

**Dual interacting cosmologies and late accelerated expansion**Luis P. Chimento<sup>1,\*</sup> and Diego Pavón<sup>2,†</sup><sup>1</sup>*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*<sup>2</sup>*Departamento de Física, Facultad de Ciencias, Universidad Autónoma de Barcelona, 08193 (Bellaterra) Barcelona, Spain*  
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In this paper we show that by considering a universe dominated by two interacting components a superaccelerated expansion can be obtained from a decelerated one by applying a dual transformation that leaves the Einstein's field equations invariant.

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**I. INTRODUCTION**

Currently, the view that the Universe has entered a stage of accelerated expansion is widely shared by cosmologists [1] to the point that the debate has shifted to discussing when the acceleration did actually commence and if it is just a transient phenomenon or it will last forever and, above all, which is the agent behind it. Whatever the latter, usually called dark energy, it must possess a negative pressure high enough to violate the strong energy condition (SEC). A number of dark energy candidates obeying the dominant energy condition (DEC) have been proposed, ranging from an incredibly tiny cosmological constant to a variety of exotic fields (scalar, tachyon, *k*-essence, and so on) with suitably selected potentials—see Ref. [2] for reviews. However, observations seem to marginally favor some or other energy field—dubbed “phantom energy”—that violates the DEC [3] over dark energy fields that satisfy it. Likewise, lately, it has been shown the existence of dual symmetry transformations that leaves invariant the Einstein field equations for spatially flat, homogeneous, and isotropic universes [4]. These transformations prove themselves extremely useful since they allow to obtain phantom dominated expansions from contracting scenarios [5–8]. Other features of phantom cosmologies have been investigated in [9].

The aim of this paper is twofold: (i) To extend the technique of dual symmetry transformations that preserve the form of Einstein's equations to the case that the expansion of the Universe is dominated by two fluids (dark matter and dark energy) that interact with each other. The dark energy fluid may be of phantom type or not. (ii) To apply this technique to three cases in which the dark energy is a different phantom fluid.

In Section II we sketch the dual symmetry transformation when both fluids are noninteracting and then extend the transformation first to the case that they interact and both of them satisfy the DEC, and then to the case that one of them does not satisfy the DEC. Likewise, we study the

evolution of the ratio between both energy densities. It turns out that when the interaction term is proportional to the total energy density the aforesaid ratio tends asymptotically to a constant. In Section III we apply the method of Section II, successively, to the cases that the phantom component is a scalar field with negative kinetic energy, a *k*-essence field, and a tachyon field. Finally, in Section IV we summarize our conclusions and present some comments.

**II. DUAL SYMMETRY FOR INTERACTING FLUIDS SCENARIOS**

Let us consider a homogeneous, isotropic, and spatially flat universe filled by two fluids of energy densities and pressures  $\rho_i$  and  $p_i$  (with  $i = 1, 2$ ), respectively. The Friedmann equation and the energy conservation equation read

$$\begin{aligned} 3H^2 &= \rho_1 + \rho_2, \\ \dot{\rho}_1 + \dot{\rho}_2 + 3H(\rho_1 + \rho_2 + p_1 + p_2) &= 0, \end{aligned} \quad (1)$$

where we have set  $c = 8\pi G = 1$ .

It can be readily seen that in this rather general scenario there is a dual symmetry relating this cosmology to another one (with two fluids of energy densities and pressures  $\bar{\rho}_i$ , and  $\bar{p}_i$ ), generated by

$$\begin{aligned} \bar{\rho}_1 &= \alpha\rho_1 + (1 - \beta)\rho_2, & \bar{\rho}_2 &= (1 - \alpha)\rho_1 + \beta\rho_2, \\ \bar{H} &= -H, \end{aligned} \quad (2)$$

where the parameters of the transformation,

$$\alpha = \frac{\bar{\gamma}_2 + \gamma_1}{\bar{\gamma}_2 - \bar{\gamma}_1} \quad \text{and} \quad \beta = -\frac{\gamma_2 + \bar{\gamma}_1}{\bar{\gamma}_2 - \bar{\gamma}_1},$$

solely depend on the barotropic indexes of the fluids. As usual, these indexes are given by  $\gamma_i = 1 + (p_i/\rho_i)$  and parallel expressions for the  $\bar{\gamma}_i$  of the other cosmology. We define the overall barotropic index  $\gamma = (\gamma_1\rho_1 + \gamma_2\rho_2)/(\rho_1 + \rho_2)$  for the unbarred cosmology. An entirely

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parallel expression exists for  $\bar{\gamma}$  in the other cosmology. Obviously the duality transformation connects these two indexes by  $\bar{\gamma} \rightarrow -\gamma$ . This means that  $\rho_1 + \rho_2 + p_1 + p_2 \rightarrow -(\rho_1 + \rho_2 + p_1 + p_2)$ . Put another way, if the DEC is fulfilled in one cosmology, then it is violated in the other. The transformation law (2) implies  $\bar{a} = 1/a$ . Accordingly, if one cosmology (say, the unbarred one) describes a phase of contraction, the barred one describes a phase of expansion, i.e., both cosmologies are dual of each other [4].

In the remainder of this section we generalize this technique to the case that the fluids do not conserve separately but interact with each other and then investigate the consequences. We begin by writing

$$\begin{aligned} 3H^2 &= \rho_1 + \rho_2, & \dot{\rho}_1 + 3H\gamma_1\rho_1 &= -3H\Pi, \\ \dot{\rho}_2 + 3H\gamma_2\rho_2 &= 3H\Pi, \end{aligned} \quad (3)$$

where the quantity  $\Pi$  characterizes the interaction. Automatically the above dual symmetry gets restricted to the following transformation  $\rho_i \rightarrow \rho_i$ ,  $H \rightarrow -H$ ,  $\gamma_i \rightarrow -\gamma_i$ , and  $\Pi \rightarrow -\Pi$ , with the overall barotropic index transforming as  $\gamma \rightarrow -\gamma$ . Therefore, there is a duality between two cosmologies, driven by two interacting fluids through the set of Eqs. (3), that have the sign of the individual barotropic indexes reversed. This opens the possibility of considering phantom dark energy with a negative barotropic index, which characterizes a ghost or phantom cosmology, as a source of Einstein's equations.

Defining the energy density ratio  $r = \rho_1/\rho_2$  and using Eqs. (3), we obtain the evolution equation

$$\dot{r} = -3\Gamma Hr, \quad \Gamma = \gamma_1 - \gamma_2 + \frac{\rho_1 + \rho_2}{\rho_1\rho_2} \Pi. \quad (4)$$

Since, except for the sign, dual cosmologies share the same interaction term  $\Pi$  the transformation  $\Gamma \rightarrow -\Gamma$ , holds. In addition, Eq. (4) and the ratio  $r = \rho_1/\rho_2$  are invariant under the dual transformation thereby  $r$  is a well-defined quantity. In particular if  $\dot{r}$  vanishes in one cosmology, it also vanishes in the dual one, meaning that the stationary solutions  $r = r_s$  of Eq. (4) are shared by both cosmologies.

Let us now assume that both fluids satisfy the DEC,  $\rho_i + p_i > 0$  (i.e., none of them is of phantom type) but one of them (say, fluid 2) violates the SEC,  $\rho_2 + 3p_2 < 0$  (i.e., it is a dark energy fluid), while the other does not, and specialize the interaction term to  $\Pi = -c^2(\rho_1 + \rho_2)$  with  $c^2$  a small dimensionless constant. This particular choice of  $\Pi$  has proved interesting because it provides analytical solutions and leads to a fixed ratio matter/dark energy at late times whatever the initial conditions (see, e.g., [10,11]). Farther ahead in this section we shall see that this is also true when the dark energy is of phantom type.

The stationary solutions of Eq. (4) are obtained by solving  $r_s\Gamma(r_s) = 0$ . When  $\gamma_1$  and  $\gamma_2$  are constants these solutions are given by the roots of the quadratic equation

$$r_s^\pm = -1 + 2b \pm 2\sqrt{b(b-1)}, \quad b = \frac{\gamma_1 - \gamma_2}{4c^2} > 1. \quad (5)$$

These satisfy the inequalities  $r_s^+ \geq 1 \geq r_s^-$  and for this model the general solution of Eq. (4) read

$$r(x) = \frac{r_s^- + xr_s^+}{1+x}, \quad (6)$$

where  $x = (a/a_0)^{-\lambda}$  with  $\lambda \equiv 12c^2\sqrt{b(b-1)}$ . It is readily seen that  $r(x)$  is a monotonic decreasing function in the range  $r_s^- < r < r_s^+$ . Finally, near this attractor solution,  $r \approx r_s^-$ , the last two Eqs. (3) can be approximated by

$$\frac{\rho_1'}{\rho_1} \simeq \frac{\gamma_1 - c^2(1 + 1/r_s^-)}{c^2(r_s^+ - r_s^-)(r - r_s^-)}, \quad (7)$$

$$\frac{\rho_2'}{\rho_2} \simeq \frac{\gamma_2 + c^2(1 + r_s^-)}{c^2(r_s^+ - r_s^-)(r - r_s^-)}, \quad (8)$$

where the prime denotes derivative with respect to  $r$ . For nearly constant barotropic indexes,  $\gamma_1$  and  $\gamma_2$ , last equations integrate to

$$\rho_1 \propto a^{-3[\gamma_1 - c^2(1+1/r_s^-)]}, \quad \rho_2 \propto a^{-3[\gamma_2 + c^2(1+r_s^-)]}, \quad (9)$$

while from the Friedmann equation (3) the time dependence of the scale factor

$$a \propto (\pm t)^{2/3[\gamma_2 + c^2(1+r_s^-)]} \quad (10)$$

is readily obtained.

From the condition  $\Gamma(r_s^-) = 0$  it follows that the exponents in the energy densities (9), which can be considered as effective barotropic indexes, coincide. This shows that the interaction modifies the apparent physical properties of the fluids.

We now apply this model to the case that the fluid 2 violates the DEC—i.e., it is a phantom fluid with  $\gamma_2 < 0$ . From the two last expressions (9) and (10) and duality four distinct possibilities emerge (see Fig. 1):

- (i)  $\gamma_2 + c^2(1 + r_s^-) > 0$ ,  
for  $t \geq 0$ , the Universe expands from an initial singularity at  $t = 0$  with a vanishing scale factor, (A),  
for  $t \leq 0$ , the Universe contracts from the far past and ends in a big crunch at  $t = 0$ , (B).
- (ii)  $\gamma_2 + c^2(1 + r_s^-) < 0$  (the dual of (i), namely,  $\gamma_2 \rightarrow -\gamma_2$  and  $c^2 \rightarrow -c^2$ ),  
for  $t \geq 0$  the Universe contracts from an initial singularity at  $t = 0$  with an infinite scale factor, (C),  
for  $t \leq 0$ , the Universe expands from the past and ends in a big rip at  $t = 0$ , (D).

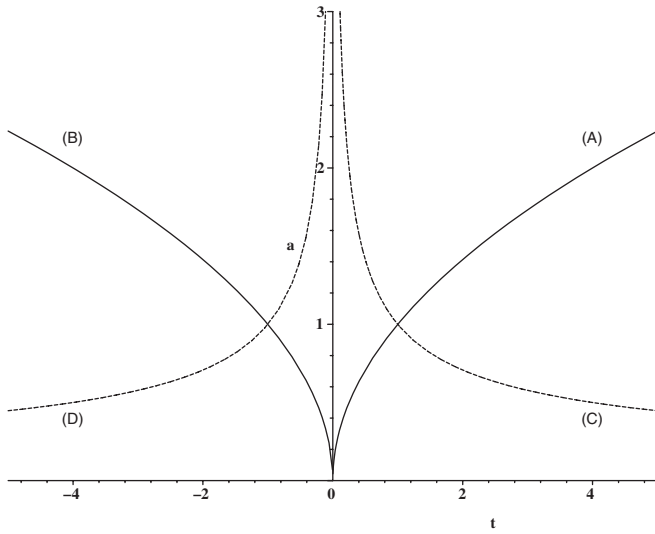


FIG. 1. The four branches of Eq. (10). The duality transformation maps curve A into C (and vice versa). Likewise, it maps curve B into D (and vice versa). Thus, curves A and C are dual of each other, the same is true for the pair B, D—see the text. The vertical axis corresponds to the scale factor  $a$ .

We have assumed, without loss of generality, that  $r$  is near the attractor  $r_s^-$ ; this facilitates the qualitative description and more readily illustrates the dual symmetry.

We wish to emphasize that one may get a superaccelerated expanding phase (i.e.,  $H > 0$  together with  $\dot{H} > 0$ ) when  $|\gamma_2| > c^2(1 + r_s^-)$  and also when  $|\gamma_2| < c^2(1 + r_s^-)$ . In the latter case the superaccelerated expanding phase is obtained by a dual transformation that reverses the signs of  $\gamma_1$ ,  $\gamma_2$ , and  $c^2$ . This interchanges the roles of both fluids and replaces the term  $\Pi$  by  $-\Pi$ .

### III. PHANTOM DARK ENERGY

In this section we apply the above method to three specific cases in which one component is matter (i.e., it satisfies the SEC) and the other component is a phantom fluid (as such, it does violate the SEC and DEC). For the latter we will consider in turn a scalar field, a  $k$ -essence field, and a tachyon field.

#### A. Scalar field cosmology

Let be an accelerated universe whose source of dark energy is a scalar field  $\varphi$  of phantom type. This type of field may arise in string theory, see [12] and references therein. We write its pressure and energy density admitting both signs for kinetic energy term, see e.g. Refs. [13,14],

$$\rho_\varphi = s\frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p_\varphi = s\frac{1}{2}\dot{\varphi}^2 - V(\varphi), \quad (11)$$

where  $s$  is a constant that may bear either sign. It follows that

$$\gamma_\varphi = s \frac{\dot{\varphi}^2}{\rho_\varphi}. \quad (12)$$

From the above equation we see that the barotropic index becomes negative in two separate cases, viz. when  $s < 0$  with a real scalar field and when  $s > 0$  with an imaginary scalar field. The dynamic equations of both interacting components are

$$\dot{\rho}_m + 3H\gamma_m\rho_m = -3H\Pi, \quad \dot{\rho}_\varphi + 3H\gamma_\varphi\rho_\varphi = 3H\Pi, \quad (13)$$

where  $\rho_m$  indicates the matter energy density. Since  $\rho_m$  and  $\rho_\varphi$  may be seen as functions of  $r = \rho_m/\rho_\varphi$ , with the help of (4), the second of Eqs. (13) can be written as  $\Pi = -r\Gamma\rho'_\varphi + \gamma_\varphi\rho_\varphi$  and in accordance with Eqs. (4) and (11) we obtain a differential equation for the potential

$$\frac{\Pi}{\rho_\varphi} = \gamma_\varphi - \Gamma r \left[ \frac{\gamma'_\varphi}{2 - \gamma_\varphi} + \frac{V'(\varphi)}{V(\varphi)} \right]. \quad (14)$$

The latter is very useful because when all the quantities that enter it, except  $V(\varphi)$  and  $V'(\varphi)$ , are known functions of the ratio  $r$  the potential  $V(r)$  can be obtained by integration. Combining it with  $r(a)$ , derived from (4), we can resort to the Friedmann's equation,  $3H^2 = \rho_\varphi(1 + r)$ , to obtain the scale factor as a function of time. Also, in virtue of the relation  $\Pi = -c^2(\rho_m + \rho_\varphi)$ , the conservation equation (13) for  $\varphi$  can be written as

$$\ddot{\varphi} + 3H\dot{\varphi} \left[ 1 + \frac{c^2(1 + r)}{\gamma_\varphi} \right] + \frac{1}{s} \frac{dV}{d\varphi} = 0. \quad (15)$$

Near the attractor dominated regime,  $r \approx r_s^-$ , and for constant barotropic indexes,  $\gamma_m$  and  $\gamma_\varphi$ , the scale factor has the power law solution  $a \propto t^{2/3\nu_\varphi}$ , given by (10), with  $\nu_\varphi = \gamma_\varphi + c^2(1 + r_s^-)$ . In this approximation, the simultaneous solution of Friedmann's equation and (15) leads to a potential that can be cast as a series expansion in the exponential potential (see Ref. [10]). Approximating  $V(\varphi)$  by this term we have

$$V(\varphi) \approx \frac{2(2 - \gamma_\varphi)}{3\nu_\varphi^2(1 + r_s^-)} e^{-sA\varphi}, \quad (16)$$

$$\varphi \approx \frac{2}{sA} \ln t, \quad A = |\nu_\varphi| \sqrt{\frac{3(1 + r_s^-)}{s\gamma_\varphi}}. \quad (17)$$

When  $s$  is negative, the parameter  $A$  is real, so both the phantom scalar field and potential become real quantities. By contrast, when  $s$  is positive, the parameter  $A$  is imaginary whence the phantom scalar field becomes imaginary but the dominant term of the potential remains real. In general, applying the dual transformation

$$\bar{V} = s\dot{\phi}^2 + V, \quad \dot{\bar{\phi}}^2 = -\frac{s}{\bar{s}}\dot{\phi}^2, \quad (18)$$

to the solution (16) and (17) we get the transformed potential and scalar field for any  $s, \bar{s}$  values. This transformation together with the change of the interaction term  $\Pi \rightarrow -\Pi$  reverses the sign of  $\nu_\phi$  and the new configuration is given by the barred quantities.

### B. $k$ -essence cosmology

Here we consider the case in which the dark energy is provided by a  $k$ -essence field,  $\phi$ , characterized by the Lagrangian  $L = -U(\phi)F(x)$ . The potential  $U(\phi)$  is a positive-definite function of the  $k$ -essence field  $\phi$  and  $F$  depends on the variable  $x \equiv g^{ik}\phi_i\phi_k$  with  $\phi_i \equiv \partial\phi/\partial x_i$ . This field arises, for instance, in open bosonic string field theory [15]. Identifying the energy-momentum tensor of the  $k$  field with that of a perfect fluid, its energy density and pressure are given by

$$\rho_\phi = U(F - 2xF_x), \quad p_\phi = -UF, \quad (19)$$

where the subscript  $x$  means  $d/dx$ .

Assuming that this perfect fluid obeys the barotropic equation of state it follows that  $\gamma_\phi = -2xF_x/(F - 2xF_x)$ , and  $\rho_\phi = UF/(1 - \gamma_\phi)$ . The  $k$ -essence field represents phantom dark energy when  $\gamma_\phi$  is negative. This requires a decreasing, positive-definite kinetic function. The Friedmann and the conservation equation for the  $k$ -essence field can be written as

$$3H^2 = \frac{UF(1+r)}{1-\gamma_\phi}, \quad r = \frac{\rho_m}{\rho_\phi}, \quad (20)$$

$$[F_x + 2xF_{xx}]\ddot{\phi} + 3HF_x\dot{\phi}\left[1 + \frac{c^2(1+r)}{\gamma_\phi}\right] + \frac{V'}{2V}[F - 2xF_x] = 0. \quad (21)$$

Again, near the attractor dominated regime and for constant  $\gamma_m$  and  $\gamma_\phi$ , the scale factor has the power law solution  $a \propto t^{2/3\nu_\phi}$  given by Eq. (10), with  $\nu_\phi = \gamma_\phi + c^2(1+r_s^-)$ . In this case the simultaneous solution of Eqs. (20) and (21) leads to a potential that can be expressed as a series expansion in inverse square potential. Approximating the potential by its leading term we write

$$U(\phi) \approx \frac{2\gamma_\phi}{3\nu_\phi^2(1+r^-)F_x(-\phi_0^2)\phi^2}, \quad (22)$$

where  $\gamma_\phi \approx 2\phi_0^2 F_x(-\phi_0^2)/[F(-\phi_0^2) + 2\phi_0^2 F_x(-\phi_0^2)]$  along with the  $k$ -essence field,  $\phi \approx \phi_0 t$ . When  $\nu_\phi > 0$  we apply a dual transformation to reverse its sign.

### C. Tachyon field cosmology

The energy density and pressure of the phantom tachyon field  $\phi$  generated by the kinetic function  $F(x) = (1+sx)^{1/2} = (1-s\dot{\phi}^2)^{1/2}$  are [6]

$$\rho_\phi = U(1-s\dot{\phi}^2)^{-1/2}, \quad p_\phi = -U\sqrt{1-s\dot{\phi}^2}, \quad (23)$$

respectively, and its barotropic index is given by  $\gamma_\phi = s\dot{\phi}^2$ . A negative barotropic index is obtained in two separate cases, viz. when  $s < 0$  with a real tachyon field and when  $s > 0$  with an imaginary tachyon field.

Assuming an interaction between the tachyon field and matter governed by Eqs. (13), with the subscript  $\varphi$  replaced by  $\phi$ , and proceeding along parallel lines to those sketched above one finds that the ratio  $r = \rho_m/\rho_\phi$  evolves from  $r_s^+$  to  $r_s^-$  and  $a \propto t^{2/3\nu_\phi}$ , where  $\nu_\phi = \gamma_\phi + c^2(1+r_s^-)$ . In this case duality, which requires that  $\rho_\phi \rightarrow \rho_\phi$ ,  $\gamma_\phi \rightarrow -\gamma_\phi$ , and  $\Pi \rightarrow -\Pi$ , leads to the following transformations for the tachyon field and its potential

$$\dot{\phi}^2 \rightarrow -\frac{s}{\bar{s}}\dot{\phi}^2, \quad U_0 \rightarrow -\frac{s\sqrt{1+s\dot{\phi}_0^2}}{\bar{s}\sqrt{1-s\dot{\phi}_0^2}}U_0. \quad (24)$$

As above, when  $\nu_\phi > 0$  we can apply a dual transformation to reverse its sign.

## IV. CONCLUDING REMARKS

We have considered a homogeneous, isotropic and spatially flat universe dominated by two fluids (pressureless matter and dark energy) that do not conserve separately but interact with each other. We have shown that in this scenario there is a dual symmetry transformation, given by  $\rho_i \rightarrow \rho_i$ ,  $H \rightarrow -H$ ,  $\gamma_i \rightarrow -\gamma_i$ , and  $\Pi \rightarrow -\Pi$ , that preserves the form of Einstein's equations irrespective of whether the dark energy is phantom or not. As a consequence, superaccelerated expansions can be obtained from decelerated ones and vice versa without affecting the field equations also in the case that matter and dark energy interact.

We observe, by passing, that if the interaction term is given by  $\Pi = -c^2(\rho_1 + \rho_2)$ , then the cosmic coincidence problem (i.e., ‘‘why are the vacuum and dust energy densities of precisely the same order today?’’ [16]) is somewhat alleviated in the sense that there is an attractor such that the energy densities of matter and dark energy tend asymptotically to a fixed ratio,  $r_s^-$ , regardless if the dark energy component is phantom or not. Obviously, this does not solve the coincidence problem in full. Its full solution would require to show, in addition, that the attractor was reached only recently—or that we are very close to it. Otherwise our approach would conflict with the tight constraints imposed by the cosmic background radiation and the standard scenario of large scale structure formation. On the other hand, the precise value of  $r_s^-$  cannot be derived at present. For the time being, it must be understood as an input parameter. This is also the case of a handful of cosmic quantities such as the present value of the cosmic background radiation temperature, or the ratio between the number of baryons and photons.

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