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Non-Abelian monopoles as the origin of dark matter

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We suggest that dark matter may be partially constituted by a dilute 't Hooft–Polyakov monopoles gas. We reach this conclusion by using the Georgi–Glashow model coupled to a dual kinetic mixing term $F\tilde{\mathcal{G}}$ where F is the electromagnetic field and $\mathcal{G}$ the 't Hooft tensor. We show that these monopoles carry both (Maxwell) electric and (Georgi–Glashow) magnetic charges and the electric charge quantization condition is modified in terms of a dimensionless real parameter. This parameter could be determined from milli-charged particle experiments.

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The detection of dark matter is one of the most important challenges on high energy physics in present days because its discovery would explain a number of very important unsolved problems in astrophysics, astronomy and particle physics.[1] Since the dark matter interacts very weakly with visible matter, it seems that the most promissory way to detect it would be through indirect methods as, for example, the detection of the products of the annihilation of pairs of dark/anti-dark matter which could produce overabundance of visible matter or energy.[2] Direct detection methods, on the other hand, has also been proposed and it is a very active field of research.[3]

The expected overabundance of visible matter could be explained via annihilation of dark matter (χ) processes of the type

$$\chi + \bar{\chi} \rightarrow \text{visible matter}$$

whose evaluation requires, of course, the precise knowledge of the production mechanism, so far unknown. The excess of gamma rays at the galactic center is, for example, a possible signal of a dark matter annihilation process,[4] and other effects[5] might be understood as a manifestation of some relevant non-perturbative mechanism presumably not taken into account until now.

In this paper, we would like to explore a model in which the dark sector contains a non-Abelian field theory admitting massive topologically stable classical solutions: monopoles which weakly interact with ordinary fields. These configurations are characterized by a *topological charge*, an additive magnitude which can take both signs in such a way that two such solutions can be smoothly merged into a single object with the sum of their topological charges. We will assume that it is energetically favorable to have configuration with the same topological charge sign separated far away. Then, we can consider a dilute gas of monopoles of unit topological charge (of both signs), neutral in the mean, created at a very energetic event in the past.

Under these conditions, the most relevant interaction for such objects would be the annihilation of a monopole (charge $+1$) with an anti-monopole (charge -1), producing a non-topological object and emitting their energy in the form of dark and ordinary particles. Being very massive, these objects could also be gravitationally attracted by the galaxies, enhancing the probability of such annihilation process in their immediacies. If the result includes the emission of normal photons, this scheme would fit in the excess of luminosity of galaxies centers.[4]

There is also a simple experimental example which provides an analogy of the aforementioned scenario. Indeed, in a ripple tank with two vortices produced in the water with opposite sense of rotation, the collision between these two *topological defects* produces a disturbance in water in the form of waves whose amplitude will depend on the vortex energies,[6,7] in virtue of the energy and angular momentum conservation.

Our discussion is based on a simple application of the Georgi–Glashow model[8] for the dark sector with a suitable gauge-invariant *kinetic mixing term* between the non-Abelian (dark) fields and the usual electromagnetic field. The coupling between both sectors is realized by adding to the Lagrangian a term proportional to the electromagnetic strength tensor times the gauge-invariant 't Hooft tensor.[9–11]

In order to establish our notation, let us start by describing the Lagrangian of the Georgi–Glashow model, an SU(2) non-Abelian gauge theory with a triplet of scalar fields in the adjoint representation,

$$\mathcal{L}_{GG} = -\frac{1}{4} G_{a,\mu\nu}(X) G_a^{\mu\nu}(X) - \frac{1}{2}(D_\mu[X]\phi)_a (D^\mu[X]\phi)_a - V(\phi_a\phi_a)\,, \qquad (1)$$

where an implicit summation over $a, b, \ldots = 1, 2, 3$ is understood each time an index is repeated in a term. Here, the covariant derivative of the scalars in the adjoint representation is given by

$$(D_\mu \phi)_a = \partial_\mu \phi_a + q\, \epsilon_{abc} X_{b,\nu} \phi_c \,, \tag{2}$$

where $X_{a,\mu}$ is the (non-Abelian) gauge field and the strength tensor is given by

$$G_{a,\mu\nu}(X) = \partial_\mu X_{a,\nu} - \partial_\nu X_{a,\mu} + q \epsilon_{abc} X_{b,\mu} X_{c,\nu} \,, \tag{3}$$

where ϵ_{abc} is completely antisymmetric with $\epsilon_{123} = 1$.

Note that both the strength tensor and the triplet of scalars can be represented as elements in the Lie algebra of SU(2) as

$$G_{\mu\nu} = G_{a,\mu\nu} T_a \,, \qquad \Phi = \phi_a T_a \,, \tag{4}$$

where $T_a = \frac{\sigma_a}{2}$, $a = 1, 2, 3$, are the SU(2) generators which satisfy $[T_a, T_b] = i\epsilon_{abc} T_c$ and $\mathrm{tr}\{T_a T_b\} = \frac{\delta_{ab}}{2}$. Under a gauge transformation $U(x) \in \mathrm{SU}(2)$ these elements transform as

$$G_{\mu\nu} \to U G_{\mu\nu} U^\dagger \,, \qquad \Phi \to U \Phi U^\dagger \,. \tag{5}$$

We will assume that the potential $V(\phi_a \phi_a) \geq 0$ has its absolute minima at $\phi_a \phi_a = v^2$. Then, the symmetry is spontaneously broken to U(1). We also adopt the temporal gauge, $X_{a,0} = 0$.

We will be interested in *background* nontrivial static configurations of finite energy, condition which requires that the scalar fields tend to a minimum of the potential sufficiently fast for $\mathbf{x}^2 = r^2 \to \infty$. Since the potential minima belong to a 2-sphere (of radius v), $\mathcal{S}^2_v$, the scalar field at infinity establishes an application of $\mathcal{S}^2_\infty = \partial\mathbb{R}^3$ onto $\mathcal{S}^2_v$, which is characterized by the *winding number*, the (integer) number of times the application involves the sphere of minima $\mathcal{S}^2_v$, with a sign determined by the sense of this covering. 't Hooft[9] and Polyakov[10] have shown that there exist such static and finite energy solutions with nontrivial winding number, which are *stable* as a consequence of their topology.

't Hooft[9] has also constructed a gauge invariant tensor given by

$$\begin{aligned} \mathcal{G}_{\mu\nu} &= 2\ \mathrm{tr}\left\{ G_{\mu\nu}\hat{\Phi} + \frac{i}{q}[D_\mu\hat{\Phi}, D_\nu\hat{\Phi}]\hat{\Phi} \right\} \\ &= G_{a,\mu\nu}\hat{\phi}_a - \frac{1}{q}\epsilon_{abc}\hat{\phi}_a (D_\mu\hat{\phi})_b (D_\nu\hat{\phi})_c \,, \end{aligned} \tag{6}$$

where $\hat{\Phi} = \hat{\phi}_a T_a$, with $\hat{\phi}_a = \phi_a / \sqrt{\phi_b \phi_b}$. Note that $\mathcal{G}_{\mu\nu}$ has dimension of (mass)^2. This tensor can be brought to coincide with $G_{3,\mu\nu}$, for example, in any *bounded* region of $\mathbb{R}^3$ through a suitable (smooth) gauge transformation, without changing the winding number. These considerations justify the interpretation of $\mathcal{B}_i := \frac{1}{2}\epsilon_{ijk}\mathcal{G}_{jk}$ as the *magnetic field* associated with the unbroken U(1) symmetry,[11] and

$$g := \frac{1}{8\pi} \oint_{\mathcal{S}^2_\infty} \mathcal{B}_k \, dS_k = \frac{-1}{8\pi q} \epsilon_{ijk} \epsilon_{abc} \oint_{\mathcal{S}^2_\infty} \hat{\phi}_a \partial_\mu \phi_b \partial_\nu \hat{\phi}_c \, dS_k \tag{7}$$

as the *magnetic charge* of the topological configuration of this non-Abelian field.

So defined, g is a topological invariant[9–11] which equals the winding number divided by the constant q,[12] $g = n/q$. Moreover, since it is a surface integral, g is an additive quantity for well-separated topological configurations, and its value remains invariant when these configurations are smoothly brought together. In this sense, these classical configurations of the non-Abelian theory are *monopoles* of *magnetic charge* quantized in units of q^{-1}.

The static solution with $n = \pm 1$ is the 't Hooft–Polyakov monopole,[9,10] a regular configuration free of the *Dirac singularities* present in the description of monopoles in an Abelian gauge theory. These are *massive* configurations which minimize the energy functional given by

$$E[\mathbf{X}, \Phi] = \int_{\mathbb{R}^3} \left\{ \frac{1}{2} G_{a,ij} G_a^{ij} + \frac{1}{2} (D_i \phi)_a^2 + V(\phi_a \phi_a) \right\} d^3x \,, \tag{8}$$

with $V(\phi_a \phi_a) = \frac{\lambda}{8}(\phi_a \phi_a - v^2)^2$.

The numerical solution of the variational problem for these minima[13] shows that there is a *core* of radius $R_c \approx M_X^{-1} = 1/qv$ outside which the massive gauge bosons (of mass $M_X = qv$) rapidly approach their asymptotic value, while the scalar field approaches its asymptotic value outside a region of dimension $R_H = M_H^{-1} = 1/v\sqrt{\lambda}$ (where $M_H = v\sqrt{\lambda}$ is the scalar (*Higgs*) mass), less than R_c for sufficiently large λ. The main contributions to the energy come from the *magnetic field* $\mathcal{B}$ outside the core and from the gradient of the scalar field inside it. 't Hooft has shown[9,13] that the monopole mass is

$$M_{\text{mon}} = \frac{4\pi v}{q} f(\lambda/q^2) = \frac{4\pi}{q^2} M_X f(\lambda/q^2) \,, \tag{9}$$

where $f(x)$ is an O(1) monotonically increasing function,[13] $1 \leq f(x) < 2$ for $x \in \mathbb{R}^+$.

After this brief description of known results, about one of the most relevant non-perturbative developments obtained in the '70s, we proceed to employ this non-Abelian gauge theory as a model for a dark matter sector, which we put in interaction with the *visible* Maxwell electromagnetic field A_μ by adding to the Lagrangian a (gauge invariant) *dual kinetic mixing term*[14]

$$\mathcal{L}_1 = \gamma F_{\mu\nu}(A) \tilde{\mathcal{G}}^{\mu\nu}(X) \,. \tag{10}$$

Here γ is a dimensionless constant, $F_{\mu\nu}(A)$ is the strength tensor of the electromagnetic field and the dual strength tensor of the non-Abelian field is $\tilde{\mathcal{G}}^{\mu\nu}(X) = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathcal{G}_{\rho\sigma}(X)$, with $\mathcal{G}_{\mu\nu}(X)$ defined in (6).

It would be noted that this coupling between the dark sector and the electromagnetic field breaks the CP-symmetry but, as we will see, it also gives rise to the possibility of having massive milli-charged objects whose detection would be an indication in favor of this model. In fact, an independent argument of indirect CP-violation in the dark sector has been recently proposed in Ref. 15.

Our Lagrangian is then

$$\begin{aligned}\mathcal{L}_0 &= \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{GG} + \gamma F_{\mu\nu}(A)\tilde{\mathcal{G}}^{\mu\nu}(X) \\ &= -\frac{1}{4}F_{\mu\nu}(A) - \frac{1}{4}G_{a,\mu\nu}(X)G_a^{\mu\nu}(X) - \frac{1}{2}(D_\mu[X]\phi)_a(D^\mu[X]\phi)_a \\ &\quad - V(\phi_a\phi_a) + \gamma F_{\mu\nu}(A)\tilde{\mathcal{G}}^{\mu\nu}(X)\,, \end{aligned} \tag{11}$$

where we will assume that the visible and dark sectors are weakly coupled ($\gamma \ll 1$).

It is worth to remark that in Ref. 16 a similar model of dark monopoles in interaction with the visible sector has been considered. In that paper, the interaction does not break parity and, as a consequence, the monopoles are not a source for the electromagnetic field. In the following, we will show that the coupling we propose in (10) makes the monopoles to appear as massive charged objects, which effectively are sources for the visible electromagnetic field.[17]

Indeed, the Euler–Lagrange equations for the electromagnetic field derived from Eq. (11) are

$$\partial_\mu \frac{\partial L}{\partial(\partial_\mu A_\nu)} = \partial_\mu\{-F^{\mu\nu} + 2\gamma\tilde{\mathcal{G}}^{\mu\nu}\} = 0\,, \tag{12}$$

which implies that the content of electric charge of the topological configuration in Eq. (6) is

$$\begin{aligned} Q_{\text{mon}} &= \int_{\mathbb{R}^3} \partial_i F^{i0}\, d^3x = \gamma\epsilon^{i0jk}\int_{\mathbb{R}^3} \partial_i \mathcal{G}_{jk}\, d^3x \\ &= 2\gamma \oint_{\partial\mathbb{R}^3} \mathcal{B}_i\, dS_i = 16\pi\gamma g = \frac{16\pi\gamma}{q} n\,. \end{aligned} \tag{13}$$

Therefore, these monopoles also carry an electric charge proportional to the winding number, quantized in units of $16\pi\gamma/q$.

The next step is to physically interpret the model, in which we are essentially assuming that the electromagnetic field is weakly coupled to the non-Abelian sector ($\gamma \ll 1$) and the monopoles are very massive background configurations, which requires that $q < 1$ (see Eq. (9)). On the other hand, the monopole electric charge in Eq. (13) must be small in order for they remain *dark* to the electromagnetic interaction; then, $0 < \gamma \ll q < 1$. Taking into account the additivity of the magnetic charge, we have also assumed that it is energetically favorable to have monopoles of winding number ± 1.

These monopoles are topologically stable classical configurations which cannot individually decay through the emission of dark or visible particles, since these processes do not change their winding number. Its decay can only occur when a pair monopole–anti-monopole meet each other and, due to the additivity of the magnetic charge, they merge into an electrically neutral configuration with vanishing winding number. In this case, an energy equal to twice the monopole mass can be emitted as visible and dark particles. This possibility presents as an interesting route to explore in the context of the observed photon excess at the center of the galaxy.[4]

The precise mechanism of annihilation of dark matter still remains unknown, but the possibility that it decays in cascades until finally reaching a pair of particle–anti-particle of the Standard Model is an interesting prospect to explore in order to get some numerical bounds. In this context, it would be worth to consider the model previously discussed.

As previously mentioned, the CP-breaking kinetic mixing approach in Eq. (10) for the interaction between the Maxwell field and a non-Abelian SU(2) gauge theory for the dark sector leads to the appearance of 't Hooft–Polyakov monopoles in the dark sector which, additionally to their (non-Abelian) magnetic charge, present in the visible sector as milli-charged massive particles, whose detection would give support for this proposal.[18]

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