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COLLAPSE OF K-Rb FERMI-BOSE MIXTURES IN OPTICAL LATTICES

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We study a confined mixture of Rb and K atoms in a one dimensional optical lattice, at low temperature, in the quantal degeneracy regime. This mixture exhibits an attractive boson-fermion interaction, and thus above certain values of the number of particles the mixture collapses. We investigate, in the mean-field approximation, the curve for which this phenomenon occurs, in the space of number of particles of both species. This is done for different types of optical lattices.

Keywords: Atomic gases; optical lattice; mixture.

1. Introduction

Recent experiments on degenerate Fermi-Bose mixtures have opened the possibility of studying in a direct way the effects of quantum statistics in Bose-Einstein condensates (BECs).

From a theoretical point of view a considerable amount of work has been done to study boson-fermion mixtures.[1–3] In particular, an exhaustive and systematic study of the structure of binary mixtures has been performed by Roth in Ref. 1. In this work he has discussed all posible sign combinations of scattering lengths between boson-boson and boson-fermion s-wave interactions.

The aim of this work is to study the stability of the K-Rb mixture being the atoms confined within a harmonic trap plus a one dimensional optical lattice. This mixture exhibits an attractive interaction between fermions and bosons, and thus there exists an upper limit, in the number of particles, for the mixture to be stable. For numbers of particles above some critical ones the system collapses. In a previous work[4] we have analysed some properties of these mixtures. Here we concentrate ourselves on the study of the phenomenon of collapse when this mixture is in the presence of an optical lattice. We observe the effect of varying the height of the barriers and the spacing between consecutive wells.

2. The model

We consider a mixture of condensed bosons (B) and degenerated fermions (F) at zero temperrature confined in an external potential given by the sum of magnetic origin potentials, V_B^H and V_F^H, for bosons and fermions respectively, and of a stationary optical potential V_{opt} modulated along the z axis. The axially symmetric harmonic traps in cylindrical coordinates (r, ϕ, z) read,

$$V_B^H = \frac{1}{2} M_B \left[\omega_{rB}^2 r^2 + \omega_{zB}^2 z^2\right] \tag{1}$$

and

$$V_F^H = \frac{1}{2} M_F \left[\omega_{rF}^2 r^2 + \omega_{zF}^2 z^2\right] \quad , \tag{2}$$

where ω_{rB}, ω_{zB} and ω_{rF} ω_{zF} are the radial (axial) angular trapping frequencies for bosons and fermions, respectively. M_B and M_F are the corresponding masses.

The optical potential has the following expression:

$$V_{opt} = s\ E_r \sin^2(\frac{2\pi z}{d}), \tag{3}$$

where E_r denotes the recoil energy, s an adimensional amplitude parameter and d the spatial period of the lattice.

Thus the total potential for either bosons or fermions is $V_B = V_B^H + V_{opt}$ and $V_F = V_F^H + V_{opt}$, respectively.

Since the number of fermions we are considering is very large, we can use the Thomas-Fermi-Weizsäcker (TFW) approximation to write their kinetic energy density per unit volume as a function of the fermionic local density n_F and its gradients, for more details see Ref. 4. For fully polarized spin 1/2 fermions, it reads:

$$\tau_F(\vec{r}) = \frac{3}{5}(6\pi^2)^{2/3} n_F^{5/3} + \beta \frac{(\nabla n_F)^2}{n_F} \tag{4}$$

in terms of which the femionic kinetic energy is

$$T_F = \frac{\hbar^2}{2M_F} \int d\vec{r}\, \tau_F(\vec{r}) \tag{5}$$

The value of the β coefficient in the Weizsäcker term is fixed to 1/18. This term contributes little to the total fermion kinetic energy, and it is usually ignored.[1]

Neglecting all p-wave interactions, the energy density per unit volume has the form

$$\mathcal{E}(\vec{r}) = \frac{\hbar^2}{2M_B} |\nabla \Psi|^2 + V_B n_B + \frac{1}{2} g_{BB} n_B^2 + g_{BF} n_F n_B + \frac{1}{2M_F} \tau_F + V_F n_F \quad , \tag{6}$$

where $n_B = |\Psi|^2$ denotes the local density of bosons. The boson-boson and boson-fermion coupling constants g_{BB} and g_{BF}, are written in terms of the s-wave scattering lengths a_B and a_{BF} as $g_{BB} = 4\pi\, a_B \hbar^2/M_B$ and $g_{BF} = 4\pi\, a_{BF} \hbar^2/M_{BF}$, respectively. We have defined $M_{BF} \equiv 2M_B M_F/(M_B + M_F)$.

To obtain the ground state for this system, we solve the Gross-Pitaevskii (GP) equation for the bosons coupled to the Thomas-Fermi-Weizsäcker equation for the fermions which arise from the variation of $\mathcal{E}$ with respect to n_B and n_F keeping the particle numbers constant [Euler-Lagrange (EL) equations]:

$$\left(-\frac{\hbar^2 \nabla^2}{2M_B} + V_B + g_{BB}\, n_B + g_{BF}\, n_F \right) \Psi = \mu_B \Psi \tag{7}$$

$$\frac{\hbar^2}{2M_F} \left[(6\pi^2)^{2/3} n_F^{5/3} + \beta \frac{(\nabla n_F)^2}{n_F} - 2\beta \Delta n_F \right] + V_F n_F + g_{BF} n_B n_F = \mu_F n_F \quad , \tag{8}$$

where μ_B and μ_F denote the boson and fermion chemical potentials, respectively.

Note that the solution to Eq. (8) is not more complicated than that of the GP equation. This can be readily seen by writing the later in terms of n_B:

$$\frac{\hbar^2}{2M_B} \left[\frac{1}{4} \frac{(\nabla n_B)^2}{n_B} - \frac{1}{2} \Delta n_B \right] + V_B n_B + g_{BB} n_B + g_{BF} n_B n_F = \mu_B n_B \quad , \tag{9}$$

which is formally equivalent to Eq. (8).

We have employed an imaginary time method to find the solution of these expressions written as imaginary time diffusion equations.[5]

3. Numerical results

The system we take into consideration is a ^{87}Rb+ ^{40}K mixture. We have numerically solved Eqs. (9) and (8) using the set of scattering lengths reported by Modugno et al.,[6] namely $a_B = 98.98\, a_0$, and $a_{BF} = -395\, a_0$, being a_0 the Bohr radius. We have used spherically symmetric traps both for bosons and fermions, but due to the one dimensional optical lattice, the mixture only has axial symmetry around the z axis. For bosons we have taken $\omega_B = 2\pi \times 100$ Hz, while for fermions we have taken $\omega_F = \sqrt{M_B/M_F}\, \omega_B$.

In Fig. 1 we display the particle density profiles of bosons (n_B) and fermions (n_F) as functions of z, within a harmonic trap. The typical radius of the Bose condensates we have worked with is $\sim R_B = 5\,\mu$m. The density profiles plotted in this figure correspond to a number of fermions $N_F = 1 \times 10^4$ and the number of bosons is $N_B = 6 \times 10^4$. It may be seen that even for a rather small N_F value, the shape of the boson density differs from the parabolic-type profile yielded by the TF approximation for a fermion-free condensate.

In Fig. 2 (a) and (b) we plot the density profiles for the same number of particles as those used in Fig. 1, but we have added the optical lattice to the trapping potential. The parameters of the optical lattice in Fig. 2 (a) [Fig. 2 (b)] are $s = 4$ and $d = 1.35$ μm ($d = 4$ μm). By comparing these two graphs, it may be seen that the density profile turns out to be much more distorted with respect to that shown in Fig. 1, when the spacing between wells (d) is larger.

Finally, the results related to the collapse are presented in Fig. 3. In this figure we have plotted the stability diagram in the $N_B - N_F$ plane. We show our theoretical prediction for critical numbers (N_F^c, N_B^c), considering the following cases: when the external potential is only given by the harmonic trap (trapping potential), whose

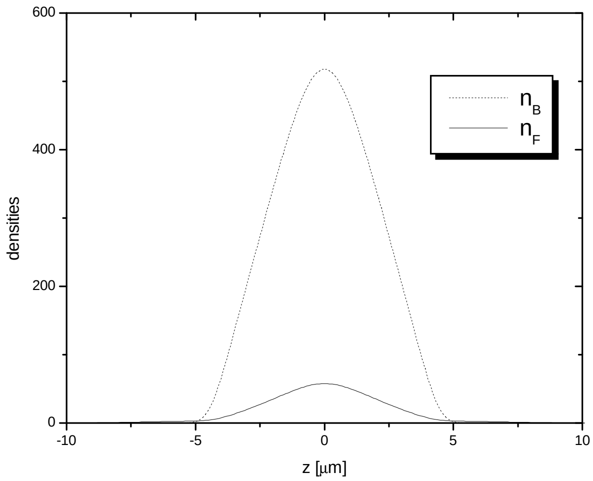

Fig. 1. Boson and fermion density profiles when considering as external potential only the harmonic trap. The number of particles are $N_B = 6 \times 10^4$ and $N_F = 1 \times 10^4$.

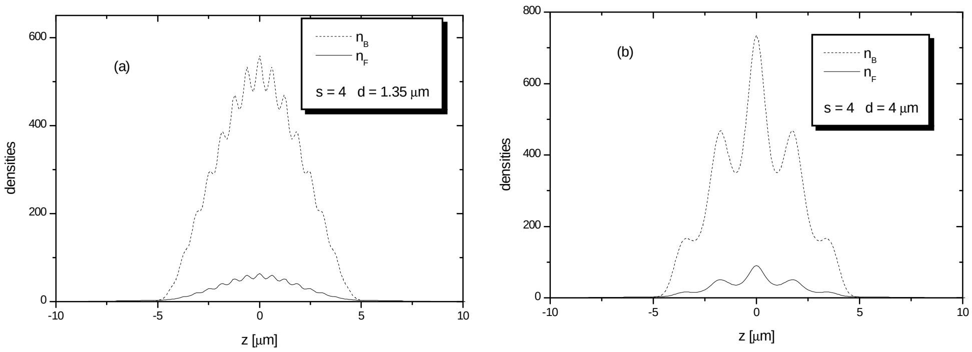

Fig. 2. Boson and fermion density profiles with the external potential including the optical lattice, for the same number of particles as in Fig. 1.

points are indicated by squares in the graph, and when an optical lattice is added with parameters $s = 4,\ d = 1.35\ \mu$m ($s = 4,\ d = 4\ \mu$m), the respective points being indicated by crosses (triangles).

It may be observed that for a spacing between consecutive wells $d = 4\ \mu$m and a height of barriers $s = 4$, the critical number decrease about 20 % with respect to the system without an optical lattice, and thus the system is more unstable. Whereas for the same barrier and a spacing of d= 1.35 μm, the curve turns out to be practically superimposed to the previous one. The same observation holds for other values of barrier heights. In summary, the degree of instability turns out to be significantly affected by the spacing of the lattice wells.

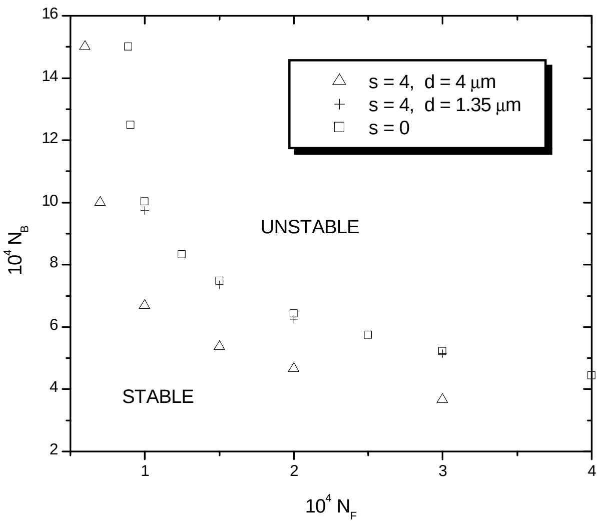

Fig. 3. Stability diagram in the $N_B - N_F$ plane.

In conclusion, we have observed that the inclusion of an optical lattice makes the system to become more unstable, and thus the number of particles should be lowered in a considerable amount, which depends not only on the height of the barrier but also on the spacing between consecutive wells.

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