Isoscaling: Geometry, correlations and symmetry energy

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This work uses a simple model to understand the properties of isoscaling. Using a generalized percolation model, it is first shown that isoscaling is a general property of fragmenting systems. In particular, it is found that the usual isoscaling property can be seen as a limit case of bond percolation in lattices in D dimensions, with N colors, with independent probabilities for each color, and for any regular topology. The effect of introducing correlations is also studied.

I. INTRODUCTION

The isoscaling property has been found in the analysis of multifragmentation experiments in nuclear physics. Nuclei are "two component systems", i.e., they are composed of protons and neutrons. Nuclei can be characterized by their mass A and their proton content Z. The neutron content is then given by $N = A - Z$. If one performs two sets of nucleus-nucleus collisions at a given energy E, in which the first set involves partners characterized by $A_1(Z_t)$ and the second by $A_2(Z_t)$ with, say, $A_2 > A_1$ (notice that being Z_t the same in both cases $N_2 > N_1$), it is found that the following relation holds:

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} \propto \exp(\alpha N + \beta Z) \qquad (1)$$

with $Y_i(N, Z)$ representing the number of fragments characterized by (N, Z) that are produced in the nucleus collisions involving partners characterized by $A_i(Z_t)$; α and β are fitting parameters.

In the past, this power law expression for R_{21} has been linked, under diverse approximations, to primary yields produced by disassembling infinite equilibrated systems in microcanonical and grand canonical ensembles [1,2], as well in canonical ensembles [3], and it has also been observed in the framework of the grand-canonical limit of the statistical multifragmentation model [4], in the expanding-emitting source model [1], and in the antisymmetrized molecular dynamics model [6]. Furthermore, under these approximations, the isoscaling parameters α and β have been found to be related to the symmetry term of the nuclear binding energy [4,5], to the level of isospin equilibration [7], and to the values of transport coefficients [8].

More recently, however, it has been determined through the use of molecular dynamics [9] that isoscaling can exist in purely classical systems, and that it can be created in systems out of equilibrium. It was also found, among other things, that R_{21} can maintain the power-law behavior even when it contains yield contributions generated at different times and corresponding to diverse thermodynamic conditions.

This communication presents results of the isoscaling analysis performed in the framework of simple geometrical models. It studies the ratio of the number of fragments produced by breaking bonds of two sets of lattices in D dimensions, of sizes A_1 and A_2 in which the nodes are

assigned a given color out of N colors (this is usually referred as polychromatic percolation [10]). The determination of the colors of the nodes can be performed in an independent way or in a correlated way. It is found that when the bond breaking probabilities are the same for both lattices a generalized version of Eq. (1) is obtained. It is shown that the presence of correlations affects the numerical values of the exponents in Eq. (1). In order to get analytical expressions a development in terms of bond lattice animals is performed.

In Sec. I, it is shown that the functional relationship R_{21} can be obtained in a simple generalized percolation problem, as the limit for two dimensions and two colors of the problem of calculating the ratio of yields of fragments of D dimensional, N colors and any regular topology for the corresponding lattices.

In Sec. II the effects of introducing correlations in the assignment of colors to the nodes in the percolation problem is explored. For this purpose, the study focuses on a simple square lattice in two dimensions and two colors (hereafter we replace the two colors notation for "protons" and "neutrons" to recall the nuclear origin of this problem). This is accomplished by performing a bond percolation problem on this lattice for which the distribution of protons and neutrons is generated using a lattice gas approach. The lattice gas is defined via different values of the interaction terms for the different isospin pairs present in the system, i.e., V_{nn} the interaction between two nearest neighboring neutrons, V_{pp} for two protons and V_{np} for a neutron proton pairs. For this purpose Metropolis Monte Carlo simulations are performed at different temperatures. It is shown that the strength of the correlation between neutrons and protons is a function of the temperature and the strength of the interaction between "protons" and "neutrons". It is shown that the isoscaling property is preserved and that the effect of the correlations shows in the value of the exponents α and β.

II. GEOMETRY

A. *G*-isoscaling

In this section we will derive exactly the relation R_{21} for the fragmentation of two two-dimensional lattices with nodes occupied with "protons" and "neutrons". This case, proposed in Ref. [9], is a limiting case of the more general problem of bond percolation of two D-dimensions regular lattices of any topology with nodes labeled by N colors.

Considering a square D-dimensional lattice with nodes occupied by "particles" of N colors. The total number of nodes is A and $A = N + Z + R + \ldots$ with $N, Z, R, \ldots$ denoting the number of nodes of given colors. Each color is assigned with a given probability p_Z, p_N, p_R, etc., with $1 = p_Z + p_N + p_R + \cdots$. At first these probabilities are taken to be independent and homogeneous.

Fragments are produced when we perform a bond breaking process characterized by a bond breaking probability b. A fragment is defined as a subset of nodes such that for any member of this set there is at least another member of the set such that both are linked by an active bond. Moreover, there are no active bonds linking any member of the set with other nodes in the lattice. In this way fragments can be characterized by the number of nodes that compose the fragment. The total number of fragments of size s will be denoted as N_s. For an infinite lattice the number of fragments of size s will diverge. In this limit it is convenient to consider the quantity n_s which is the number of clusters of size s per site, which can be written

$$n_s = \lim_{L \to \infty} \frac{N_s}{L^d}. \tag{2}$$

With L the linear dimension of the lattice under consideration measured in "*bonds*."

Disregarding the coloring of the nodes, n_s can be written as

$$n_s = \sum_{a,t} g_{sat} (1 - b)^a b^t \tag{3}$$

with t the number of bonds that are to be broken in order to isolate the s nodes composing a fragment, a the number of bonds that are to be kept in order that the resulting fragment of size s is connected and g_{sat} is the number of geometrical ways in which such a fragment can be built in the lattice.

When the nodes are colored, the previous expression adopts the form

$$n_s = \left[\sum_{a,t} g_{sat}(1-b)^a b^t\right]\left[\sum_{\substack{\{Z,N,R...\} \\ Z+N+R+...=s}} \alpha_{Z,N,R...}(p_Z)^Z (p_N)^N (p_R)^R \cdots\right] \tag{4}$$

with $\alpha_{Z,N,R...} = (s!/N!Z!R!\ldots)$ being the degeneracy of the configurations when all possible colors combinations are considered.

Considering now those fragments characterized by a given number of nodes of color $Z, N, R \ldots$

$$n_s(Z,N,R,\ldots) = \left[\sum_{a,t} g_{sat}(1-b)^a b^t\right]\left[\alpha_{Z,N,R...}(p_Z)^Z (p_N)^N \ldots\right]. \tag{5}$$

The general expression of the isoscaling coefficient R_{21} adopts the form

$$\frac{n_{s_2}(Z, N, R, \ldots)}{n_{s_1}(Z, N, R, \ldots)}$$
$$= \frac{\left[\sum_{a,t} g_{sat}(1-b)^a b^t\right]\left[(p_{Z_2})^Z (p_{N_2})^N (p_{R_2})^R \ldots\right]}{\left[\sum_{a,t} g_{sat}(1-b')^a b'^t\right]\left[(p_{Z_1})^Z (p_{N_1})^N (p_{R_1})^R \ldots\right]}. \tag{6}$$

In this general expression the bond breaking probabilities are different for the two lattices considered.

If the bond breaking probability (this will be referred as the equilibrium solution) is fixed to be the same for each of the two lattices, i.e., $b = b'$, it is obtained

$$\frac{n_{s_2}(Z, N, R, \ldots)}{n_{s_1}(Z, N, R, \ldots)} = \frac{(p_{Z_2})^Z (p_{N_2})^N (p_{R_2})^R \ldots}{(p_{Z_1})^Z (p_{N_1})^N (p_{R_1})^R \ldots} \tag{7}$$

which can be rewritten as

$$R_{21}(N, Z, R, \ldots) = \left[\frac{p_{Z_2}}{p_{Z_1}}\right]^Z \left[\frac{p_{N_2}}{p_{N_1}}\right]^N \left[\frac{p_{R_2}}{p_{R_1}}\right]^R \ldots \tag{8}$$

which can further be written as

$$R_{21}(N, Z, R, \ldots) = \exp(Z\beta + \alpha N + \delta R + \ldots) \tag{9}$$

with

$$\beta = \ln\frac{p_{Z_2}}{p_{Z_1}}; \quad \alpha = \ln\frac{p_{N_2}}{p_{N_1}}; \quad \delta = \ln\frac{p_{R_2}}{p_{R_1}}; \cdots.$$

For the finite case, the number of fragments of size s in the system of size l^d can be approximated as An_s to yield

$$R_{21}(N, Z, R, \ldots) = \frac{A_2}{A_1}\frac{n_{s_2}(N, Z, R \ldots)}{n_{s_1}(N, Z, R \ldots)} \tag{10}$$

which upon multiplying and dividing by Z_t, with Z_t the number of nodes with color Z, gives

$$R_{21}(N, Z, R, \ldots) = \frac{p_{Z_1}}{p_{Z_2}}\frac{n_{s_2}(N, Z, R \ldots)}{n_{s_1}(N, Z, R \ldots)}. \tag{11}$$

From which the value of the constant of proportionality between R_{21} and the quotient of the yields can be readily obtained

$$R_{21}(N, Z, R, \ldots) = C \exp(Z\beta + \alpha N + \delta R + \ldots) \tag{12}$$

with

$$C = \frac{p_{Z_1}}{p_{Z_2}}.$$

This relation (here denoted as gisoscaling to indicate that it is a generalization of the standard isoscaling property) reduces to the usual isoscaling expression when we consider two colors.

$$R_{21}(N, Z) = C \exp(\beta Z + \alpha N). \tag{13}$$

It is convenient to recall at this point the main assumptions used so far in order to get the usual isoscaling law:

(i) the probabilities of color assignation to the nodes are independent,

(ii) the bond breaking probabilities are the same for the two lattices under consideration.

B. Correlations

The previous section presented a derivation of the isoscaling relation with the simple assumption that the isospin character of each node is assigned in an homogeneous independent way. Here, the consequences of introducing correlations in the model are explored.

For this purpose we perform the bond percolation problem on lattices for which the isospin character of each node is obtained in the frame of the lattice gas model. In other words, we perform the bond breaking fragmentation on the one hand, and then (the order of the operations in irrelevant) we assign its isospin character from the outcome of a Monte Carlo calculation at temperature T on an equivalent lattice gas model. It should be emphasized at this point that the role of temperature T in this calculation is only to control the degree of correlation in the isospin character assignment. We then adopt the following scheme: protons and neutrons interact according to the following prescription for nearest neighbors (the values are standard for the nuclear case):

$$V_{np} = -5.33, \quad V_{nn} = V_{pp} = -a.$$

V_{nn}, V_{pp} and V_{np} are measured in MeV.

The size of the two lattices needed to perform the ratios is fixed as $1 \rightarrow 16 \times 16$ with 128 protons, this giving a relative occupation of 50% and $2 \rightarrow 20 \times 20$, and then the relative population of protons is 32%.

The isoscaling for $a = 0$ and different temperatures is now calculated; Fig. 1 shows a typical isoscaling result.

Figure 2 shows the mean value of the number of nearest neighboring protons of a proton $\langle pp \rangle_{nn}$. It can be seen that as the temperature is increased, the value of $\langle pp \rangle_{nn}$ approaches 2 for the $(16 \times 16)_{128}$ lattice, i.e., converges to the uncorrelated limit. On the other hand as the temperature is reduced it approaches the value 0 which corresponds to the minimum energy at $T = 0$.

This can be correlated with the value of the exponents α and β as displayed in Fig. 3.

It is worth noticing at this point that this calculation at different temperatures is equivalent to calculating the lattice gas problem at fixed temperature but changing the value of, for example, a.

Figure 4 compares the isoscaling behavior for $a = 0$ (open symbols) and $a = -4$ (full symbols) (see figure caption for details). In this case the temperature of the underlying lattice gas has been fixed to $T = 10$. It is immediate that when a grows, which means that the difference in binding energy
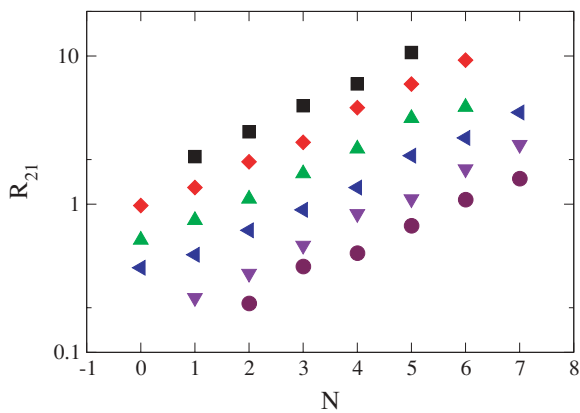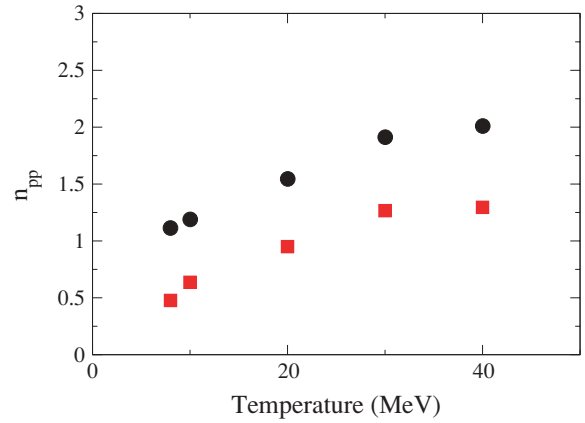

FIG. 2. (Color online) Mean number of nearest neighboring protons to a proton. Black circles for a 16X16 lattice and 128 protons. Red square for a 20X20 lattice with the same number of protons.
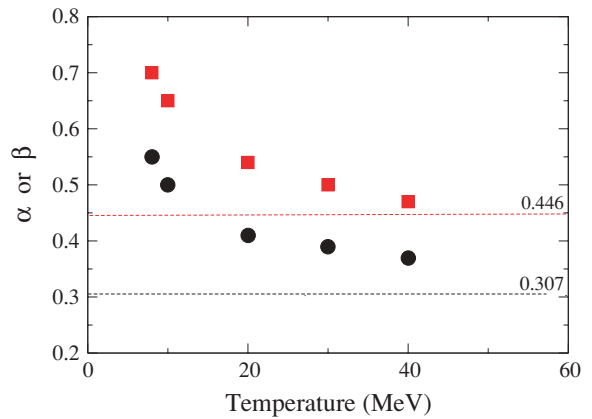

FIG. 3. (Color online) α (black circles) and absolute value of β (red squares) coeficients as a function of temperature, dashed lines correspond to the values of the coefficients for the homogeneous percolation problem or to the infinite temperture limit.
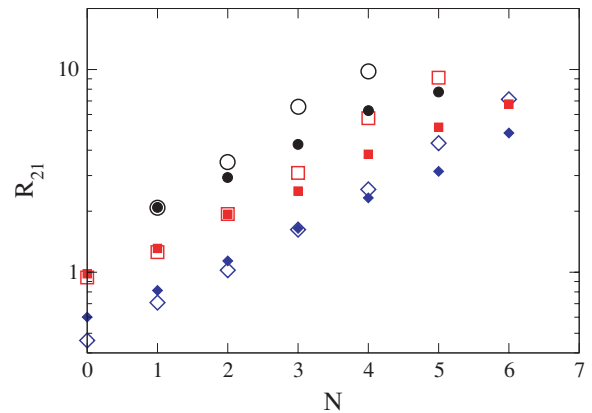

FIG. 1. (Color online) Typical result of the calculation of the quotients in R_{21}.

FIG. 4. (Color online) Isoscaling deppendence on a. Open symbols stand for $a = 0$, full symbols represent $a = -4$.
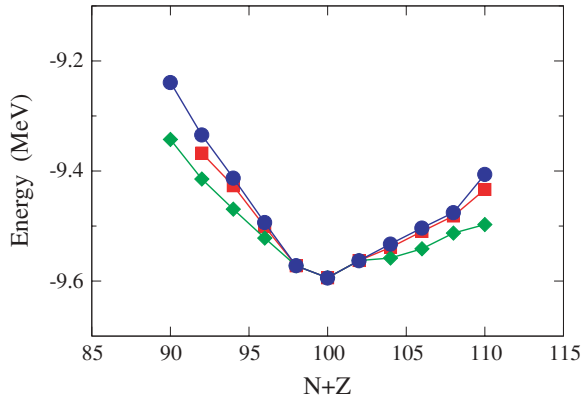
FIG. 5. (Color online) Binding energies for different values of a for $Z = 50$ and $40 < N < 60$. Blue circles stand for results with $a = 0$, red squares for $a = -3$ and brown diamonds for $a = -4$.

between pairs neutron-proton, proton-proton, etc., is lowered, the isoscaling coefficients decrease, as happens when the temperature is raised.

As the previous results can be associated to the "symmetry effect," the variation of the ground state energy will be studied with fixed number of "protons" and varying the number of "neutrons."

In order to calculate the ground state energy for given values of (N, Z) a calculation in the spirit of simulated annealing was done. In such a calculation a Markov chain in the space of configurations is performed. The Markov chain is composed of steps consisting in the exchange of the "state" of two randomly chosen cells of a lattice with $M \gg (N + Z)$ cells. The "state" of a cell can be occupied by a proton, occupied by a neutron or empty. Using the same transition probabilities as in standard Metropolis Monte Carlo calculations, this Markov chain develops with an effective temperature parameter that is lowered in an exponential way ($\tau' = \alpha\tau$, with $\alpha \lesssim 1$). The

calculation is stopped when a lower limit in τ is reached, or when no better solution is obtained in a given number of steps.

Figure 5 shows that the binding energy curve gets flatter when the value of a is increased thus indicating a smaller "symmetry effect."

III. CONCLUSIONS

In this work the generalized isoscaling property of fragmenting systems has been explored. It has been shown that the property initially detected for nuclear collisions is in fact a particular case of a general property of fragmenting systems. Likewise, it has been shown that it can be analytically obtained from the calculation of the ratios of yields of "fragments" produced in d-dimensional lattices of different sizes and nodes labeled with arbitrary colors when a bond percolation mechanism is applied. Two conditions are to be satisfied for this to be true: (a) the bond braking probabilities are to be the same in both lattices, and (b) the probabilities of assignation of the labels (colors) are independent.

It has also been explored what happens when condition (b) is not fulfilled. When the occupation probabilities are not uncorrelated, but homogenous, it is observed that the property is preserved, but the values of the parameters α and β are a function of the correlations appearing in the system. These correlations are a function of the temperatures of the system as well as of the interaction energy in the corresponding lattice gas used to generate the labeling. In the second case it is clear that the values of the parameters in the isoscaling relation are related to the "symmetry" energy term.

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