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# q-Gamow states as continuous linear functionals on analytical test functions

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## Abstract

We define here q-Gamow states corresponding to Tsallis' q-statistics. We compute for them their norm, mean energy value and the q-analogue of the Breit–Wigner distribution (a q-Breit–Wigner).

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## 1. Introduction

In four previous papers [1–4] we have shown that Gamow-states [5,6] can be interpreted as Sebastiao e Silva's Ultradistributions [7–9], whose proper treatment appeals to Rigged Hilbert Space [10–13].

There exist a large number indeed of high energy experiments amenable to successful interpretation through Tsallis' q-statistics [14]. In particular, this happens for LHC experiments in what respects to distributions linked to stationary states. The q-statistics appears to adequately describe the transverse momentum distributions of different hadrons. The 4 LHC experiments yielded publications involving such distributions that seem to be properly fitted using the q-exponential function. The associated q-value is of the order of 1.15, clearly a departure from the Gibbs–Boltzmann's  $q = 1-$  value. Accordingly, stationary states before hadronization are not

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thermal equilibrium-ones. Remark that measuring the  $p_T$  distribution over a logarithmic range of fourteen decades shows that  $q = 1.15$  fits the data over such immense range [15,16].

These circumstances strongly motivate us to investigate complex energy states related to the  $q$ -exponential distributions, that is,  $q$ -Gamow states (or  $q$ -resonances), and establish their relation with Gamow-states. *These  $q$ -Gamow states are NOT solutions of Schroedinger's equation but of its non-linear  $q$ -version*, proposed by F.D. Nobre, M.A. Rego-Monteiro, and C. Tsallis in [17] and deduced in [21]. (For a brief review about usual Gamow states see [22].) We focus attention then on decay states at a great distance from the dispersion center and ascertain that a  $q$ -Gamow representation is adequate. **The reader should note that the use of  $q$ -exponentials (respectively, exponentials) to represent a  $q$ -resonance (respectively, resonance) is an approximation valid only at a great distance from the dispersion center.**

We begin our considerations with a review of the work of Refs. [1–3] on Gamow states. *The reader is strongly urged to peruse these papers.*

## 2. Review of the work of Refs. [1–3] on Gamow states

A brief look at is indispensable in order to properly follow our developments. Indeed, following [1–3], and using the notation of [7], we define a Gamow-state in a free space [this means at a great distance from the dispersion center], as

$$|\psi_G\rangle = \int_{-\infty}^{\infty} \{\mathcal{H}[\Im(p)]\mathcal{H}(x) - \mathcal{H}[-\Im(p)]\mathcal{H}(-x)\} e^{\frac{ipx}{\hbar}} |x\rangle dx, \quad (2.1)$$

or

$$\psi_G(x) = \{\mathcal{H}[\Im(p)]\mathcal{H}(x) - \mathcal{H}[-\Im(p)]\mathcal{H}(-x)\} e^{\frac{ipx}{\hbar}}, \quad (2.2)$$

where  $\mathcal{H}(x)$  is the Heaviside's step function.

The norm-squared for such a state reads [6]

$$\langle\psi_G|\psi_G\rangle = \int_0^{\infty} \mathcal{H}[\Im(p)] e^{\frac{i(p-p^*)x}{\hbar}} dx - \int_{-\infty}^0 \mathcal{H}[-\Im(p)] e^{\frac{i(p-p^*)x}{\hbar}} dx. \quad (2.3)$$

These integrals can be easily evaluated. One finds

$$\langle\psi_G|\psi_G\rangle = \{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\} \frac{\hbar}{i(p^* - p)} = \frac{\hbar}{2|\Im(p)|}. \quad (2.4)$$

Accordingly, the normalized Gamow-state  $\phi_G$  becomes [6]

$$|\phi_G\rangle = \sqrt{\frac{2|\Im(p)|}{\hbar}} |\psi_G\rangle. \quad (2.5)$$

Since [we are using here a special notation employed in the classical text-book by Messiah on quantum mechanics [25]]

$$\langle\phi_G|(H|\phi_G\rangle) = \frac{p^2}{2m}, \quad (2.6)$$

$$(\langle\phi_G|H)|\phi_G\rangle = \frac{p^{*2}}{2m}, \quad (2.7)$$

one encounters, for the mean-energy, a real value that agrees with experiment [1–3] and reads

$$\langle H \rangle = \frac{1}{2} [(\phi_G | H | \phi_G) + ((\phi_G | H) | \phi_G)] = \frac{p^2 + p^{*2}}{4m} = \frac{\Re(p^2)}{2m}. \tag{2.8}$$

In order to obtain the probability distribution associated to a Gamow state we start by the looking at scalar product between this state and a free one:

$$\langle \phi | \phi_G \rangle = \frac{1}{\hbar} \sqrt{\frac{|\Im(p)|}{\pi}} \left\{ \int_0^\infty \mathcal{H}[\Im(p)] e^{\frac{i(p-k)x}{\hbar}} dx - \int_{-\infty}^0 \mathcal{H}[-\Im(p)] e^{\frac{i(p-k)x}{\hbar}} dx \right\}. \tag{2.9}$$

Thus,

$$\langle \phi | \phi_G \rangle = \frac{i \sqrt{\frac{|\Im(p)|}{\pi}}}{p - k}. \tag{2.10}$$

The ensuing probability distribution is the Breit–Wigner one [6]

$$|\langle \phi | \phi_G \rangle|^2 = \frac{|\Im(p)|}{\pi \{[\Re(p) - k]^2 + \Im(p)^2\}}. \tag{2.11}$$

### 3. q-Gamow states

According to the q-statistics strictures (this word exists!) we must replace everywhere ordinary exponentials by so-called q-exponentials  $e_q(x)$  [14]

$$e_q(x) = [1 + (1 - q)x]^{1/1-q}; \quad q \in \mathcal{R}, \tag{3.1}$$

that becomes the ordinary exponential at  $q = 1$ . Accordingly,

$$|\psi_{qG}\rangle = \int_{-\infty}^\infty \{ \mathcal{H}[\Im(p)] \mathcal{H}(x) - \mathcal{H}[-\Im(p)] \mathcal{H}(-x) \} \otimes \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} |x\rangle dx, \tag{3.2}$$

or

$$\psi_{qG}(x) = \{ \mathcal{H}[\Im(p)] \mathcal{H}(x) - \mathcal{H}[-\Im(p)] \mathcal{H}(-x) \} \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}}. \tag{3.3}$$

The norm of a q-Gamow state is

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \int_0^\infty \mathcal{H}[\Im(p)] \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \left[ 1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \\ &\quad + \int_{-\infty}^0 \mathcal{H}[-\Im(p)] \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \left[ 1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx, \end{aligned} \tag{3.4}$$

or equivalently,

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \int_0^\infty \mathcal{H}[\Im(p)] \left[ 1 + \frac{2(q-1)\Im(p)x}{\hbar\sqrt{2(q+1)}} + \frac{(q-1)^2|p|^2x^2}{\hbar^22(q+1)} \right]^{\frac{2}{1-q}} dx \\ &+ \int_0^\infty \mathcal{H}[-\Im(p)] \left[ 1 - \frac{2(q-1)\Im(p)x}{\hbar\sqrt{2(q+1)}} + \frac{(q-1)^2|p|^2x^2}{\hbar^22(q+1)} \right]^{\frac{2}{1-q}} dx, \end{aligned} \quad (3.5)$$

that can be recast as

$$\langle \psi_{qG} | \psi_{qG} \rangle = \int_0^\infty \left[ 1 + \frac{2(q-1)|\Im(p)|x}{\hbar\sqrt{2(q+1)}} + \frac{(q-1)^2|p|^2x^2}{\hbar^22(q+1)} \right]^{\frac{2}{1-q}} dx. \quad (3.6)$$

We effect now the change of variables  $y = \frac{(q-1)|p|x}{\hbar\sqrt{2(q+1)}}$  and obtain

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar\sqrt{2(q+1)}}{(q-1)|p|} \int_0^\infty \left[ 1 + \frac{2|\Im(p)|y}{|p|^2} + y^2 \right]^{\frac{2}{1-q}} dy. \quad (3.7)$$

After a new change of variables  $z = y + \frac{|\Im(p)|}{|p|}$  we find

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar\sqrt{2(q+1)}}{(q-1)|p|} \int_{\frac{|\Im(p)|}{|p|}}^\infty \left\{ z^2 + \frac{[\Re(p)]^2}{|p|^2} \right\}^{\frac{2}{1-q}} dz. \quad (3.8)$$

Finally, after a third change of variables  $s = z^2$  we get for our norm

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar\sqrt{2(q+1)}}{2(q-1)|p|} \int_{\frac{|\Im(p)|^2}{|p|^2}}^\infty s^{-\frac{1}{2}} \left\{ s + \frac{[\Re(p)]^2}{|p|^2} \right\}^{\frac{2}{1-q}} ds. \quad (3.9)$$

Using the result given in [19] we arrive at:

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \frac{\hbar}{5-q} \frac{\sqrt{2(q+1)}}{|p|} \left\{ \frac{[\Im(p)]^2}{|p|^2} \right\}^{\frac{q-5}{2(q-1)}} \otimes \\ &F \left( \frac{2}{q-1}, \frac{5-q}{2(q-1)}; \frac{3+q}{2(q-1)}; -\frac{[\Re(p)]^2}{[\Im(p)]^2} \right). \end{aligned} \quad (3.10)$$

It is shown in [18] that

$$\begin{aligned} F \left( \frac{2}{q-1}, \frac{5-q}{2(q-1)}; \frac{3+q}{2(q-1)}; -\frac{[\Re(p)]^2}{[\Im(p)]^2} \right) &= \\ \left\{ \frac{|p|^2}{[\Im(p)]^2} \right\}^{\frac{q-5}{2(q-1)}} F \left( \frac{1}{2}, \frac{5-q}{2(q-1)}; \frac{3+q}{2(q-1)}; \frac{[\Re(p)]^2}{|p|^2} \right), \end{aligned} \quad (3.11)$$

which yields for the norm the expression

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar}{5-q} \frac{\sqrt{2(q+1)}}{|p|} \otimes F\left(\frac{1}{2}, \frac{5-q}{2(q-1)}; \frac{3+q}{2(q-1)}; \frac{[\Re(p)]^2}{|p|^2}\right) = [A(q, p)]^2, \tag{3.12}$$

so that the normalized q-Gamow state becomes

$$|\phi_{qG}\rangle = [A(q, p)]^{-1} |\psi_{qG}\rangle. \tag{3.13}$$

Noticing that

$$\lim_{q \rightarrow 1} F\left(\frac{1}{2}, \frac{5-q}{2(q-1)}; \frac{3+q}{2(q-1)}; \frac{[\Re(p)]^2}{|p|^2}\right) = F\left(\frac{1}{2}, 4; 4; \frac{[\Re(p)]^2}{|p|^2}\right) \tag{3.14}$$

and using a result of [20] one has

$$F\left(\frac{1}{2}, 4; 4; \frac{[\Re(p)]^2}{|p|^2}\right) = \left[\frac{[\Im(p)]^2}{|p|^2}\right]^{-\frac{1}{2}}, \tag{3.15}$$

and

$$\lim_{q \rightarrow 1} [A(q, p)]^2 = \frac{\hbar}{2|\Im(p)|}. \tag{3.16}$$

Using now a result derived in [21] for the q-generalization of Schroedinger’s equation of Ref. [17], i.e.,

$$H\phi_q(x) = \frac{p^2}{2m} [\phi_q(x)]^q, \tag{3.17}$$

we encounter

$$\begin{aligned} \langle \phi_{qG} | (H | \phi_{qG}) \rangle &= [A(p, q)]^{-2} \frac{p^2}{2m} \otimes \left\{ \int_0^\infty \mathcal{H}[\Im(p)] \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right. \\ &\quad \left. + \int_{-\infty}^0 \mathcal{H}[-\Im(p)] \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right\}, \end{aligned} \tag{3.18}$$

or equivalently,

$$\begin{aligned} \langle \phi_{qG} | (H | \phi_{qG}) \rangle &= [A(p, q)]^{-2} \frac{p^2}{2m} \otimes \left\{ \int_0^\infty \mathcal{H}[\Im(p)] \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right. \\ &\quad \left. + \int_0^\infty \mathcal{H}[-\Im(p)] \left[1 + \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 - \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right\}. \end{aligned} \tag{3.19}$$

We can recast (3.19) as

$$\begin{aligned}
 \langle \phi_{qG} | (H | \phi_{qG}) \rangle &= [A(p, q)]^{-2} \frac{p^2}{2m} \otimes \\
 &\left\{ \mathcal{H}[\Im(p)] \left[ \frac{i(q-1)p^*}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ \frac{-i(q-1)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \otimes \right. \\
 &\int_0^\infty \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ 1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} dx \\
 &+ \mathcal{H}[-\Im(p)] \left[ \frac{-i(q-1)p^*}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ \frac{i(q-1)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \otimes \\
 &\left. \int_0^\infty \left[ 1 + \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ 1 - \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} dx \right\}. \tag{3.20}
 \end{aligned}$$

We use now a result from [23] and obtain

$$\begin{aligned}
 \langle \phi_{qG} | (H | \phi_{qG}) \rangle &= -\frac{\hbar}{i[A(p, q)]^2} \frac{p}{2m} \mathcal{B}\left(1, \frac{3+q}{q-1}\right) \frac{\sqrt{2(q+1)}}{(q-1)} \otimes \\
 &\{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\} F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p}\right), \tag{3.21}
 \end{aligned}$$

or equivalently,

$$\begin{aligned}
 \langle \phi_{qG} | (H | \phi_{qG}) \rangle &= -\frac{\hbar}{i[A(p, q)]^2} \frac{p}{2m} \frac{\sqrt{2(q+1)}}{(3+q)} \otimes \\
 &\{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\} F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p}\right). \tag{3.22}
 \end{aligned}$$

In analogous fashion we find

$$\begin{aligned}
 \langle (\phi_{qG} | H) | \phi_{qG} \rangle &= \frac{\hbar}{i[A(p, q)]^2} \frac{p^*}{2m} \frac{\sqrt{2(q+1)}}{(3+q)} \otimes \\
 &\{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\} F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p}{p^*}\right). \tag{3.23}
 \end{aligned}$$

Thus, according to [1–3] we obtain for the mean energy value

$$\langle H \rangle_q = \frac{1}{2} [\langle \phi_{qG} | (H | \phi_{qG}) \rangle + \langle (\phi_{qG} | H) | \phi_{qG} \rangle]. \tag{3.24}$$

Additionally, since

$$\lim_{q \rightarrow 1} F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p}\right) = \frac{2p}{p-p^*}, \tag{3.25}$$

we have

$$\lim_{q \rightarrow 1} \langle H \rangle_q = \frac{\Re(p^2)}{2m} = \langle H \rangle. \tag{3.26}$$

We investigate now the  $q$ -analogue of the Breit–Wigner distribution tackling

$$\langle \phi | \phi_{Gq} \rangle = \frac{1}{\sqrt{2\pi \hbar A(q, p)}} \left\{ \mathcal{H}[\Im(p)] \int_0^\infty e^{-ikx} \left[ 1 + \frac{i(1-q)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx - \mathcal{H}[-\Im(p)] \int_0^\infty e^{ikx} \left[ 1 - \frac{i(1-q)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \right\}, \tag{3.27}$$

and rewrite it as

$$\langle \phi | \phi_{Gq} \rangle = \frac{1}{\sqrt{2\pi \hbar A(q, p)}} \left\{ H[\Im(p)] \left[ \frac{i(1-q)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \otimes \int_0^\infty e^{-ikx} \left[ x + \frac{\hbar\sqrt{2(q+1)}}{i(1-q)p} \right]^{\frac{2}{1-q}} dx - H[-\Im(p)] \left[ -\frac{i(1-q)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \otimes \int_0^\infty e^{ikx} \left[ x - \frac{\hbar\sqrt{2(q+1)}}{i(1-q)p} \right]^{\frac{2}{1-q}} dx \right\}. \tag{3.28}$$

We appeal now to a result of [24] and obtain

$$\langle \phi | \phi_{Gq} \rangle = -i \sqrt{\frac{\hbar}{2\pi A(q, p)}} \left[ \frac{\sqrt{2(q+1)}}{(1-q)p} \right]^{\frac{2}{q-1}} k^{\frac{3-q}{q-1}} e^{\frac{\sqrt{2(q+1)}k}{(1-q)p}} \otimes \Gamma \left[ \frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}k}{(1-q)p} \right], \tag{3.29}$$

which leads to the  $q$ -Breit–Wigner result

$$|\langle \phi | \phi_{Gq} \rangle|^2 = \frac{\hbar}{2\pi A(q, p)} \left[ \frac{2(q+1)}{(1-q)^2 |p|^2} \right]^{\frac{2}{q-1}} k^{\frac{2(3-q)}{q-1}} e^{\frac{\sqrt{2(q+1)}k(p+p^*)}{(1-q)|p|^2}} \otimes \Gamma \left[ \frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}k}{(1-q)p} \right] \left\{ \Gamma \left[ \frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}k}{(1-q)p} \right] \right\}^*. \tag{3.30}$$

Note that (3.27) converges uniformly for  $q \rightarrow 1$  [since the  $q$ -exponential converges in that way to the ordinary one], i.e.,

$$\lim_{q \rightarrow 1} |\langle \phi | \phi_{Gq} \rangle|^2 = \frac{|\Im(p)|}{\pi \{ [\Re(p) - k]^2 + \Im(p)^2 \}}. \tag{3.31}$$

#### 4. Conclusions

The  $q$ -statistics appears to adequately describe the transverse momentum distributions of different hadrons. The 4 LHC experiments yielded publications involving such distributions that seem to be properly fitted using the  $q$ -exponential function. The associated  $q$ -value is of the order of 1.15, clearly a departure from the Gibbs–Boltzmann’s  $q = 1 -$  value. Accordingly, stationary states before hadronization are not thermal equilibrium-ones [15,16].

In this work we have introduced  $q$ -Gamow states, associated to the  $q$ -statistics, that are solutions to a non-linear,  $q$ -generalization of Schrodinger's equation [17,21].

For such a purpose we have computed their norm, the mean energy value, and the concomitant  $q$ -Breit–Wigner distributions. In all instances, results tend to the customary ones when the all important  $q$ -parameter of Tsallis' obeys  $q \rightarrow 1$ . It is hoped that these  $q$ -Gamow states may be found by recourse to the extant LHC data.

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