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Analysis of intermittence, scale invariance and characteristic scales in the behavior of major indices near a crash

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Abstract

This work is devoted to the study of the relation between intermittence and scale invariance. We find the conditions that a function in which both effects are present must satisfy, and we analyze the relation with characteristic scales. We present an efficient method that detects characteristic scales in different systems. Finally we develop a model that predicts the existence of intermittence and characteristic scales in the behavior of a financial index near a crash, and we apply the model to the analysis of several financial indices.

Keywords: Econophysics; Scale invariance; Intermittence; Stock market prices; Latin American indices

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1. Introduction

The presence of log-periodic structures in data analysis suggests that the system has characteristic scales. During the last years the study of this phenomena and its relation with the concept of scale invariance had grown, due to the great amount of physical systems presenting log-periodic structures: fluid turbulence [1,2], diamond Ising model [3], earthquakes [4], materials rupture [5], black holes [6] and gravitational collapses [7] among others. In a mathematical context, we recall constructions as the Cantor Fractal [3,8], with a discrete scale changes invariant. This phenomenon produces a non-real term in the fractal dimension.

The presence of logarithmic periods in physical systems was noted by Novikov in 1966 [9], with the discovery of intermittence effect in turbulent fluids. The relation between both effects has been deeply studied, but it has not been formalized yet.

At the same time a new discipline: Econophysics, has been developed [10]. This discipline studies the application of mathematical tools that are usually applied to physical models, to the study of financial models. Simultaneously, there has been a growing literature in financial economics analyzing the behavior of major stock indices [10–14].

The Statistical Mechanics theory, like phase transitions and critical phenomena have been applied by many authors to the study of the speculative bubbles preceding a financial crash (see for example Refs. [15,16]). In these works the main assumption is the existence of log-periodic oscillation in the data. The scale invariance in the behavior of financial indices near a crash has been studied in Refs. [17,18].

We first study the relation between intermittence and scale invariance. We give the conditions that a function has to satisfy when both effects are present, and we analyze the relation with characteristic scales. We present a new method that detects characteristic scales in different systems using the previous results. Finally we develop a model that predicts the existence of intermittence and characteristic scales in the behavior of a financial index near a crash, and we apply the model to the analysis of the behavior of several financial indices: The NASDAQ index near the crash in April 2000, the S&P500 index near the October1987 crash, and the Hong Kong HSI index as well as the Brazil BOVESPA index, the Mexico MMX index, and the Turkey XU100 index near the October 1997 Asian crash.

2. Scale invariance

A function A, that depends on a variable "x", is *invariant for the scale change* "λx" when

$$A(x) = \mu A(\lambda x)\,, \tag{1}$$

where μ is a constant independent of x.

For a better understanding of condition (1), we can present it in the following way:

$$\mu A'(x) = A(x) \tag{2}$$

with $A'(x/\lambda) = A(x)$. Then, for two values x_1, x_2 the following equality must hold:

$$\frac{A(x_1)}{A(x_2)} = \frac{A'(x_1)}{A'(x_2)} \tag{3}$$

or equivalently,

$$\frac{A(x_1) - A(x_2)}{A(x_1)} = \frac{A'(x_1) - A'(x_2)}{A'(x_1)} \tag{4}$$

for all x_1, x_2.

At this point we recall that the scale invariance can be defined as the *scale change $x \to \lambda x$ verifying that the percentage variation of the observable $A(x)$ remains invariant*.

It can be easily proved that the function $A(x) = x^2$ remains invariant for any scale change $x \to \lambda x$ where λ is any real number. This is an example of *continuous scale invariance*.

In order to find the functions that are invariant for scale changes, we shall work with Eq. (1).

The general solution to this equation is

$$A(x) = Cx^{\alpha}\,, \tag{5}$$

where α is a complex number defined by:

$$\alpha = -\log_{\lambda}\mu + \frac{\mathrm{i}2\pi n}{\ln\lambda}, \quad n \in Z\,. \tag{6}$$

From (5) and (6) we conclude that any observable which remains invariant for the scale change $x \to \lambda x$ can be expressed as

$$A(x) = x^{-log_{\lambda}\mu} \sum_{n=-\infty}^{n=+\infty} a_n \mathrm{e}^{\mathrm{i}2\pi n \frac{\ln x}{\ln\lambda}}\,. \tag{7}$$

Now we recall the difference between discrete and continuous invariance.

When the observable given by (7) presents *continuous scale invariance*, for any real number λ there exists μ such that condition (1) is fulfilled. We can deduce that in this case $-\log_{\lambda}\mu$ does not depend on λ, and $a_n = a_0\delta_{\mathrm{on}}$.

When the observable $A(x)$ verifies Eq. (1) only for numerable values of the λ's, it presents a *discrete scale invariance*.

Two crucial remarks have to be done: the continuous scale invariance implies that the exponent in Eq. (1) has to be a real number. On the other hand, if an observable remains invariant for a scale change $x \to \lambda_0 x$ from (7) we can conclude that Eq. (1) holds for any $\lambda = \lambda_0^n, n \in Z$.

3. Intermittence and discrete scale invariance

In the previous section we studied the general structure of functions with discrete scale invariance. In this section we shall analyze a particular type of scale invariance:

the one arising in the existence of intermittences or ‘‘stationary intervals’’, constant in the logarithm of the independent variable.

The functions that can be obtained from this analysis are:

$$f^F(x) = \beta e^{\alpha F(\log_a x)} , \tag{8}$$

$$f^C(x) = \beta e^{\alpha C(\log_a x)} , \tag{9}$$

where β and α are real numbers, ‘‘α’’ is positive and $F(x) = I(x)$ and $C(x) = I(x) + 1$ are the Floor and Ceiling functions, respectively.

Hence, the value obtained when applying the Floor function to a variable x will be the nearest entire number to x from the left, and the value obtained by the Ceiling function will be the nearest entire number to x from the right.

These two functions are discrete scale invariants, and more specifically, they satisfy Eq. (1) only when

$$\lambda = a^n, \quad n \in Z . \tag{10}$$

This can be proved as follows:

From (1), we have

$$\frac{f^F(\lambda x_1)}{f^F(\lambda x_2)} = \frac{f^F(x_1)}{f^F(x_2)}, \quad \forall x_1, x_2 ,$$

which is

$$\frac{e^{\alpha F(\log_a \lambda x_1)}}{e^{\alpha F(\log_a \lambda x_2)}} = \frac{e^{\alpha F(\log_a x_1)}}{e^{\alpha F(\log_a x_2)}}, \quad \text{i.e.,} \ e^{\alpha(F(\log_a \lambda x_1) - F(\log_a \lambda x_2))} = e^{\alpha(F(\log_a x_1) - F(\log_a x_2))} .$$

Thus we need

$$F(\log_a \lambda x_1) - F(\log_a \lambda x_2) = F(\log_a x_1) - F(\log_a x_2) ,$$

i.e.,

$$I(\log_a x_1 + \log_a \lambda) - I(\log_a x_2 + \log_a \lambda) = I(\log_a x_1) - I(\log_a x_2) .$$

Then we must have

$$\log_a \lambda = n, \quad n \in Z .$$

Therefore we can conclude that these functions have characteristic scales.

The answer to this question will help to identify the systems in which this behavior will take place.

Now we want to know the conditions for a function to have discrete scale invariance, after we know that the system has intermittences. The conditions are the following:

(i) The intermittence intervals must be constant in logarithmic scale, i.e., the steps have to be discrete:

$$d(\ln x) = K , \tag{11}$$

where K is a positive number.

Let $a = e^K$, then we have

$$d(\log_a x) = 1 \tag{12}$$

and

$$I(\log_a x_1 + \log_a \lambda) - I(\log_a x_2 + \log_a \lambda) = I(t_1 + n) - I(t_2 + n) = I(t_1) - I(t_2)\,.$$

Then, the interval of time of the intermittence is such that the logarithm of the variable x, in a basis a, has advanced one unit (in that period of time).

Hence, we can conclude that due to the longitude of the intermittences there exists a basis in which the logarithm of the variable is equal to one.

(ii) The stationary intervals are consecutive, when one finishes, begins the next one.

(iii) The function such that its variable is discretized with the rule $d(\log_a x) = 1$ is a power law, and the beginning and the end of a stationary interval have both to be a point of the function. We will call this function the basis function, which can be illustrated as follows.

4. A new data analysis method

Now we will apply the tools given by the theory previously developed in order to analyze data presenting characteristics similar to those mentioned above. Therefore, the data basis function will be a power law.

First, we will find this law; the second step will be to reduce the free parameters to only one: the logarithmic basis. Finally, we will use (8) or (9) in order to find the value of "a" minimizing the distance with the data.

Hence, we obtain the system characteristic scales. The estimation of the power law is crucial in this method because it will be the basis of the function that will be used for fitting the data. We remark that one of the mistakes that have to be avoided is to estimate the power law with all the data range. Trying to fit the data very close to the crash will be largely fitting the noise. The estimation must be done using only data points as the ones in Fig. 1.

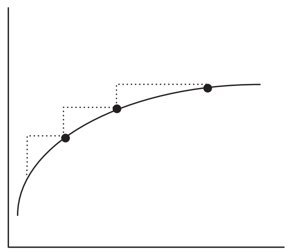

Fig. 1. Basic scheme showing the relation between a power law (in solid line) and a function like the one defined in (10) (dash line). The black points are the intersection points of both functions.

5. Financial indices prices near a crash

The evolution of a financial index represents the changes of a portfolio [19]. There is a simple model that takes account of the evolution of an asset price in the market [20], this model considers two contributions to the percentage variation in the asset price: one deterministic, and one stochastic:

$$\frac{\mathrm{d}S}{S} = \mu\,\mathrm{d}T + \sigma\,\mathrm{d}X \,, \tag{15}$$

where S is the asset price, μ a constant (called drift), $\mathrm{d}T$ the interval of time, σ another constant (called volatility) and $\mathrm{d}X$ a random variable. It is assumed that $\mathrm{d}X$ has normal probability distribution.

We focus our attention in the deterministic contribution:

$$\frac{\mathrm{d}S}{\mathrm{d}T} = \mu S \,. \tag{16}$$

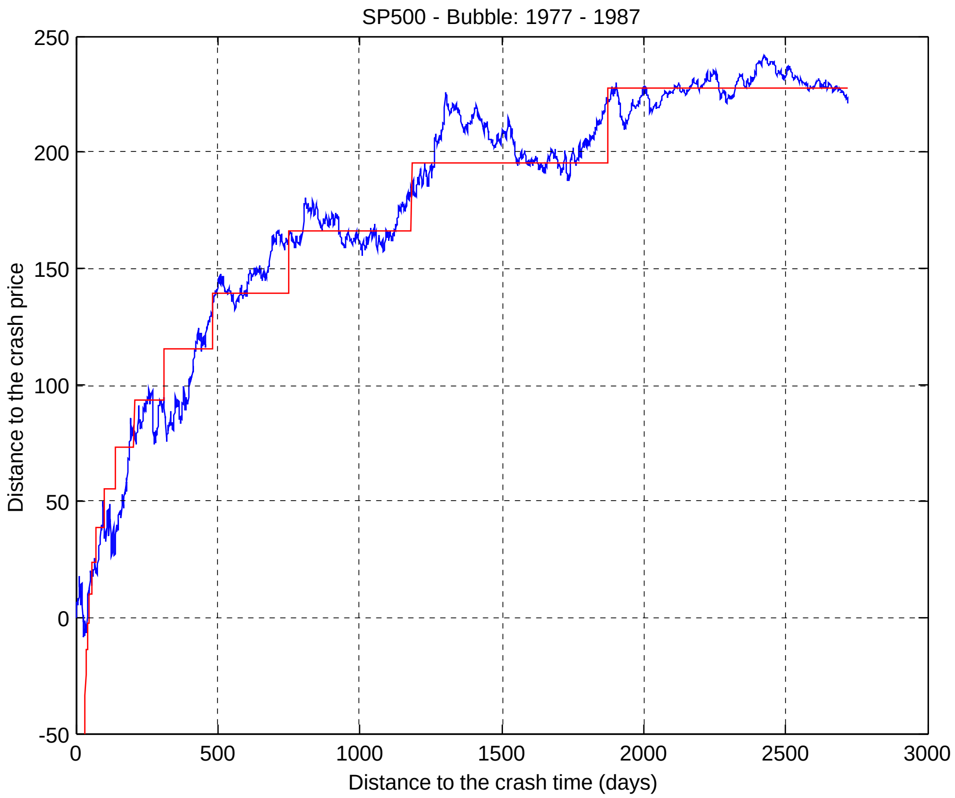

Fig. 2. The temporal dependence of the S&P500 index near the October 1987 crash. The variables are the distance to the crash price and the distance to the crash time, respectively. The solid line represents the best fit with Eq. (9), for: $a = 1.6$, $\alpha = 0.095$, and $\beta = 77.56$.

As a financial index can be considered an asset, its (deterministic) financial behavior will be given by (16). Our hypothesis is that near a crash Eq. (16) is modified:

$$\frac{\mathrm{d}S}{\mathrm{d}T} = \mu \frac{(S_c - S)}{(T_c - T)} , \tag{17}$$

where T_c and S_c are, respectively, the time and the price for which the crash takes place. The heuristic analysis is as follows: near a crash there is a factor that produces a considerable increase in the index price, by the other hand, when the price is very near to the crash price, it has to exist another factor smoothing those variations, otherwise the crash would take place before the real date. Changing the variables in (17) we obtain that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mu \frac{P}{t} , \tag{18}$$

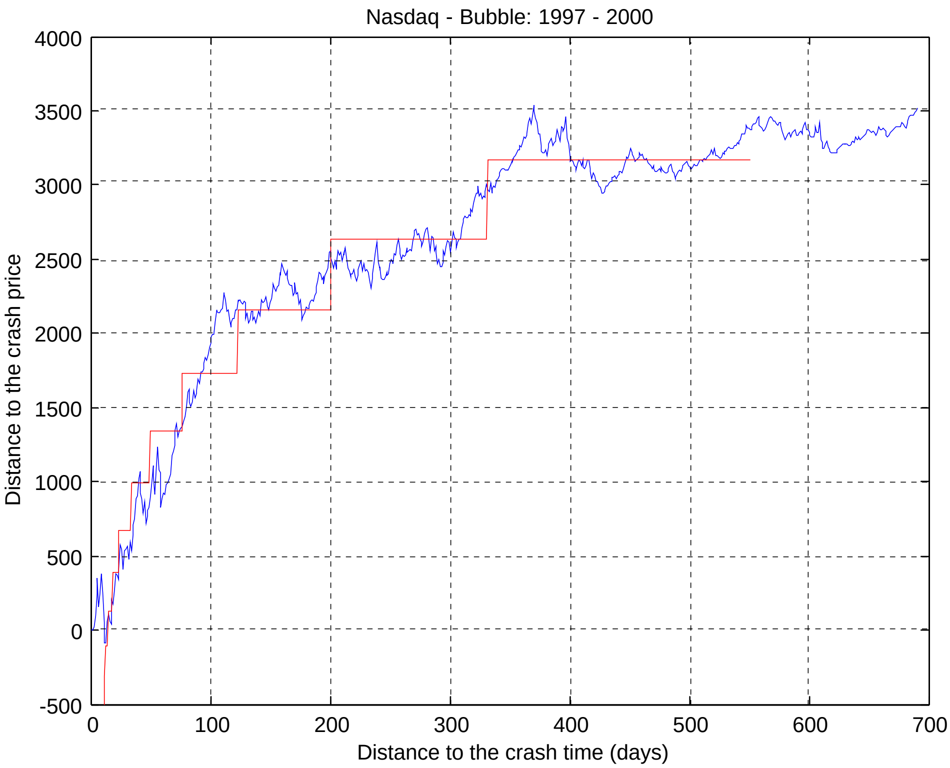

Fig. 3. The temporal dependence of the NASDAQ index near the April 2000 crash. The variables are the distance to the crash price and the distance to the crash time, respectively. The solid line represents the best fit with Eq. (9), for: $a = 1.69$, $\alpha = 0.1034$, and $\beta = 1731$.

where the variables are not anymore absolute data of the system: t and P are the distance to the critical time and critical price, respectively.

The second assumption will be that the temporal steps are discrete; therefore, the index evolution is not continuous and is given by

$$\mathrm{d}t = Kt\,. \tag{19}$$

Then the frequency in the index price changes is proportional to the distance to the date in which the crash takes place. Eq. (19) implies that

$$\mathrm{d}(\ln\, t) = K\,. \tag{20}$$

From Eqs. (18) and (20) we arrive to functions like (8) or (9). In this case we will work with function (9), due to the fact that the intermittences must take into account that the time approaches to the critical time from the right, because of the change of variables (18).

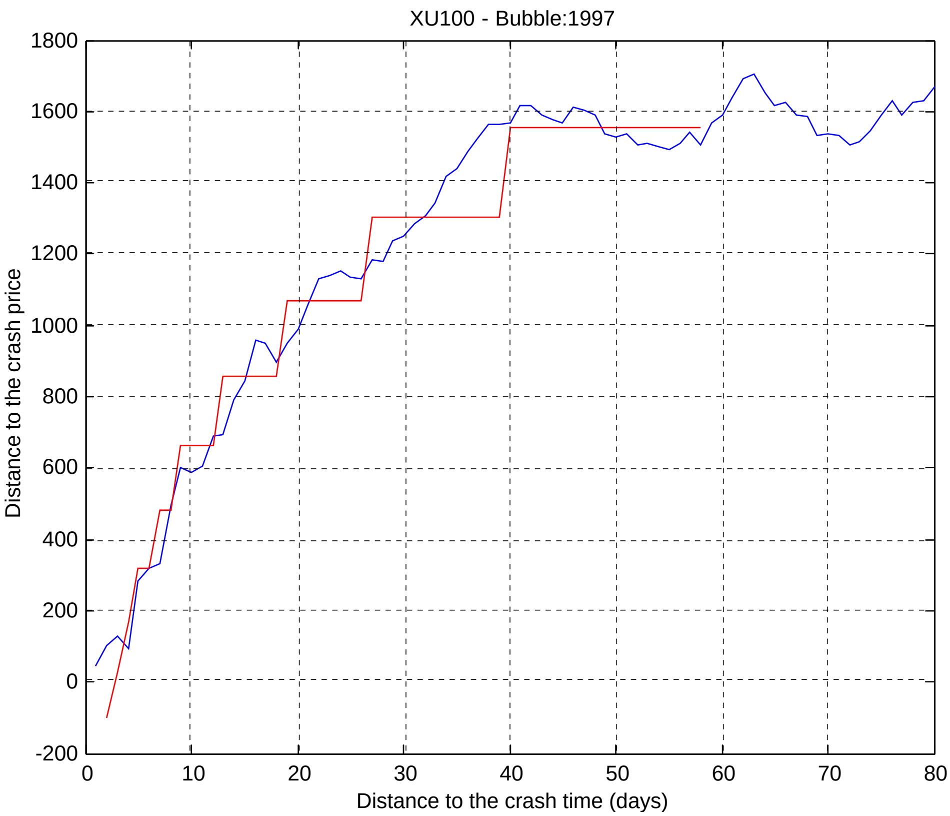

Fig. 4. The temporal dependence of the Turkey XU100 index near the October 1997 crash. The variables are the distance to the crash price and the distance to the crash time, respectively. The solid line represents the best fit with Eq. (9), for: $a = 1.5$, $\alpha = 0.0847$, and $\beta = 1447$.

6. The NASDAQ Index crash in April 2000, the S&P500 Index crash in October 1987, and the Asian crash of 1997

NASDAQ is an electronic financial index. It has been traded in the market from 1971 and it is the American Index with greatest increases rate. It includes more companies that the ones that are traded in the *New York Stock Exchange*, and it takes account of more of the 50% of the trading operations that take place in the US. In the last 10 years the number of companies that quote in NASDAQ have increased a lot. Principally the technological companies are the ones which quote in NASDAQ: Microsoft, Intel, MCI, Cisco System, Oracle, Sun Microsystems, and also important companies as: SAFECO Insurance and Northwest Airlines.

Between February and May 2000 the NASDAQ price fell down in a 37%. The most abrupt fall was in April, but the index decrease began some months before. We recall that in our analysis the date in which the crash takes place is the one in which

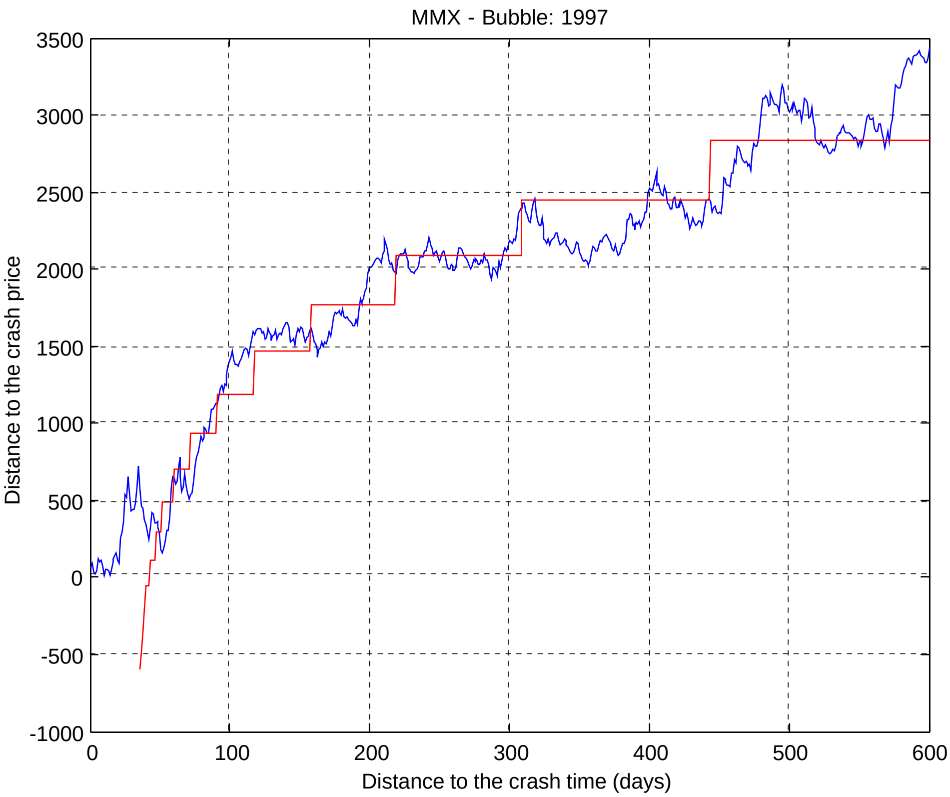

Fig. 5. The temporal dependence of the Mexico MMX index near the October 1997 crash. The variables are the distance to the crash price and the distance to the crash time, respectively. The solid line represents the best fit with Eq. (9), for: $a = 1.493$, $\alpha = 0.0837$, and $\beta = 1368$.

the slope in the index price begins, and not the one in which the fall is maximum. Hence, the critical time is in February 2000.

We have also analyzed the S&P500 index near the October1987 crash, and the Hong Kong HSI index as well as the Brazil BOVESPA index, the Mexico MMX index, and the Turkey XU100 index near the October 1997 Asian crash. We recall that the Asian crisis of October 1997 was originated between July and August when the governments of Thailand, Malaysia, Vietnam and Philippines devaluated their currencies. This devaluation resulted in global concerns in all major markets that would later be reflected in drops of the indices worldwide (Figs. 2–7).

7. Results

In this section, we present the results of the fits realized with Eq. (9). We recall that our main interest is to identify characteristic scales in the data, i.e., to find the value of "a" minimizing the distance with the data using Eq. (9).

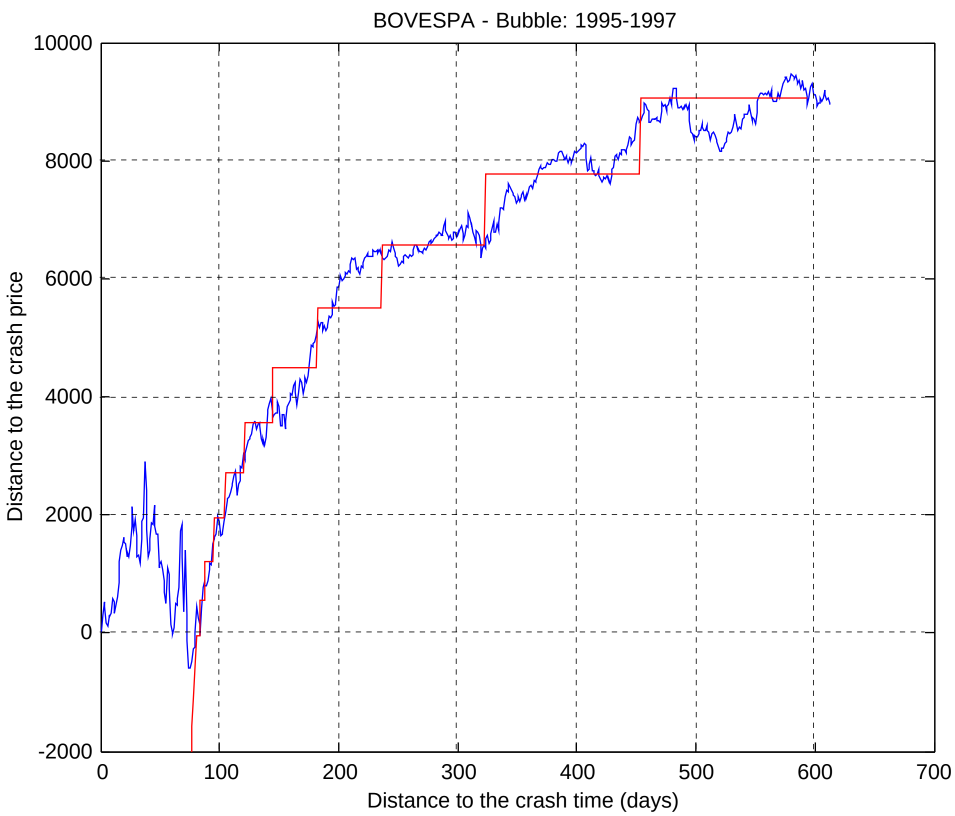

Fig. 6. The temporal dependence of the BOVESPA index near the October 1997 crash. The variables are the distance to the crash price and the distance to the crash time, respectively. The solid line represents the best fit with Eq. (9), for: $a = 1.528$, $\alpha = 0.0846$, and $\beta = 4876$.

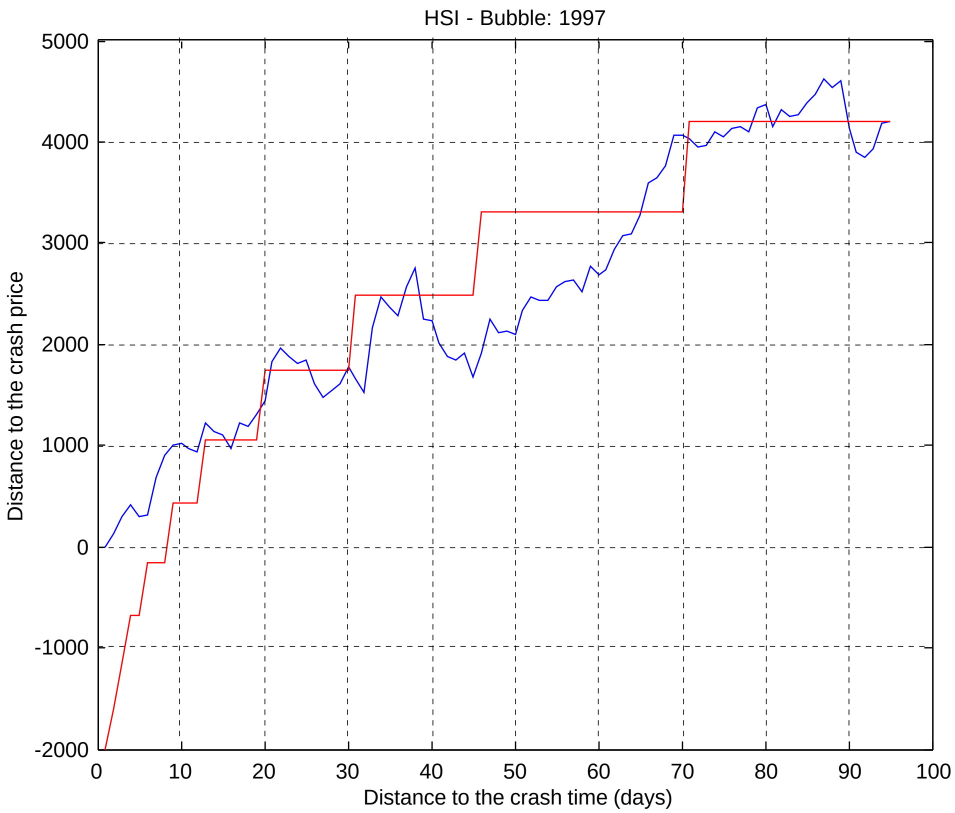

Fig. 7. The temporal dependence of the Hong Kong HSI near the October 1997 crash. The variables are the distance to the crash price and the distance to the crash time, respectively. The solid line represents the best fit with Eq. (9), for: $a = 1.53$, $\alpha = 0.0887$, and $\beta = 4336$.

8. Conclusions

The effects of certain local crisis on various and distant markets have largely been cited. The collapse of the crashes of 1987 (S&P500) dragged the collapse of markets worldwide, as did 2000 (NASDAQ). However not every crisis has sufficient strength as to drag the fall of leading indices in other countries.

In Ref. [16] it has been shown that the crashes of Asian indices had consequences on emergent markets: the Asian crisis had sufficient strength as to drag the fall of leading Latin American indices as well as the fall of the Turkey XU100 index.

Clearly all these indices crashed in similar dates due to a dragging correlated effect, which most likely started with the instability of the HSI index. So one would expect to obtain similar parameters for all these indices.

The parameters a and α obtained for the Hong Kong HSI index, the Brazil BOVESPA index, the Mexico MMX index, and the Turkey XU100 index near the

Table 1
A comparison table of the coefficients a and α for developing markets

	XU100	MMX	BVSP	HSI
a	1.50	1.493	1.528	1.53
α	0.0847	0.0837	0.0846	0.0887

October 1997 Crash are very similar (Table 1). This signals the likelihood of the events in different markets and different economic realities which strengthens the hypothesis of imitation and long range correlations among traders.

The good results obtained when fitting Eq. (9) validate the index price model presented in Sections 4 and 5, and therefore, the existence of characteristic scales in this type of systems.

An important consequence that results from Eq. (9) is that the scale invariance for changes $t \to \lambda t$, with $\lambda = a^n$ and $n \in Z$, implies (because of (4)) that the percentage change in P in scale "t" and in scale "λt", is the same after an interval of time "t" or after an interval of time "λt".

The method developed in this work is of easy implementation and of great efficiency. It is also a general method that can be applied to other similar systems.

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