

Resonances on metallic compound transmission gratings with subwavelength wires and slits

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Abstract

We investigate the response of gratings formed by subwavelength wires in a dielectric medium. For gratings with a single wire per period, most of the incident power is transmitted through the structure. If the distance between adjacent wires is small enough, the excitation of waveguide modes can lead to reflection resonances. However, only when the period is formed by several wires (compound grating), new degrees of freedom are introduced for the phase distribution of the field inside the slits, allowing a new class of resonances analyzed in this letter. We give numerical examples illustrating the combined effect produced by both types of resonances in the grating response.

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1. Introduction

In the last few years, owing to experimental evidence of extraordinary transmission in thick metallic plates with holes [1,2], much research has been reported on the transmission response of gratings with subwavelength slits. To better understand the origin of this phenomenon, different approaches considering 1D structures and single slits have been proposed [3–10]. Within this frame, metallic gratings with narrow longitudinal slits have been analyzed, and two main mechanisms were identified as responsible for the enhanced transmission: surface plasmon excitations and coupling to waveguide modes of the slits [3,6]. However, when the period of the grating comprises several wires and slits (compound grating), phase resonances can occur [11]. These resonances, first reported in structures comprising a finite number of cavities on a perfect conductor [12,13], have been recently shown to significantly modify the transmission response of compound gratings, especially

near waveguide resonances [14]. For particular wavelengths, the field inside the slits may adopt special configurations where the phases in adjacent cavities are equal or opposite to each other, and, at the same time, the inner field is maximized. In the case of a simple grating – formed by a single groove or slit in the period – the pseudoperiodic condition implies that, with the exception of a phase factor, all the grooves have the same field. The addition of cavities/slits to the period of the grating introduces new degrees of freedom regarding the possible near field configurations. Thus, a compound grating allows for different phase configurations inside each period, a fact that can lead to resonances and to sudden variations of the diffracted efficiency. It has been shown that phase resonances appear not only in perfectly conducting gratings [11,15], but also in highly (but finitely) conducting structures [16].

In this paper, we analyze the reflected and transmitted response of a wire compound grating. In particular, we study the influence of two different mechanisms on the diffraction efficiencies: (i) waveguide modes and (ii) phase resonances excited in the structure. We show that coupling to waveguide modes, which can only occur for gratings with at least two wires per period, produces a minimum in the transmitted intensity (a maximum in the reflection). This minimum could

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even be zero. When the number of wires/slits increases, a splitting of the waveguide resonance occurs. This phenomenon is explained in terms of phase resonances, i.e., particular distributions of the phase and the magnitude of the field in adjacent slits. As a net effect, the transmission minimum broadens, but the broadening is accompanied by the appearance of maxima in the same spectral region. We also show that when more wires/slits are added to the period of the structure, more phase configurations are allowed, producing more minima within each resonant dip.

2. Configuration and modal approach

The structure under study is a periodic grating (period d), and each period comprises several wires (and slits), as shown in Fig. 1. If J denotes the number of wires in the period, this structure has $J-1$ equal-width slits plus another slit with a different width. The height of the wires is h , their width is a and the spacing between wires (the width of the slits) is c . In grating (a) there is a single wire in the period, and we call it simple grating. For the gratings considered here, the width of the wires is much smaller than the period, and then most of the structure is empty. In grating (b) there are two wires in the period, such that a subwavelength slit is formed between them. More slits are formed as J increases, as shown in cases (c) and (d). The maximum number of slits that can be added is determined by the period, the wire width and the separation between wires.

The compound characteristic of the period gives rise to the phase resonances. These structures differ from those studied in Ref. [14] in the composition of the unit cell: while in the present case the array is formed by subarrays of equal wires separated by a wide hole, in Ref. [14] each period comprises several subwavelength slits in a thick metallic

screen. Moreover, while in the present case we deal with structures which are essentially transmitting, the structures considered in Ref. [14] are essentially reflecting.

To solve the diffraction problem, we use an extension to compound metallic gratings of the modal method developed in [17] for simple transmission wire gratings. Basically, the method consists in expanding the fields inside the slits in terms of their eigenfunctions, and the field outside the structure in terms of Rayleigh expansions. As in the case of simple transmission wire gratings [17], the surface impedance boundary condition (SIBC) is applied to account for losses in the metallic walls [18,19]. Finding the modes of the slits requires the numerical solution of an eigenvalue equation [17]. After matching the fields at the upper and lower interfaces, the system of equations is projected in convenient bases, and a matrix system for the unknown reflected and transmitted amplitudes is obtained.

3. Results

We consider a p-polarized plane wave of wavelength λ incident upon the grating. In Figs. 2 and 3, we show a set of curves corresponding to the transmitted (Fig. 2) and the reflected (Fig. 3) zero-order intensity for a normally incident p-polarized plane wave and for different numbers of wires in the period (J). The parameters of the gratings are: $a/d = 0.05$, $h/d = 0.7$, $c/d = 0.04$ (for $J > 1$), and the metal refraction index $\nu = 0.15 + i4.9$. The lower curve in Fig. 2(a) shows an almost perfect transmission (greater than 97%) for the simple grating in the entire range of the wavelengths considered, a result that could have been expected since this grating has only a single subwavelength wire per period, the separation between wires being $d \gg a$. However, when a subwavelength slit is formed in the grating by adding a second wire, two well-defined minima appear in the transmitted response, while two well-defined maxima appear in the reflected response. These minima in the transmission response correspond to waveguide resonances of the slit, and can give basically null transmission and high reflectivities (part of the power is absorbed by the metallic structure at those wavelengths). If a perfectly conducting metal had been considered, these minima would have been located at $\lambda = 2h/n$, with n being a positive integer. In the wavelength range considered in Fig. 2, we observe the minima corresponding to the first two modes (longer wavelengths). Within this simplified model, the minima in this example should be at $\lambda/d \approx 1.4$ and 0.7 . From the results in Fig. 2, it is evident that this is not a good estimate for the resonant wavelengths when a finitely conducting structure is considered. In this case, a better estimate of the resonant wavelengths can be obtained if we require

$$\text{Re}\{v_m h\} = n\pi, \quad (1)$$

where n is an integer, v_m is the separation constant of the Helmholtz equation in the y direction:

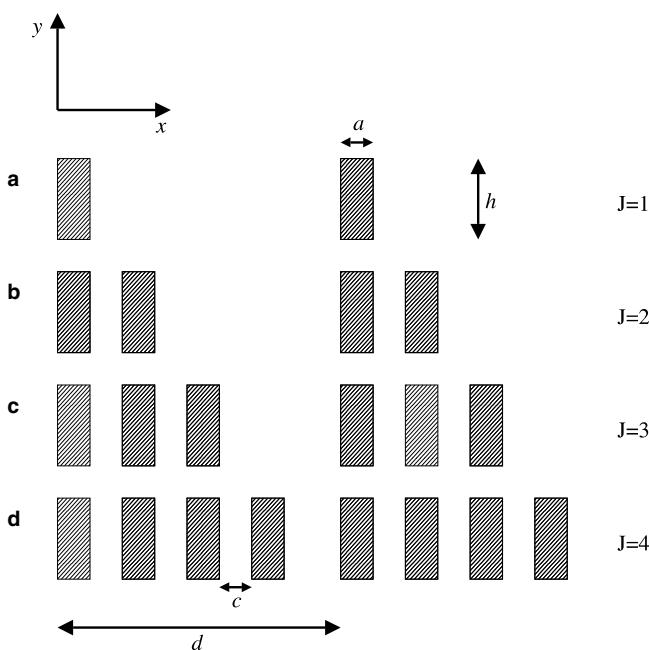


Fig. 1. Scheme of the simple and compound gratings.

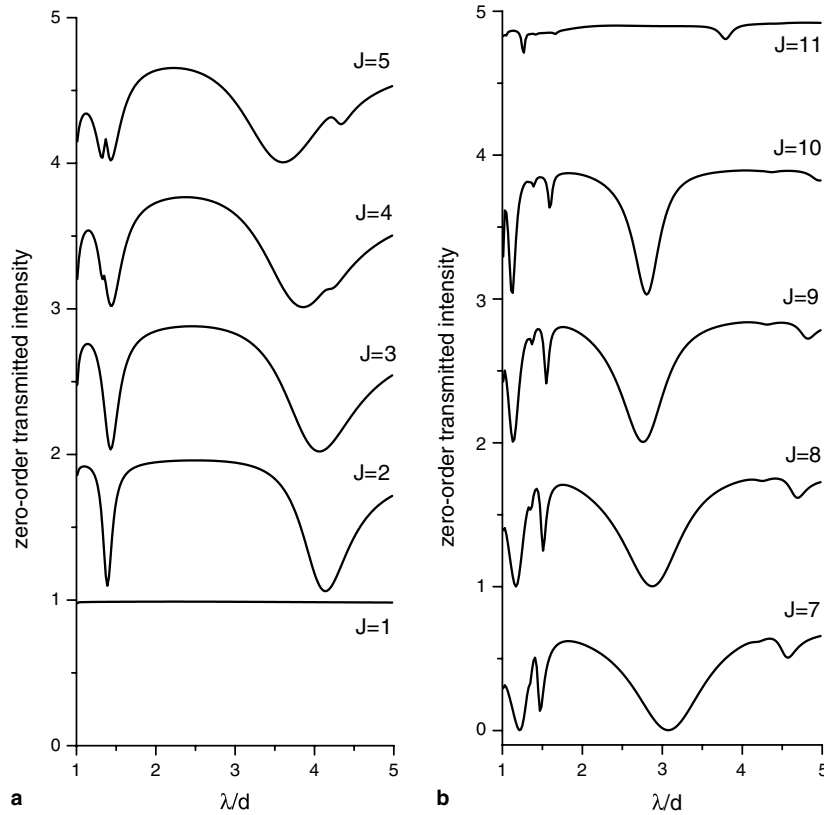


Fig. 2. Zero-order transmittance as a function of the wavelength for a normally incident, p-polarized plane wave impinging on a metallic grating with $a/d = 0.05$, $c/d = 0.04$ and $h/d = 0.7$. The refraction index of the metal is $\nu = 0.15 + i4.9$. The different curves correspond to different numbers of slits in the period: (a) $J = 1, 2, 3, 4, 5$; (b) $J = 7, 8, 9, 10, 11$.

$$v_m = \sqrt{k^2 - u_m^2} \quad (2)$$

and u_m are the solutions of the eigenvalues equation resulting from the application of the SIBC to the fields on the walls of the slit [17]:

$$\tan u_m a = \frac{2\eta u_m}{u_m^2 - \eta^2}, \quad (3)$$

where $\eta = k/(iv)$, $k = 2\pi/\lambda$. For the subwavelength slits, only the first mode ($m = 1$) is a propagating mode. Within this model, the resonant wavelengths for this case are $\lambda/d = 3.72$ ($m = 1, n = 1$) and $\lambda/d = 1.21$ ($m = 1, n = 2$). Although these values do not correspond exactly to the location of the minima in Fig. 2(a), they give a much better estimate. The curves for $J = 3$ (three wires, two slits) in Figs. 2 and 3 are very similar to those for $J = 2$. However, for $J = 4$ (three slits) the curves exhibit a different feature: each resonant dip/peak slightly widens and starts to split into two. For larger values of J , this effect becomes even more pronounced, as shown in the upper curve of Fig. 2(a) (near $\lambda/d \approx 1.5$ and 4) and in Fig. 2(b). Furthermore, the waveguide resonances split into two or more resonances which seem to repel each other.

The physical origin of these splittings can be explained in terms of phase resonances, as in the case of compound transmission gratings with all subwavelength slits [14]. For a perfectly periodic simple grating, the fields in all slits/cavities are

basically equal, i.e., with the same amplitude and with phases cascaded according to the Floquet condition. However, when the period is formed by more than one slit, different amplitude and phase configurations of the field within the slits are allowed. These phase configurations give rise to the splitting of the resonances, and also to field enhancements inside the slits. Adding more slits increases the number of possible phase configurations, producing more splittings in the resonances, as can be observed in Figs. 2 and 3. For the normal incidence case considered here, not all the combinations of phases are allowed. The possible phase configurations are restricted to those that are symmetric with respect to the symmetry axis of the period parallel to the y -axis. For two slits ($J = 3$), the phases must still be equal, and there is no splitting in the transmitted and reflected responses of the structure (see Figs. 2(a) and 3). However, for three slits in the period ($J = 4$), we can have a phase mode with the same phase in the external slits, and a different phase in the central one. If the phase difference between adjacent slits is π , the phase resonance is called π -resonance, and in this case the influence of the resonance on the response of the structure is usually maximized [13,11]. For $J = 4$, we have the first manifestation of the phase resonances in the curves. For $J = 5$, the effect is more pronounced, but there is still only one phase configuration allowed: the phases in the external slits are equal to each other but different from those at the central slits. For $J > 5$, more symmetric configurations can

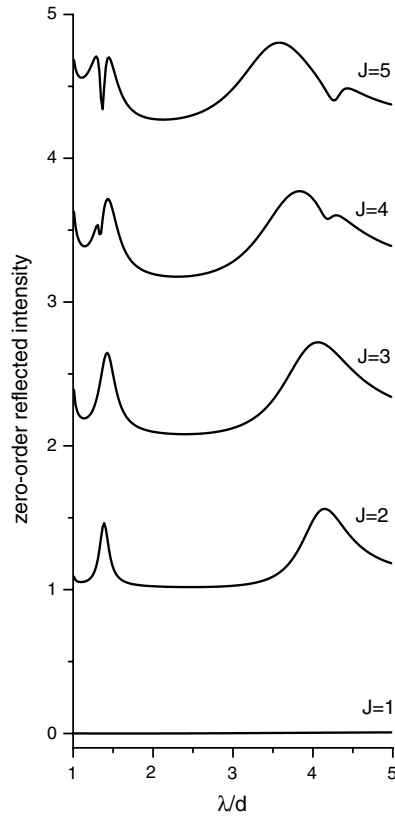


Fig. 3. Zero-order reflectance as a function of the wavelength for the same parameters of Fig. 2, and for $J = 1, 2, 3, 4, 5$.

be found, and this fact is exhibited in Fig. 2(b), where it can be observed that for $J = 7$, a new anomaly is found between each pair of splitted dips corresponding to the original waveguide resonances. Notice that this anomaly becomes more marked as J increases, forming a new dip for $J > 8$. For $J = 11$, the grating becomes “almost” a simple grating, i.e., almost the entire period is filled with wires and slits, and then the true period of the grating is now $d' = a + c \approx d/11$.

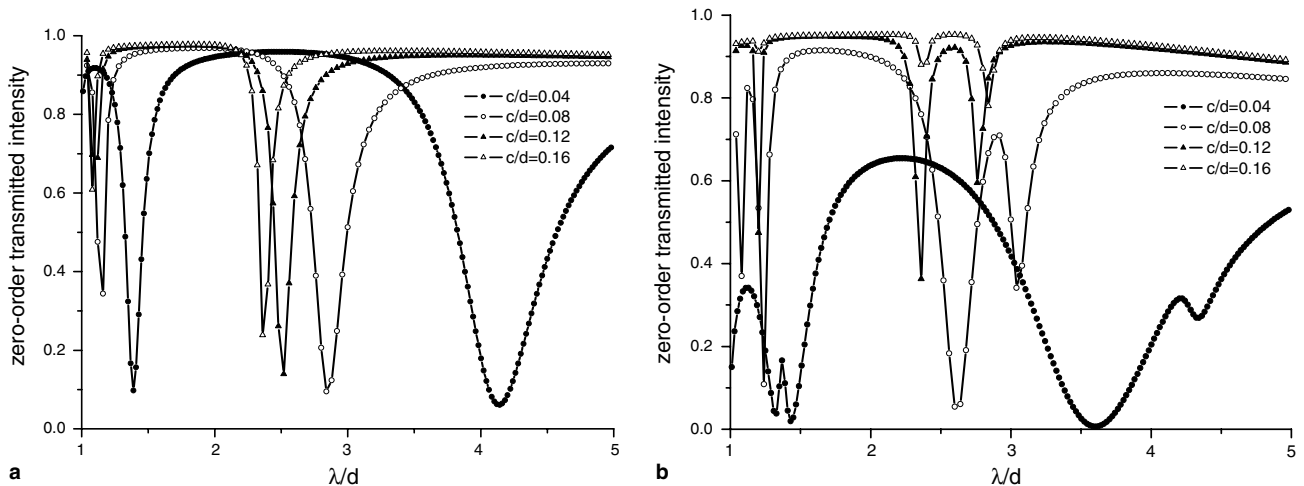


Fig. 4. Zero-order transmittance as a function of the wavelength for $a/d = 0.05$, $h/d = 0.7$, $v = 0.15 + i4.9$, and for different slit widths c/d . (a) $J = 2$; (b) $J = 5$.

Therefore, no phase resonances are allowed, and the structure becomes essentially transmitting again.

To get some insight into how the variations in the geometrical parameters affect the phenomenon of phase resonances, we performed transmittance curves for varying slit widths (Fig. 4) and thicknesses (Fig. 5), keeping the period d fixed. When no phase resonances are allowed, such as in the $J = 2$ case, where the period comprises two wires and a single thin slit, an increase in the slits width produces a shift and a narrowing of the waveguide resonances, as observed in Fig. 4(a). This shift can be understood if we take into account that the separation constants v_m , which actually govern the location of the minima, vary with the width, as explained above (see Eq. (1)). The quantity $v_m h$ takes into account the losses and the penetration of the field inside the metal. In Fig. 4(b), we show the transmittance curve but for $J = 5$, in which case phase resonances

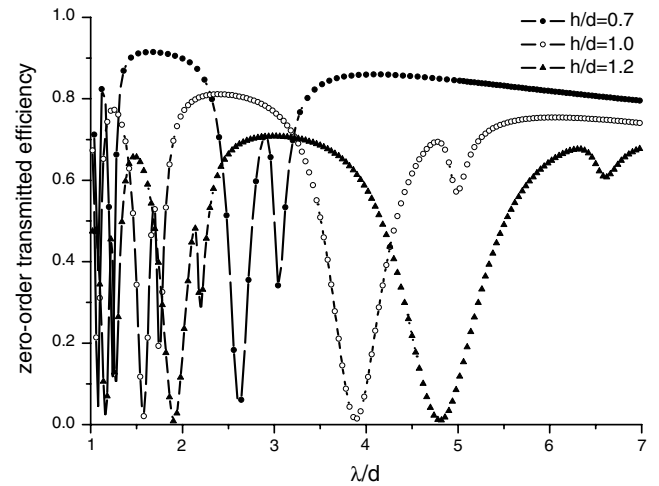


Fig. 5. Zero-order transmittance as a function of the wavelength for $J = 5$, $a/d = 0.05$, $c/d = 0.08$, $v = 0.15 + i4.9$, and for different thicknesses h/d .

occur. It is clear that the splitting of the waveguide resonances appear in all the cases, suggesting that this phenomenon is not highly dependent on the slit width, within a subwavelength range. Moreover, the maximum inside the dip can reach up to 90% of the incoming power, as for $c/d = 0.12$ and 0.16 (filled and empty triangles in Fig. 4(b), respectively). Notice that in the latter case the five wires and the four narrow slits occupy most of the period ($(5a + 4c)/d = 0.89$) and then, the remaining slit is even narrower than the four equal slits.

The dependence of the resonance on the thickness of the wires is illustrated in Fig. 5, where we show curves of transmitted intensity vs. wavelength for $J = 5$ and for various values of h/d . As the thickness increases, the waveguide resonance dips – which roughly correspond to an integer number of half-wavelengths contained in the vertical slits – shift towards longer wavelengths. Consequently, since the phase resonances always appear within these dips, they also move. It can be observed that the resonance minima widen as h increases, and the split dips separate from each other. Even though the shape of the curve is modified, the phase resonances are present for all the values considered, suggesting that the structure could be designed for the obtention of a particular response. For smaller thicknesses, the resonances shift to shorter wavelengths until a limit value is reached, at which the holes can no more be regarded as slits but as apertures in a metallic thin sheet, with their depth comparable to their width, and then vertical waveguide modes are not allowed (not shown).

In Fig. 6, we analyze the influence of the refractive index of the metal in the generation of phase resonances. It is evident that both the real and the imaginary parts of ν modify the location as well as the width and depth of the resonances. For small values of $\text{Im}(\nu)$, the absorption of the structure increases, producing a general decrease in the transmitted response. However, the phenomenon of phase resonances is still present when the conductivity of the metal is high enough.

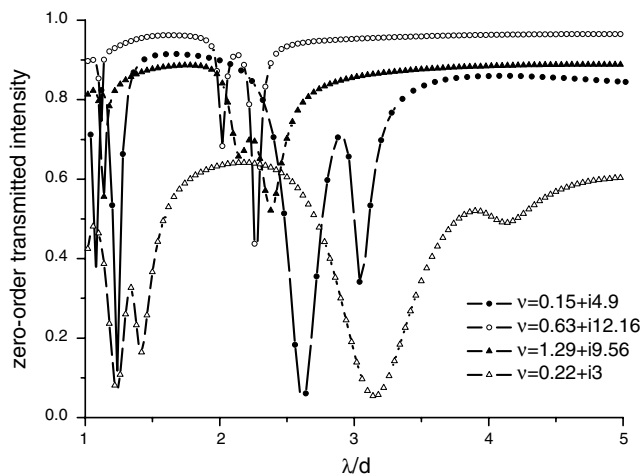


Fig. 6. Zero-order transmittance as a function of the wavelength for $J = 5$, $a/d = 0.05$, $c/d = 0.08$, $h/d = 0.7$, and for different metal refractive indices ν .

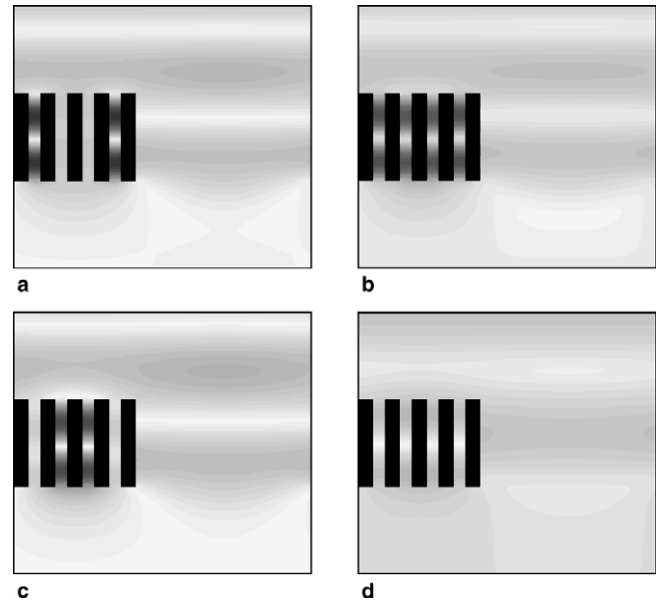


Fig. 7. Magnitude of the tangential magnetic field for a normally incident p-polarized plane wave impinging on a metallic grating with $a/d = 0.05$, $c/d = 0.04$, $h/d = 0.7$, and $J = 5$. The refractive index of the metal is $\nu = 0.15 + i4.9$. (a) $\lambda/d = 1.33$, (b) $\lambda/d = 1.37$, (c) $\lambda/d = 1.43$ and (d) $\lambda/d = 2$.

To visualize the field distributions that can be obtained with compound gratings, in Fig. 7 we show contour plots of the near magnetic field in the whole period, for the case $J = 5$, i.e., when the period comprises five wires, four equal subwavelength slits, and a wide hole (these parameters correspond to the upper curve of Fig. 2(a) and Fig. 3). We show the field maps at different wavelengths within the second waveguide resonance ($\lambda/d \approx 1.4$). Black zones represent the most intense fields, and the same grey scale is kept in the four maps for the sake of comparison. For the first minimum in the transmitted intensity ($\lambda/d = 1.33$), there is an enhancement of the field in the external slits with respect to the central ones (Fig. 7(a)). In this case, the phases of the field at the outer and inner slits are opposite to each other. For $\lambda/d = 1.37$ (peak in the transmission response, see Fig. 2(a)), the magnitude of the field is similar in all slits (Fig. 7(b)), and the phase distribution is similar to that of the previous case. For the second minimum ($\lambda/d = 1.43$), the fields at all slits are in phase, and there is an enhancement of the field in the central slits (Fig. 7(c)). Finally, for $\lambda/d = 2$ (non-resonant wavelength) we can observe that the field inside the narrow slits is much weaker than for all the other wavelengths.

4. Summary and conclusions

We have investigated the response of gratings formed by subwavelength wires in a dielectric medium. We have shown that, in the vicinity of waveguide mode resonances of the structure, new peaks and dips are generated and, at the same time, the field is enhanced within the slits. These features have been explained in terms of phase resonances, which allow for different phase and amplitude

distributions of the field inside the slits. We have shown that transmission and reflection responses may exhibit particular characteristics that are not present in simple gratings, thus adding new degrees of freedom for the design of optical devices. Based on the mechanism of phase resonances, structures with a prescribed number of resonances at specified wavelengths could be designed.

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