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Field Interaction and Anomalies in Astrophysical and Cosmological Phenomena

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The WKB expansion of bosonic and fermionic interacting fields in general curved spacetimes is computed. The explicit expression of the loss of energy in terms of macroscopic quantities like densities and velocities, according to the corresponding spinorial structure, is obtained. Analyzing experimental data of the PSR 1913+16 pulsar, we propose an interpretation of the rotational energy decay and estimate the possible bosonic-fermionic interaction strength capable of producing it. We use these results to discuss some cosmological anomalies related to red-shifts and energy decays giving alternative interpretations.

KEY WORDS : Energy decay ; field interaction ; pulsar

1. INTRODUCTION

According to the Friedmann–Robertson–Walker model (FRW) clusters follow geodesics corresponding to a geometrical background in expansion. This successful model nevertheless has well-known anomalies. The aim of this article is to apply some previous results obtained in non-Riemannian geometries and to put forward a theoretical scheme in which one astrophysical phenomenon and some cosmological anomalies could be better understood.

In [1,2] we introduced an energy-momentum density vector which is independent of the affine structure of the manifold, defined Hamilton-

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ian and momentum and solved some ambiguities in the definition of local energy-momentum. As a consequence, we were able to define an Equivalence Principle for manifolds with torsion as a natural extension of the Equivalence Principle of General Relativity which predicts, in any scenario, geodesic motion for freely falling particles; therefore, any deviation from geodesic trajectories should be attributed to a non-conservative behavior. Our idea is to discuss some well-known dispersions from standard trajectories in cosmological phenomena, attributable to the presence of spinorial matter interacting with the cosmic geodesic dust taken to be the source of the Einstein equation. To face this problem we need to determine the strength with which different spin matter fields may couple in curved space-time. We choose the case of the binary pulsar applying to this orbiting system our non-conservative property by linking accelerated observers with observable parameters such as orbital periods and energy decays and recognizing spin-0–spin-$\frac{1}{2}$ fields interactions as the cause of some kind of energy loss. So, from the well-known data of the pulsar PSR 1913+16, we obtain the coupling constant corresponding to this type of interaction whose value we suppose to be the same in any interaction of this class while taking place in a cosmological level.

2. ENERGY-MOMENTUM DENSITY AND LOCAL CONSERVATION

In [1] we obtained, for the case of very general manifold, the expression of the total energy-momentum density $\mathcal{F}_\varepsilon$ measured by a system of observers with four-velocity $\bar{u}(x) = \bar{\varepsilon}(x)$:

$$\mathcal{F}_\varepsilon = T^{\mu\nu} \nabla^{\{\}}_{\mu}\, \varepsilon_\nu(x) = T^{\mu\nu} \mathcal{L}_\varepsilon g_{\mu\nu}\,, \tag{1}$$

where $\varepsilon(x)$ is a time-like vector field which defines the observer, $\nabla^{\{\}}$ is the covariant derivative built up with the Christoffel symbols (in the case of non-Riemannian geometries these symbols do not necessarily represent the complete affine connection of the manifold, which may be not symmetric and may include torsion), $\mathcal{L}$ is the Lie derivative and $T^{\mu\nu}$ is the energy-momentum tensor which is independent of the affine structure of the manifold. $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = \frac{1}{g^{1/2}} \left(\frac{\partial L_{\mathrm{M}}}{\partial \partial_\mu \Psi^\alpha_a}\, \nabla^{\{\}}_\mu \Psi^\alpha_a - L_{\mathrm{M}} g^{\mu\nu} \right), \tag{2}$$

where L_{M} is the matter density Lagrangian. In [2] we applied the same formalism to the general case of an action including surface terms. Integrating (2) over time-like surfaces, we were able to define Hamiltonian and momentum over manifolds with torsion which admit global foliation. These results coincided with the corresponding ADM definitions for the case of irrotational Riemannian manifolds [3].

We also introduced a natural extension of the Equivalence Principle to non-Riemannian geometries, the *local holomicity property*:

$$\mathcal{L}_{\hat{\varepsilon}_A}\hat{\varepsilon}_B(x_0) = 0 \qquad \text{or equivalently} \qquad \mathcal{L}_{\hat{\varepsilon}_A} g_{\mu\nu}(x_0) = 0, \tag{3}$$

where $\hat{\varepsilon}_A$ is the tetrad. Property (3) does not imply a locally null connection but guarantees local energy-momentum conservation and avoids the problem of nonvanishing local complete affine connection due to non null torsion (see Refs. 4,5). The Equivalence Principle (3) predicts, for any case, geodesic motion for freely falling particles ($u^\mu \nabla_\mu^{\{\}} u^\nu = 0$) with $\mathcal{F}_\varepsilon = 0$; i.e. local energy-momentum conservation. So, any deviation from the geodesic trajectory must be attributed to non-conservative behaviors. (Instead, if freely falling particles were related to an Equivalence Principle based on a local vanishing of the tetrad connection ($\Gamma^C_{AB} = 0$), particles would follow autoparallels ($u^\mu \nabla^\Gamma_\mu u^\nu = 0$) [2] which coincide with geodesics in the case of null torsion.)

3. BOSONIC-FERMIONIC INTERACTION IN CURVED SPACE-TIME

The Lagrangian of a free integer (half-integer) spin field is of second (first) order in its derivatives. This means that in the highest — classical — order in its Laurent expansion, the corresponding Lagrangian will be of $\hbar^{-2}$ ($\hbar^{-1}$) order in the Planck constant. As a consequence, if in the classical level the interaction term is of order $\hbar^{-2}$ then trajectories of spin-0 particles will remain geodesics whereas trajectories of spin-$\frac{1}{2}$ particles will not [6]. This situation occurs if the interacting Lagrangian is of first order in one of the fields. In this case, this Lagrangian is of the form

$$\Lambda = \bar{k}^\mu(x)\lambda\partial_\mu\phi, \tag{4}$$

where the spinorial-vector functions components $\bar{k}^\mu(x)$ include the corresponding coupling constant and the γ's Dirac matrices in their definition. In order to guarantee the Lagrangian and the corresponding energy-momentum tensor to be real, we must choose the scalar field ϕ in (4) to be real and take a real representation of the Dirac matrices (i.e. $\gamma^1\gamma^2\gamma^3$ real, $\gamma^4 = i\gamma^0$ purely imaginary and $\gamma_5 = \gamma^1\gamma^2\gamma^3\gamma^4$ Hermitian and imaginary) so that $(\bar{k}\lambda)^* = \bar{k}\lambda$, with $\bar{k} = k^\dagger\gamma^4$. Due to the derivative in (4), Lagrangians are of $\hbar^{-1}$ order and only produce some effect in the motion of those particles associated with spin-$\frac{1}{2}$ fields. An important example of interaction (4) is the chiral supersymmetric Wess–Zumino matter multiplet (A, B, λ) in curved space-time. For this case $\bar{k}^\mu(x) = \frac{1}{2}\kappa\bar{\psi}_\sigma\gamma^\mu\gamma^\sigma$ [7] and Λ reads

$$\Lambda = \tfrac{1}{2}\kappa\bar{\psi}_\sigma\gamma^\mu\gamma^\sigma\lambda\partial_\mu(A + i\gamma_5 B), \tag{5}$$

where A and B are scalar and pseudo-scalar real fields respectively; λ is the spin-$\frac{1}{2}$ field, ψ is a spin-$\frac{3}{2}$ gravitino field and $\kappa^2 = 8\pi G$, with G being the gravitational constant.

4. THE ENERGY-MOMENTUM FLUX COMPUTATION

Via WKB expansions, in [6,8] we have related Quantum Field Theory in general curved space-time to perfect fluids variables such as densities and four-velocities. Classical Lagrangians corresponding to interacting spin-0–spin-$\frac{1}{2}$ fields were also determined. This last result, plus the one expressed in (1), will enable us to study, in the classical limit, the energy-momentum exchange between different matter fields defined over a curved geometry with any affine connection (symmetric or non-symmetric).

Let us consider the family of interactions (4). The energy-momentum term $T_{\rm I}^{\mu\nu}$ obtained from the corresponding interaction Lagrangian reads

$$T_{\rm I}^{\mu\nu} = \tfrac{1}{2}\bar{k}^{(\mu}(x)\lambda\partial^{\nu)}\phi - g^{\mu\nu}\bar{k}^\sigma(x)\lambda\partial_\sigma\phi. \tag{6}$$

On the other side, the WKB expansions corresponding to spin-0 and spin-$\frac{1}{2}$ fields read

$$\phi = \sum_{n=0}(-i\hbar)^n\phi_n e^{iS_{\phi_{\rm cl}}/\hbar} + \text{h.c.}, \tag{7}$$

$$\lambda = \sum_{n=0}(-i\hbar)^n\lambda_n e^{iS_{\lambda_{\rm cl}}/\hbar} + \text{h.c.}, \tag{8}$$

where h.c. indicates the Hermitian conjugate and $S_{\phi,\lambda_{\rm cl}}$ are the classical actions. Replacing these expansions in the respective kinetic and mass terms and in the interaction term (6) of the total energy-momentum tensor and keeping only the highest order, we obtain after a straightforward computation the classical energy-momentum tensor density term:

$$T_{\rm cl}^{\mu\nu} = \rho_{(\phi)}u^\mu_{(\phi)}u^\nu_{(\phi)} + \rho_{(\lambda)}u^\mu_{(\lambda)}u^\nu_{(\lambda)} + \tilde{\rho}^{1/2}_{(\phi)}(\tfrac{1}{2}\bar{k}^{(\mu}\tilde{\lambda}_0 u^{\nu)}_{(\phi)} - g^{\mu\nu}\bar{k}^\sigma\tilde{\lambda}u_{(\phi)\sigma}, \tag{9}$$

where u^μ is the four-velocity,

$$\tilde{\rho} = 2\rho\cos(S_{(\phi)}/\hbar) = 2m^2_{(\phi)}\phi_0^2\cos(S_{(\phi)}/\hbar)/\hbar^2,$$

$\tilde{\lambda}_0 = \lambda_0 e^{iS_{(\lambda)}/\hbar}$, with $\rho_{(\phi)} = m^2\phi_0\phi_0^*/\hbar^2$ and $\rho_{(\lambda)} = i\bar{\lambda}_0\lambda_0 m_{(\lambda)}/\hbar$; quantities $\rho_{(\phi,\lambda)}$ represent the energy densities and $\partial_\mu S_{(\phi,\lambda)}$ the canonical four-momentum of the scalar and spinorial fluids. In the free case, $\partial_\mu S_{(\phi,\lambda)} = m u^u_{(\phi,\lambda)}$, coinciding with the ordinal momentum. So flux (1) which is measured in the reference system fixed to the geodesic scalar fluid, reduces to

$$\mathcal{F} = (\rho_{(\lambda)} u^\mu_{(\lambda)} u^\nu_{(\lambda)} \tilde{\rho}^{1/2}_{(\phi)} \tilde{\lambda}_0 g^{\mu\nu} \bar{k}^\sigma u_{(\phi)\sigma}) \nabla^{\{\}}_\mu u_{(\phi)\nu} \,. \tag{10}$$

It represents a non-geodesic behavior of the fermionic fluid λ as it is measured in a geodesic reference system of observers satisfying (1). (We have used the geodesic equation $u^\nu \nabla^{\{\}}_\nu u_\mu = 0$ for the scalar field and the fact that $u^\nu \nabla^{\{\}}_\mu u_\nu = 0$.) In the particular case in which the interaction Lagrangian is null, the associated fluids to the scalar and the spinorial fields follow geodesics and $\mathcal{F}_{u_{(\phi)}}$ is zero. This result is expected due to the annihilation of both $\bar{k}$ and factor $u^\nu_{(\lambda)} \nabla^{\{\}}_\mu u_{(\phi)\nu}$.

5. THE BINARY PULSAR

Now we apply our results to the case of the binary pulsar PSR 1913+16. This pulsar shows two phenomena of energy decay. The first one is well described through emission of gravitational waves and it is related to a decrease of its orbital period P_b. The other one is the variation of the pulse emission itself due to the intrinsic rotational energy loss which implies an increase of the pulse period P_{P}. The causes of this last process are poorly understood. They are attributed, for example, to small breaking torques caused by magnetic Lorentz forces, to various forms of emission (particles, electromagnetic radiation, etc.) or, in general, to some kind of interaction between the pulsar and its companion.

Let us consider the latter mechanism analyzing this effect by using the spinorial-scalar matter interaction as previously formulated. We assume that the pulsar is a neutron star composed by spinorial matter described macroscopically by a sphere of radius r and density $\rho(\lambda)$. This star is supposed to be moving in the inner regions of the accretion disk of the companion — composed by scalar matter — with density $\rho_{(\phi)}$. Now we compare the energy decay of the pulsar with our theoretical value $\mathcal{F}$ in the r.h.s. of (10). In our approach, the interaction of the pulsar with its companion causes the rotational energy loss as given by (4). So we must consider $\mathcal{F} = \mathcal{F}_b + \mathcal{F}_{\mathrm{P}} + \mathcal{F}_{\mathrm{Pot}}$, where $\mathcal{F}_b$ and $\mathcal{F}_{\mathrm{P}}$ are the orbital and the rotational kinetic energy terms respectively, and $\mathcal{F}_{\mathrm{Pot}}$ is the potential energy term. A straightforward computation shows that both the orbital kinetic and the potential energy contributions, are compensated in the r.h.s. of (10) by the corresponding kinetic term ($\rho u^\mu u^\nu$) and the gravitational potentials (through the Christoffel symbols corrections in the covariant derivative) respectively, i.e.

$$\mathcal{F}_b + \mathcal{F}_{\rm Pot} \sim (\rho_{(\lambda)} u^\mu_{(\lambda)} u^\nu_{(\lambda)}) \nabla^{\{\}}_\mu u_{(\phi)\nu} \,. \tag{11}$$

As a consequence, the way to explain the intrinsic rotational energy decay of the pulsar is through the remaining terms related to the coupling quantity k in (10), i.e.

$$\mathcal{F}_{\rm P} \sim \tilde{\rho}^{1/2}_{(\phi)} \tilde{\lambda}_0 \, g^{\mu\nu} \bar{k}^\sigma u_{(\phi)\sigma} \nabla^{\{\}}_\mu u_{(\phi)\nu} \,, \tag{12}$$

which in terms of the observational parameters leads to

$$\rho_{(\lambda)} r^2 \omega \dot{\omega}/c \sim k(\rho_{(\phi)} \rho_{(\lambda)} \hbar m_{\rm N})^{1/2} \Omega, \tag{13}$$

where $\rho_{(\lambda)} \sim 10^{14}$g/cm^3 is the density of the pulsar, $r \sim 10^6$cm is the radius of the pulsar, $\omega = 2\pi/P_{\rm P} \sim 10^2$s^{-1} and $\dot{\omega} \sim 10^{-14}$s^{-2} with $P_{\rm P}$ the pulse period, $c \sim 10^{10}$cm/s is the speed of light, $\rho_{(\phi)} \sim 10^{-1}$g/cm^3 is the estimated disk accretion of the companion ($M_{\rm Comp.} \sim 1.4 M_{\rm Sun}$), $m_{\rm N} \sim 10^{-24}$g is the neutron mass and $\Omega \sim 10^{-4}$s^{-1} is the angular velocity of the pulsar. All these values were obtain from the well-known data of the PSR 1913+16 [9,10].

From these values we conclude that the lower bound interaction term k compatible with relation (13) and the experimental data corresponding to the binary pulsar PSR 1913+16, is

$$k \sim 10^4 {\rm s}^{-1}. \tag{14}$$

6. CONCLUSIONS AND SOME POSSIBLE APPLICATIONS TO COSMOLOGY

We introduced some previous results — specifically the value of energy flux in general space-times as measured by observers — in order to study one kind of interaction that can be attributed to the coupling of different spinorial matter. Using this scheme we studied the case of the binary pulsar which is assumed to be a typical and relevant example of energy loss and found the lower bound value of the interaction coupling "constant" to be between spin-0 and spin-$\frac{1}{2}$ matter. Now, as a result, we are able to discuss qualitatively some possible applications to other astrophysical and cosmological problems.

In the present matter-dominated era the source of the Einstein equation is represented by scalar matter in a geometric background in expansion. In the FRW scheme it is possible to use an interaction term of the form expressed in (4) whose origin may be thought as a remainder of a supersymmetric coupling between scalar and spinorial matter. The interaction strength k, estimated for the case of the binary pulsar, could be present at a cosmological leved, in the same way as the gravitational coupling constant G acts on both, astrophysical and cosmological levels. As was shown in [6,8], in the classical limit, free matter represents a perfect fluid without pressure — i.e. a free dust — while the energy-momentum tensor corresponding to spinorial matter is of higher order in $\hbar$. So, spinorial matter would be negligible as a source in the r.h.s. of the Einstein equation but not in the spin-$\frac{1}{2}$ matter field equation that leads to its corresponding motion law. A dissipative system described in the scheme of the classical standard model could be then attributed to an interaction whose strength should be given by quantity k found in Section 5, which does not modify the geodesic motion of the galaxies' clusters. As the non-geodesic motion law of spinorial matter should differ from the corresponding cosmic matter law, we may suppose that the former one is moving "over" the expansion of the background. So its velocity, given by the red-shift, does not, necessarily represent its distance to our position: it is due to its interaction with the background in addition to the expansion of the universe. All these considerations become interesting if we take into account that there are examples of some important anomalies that are not satisfactorily solved:
— distances computed through red-shift and the Hubble law leading to paradoxical conclusions such as galaxies showing red-shift which, according to its structure, should be older than the age attributed to the universe,
— the enormous distances and velocities of quasars,
— the dark matter introduced to explain the unexpected rigid rotation law shown by certain spiral galaxies.

In principle, our dynamical approach could be applied to all these subjects and so the interaction between scalar and fermionic matter — whose strength was estimated for the case of the binary pulsar — could be useful in the study of the problems mentioned to explain the anomalous dynamical behavior of these objects. A forthcoming article will be devoted to these subjects.

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