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Evaluation of the spreading thermal resistance for rough spheres

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ABSTRACT

The solid–solid contact of rough spherical particles in packed beds takes place by the deformation of individual asperities that transmit the stresses to the main body of the particles. This causes an elastically deformed area (a disk of radius a, most usually much smaller than the particle radius) enclosing the micro-contacts. Micro thermal resistances around the contact spots can be described in terms of a profile of thermal conductance $h(r)$ that decreases toward the disk edge. An additional thermal resistance, the spreading resistance, is caused by the convergence of flux lines in the bulk of the spheres towards the reduced section of the disk. The described mechanism has been modeled by Bahrami et al. (2006) and a formulation was provided to predict the contribution of contact areas to the effective thermal conductivity. The spreading resistance Ω_a is re-evaluated in this work on account of the $h(r)$ profile, by means of numerical calculations. It is found that the shape of $h(r)$ has a large impact on Ω_a and a weaker, but still significant, effect on the overall contact resistance. The results of Ω_a have been suitably correlated for a conductance profile of the form $h(r) = h_0[1 - (r/a)^2]^p$.

1. Introduction

The effective thermal conductivity of packed beds is known to be determined by a large number of factors (e.g. [1]). This paper is concerned with the contribution of the solid–solid contact areas that play a relevant role under vacuum conditions or for particles with high thermal conductivities, such as metals, alloys or other materials like high-density alumina and carbon.

Compression forces increase the areas of solid contacts. The own weight of the bed in industrial applications (bed heights are in the order of meters) will build enough pressure to enhance significantly the contact contribution to the effective thermal conductivity. The assumption of ideally smooth surfaces simplifies greatly the estimation of such a contribution, but unfortunately most particles of practical use show rough surfaces and real contact spots occur by deformation of the asperities. The general problem of contact between rough surfaces and the associated heat transfer effects have been extensively studied in the past decades [2]. Although the basic features of the problem seem to be fairly well understood, there is not at present a procedure generally accepted to predict the magnitude of the heat transfer rate through rough surfaces in contact, in particular in the case of granular beds. This is a consequence of the interplay between the mechanical, geometric and thermal aspects of the problem, and perhaps most important, the distributed nature of the size and shape of asperities that in turn are specific for each particle surface. Among a number of models to evaluate the thermal contact contribution (some of them have been summarized and evaluated in the recent publication by Abyzov et al. [3]), we appraise the approach of Bahrami et al. [4] as a relevant alternative to predict the effect of the solid contact contribution to the effective thermal conductivity. Based on an ample body of previous theoretical and experimental studies, the authors have been able to formulate a relatively simple model requiring a basic characterization of roughness. The contact contribution, or alternatively expressed as a thermal contact resistance, is evaluated along with the gas contribution, but the former can be implemented with alternative formulations for the latter, as in the model of van Antwerpen et al. [5]. The overall contact resistance according to Bahrami et al. [4] is expressed for spherical particles in contact by adding the micro-contact thermal resistance due to the deformation of the asperities in the plastic regime to the macro thermal resistance due the spreading/constriction of flux lines from/towards the area enclosing all micro-contacts (a disk), as defined by the elastic deformation of the bulk material.

This paper presents a re-assessment of the macro-thermal resistance (herein referred to as spreading resistance for short) in the Bahrami et al. [4] formulation to account for the usually strong

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variation of the local thermal resistance within the disk of micro-contacts. After outlining the Bahrami et al. [4] model and giving examples of its application, the spreading resistance is evaluated correctly from numerical calculations, the results are expressed in simple correlations and the effect on the Bahrami et al. formulation is finally assessed.

2. Bahrami et al. [4] formulation for the contribution of solid contact areas to the effective thermal conductivity

Even when the Bahrami et al. [4] model to predict the overall contact resistance can be taken as a starting point to more general cases, it has been specifically formulated for contact between spheres. This case will be treated here, and further, to simplify the analysis, spheres of the same size and material will be considered without restricting the conclusions.

For a brief outline of their model, only the evaluation of the thermal contact resistance regarding a single contact zone between two spheres will be considered, while the evaluation of heat transfer through the interstitial gas is disregarded.

Around a nominal contact point between the spheres subject to a compression force F, the main bodies suffer a macro-deformation of elastic nature covering a disk of radius a, which except if F is unusually high, will be typically of the order of a hundredth of the sphere radius R_p. Considering rough spheres, the contact within the disk is not uniform, but takes place by means of isolated spots generated by the deformation of the asperities that come into contact to sustain the compressive force. Plastic asperity deformations are considered in the Bahrami et al. [4] model.

For each hemisphere in contact, the (macro) spreading resistance Ω_a is evaluated by the following expression corresponding to the case in which the temperature over the disk is uniform ($\Omega_{a,T}$):

$$\Omega_{a,T} = 1/(4a\lambda_s) \tag{1}$$

where λ_s is the solid thermal conductivity. It should be noted that the Bahrami et al. [4] formulation is given for the joint resistance exerted by both hemispheres in contact. Instead, we will write expressions for only one of the hemispheres, *i.e.* as for the heat exchange between such a hemisphere and the hypothetical plane tangent to the surfaces in contact. Therefore, expressions for thermal resistances given here (*e.g.* Eq. (1)) are half of those in Bahrami et al. [4].

The evaluation of the radius a follows from the solution of the elastic deformation equations applied to the surfaces subject to an axisymmetric pressure distribution resulting from the plastic deformations of the asperities. The results are primarily obtained via a numerical solution [6], but the authors developed approximate explicit expressions for a and for the pressure distribution $P(r)$, in terms of Hertzian parameters (contact of ideally smooth surfaces). In the form given by Bahrami et al. [4] the relationships are:

$$a = a_H \times \begin{cases} 1.605/\sqrt{P_0^*}, & \text{if } 0.01 \leqslant P_0^* \leqslant 0.47 \\ 3.51 - 2.51P_0^*, & \text{if } 0.47 \leqslant P_0^* \leqslant 1 \end{cases} \tag{2a}$$

$$P_0^* = P_0/P_{H,0} = \left(1 + 1.22\alpha\chi^{-0.16}\right)^{-1} \tag{2b}$$

$$P(r) = P_0[1-(r/a)]^p; \quad p = 1.5P_0^*(a/a_H)^2 - 1 \tag{2c}$$

In Eqs. (2) P_0 is the (maximum) pressure at $r = 0$, $P_{H,0} = 1.5F/(\pi a_H^2)$ is the Hertz pressure at $r = 0$, $a_H = (0.75\ FR'/E')^{1/3}$ is the Hertzian disk radius, and the parameters in Eq. (2b) are $\alpha = \sigma' R'/a_H^2$, $\chi = (H_{162}/E')(R'/\sigma')^{0.5}$. Furthermore, for two identical spheres, $R' = R_p/2$ and $1/E' = 2(1 - \upsilon^2)/E$, where υ is the Poisson ratio and E is the Young's modulus. H_{162} is an effective micro-hardness evaluated as $H_{162} = c_1[1.62\ (\sigma'/m')/\sigma_1]^{c2}$, where c_1 and c_2 are micro-hardness constants depending on the sphere material and the state of its surface, $\sigma_1 = 1\ \mu m$, $\sigma' = \sqrt{2}\ \sigma$, $m' = \sqrt{2}\ m$, with σ and m the root mean roughness and mean asperity slope, respectively, of the sphere surface.

As the deformed asperities within the disk are assumed to be far enough apart from each other, each contact spot will also induce a micro-spreading resistance. In terms of the local conductance $h(r)$, Bahrami et al. [7] expressed

$$h(r) = 2\lambda_s P(r)/(0.565H'\sigma'/m') \tag{3}$$

In Eq. (3) the true micro-hardness and the true mean contact radius are assumed to be proportional to $H' = c_1[(\sigma'/m')/\sigma_1]^{c2}$ and (σ'/m') respectively, while the numerical coefficient 0.565 is an empirical constant obtained from comparison with experimental data.

It is noted that an underlying assumption in writing Eq. (3) is that the fraction of the local nominal area in contact f_A is small.

Assuming that the local thermal driving force is constant on the disk of radius a, the heat transfer rate will be proportional to the sum of local contributions within the disk. Thus, the micro contact resistance is defined as

$$\Omega_c = \frac{1}{2\pi\int_0^a h(r)rdr} \tag{4}$$

Since $F = 2\pi\int_0^a P(r)rdr$, Eq. (4) can be written as

$$\Omega_c = \frac{0.565}{2}\frac{1}{\lambda_s}\frac{H'}{F}\frac{\sigma'}{m'} \tag{5}$$

Finally, Bahrami et al. [4] evaluated the overall contact resistance by adding the spreading resistance caused by the disk of radius a, $\Omega_a \equiv \Omega_{a,T}$ (Eq. (1)), and Ω_c from Eq. (5). The result denoted Ω_T is then

$$\Omega_T = \Omega_{a,T} + \Omega_c \tag{6}$$

The contribution of solid contacts to the effective thermal conductivity of the bed (λ_{eff}) will be inversely proportional to the product ($R_p\ \Omega_T$).

Employing Eq. (2c) for $P(r)$, it will be useful in the next sections to rewrite Eq. (3) in the form

$$h(r) = h_0\left[1-(r/a)^2\right]^p \tag{7}$$

where $h_0 = h(0) = 2\lambda_s P_0/(0.565\ H'\sigma'/m')$ and P_0 is evaluated from (2b). Then, Ω_c becomes alternatively written as

$$\Omega_c = \frac{1+p}{h_0\pi a^2} \tag{8}$$

3. Numerical examples from Bahrami et al. [4] formulation

Table 1 lists results from the formulation summarized in Section 2 applied for spheres made of a relatively soft metal, with properties similar to those of a bronze. The thermal resistances Ω_a, $\Omega_{a,0}$, Ω and Ω_0, also listed in Table 1, will be defined and evaluated as explained in Sections 4 and 5. Besides, the fraction of area in actual contact at $r = 0$ is calculated as $f_{A0} = P_0/H_{162}$, and the significance of parameter $(a/\lambda_s)h_0/(p + 1)$ will be discussed in Sections 5 and 6.

The effects of three variables have been analyzed: particle radius (1.5 and 15 mm), the compression force, assumed imposed by the own weight (light load, experimental bed of height $L_{bed} = 0.15$ m and high load, full-size bed, $L_{bed} = 2.5$ m), and the surface roughness (σ'/m': 1 μm/0.07 and 10 μm/0.25). According to

Table 1
Values of thermal resistances for E = 100 GPa, υ = 0.35, c_1 = 4 GPa, c_2 = −0.26; δ = 7000 kg/m^3, λ_s = 100 W/(m K).

Run#	1 L_{bed} = 0.15 m	2 L_{bed} = 0.15 m	3 L_{bed} = 2.5 m	4 L_{bed} = 2.5 m	5 L_{bed} = 2.5 m
R_p [mm]	1.5	15	1.5	15	15
F [N]	0.065	6.5	1.08	108	108
σ' [μm]	1	1	1	1	10
m'	0.07	0.07	0.07	0.07	0.25
P_0^*	0.073	0.49	0.34	0.86	0.33
a/a_H	5.93	2.29	2.75	1.35	2.79
f_{A0}	0.017	0.115	0.205	0.518	0.260
p	2.86	2.82	2.86	1.35	2.86
Ω_c [K W^{-1}]	1251	12.5	75.0	0.75	1.61
$\Omega_{a,T}$ [K W^{-1}]	48.9	12.7	41.3	8.43	4.07
Ω_a [K W^{-1}]	81.2	19.8	65.7	10.4	6.14
$\Omega_{a,0}$ [K W^{-1}]	82.5	21.3	69.7	11.7	6.87
Ω_H [K W^{-1}]	290	29.0	114	11.4	11.4
Ω_T [K W^{-1}]	1300	25.2	116	9.18	5.68
Ω [K W^{-1}]	1332	32.3	141	11.1	7.75
Ω_0 [K W^{-1}]	1333	33.8	145	12.4	8.47
$(a/\lambda_s)h_0/(p+1)$	0.05	1.29	0.7	14.3	3.22

our estimations, the force on each particle increases proportionally to L_{bed} and R_p^2.

The solid conductivity λ_s is not relevant for the present evaluations, as heat conduction through the fluid is neglected and all thermal resistances involving solid contact depend on λ_s^{-1}. The value λ_s = 100 W/(mK) is assumed for the reported thermal resistances.

The Hertzian contact resistance, $\Omega_H = 1/(4a_H\lambda_s) = [E'/(0.75FR')]^{1/3}/(4\lambda_s)$, for ideally smooth surfaces can be taken as a reference for the data in Table 1. Accordingly, $\Omega_H \propto 1/(L_{bed}^{1/3}R_p)$ and $\lambda_{eff} \propto L_{bed}^{1/3}$, while being independent of particle size (R_p).

The effect of particle size (R_p) under a light load (L_{bed} = 0.15 m) can be appreciated by comparing runs #1 and 2. The micro contact resistance Ω_c is reduced by 100 times when R_p increases from 1.5 to 15 mm, because the force F between particles is raised. The spreading resistance $\Omega_{a,T}$ also decreases, although moderately, due to a larger contact radius a. As a net result, the overall contact resistance Ω_T (Eq. (6)) decreases by a factor of around 50. Noticeably, Ω_T is even somewhat lower than Ω_H for R_p = 15 mm. This feature, which is also shared by run#4 and 5 discussed later on, is not intuitively evident, since the overall contact area is always larger for smooth surfaces. However, the contact spots between the rough spheres are distributed on a larger area ($a/a_H > 1$) in the case of rough particles, a fact that facilitates the heat distribution in the main body of the particles ($\Omega_{a,T} < \Omega_H$). If in addition Ω_c is low enough (as in run #2), the result $\Omega_T < \Omega_H$ can arise.

At the high load (L_{bed} = 2.5 m), the micro contact resistance Ω_c is relatively low, even for the small particle (run#3), and the variation of Ω_T as R_p is raised (runs#3 and 4) follows closely that of Ω_H.

The results for run#5 have been obtained with the remaining parameters as for run#4, but with a rougher surface σ' = 10 μm and m' = 0.25 ($m \propto \sigma^{0.52}$ was considered, according to Lambert and Fletcher [8]). The larger value of σ' in run#5 causes the enlargement of radius a with respect to that of run#4, reducing the value of the spreading resistance $\Omega_{a,T}$ to less than half. As $\Omega_{a,T}$ is the dominant resistance in runs#4 and 5, its effects outweighs the larger micro contact resistance Ω_c in run#5, and a lower overall contact resistance Ω_T arises for the rougher surface. In addition, Ω_T in run#5 is half the value of Ω_H for a smooth surface.

As in the above discussed examples, the role of the spreading resistance will be frequently most relevant, except for relatively light loads and small particles (e.g., run#1). As recalled in Section 2, the spreading resistance is evaluated in the Bahrami et al. [4] approach by assuming that the temperature over the contact disk is uniform ($\Omega_{a,T}$ in Eq. (1)). To examine this assumption, the shape of the thermal conductance $h(r)$ over the contact disk, as defined by the exponent p in Eq. (7), should be further analyzed. If $P_0^* < 0.47$, p = 2.864 (from Eq. (2a), as happens in runs#1, 3 and 5), while for $0.47 < P_0^* < 1$, p decreases down to 0.5. High values of p means that $h(r)$ will decrease rapidly from the maximum (h_0) at r = 0, and will attain very low values even far from the edge $r = a$ (Fig. 1). It can be visualized in Fig. 1 that when p is high the "effective" size of heat transfer zone will be smaller than the disk of radius a, and consequently the actual spreading resistance will be higher than evaluated by Eq. (1). In other terms, the temperature profile inside the disk of radius a cannot be expected to be uniform, as assumed by Eq. (1), and $\Omega_{a,T}$ (Eq. (1)) will provide a low estimate of the spreading resistance. The magnitude of this effect will be found to be significant in Sections 5 and 6, where results are presented to evaluate correctly the spreading resistance by considering the distributed heat flux over the disk, as arises from the profile of the heat conductance $h(r)$ given in Eq. (7).

Before undertaking such a task, it should be noted from Table 1 that the fraction of surface in actual contact close to r = 0 becomes "large". Except for run#1, such fraction just at r = 0, f_{A0}, is larger than 0.1. The significance of this fact is that the assumption of

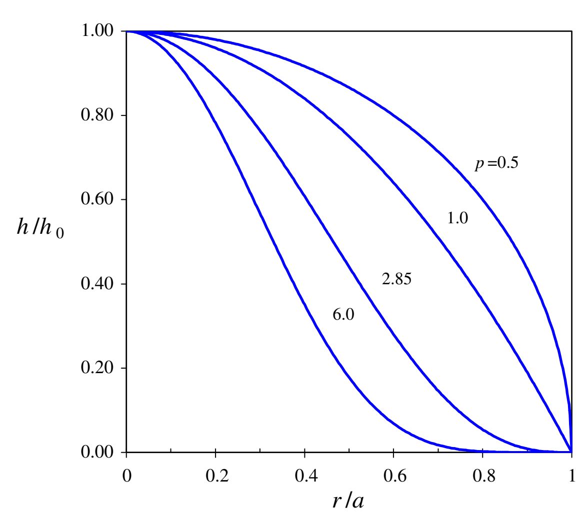

Fig. 1. Conductance profile in a disk of radius a according to Eq. (7) for different values of p.

isolated micro-contacts starts to be unsound. Thus, it has been frequently proposed (see. e.g. Yovanovich and Marotta [9]) that the micro-spreading resistance should decrease approximately by a factor $\left(1-f_A^{0.5}\right)^{1.5}$. When $f_A = 0.1$, such factor is ≈0.57 and the conductance h will be nearly twice the value in Eq. (3). This effect would lead to a further increase of an effective exponent p in Eq. (7). However, at this stage it cannot be directly introduced in the Bahrami et al. [4] formulation, as their basic Eq. (5) for Ω_c does not account for other possible effects, such as the variation of the micro-contact size with the local load $P(r)$, which would partially counteract the effect of $\left(1-f_A^{0.5}\right)^{1.5}$.

4. Discussion on the definition of the spreading resistance Ω_a

Mikic [10] gave a definition of the spreading resistance based on a cylindrical flux-tube (radius b) that exchanges heat through its base ($z = 0$). Here, we will limit the definition to the case when the heat exchange only takes place through a disk of radius a at the cylinder base, while the remaining of the base is maintained adiabatic. The disk itself exchanges heat with an external source/sink kept at uniform temperature T_0 at a local rate

$$q = h(r)(T_c - T_0);\ \text{at}\ \ z = 0\ \ \text{over}\ \ 0 < r < a \tag{9}$$

where $T_c(r)$ is the temperature of the flux tube at $z = 0$. Eq. (9) is a Robin (or 3rd type) boundary condition on $0 < r < a$. The heat exchange rate will be

$$Q = 2\pi \int_0^a h(r)(T_c - T_0)rdr \tag{10}$$

The tube is assumed long enough so that at high enough values of z the temperature gradient can be regarded as being uniform, $(dT/dz)_\infty$. Defining T_s as the hypothetical temperature obtained at $z = 0$ by the extrapolation of the linear profile with slope $(dT/dz)_\infty$, Eq. (10) can be re-written as

$$Q = 2\pi(T_s - T_0)\int_0^a h(r)rdr - 2\pi\int_0^a h(r)(T_s - T_c)rdr \tag{11}$$

The spreading resistance Ω_a is then defined by expressing the overall contact resistance as

$$\Omega_c + \Omega_a = (T_s - T_0)/Q \tag{12}$$

where the micro contact resistance Ω_c is set as

$$\Omega_c = \frac{1}{2\pi\int_0^a h(r)rdr} \tag{13}$$

From Eqs. (11)–(13), Ω_a is expressed as

$$\Omega_a = \frac{1}{Q}\frac{\int_0^a h(r)(T_s - T_c)rdr}{\int_0^a h(r)rdr} \tag{14}$$

When $h(r) = h_0 f(r)$, where $f(r)$ is a shape function such as $[1-(r/a)^2]^p$ in Eq. (7) and h_0 a constant, the situation $h_0 \to 0$ (e.g., when a low force is applied) can be considered. In this case, the temperature field T in the flux-tube will be nearly uniform. Introducing a reference value T_L far enough from the disk, both T_c and T_s will not depart significantly from T_L. Then, the heat flux through the disk can be written as $q = h(r)(T_L - T_0)$. Therefore, this case corresponds to a *variable and prescribed flux problem*. As such, the difference $(T - T_0)$ is proportional to h_0, and particularly $(T_s - T_c)$ will be so. Having this fact in mind and re-writing Eq. (10) as

$$h_0 \to 0: \quad Q = (T_L - T_0)2\pi\int_0^a h(r)rdr, \tag{15}$$

Eq. (14) for Ω_a becomes

$$\Omega_{a,0} = \frac{\int_0^a h(r)(T_s - T_c)rdr}{2\pi(T_L - T_0)[\int_0^a h(r)rdr]^2} \tag{16}$$

where on account of the specific case $h_0 \to 0$, the corresponding value of the spreading resistance is denoted $\Omega_{a,0}$. As $(T_s - T_c) \propto h_0$, $\Omega_{a,0}$ does not depend on h_0.

In his work, Mikic [10] presented a solution for the prescribed flux condition $q \propto h_0 f(r)$, without considering the magnitude of h_0. The result for the thermal resistance is the same as that in Eq. (16), but the analysis made above shows that $\Omega_{a,0}$ represents a limiting value of Ω_a (Eq. (14)) when $h_0 \to 0$, for the Robin boundary condition Eq. (9) with $h(r) = h_0 f(r)$. On the other hand, when $h_0 \to \infty$ (e.g. when a high force is applied), $(T_c - T_0)$ should tend to zero to keep q bounded and the problem will correspond to a Dirichlet boundary condition with temperature T_0 prescribed on the disk. The solution for Ω_a is the one already given in Eq. (1), $\Omega_{a,T} = 0.25/(a\lambda_s)$. For any finite value of h_0, Ω_a will be a function of h_0, further depending on the shape function $f(r)$, and we will see in the Section 5 that $\Omega_{a,T} < \Omega_a < \Omega_{a,0}$.

It is relevant to note that a definition of Ω_a different to that of Mikic [10] is customarily found in the literature (*e.g.* Yovanovich and Marotta [9]). Instead of Eq. (12),

$$\Omega_a = (T_s - \bar{T}_c)/Q \tag{17}$$

where

$$\bar{T}_c = \frac{2}{a^2}\int_0^a T_c(r)rdr \tag{18}$$

Then, Eq. (17) is alternatively written as

$$\Omega_a = \frac{\frac{2}{a^2}\int_0^a [T_s - T_c(r)]\,rdr}{Q} \tag{19}$$

When Eq. (9) is prescribed and Q is given by Eq. (10), the definitions in Eqs. (14) and (19) will render the same Ω_a only if $h(r) \to \infty$ over $0 < r < a$ (case in which the solution $(a\lambda_s)\,\Omega_{a,T} = 0.25$ is again retrieved) or when h is uniform. In particular, when $h \to 0$ uniformly on $0 < r < a$, the very well known result $(a\lambda_s)\,\Omega_{a,0} = 8/(3\pi^2) \approx 0.2702$ is obtained from both definitions.

Coming back to the specific case of interest here for $h(r)$ in Eq. (7), Yovanovich and Marotta [9] reported for $h_0 \to 0$ and $p = 0.5$ the value $(a\lambda_s)\,\Omega_{a,0} = 0.281$ from Eq. (19), and the calculation based on Eq. (14) renders $(a\lambda_s)\,\Omega_{a,0} = 0.297$. The difference at $p = 0.5$ is low, but it increases as p increases. Thus, for $p = 2.85$, $(a\lambda_s)\,\Omega_{a,0} = 0.298$ from Eq. (19) and $(a\lambda_s)\,\Omega_{a,0} = 0.422$ from Eq. (14).

In the general case with a finite value h_0, Ω_a from both definitions will depend on h_0. However, knowing just the value of Ω_a according to Eq. (19) does not suffice to evaluate the rate Q in a given case, as $\bar{T}_c$ should also be known to use Eq. (17) (Eq. (10) is not an option either, as the profile $T_c(r)$ is required). Instead, knowing Ω_a from (14) suffices to evaluate Q, using Eqs. (12) and (13). Therefore, it seems that the definition (14) is more practical when a Robin condition with variable $h(r)$ (*e.g.* Eq. (7)) is undertaken, and the concept of a spreading resistance is to be applied. The definition (14) will be employed in Section 5 to evaluate Ω_a for $h(r)$ defined in Eq. (7).

5. Assessment of Ω_a from Eq. (14) and $h = h_0[1-(r/a)^2]^p$

According to our knowledge, a general analytical solution of the Laplace equation $\nabla^2 T = 0$ in the flux tube and boundary condition (9) is not available. Therefore, we have evaluated the temperature field numerically by employing the Comsol Multiphysics platform. This task was carried out by taking a finite height of the tube L, $T(L) = T_L$ uniform over $0 < r < b$, and the adiabatic condition at $r = b$ over $0 \leqslant z \leqslant L$ and at $z = 0$ over $a < r < b$. The calculations were

Table 2
Comparison of $(a\lambda_s)\ \Omega_{a,0}$ using Comsol Mutiphysics and from Eqs. (20) and (21).

p	$(a\lambda_s)\ \Omega_{a,0}$	
	Numerical values	Eqs. (20) and (21)
0	0.270	0.267
0.5	0.297	0.299
1	0.327	0.328
2.85	0.422	0.421
6	0.549	0.546

made with ratios $L/b = 1/4$ and $b/a = 200$. It has been found that the results are insensitive for larger values of these ratios. Therefore, values of Ω_a (Eq. (14)) presented here correspond to $a/b \to 0$. The mesh was refined so as to achieve values of Ω_a within a precision of 0.5%. The results are expressed as

$$\Omega_a = \Omega_{a,T} + \Delta\Omega_a, \tag{20}$$

recalling that $\Omega_{a,T} = 1/(4a\lambda_s)$ is the value when T_c is constant on $0 < r < a$, or equivalently when $h_0 \to \infty$ ($T_c = T_0$).

The range chosen for parameter p was $0 < p < 6$. The upper bound was chosen on account that $p = 2.864$ is a typical value for the Bahrami et al. [4] formulation, but even higher values cannot be ruled out if some other effects are included in the evaluation of $h(r)$.

The case $h_0 \to 0$ (i.e. prescribed non-uniform flux on the disk) is first undertaken. The corresponding values of the spreading resistance obtained by using Comsol Multiphysics are expressed, according to Eq. (20), as $\Omega_{a,0} = \Omega_{a,T} + \Delta\Omega_{a,0}$, and $\Delta\Omega_{a,0}$ was approximated by means of the following expression

$$(a\lambda_s)\Delta\Omega_{a,0} = 0.446\ ln(1.04 + 0.15p), \tag{21}$$

which compares quite well with the numerical results (Table 2). Note the significant variation of $\Omega_{a,0}$ (up to 100%) in the range $0 < p < 6$.

For finite values of h_0 (*i.e.* for a Robin boundary condition on the disk), the following approximate expression was developed on the basis of the numerical results obtained from Comsol Multiphysics

$$\Delta\Omega_a = \frac{\Delta\Omega_{a,0}}{1 + [0.04(a/\lambda_s)h_0/(p+1)]^{1/\sqrt{p+1}}} \tag{22}$$

When the heat exchange rate Q is calculated using Eq. (12), Eq. (8) for Ω_c and Eqs. (20)–(22) for Ω_a, the results deviate by a maximum of 4% from the numerical values calculated using Comsol Multiphysics within the ranges $0 < p < 6$, $0 < h_0 < \infty$. Such a deviation is acceptable for most practical purposes.

Defining from Eq. (12) the overall contact resistance Ω

$$\Omega = \Omega_c + \Omega_a, \tag{23}$$

the dimensionless heat-exchange rate $Q/[a\lambda_s(T_s - T_0)] = [a\lambda_s\ \Omega]^{-1}$ is plotted in Fig. 2 as a function of parameter $(4/\pi)\ \Omega_{a,T}/\Omega_c = (a/\lambda_s)h_0/(p+1)$, using numerical and approximate (Eqs (20)–(22)) values of Ω_a. Fig. 2 illustrates the good agreement between approximate and numerical results.

Results by taking $\Omega_a \equiv \Omega_{a,T}$, i.e. as employed by Bahrami et al. [4] to evaluate the overall thermal resistance Ω_T (Eq. (6)), are also plotted in Fig. 2. The values of the dimensionless heat-exchange rate thus calculated approach those numerically evaluated either for low or for high values of the ratio $(4/\pi)\ \Omega_{a,T}/\Omega_c$. This behavior

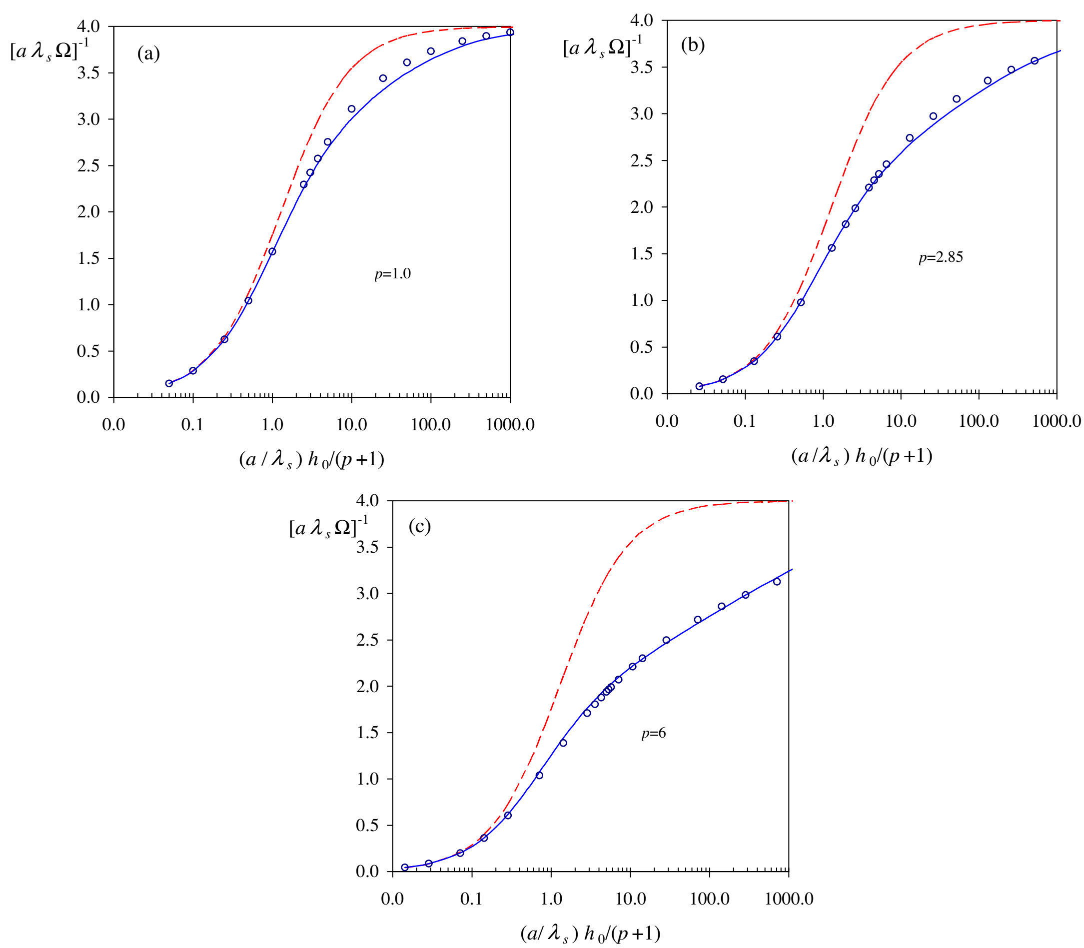

Fig. 2. Values of $Q/[a\lambda_s(T_s - T_0)] = [a\lambda_s\Omega]^{-1}$ evaluated for different values of p (cases a, b, c) and different alternatives for the spreading resistance. Symbols (○): results using Comsol Mutiphysics for Ω_a; ——: results using Eqs. (20)–(22) for Ω_a; – – –: results using $\Omega_a \equiv \Omega_{a,T}$ (Eq. (6)).

reflects the fact that when this ratio is low Ω_c controls the rate of heat transfer irrespective of the spreading resistance (Ω_a or $\Omega_{a,T}$), while when the ratio is large the overall heat transfer process becomes controlled by nearly the same spreading resistance, as $\Omega_a \to \Omega_{a,T}$. At intermediate values of $(4/\pi)\ \Omega_{a,T}/\Omega_c$, very significant differences arise when p is large enough (e.g. $p = 2.85, 6$).

The results here provided correspond to $h(r)$ defined in Eq. (7). For a more general expression $h(r) = h_0 f(r)$, where the shape function monotonically decreased from $r = 0$ to $r = a$, it is tentatively proposed employing Eqs. (21) and (22) with an effective value of p evaluated from

$$2\int_0^a f(r) r dr = \frac{a^2}{p+1} \tag{24}$$

In addition, when a/b is finite in a flux tube or for a sphere with $b \equiv R_p$, the spreading resistance, say $(\Omega_a)_{\text{finite }(a/b)}$, will also depend on the ratio a/b. For an isothermal contact disk, the approximate expression $(\Omega_{a,T})_{\text{finite}(a/b)} = (1 - a/b)^{1.5}\Omega_{a,T}$, with $\Omega_{a,T}$ given by Eq. (1), has been proposed (*e.g.* Yovanovich and Marotta [9]). In the more general case considered here, it was discussed in Section 3 that the "effective" size of the contact spot will be smaller than a, in relation to Eq. (1). If we evaluate such a size as $(a\ \Omega_{a,T}/\Omega_a)$, with Ω_a from Eqs. (20)–(22), the following approximation is tentatively proposed to estimate the spreading resistance for finite ratios (a/b):

$$(\Omega_a)_{\text{finite}(a/b)} = \left(1 - \frac{\Omega_{a,T}}{\Omega_a}\frac{a}{b}\right)^{1.5}\Omega_a$$

6. Further analysis of the examples from Bahrami et al. [4] formulation

Table 1 includes values of $\Omega_{a,0}$ (Eqs. (20) and (21)), Ω_a (Eqs (20)–(22)) and Ω (Eq. (23)) for the different runs. According to the considerations in this work, the correct spreading resistances Ω_a largely exceed the values of $\Omega_{a,T}$ assumed in the Bahrami et al. [4] formulation. For the four cases with $p = 2.85$, the increase ranges from 50% to 66%. Nonetheless, the overall resistances, as expressed by Ω_T or Ω, should be ultimately compared. For run#1, the micro contact resistance is definitely dominant (parameter $(4/\pi)\ \Omega_{a,T}/\Omega_c = (a/\lambda_s)h_0/(p+1)$ also given in Table 1 reflects this fact) and Ω is close to Ω_T. For the remaining four runs, the overall contact resistance Ω, is 20–36% larger than Ω_T in Bahrami et al. [4] formulation. Clearly, these differences are significant. The largest difference (36%) is reached for run#5 at an intermediate value of $(a/\lambda_s)h_0/(p+1)$, which according to the discussion in Section 5 should lead to larger differences between both ways to evaluate the overall thermal resistances or, equivalently, between the heat exchange rates. It is also noted that only for run#5 Ω becomes significantly lower than the Hertz value Ω_H.

It is finally remarked that, at least for the range of conditions spanned in Table 1, the limiting spreading resistance $\Omega_{a,0}(h_0 \to 0)$ provides a better approximation than $\Omega_{a,T}(h_0 \to \infty)$, as values $\Omega_0 = \Omega_{a,0} + \Omega_c$ are in closer agreement with Ω than values $\Omega_T = \Omega_{a,T} + \Omega_c$.

7. Conclusions

The solid–solid contact of rough spherical particles in packed beds takes place by the deformation of individual asperities that transmit the stresses to the main body of the particles, causing a usually elastically deformed area (a disk) enclosing the micro-contacts. To evaluate the effect of the solid–solid contact on the effective thermal conductivity of packed beds, Bahrami et al. [4] integrated the local conductance $h(r)$, as defined by the micro-contacts, over the disk enclosing them, leading to a micro contact resistance Ω_c. Another thermal resistance, called spreading resistance Ω_a, arises due to the convergence of flux lines in the bulk of the spheres towards the disk. Thus, the overall contact resistance Ω can be calculated as $\Omega_c + \Omega_a$. In the quoted reference the spreading resistance is evaluated as if the temperature were uniform within the disk, $\Omega_{a,T}$. However, $h(r)$ according to the own formulation of Bahrami et al. [4] will usually drop to very small values far from the disk edge and consequently $\Omega_{a,T}$ underestimates the actual spreading resistance. Considering that under normal conditions the size of the disk is much smaller than the radius of the spheres and recognizing that the flux on the disk imposes a Robin type of boundary condition for the Laplace equation inside the spheres, the correct spreading resistance Ω_a was numerically evaluated for the conductance distribution $h(r) = h_0\ [1 - (r/a)^2]^p$ proposed in the work of Bahrami et al. [4].

The spreading resistance Ω_a depends on the exponent p and on the parameter $(4/\pi)\Omega_{a,T}/\Omega_c = (a/\lambda_s)h_0/(p+1)$. Values of Ω_a can be noticeably higher than $\Omega_{a,T}$ even for $p = 0.5$, while for the largest exponent tried ($p = 6$) Ω_a can reach twice the value of $\Omega_{a,T}$. In terms of the overall contact resistance, the differences between $\Omega_{a,T}$ and Ω_a will not have a large impact when the micro contact resistance Ω_c is dominant, i.e. for low compression forces, but the overall contact resistance can become significantly underestimated in the case of relatively large loads, in particular when rough particles are involved.

The results of Ω_a have been satisfactorily correlated according to Eqs. (20)–(22) in terms of p and parameter $(4/\pi)\ \Omega_{a,T}/\Omega_c = (a/\lambda_s)h_0/(p+1)$.

Conflict of interest

None declared.

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