. Macrobius thus affirmed that the outer planets revolved around the Earth and that the Earth did not rotate on its own axis or revolve around the Sun (Stahl 1942: 234). In summary, Macrobius could have described hypothesis 3a, but he did not affirm hypotheses 1 or 2, nor 3b.

Pierre Duhem (1915, iii: 62) has interpreted a text from John Scotus Eriugena (c.815–c877) as holding the Tychonic system not only for the inner planets (hypothesis 3a), like Theon, Capella or maybe Vitruvius, but also for Mars and Jupiter. Von Erhardt and von Erhardt (1942) and Eastwood (2001) critiqued this interpretation.

In Arabic astronomy, hypothesis 1 has been widely discussed (Ragep 2001: 160; Sprenger 1856) even if rarely accepted. In contrast, there is no mention at all of the annual revolution of the Earth (hypothesis 2) or the planets revolving around the Earth (hypothesis 3) (Dreyer 1953: 271).

Indian astronomy is not only interesting in itself, but also as a way of investigating pre-Ptolemaic astronomy. As Neugebauer (1956) and Pingree (1976, 1978) have shown, there are strong reasons indicating that Indian astronomy had been influenced by Greek astronomy (probably pre-Ptolemaic) and, therefore, studying Indian astronomy is a way to approach (pre-Ptolemaic) Greek astronomy (Evans 1998: 393). It is undisputable that Indian astronomers knew hypothesis 1 and that they discarded it for physical reasons similar to those of the Greeks (Plofker 2009: 112; Shukla 1976: 120; Dutta 2006: 66–68), with the probably exception of Āryabhaṭa I (476–550), who seems to assert hypothesis 1 in his *Āryabhaṭīya* (IV, 9; Clark 1930: 64).¹⁴ But it is clear that Āryabhaṭa I assumed that all the planets revolved around the Earth, which was placed at the middle of the universe, as he affirmed in *Āryabhaṭīya*, (IV, 6; Clark 1930: 64, see also Shukla 1976:118).¹⁵ Therefore, the only author who possibly held

¹³ This text had many different interpretations during Middle Ages that Eastwood (2000, 2003) studies through the figures of the manuscripts of Capella’s *De Nuptiis*. Nevertheless, all the discussions and variants involved the inner planets only, and never the outer ones.

¹⁴ Even if some of his followers, like Somesvara, tried to reinterpret the text to make it geostatic, cfr. Shukla (1976: 120).

¹⁵ Some historians, such as Basham (1954: 491), stated that Āryabhaṭa I suggested hypothesis 2 too, but this seems to me to be absolutely unjustified. See Dutta (2006): 69.

hypothesis 1 did not hold hypotheses 2 or 3. Actually, Indian astronomy was influenced by heliocentrism only as late as the end of nineteenth century, promoted during the European colonies (Plofker 2009, VIII).

Some scholars (Ramasubramanian 1998; Ramasubramanian et al. 1994) suggest that Nīlakantha Somayaji (1444–1544) affirmed the Tychonic system some decades before Copernicus and Tycho. If so, Nīlakantha would be the first case of a pre-Copernican astronomer holding hypothesis 3 (i.e., 3a and 3b). Nevertheless, as Pingree (2001) and Plofker (2009: 250–251) have shown, what Nīlakantha apparently suggested is that the centers of the epicycles of the five planets were aligned with the Sun. This is true for the inner planets in an epicycle model. If one chooses an eccentric model for the outer planets—in the way described by Ptolemy at the beginning of the IX book of the *Almagest* (Toomer 1998: 555–556)—the motion of the eccentric’s center would also be aligned with the Sun. Therefore, the moving centers of the eccenters or the epicycles of all the five planets would remain aligned with the mean Sun, i.e., their longitudes would coincide with the solar mean longitude. There is some independent evidence that this pseudo-Tychonic model was actually asserted in Ancient Greece. On the one hand, Ptolemy asserts that the eccentric model is only viable for the outer planets. As it is noted by Toomer (1998: 555, note 2), this is certainly not true, since it would also be possible to build eccentric models for the inner planets, but in this case the speed of the eccentric’s center would not be that of the mean sun. Ptolemy’s mistake might be explained if he had in mind this pseudo-Tychonic models in which the center of the eccentric of all the planets is aligned with the mean sun (Alexander Jones, personal communication). On the other hand, there is a planetary table partially preserved in a papyrus (*P.Oxy 4173*) dated on the fourth century AD that could possibly be explained if the pseudo-Tychonic model is assumed (Jones 1999, 1.166–167 and 2.152–155). This pseudo-Tychonic model could be interpreted as a previous step to Tycho’s system, and it has been suggested that Copernicus considered it in his journey toward heliocentrism (Swerdlow 1973). Nevertheless, it is clear that it is not a heliocentric system, since it does not assert hypothesis 3.

So, in summary, there is undisputed evidence that some pre-Copernican thinkers affirmed hypothesis 1, including Heraclides, Hicetas and Ecphantus. There is not such definite evidence for other authors, such as Plato. Still, none of these authors affirmed hypotheses 2 or 3. Only two astronomers, Aristarchus and Seleucus, asserted hypotheses 1 and 2, but there is no evidence that they asserted hypothesis 3. There is no known record of any astronomer who held hypothesis 3b, i.e., that Mars, Jupiter and Saturn revolved around the Sun, but there is some evidence to show that some authors postulated heliocentrism for the inner planets (hypothesis 3a), namely Theon, Martianus Capella and possibly Macrobius. None of these, however, affirmed hypotheses 1 or 2.

The conclusion, then, is clear: there is no single reference of a pre-Copernican author who attributes hypothesis 3 to all the five known planets or that attributes hypothesis 2 and hypothesis 3a at the same time. There is no single source of any single pre-Copernican author affirming that all the five planets revolved around the Sun or that at least Venus, Mercury and the Earth did.

It is usually asserted that “absence of evidence is not evidence of absence.” What can certainly be asserted is that “absence of evidence is not a proof of absence,” but applied to disciplines like history, where almost nothing can be proved anyway, the

sentence seems rather trivial. Of course, we should be careful in this case, for the existing evidence on ancient astronomy tends to be fragmentary. Nevertheless, when one has strong reasons to expect to find some evidence, as in this case (as I will argue soon), the absence of this evidence actually is evidence of absence. Greek astronomers were especially keen on proposing geometric models to save the phenomena, whether they were real or not. And they were especially attracted to exploring models that were theoretically different, but empirically equivalent, like epicycles versus eccentrics (Ptolemy proved it in *Almagest* III,3; Toomer 1998: 141–153 for a fixed eccentric and for rotating eccentrics at IV,5; Toomer 1998: 180–189), or, even, a stationary Earth with a rotating sphere of the fixed stars versus a rotating Earth with a fixed sphere of the fixed stars. Even more, Ptolemy also mentioned the possibility of both Earth and fixed stars rotating about the same axis, “in such a way as to preserve the overtaking of one by the other” (*Almagest*, I,7; Toomer 1998: 44–45). The main task of what has been called by scholars *mathematical astronomy* was to produce models that could save the phenomena.¹⁶ According to Geminus, while it was the task of the physicist to study the true causes and nature of celestial bodies, the astronomer “is not competent to perceive the cause... sometimes he does not even desire to take up the cause ... but at other times he invents by way of hypothesis and grants certain devices, by the assumption of which the phenomena will be saved. For example, why do the Sun, Moon, and planets appear to move irregularly? [The astronomer would answer] that if we assume that their circles are eccentric, or that the stars go around on an epicycle, they apparent irregularity will be saved. And it will be necessary to fully examine in how many ways it is possible for these phenomena to be brought about.” (Evans and Berggren 2006: 253–254). Theon (Dupuis 1892: 268–269) mentioned that Hipparchus said something similar: “that it is worthy of the attention of the mathematician the reason why the same phenomena follow from such different hypotheses: that of eccentric circles and that of concentric circles with epicycles.” Geminus thought that the task of the mathematical astronomers was not to find one model for saving the phenomena, but all possible models, no matter their physical plausibility. Hipparchus added that it was the task of the astronomers also to explain why these models were equivalent.

In this context, I find it extremely implausible to accept that a Greek astronomer suggested that the planets and the Earth revolved around the Sun and realized that this system offered clear advantages for explaining the retrograde motion of the planets (b) and establishing an unequivocal order of the planets (a)—two certainly great concerns to Greek astronomers—and that there is no existing record of it. The point here is not whether hypotheses 2 and 3 could be acceptable for the Greeks. They would probably not be. They would have probably been considered an absurd hypothesis from the physical point of view. Still, as Hipparchus said, it would be the task of astronomers to explain why heliocentric and geocentric systems are equivalents. Hypothesis 1, the daily rotation of the Earth, was also considered “quite ridiculous,” for example by Ptolemy, and nevertheless, he invested a whole section of the first book of the

¹⁶ I am not assuming Duhem’s instrumentalist reading of Greek astronomy (see Duhem 1908) which was definitively discredited by Lloyd (1978). Duhem’s mistake was to assume that all Greek astronomy is reduced to the mathematical tradition. It is undeniable, however, that alongside the physical tradition, the mathematical tradition also existed.

Almagest (I,7; Toomer 1998: 43–45) to discuss and to refute it, and there are many references to this hypothesis in ancient and medieval texts. Ptolemy also implicitly discusses hypothesis 2 in this section of the *Almagest*, for he shows that the Earth could not leave the center of the Universe. Moreover, the physical and metaphysical problems followed from putting the Earth into motion (hypothesis 1 and 2), but not from putting the planets to revolve around the Sun (hypothesis 3). So, the Greeks could have suggested a complete Tychonic system (hypothesis 3, without 1 and 2) that still explained the retrograde motion and established an unambiguous order of the planets. Once again, there is no single record of a Greek (complete) Tychonic system. The discussions about the possible order of the planetary distances are the natural place where we should expect to find references to the heliocentrism of the planets, had it been postulated at all. Here, again, the silence is eloquent: leaving aside Macrobius's text which, in any case, would be referring only to the inner planets, we have not even one single reference to the heliocentrism of the planets in all the extant texts that discuss the order of the planets before Copernicus.

It is impossible to prove that some idea has never been proposed, but I think that the absence of evidence is eloquent in this case: the best explanation for the absence of evidence of hypothesis 3b (or even of hypotheses 2 and 3a together) is that these systems were never proposed before Copernicus.

4 Aristarchus's heliocentrism and its Copernican interpretation

Aristarchus seems to be almost the only precursor of Copernicus. Together with Seleucus, he is the only one about whom there are testimonies saying that he affirmed the annual motion of the Earth. Of Seleucus, as I have shown, there is not much record. Thus, I will analyze here Aristarchus's proposal in some more detail.

Aristarchus affirmed hypotheses 1 and 2. But what was his opinion regarding hypothesis 3? He posited the Earth to rotate around its own axis and to revolve around the Sun, but: what about the planets? I have already mentioned three possibilities: Aristarchus could have I) left the planets revolving around the Earth; II) put the planets to revolve around the Sun; or III) put into parenthesis the planetary problem. I think that the empirical evidence shows that II) is the most unlikely, for, had he proposed II), there would be some testimony of it today. Therefore, possibilities I) and III) are left for investigation. Aristarchus could have certainly kept all the planets revolving around the Earth, when he decided to have the Earth revolve around the Sun. This proposal would be some kind of anti-Tychonic system. While in the Tychonic system all the planets (except the Moon) revolved around the Sun, which, in turn, revolved around the Earth, in this Aristarchian system all the planets (including the Moon) would revolve around the Earth, which, in turn, would revolve around the Sun. I have already shown that in the context of ancient astronomy, the step from hypothesis 2 to 3 is not necessary or evident at all. In the same way that Copernicus kept the Moon revolving around the Earth when he made the Earth revolve around the Sun, Aristarchus could have kept the *other* five planets revolving around the Earth. Of course, this would not imply advantages a) and b) that the Copernican system has over the geocentric system. Still, this is certainly a possibility, for it would be empirically equivalent to geocentrism.

Nevertheless, I think that possibility III) is the most plausible. The absence of a single mention of Aristarchus's planetary theory certainly suggests that he did not have hypothesis 3 in mind, but it could also suggest that Aristarchus never developed any planetary theory.¹⁷ The only surviving work of Aristarchus is his *Treatise on the Sizes and Distances of the Sun and Moon* (Heath 1913). In it, Aristarchus commenced from 6 hypotheses, and applying geometry he obtained the Earth–Sun and Earth–Moon distances and the relative sizes of the Sun, Moon and Earth. One can conceive the work devoted to heliocentrism Archimedes mentioned in the same way: probably Aristarchus showed in this lost work that, assuming certain hypotheses, the phenomena could be equally saved by making the Earth revolve around the Sun or the Sun revolve around the Earth. The three bodies playing a role in this discussion would be the Earth, the Sun and the sphere of fixed stars. The planets (including the Moon) would be absolutely absent from the treatise, as they are from the *Treatise on The Sizes and Distances*. Archimedes explicitly mentioned some (or maybe all) of the hypotheses of the book on heliocentrism: that the fixed stars and the Sun remained unmoved, that the Earth revolved around the Sun and that the sphere of the fixed stars was very big and also centered at the Sun (Heath 1897: 222). Thus, the only three bodies known to have been mentioned in the hypotheses were the Earth, the Sun and the sphere of the fixed stars. One does not need anything else for showing that the apparent annual motion of the Sun could be explained by putting into motion either the Sun or the Earth and that the apparent daily motion of the stars could also be explained by making all to revolve around the Earth or the Earth to rotate about its own axis.

Looking at the close relationship between Aristarchus's astronomical work and optics, it is reasonable to argue that what Aristarchus suggested in the work mentioned by Archimedes was only to apply to the Sun–Earth system, and this was what had been discussed in works on optics, i.e., that the apparent motion of an object could be explained by the motion of the object seen by a fixed observer or by a moving observer of a fixed object.

Aristarchus's *Treatise on the Sizes and Distances* belonged to the set of works known as the *Little Astronomy*, presumably a set of books used from introducing the Great Astronomy, i.e., the *Almagest* (Heath 1913: 317–318). The fact that this set of books on astronomy and spherical geometry also included one related to optics—the *Optics* of Euclid (Heiberg and Menge 1895)—shows that the astronomical and the optical traditions (if they really were two different traditions) were very close (Webster 2014: 533). The intimate relationship between optics and astronomy is reflected not only in the books, but also in the authors, particularly Aristarchus. Aristarchus's alleged teacher, Strato of Lampsacus, worked on optics (*Aëtius* i, 15.5; Diels 1879: 313) and Aristarchus himself made some contributions to optics (*Aëtius* iv, 13, 8; Diels 1879: 404, 853). Moreover, a detailed analysis of Euclid's *Optics* and Aristarchus's

¹⁷ Censorinus (*de Die natali*, XVIII. 11; Hultsch 1867) attributed a length of the Great Year to Aristarchus. Great Years usually were cycles at which all the celestial bodies, Sun, Moon and the five planets, return to the same position. So, this attribution could imply that Aristarchus developed some planetary theory or, at least, a theory of the cycles of the planets. Tannery (1888, 93–94), however, shows that the value Aristarchus proposed only implies a return of the Sun and Moon, but not of the planets (see also Heath 1913: 314–316 and Huxley 1964). So, again, this would be an argument for holding that Aristarchus did not make any contribution to planetary theory.

Treatise indicates that the relationship between the two works is really close. Webster (2014) shows that propositions 23–27 of the *Optics* are closely related to the first three propositions of Aristarchus’s *Treatise*. Moreover, he suggests that the proofs contained in these propositions “were borrowed from Aristarchus or, more likely, some common astronomical source” (Webster 2014: 47). I am uncertain whether there is enough evidence to show that the arrow of influence goes from geometrical astronomy to optics, as Webster suggests. What is unquestionable from his research, however, is that geometrical astronomy—that of Aristarchus in particular—and optics were closely related in ancient times. Webster centers his analysis on propositions 23–27, but one can see the link between Aristarchus’s *Treatise* and Euclid’s *Optics* in many other propositions: while propositions 18–21 taught how to calculate the size of an object using the shadow of the object produced by the Sun, in the second part of his *Treatise* Aristarchus calculated the size of the Earth using its shadow projected on the Moon during lunar eclipses; while proposition 22 says that the arc of a circle placed in the same plane as the eye will appear to the eye as a straight line, in propositions 3–7 of his *Treatise*, Aristarchus uses this phenomenon at the dichotomy of the Moon with respect to the circle that divides the dark and the bright portions of the moon. Finally, while proposition 45 shows that there is a certain place from which objects of unequal size appear equal, in his proposition 8 Aristarchus argues that this is exactly what happens for the Sun and Moon seen from the Earth. Therefore, it is safe to argue for a close link between Euclid’s *Optics* and Aristarchus’s *Treatise*.

There are, however, other sets of propositions in the *Optics* that seem to be linked to heliocentrism. Propositions 37–42 analyze the cases in which an object can be seen of similar (or different) size from an eye when the eye or the object is moving. A typical case is, of course, that of the object revolving around the eye. But the *Optics* also explicitly shows that exactly the same happens if the eye revolves around a fixed object. Another set of propositions (prop. 51–56) is devoted to developing the relativity of motion, i.e., that the motion of the object or of the eye could explain the apparent motion and that the same would occur with the different apparent velocities. So easy is it to link this last set of propositions with heliocentrism that Kepler, in his own *Optics* (1604; IX, 332; Donahue 2000: 342), affirmed that “in these propositions, Euclid propounded pure, unadulterated Copernican astronomy”! Kepler might be exaggerating, but it is plausible thinking that the same close relation that one can detect between some propositions of Euclid’s *Optics* and the extant Aristarchus’s *Treatise* was present between the other sets of propositions and the non-extant treatise of Aristarchus. I think, therefore, that Aristarchus’s treatise on Heliocentrism could be understood as an application of these propositions of Euclid’s on the relativity of the motion to an observer at the center of the Earth and the Sun as the (apparent) mover, projected on the sphere of the fixed stars where, again, the planets did not play any role.

It seems, then, that Aristarchus did not hold hypothesis 3. It is also the case that no ancient or medieval scholar interpreted him as holding hypothesis 3 before Copernicus. It was only at some point after Copernicus’s heliocentrism spread out that interpreters, with Copernicus’ system in mind, began to attribute the whole Copernican system to Aristarchus.

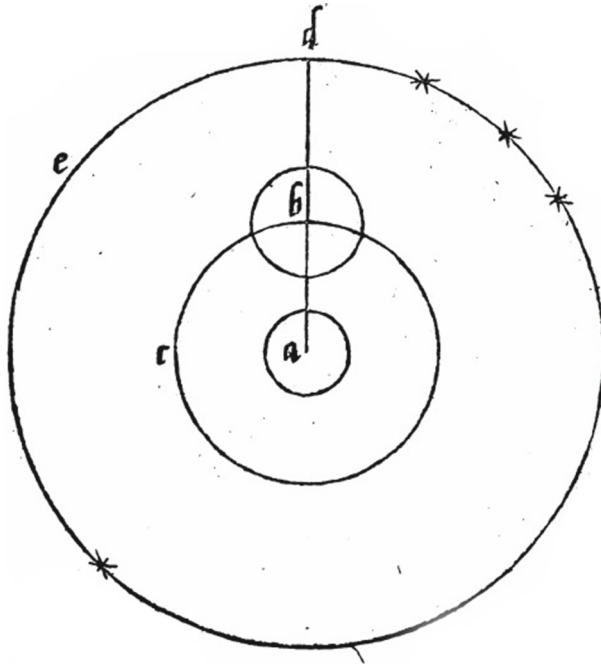


Fig. 1 Circle *de* is the sphere of the fixed stars; in a geocentric view, *a* is the center of the Earth and *b* the center of the Sun, in Aristarchian view, *a* is the center of the Sun and *b*, the center of the earth. Figure taken from Paschasius (1557: 11)

The shift in the interpretation of Aristarchus's position can be illustrated through the comments on the famous passage of the *Arenarius* where Archimedes describes Aristarchus's heliocentrism. The first printed edition with comments of the *Arenarius* was that of Hamellius (1557). Even though it was published after Copernicus's *De Revolutionibus*, it seems that Paschasius did not know the Copernican proposal. When he commented the famous text in which Archimedes attributed heliocentrism to Aristarchus, he added a figure (Fig. 1) and said that *a* was the center of the Earth, *b*, the center of the Sun, the circle *bc* was the circle around which the Sun revolves, and the circle *de*, the sphere of the fixed stars. Both circles had their daily revolution around *a*, and the Sun too had its annual revolution. By contrast, some ancients and Aristarchus among them "assigned *a* to the sun, *b* to the earth and *e* to the stars (just as before), which, together with the Sun, they claimed to be motionless. They certainly attributed to the Earth two specific motions: one, its own daily motion around center *b*, neglecting the motion of circle *bc*, the other, the annual motion on its proper sphere, both motions going to the east. They inferred from these two motions not only the years and days, but also the diurnal motion of the fixed stars. But, even if these two motions of the Earth seem to be enough for the three already mentioned motions, they are inconsistent in a non-tolerable way with too many other most certain phenomena." (Paschasius 1557: 11). Note that in Paschasius's description, neither the five planets nor the Moon are mentioned. He simply described the relationship among the Sun, the Earth and the sphere of the fixed stars.

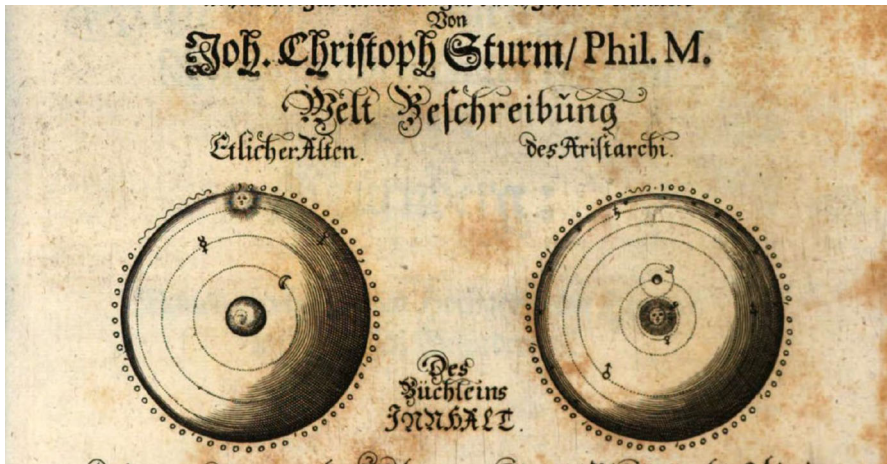


Fig. 2 Sturm (1667) representation of the Aristarchian system (at the right) and the system of the other Ancients (at the left). The Aristarchian system is indistinguishable from the Copernican proposal. Figure taken from Sturm (1667): 1.

Johann Sturm published the next commented edition of the *Arenarius* in German in 1667, the year of death of Alexander VII, who had published the *Index Librorum Prohibitorum*, in which the works of Copernicus and Galileo were included. The Copernican interpretation of Aristarchus's heliocentrism was as widespread as Copernican heliocentrism. This is clear in the figure that illustrated the book (Fig. 2), in which Sturm offered the two systems described by Archimedes. The one at the left represented the geocentric system held by the other Ancients, while the one at the right represented Aristarchus's system, in which all the planets, and not only the Earth, revolved around the fixed and central Sun.

Another Copernican interpretation of Aristarchus appeared over one hundred years before Sturm's volume, and only ten years after the publication of *De Revolutionibus*, which suggests that this kind of interpretation started as soon as Copernicus's heliocentrism began to spread out. Caspar Peucer in his *Elementa doctrinae* (1553: 100–101) said that “Aristarchus of Samos, who lived 1800 years ago, took the Earth from the center of the universe and fixed it in an orbit of its own, between the spheres of Mars and that of Venus and postulated that it rotates with its own motions around the Sun, which is fixed on the midst of the world, like the stars. And Copernicus, the most famous of all that wrote about the science of the stars after Ptolemy, added similar hypotheses to his demonstrations.” This text clearly considered Copernicus to be a follower of Aristarchus and that Aristarchus affirmed that not only the Earth, but all the planets revolved around the Sun.

The identification of Copernicus as a follower of Aristarchus—which implies at the same time a Copernican reinterpretation of Aristarchus's proposal—found in Peucer's text was also present from the very first edition of Clavius's *De Sphaera* (1570: 87; Westman 2011: 209): “Among the ancients there were some, of whom the leader was Aristarchus of Samos, four hundred years before Ptolemy—whom Nicolaus Coper-

nicus, among the moderns, follows in his work on the celestial revolutions— who fashioned this order among the bodies making up the whole universe: the Sun located immobile in the center or middle of the world, around which is Mercury’s orb, then Venus’ orb, and around that is the great orb containing the Earth with the elements and the Moon, and around that is the orb of Mars, the Jupiter’s heaven; after that, the globe of Saturn; and, at last, there follows the sphere of the fixed stars.” It is possible that Clavius’ famous volume contributed to spread the tradition that reached its peak in the work of Heath (1913) titled: “*Aristarchus of Samos: the Ancient Copernicus.*”

5 Copernicus’s originality

In the previous sections, I show that nobody before Copernicus asserted simultaneously hypotheses 2 and 3a or hypotheses 3b and that, as is shown in section 1, pre-Copernican heliocentrism (that of Aristarchus, for example) have all the disadvantages and none of the advantages of Copernican heliocentrism. Because pre-Copernican heliocentrism excluded the planets, they were not useful for calculating the distances of the celestial bodies in a non-arbitrary way (advantage a), or explaining in a non-*ad hoc* manner the retrograde motion (advantage b). At the same time, it had all the non-wanted physical consequences of putting the Earth into motion (disadvantage c) and all the non-observed astronomical consequences (e.g., annual parallax) due to changing the distance from the center of the sphere of the fixed stars (disadvantage d). These conclusions can, therefore, explain why Copernicus’s heliocentrism was accepted in 100 years, while pre-Copernican heliocentrism was adopted by but a few astronomers in over eighteen centuries.

The reasons Kuhn, Gingerich, Norton and others offered to explain this historical oddity should now be discussed under a new light. It is of course possible that some external factors played an important role in the triumph of Copernicus and the failure of pre-Copernican heliocentrism, but it is also clear from my analysis that there is a strong internal reason: Copernican heliocentrism had advantages that pre-Copernican heliocentrism did not have.

If my suggestion is correct, it also has important consequences on the dispute concerning Copernicus’s originality. Copernicus was the first astronomer in history that affirmed at the same time that the Earth *and* the five planets revolved around the Sun. Discussions regarding Copernicus’s reasons for omitting Aristarchus as his predecessor (von Erhardt and von Earhardt 1942; Neugebauer 1942; Rosen 1978; Gingerich 1985) are still interesting from a historical point of view, but it is clear that Copernicus proposed something more than Aristarchus and that this extra is essential for making his heliocentrism more advantageous than Aristarchus’s. It is no longer necessary for scholars hoping to save Copernicus’s moral integrity, like Rosen or Gingerich, to show that he was not aware of Aristarchus’s heliocentrism. The reason for attributing triumph to Copernicus’s heliocentrism, and not to Aristarchus’s, is not, as Gingerich (1985: 41) suggests, that Copernicus’s argumentation was more persuasive than that of Aristarchus, even if both proposed the same idea. In fact, the reason is that Copernicus offered a different, more complete and better heliocentrism than that of Aristarchus. In a sense, it was the Copernican reading of Aristarchus what prevented understanding that Aristarchus was not a Copernican.

Certainly, Copernicus had predecessors and his heliocentrism did not appear *ex-nihilo*: there are clear evidences of some parts of Copernican heliocentrism—hypothesis 1, hypothesis 2, hypothesis 3a—scattered throughout the entire corpus of ancient and medieval astronomy. Nevertheless, it is also true that Aristarchus was not the first Copernican. In fact, that no pre-Copernican was. The first Copernican was Copernicus.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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