Generalized Abstract Argumentation: Handling Arguments in FOL Fragments

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Abstract. Generalized argumentation frameworks relate formulae in classical logic to arguments based on the Dung's classic framework. The main purpose of the generalization is to provide a theory capable of reasoning (following argumentation technics) about inconsistent knowledge bases (KB) expressed in FOL fragments. Consequently, the notion of argument is related to a single formula in the KB. This allows to share the same primitive elements from both, the framework (arguments) and, the KB (formulae). A framework with such features would not only allow to manage a wide range of knowledge representation languages, but also to cope with the dynamics of knowledge in a straightforward manner.

1 Introduction

The formalism studied in this work is based on the widely accepted Dung's argumentation framework (AF) [1]. An AF is deemed as abstract since the language used to define arguments remains unspecified, thus, arguments in an AF are treated as "black boxes" of knowledge. In this work we go one step further into a not-so-abstract form of argumentation by proposing an argument language $\mathbb{A}\mathtt{rgs}$ in order to provide some structure to the notion of arguments while keeping them abstract. Intuitively, an *argument* may be seen as *an indivisible piece of knowledge inferring a claim from a set of premises*. Since claims and premises are distinguishable entities of any argument, we will allow both to be expressed through different sublanguages. The proposed *argument language* $\mathbb{A}\mathtt{rgs}$ is thus characterized through the interrelation between its inner components. Assuming arguments specified through $\mathbb{A}\mathtt{rgs}$ would bring about a highly versatile framework given that different knowledge representation languages could be handled through it. But consequently, some basic elements of the argumentation machinery should be accommodated, giving rise to a new kind of abstract argumentation frameworks identified as generalized (GenAF). The first approach to a GenAF in [2] was inspired by [3,4].

The GenAF here proposed aims at reasoning about inconsistent knowledge bases (KB) expressed through some fragment of first order logic (FOL). Consequently, $\mathbb{A}\mathtt{rgs}$ will be reified to the restriction imposed to the FOL KB. Thus, the maximum expressive power of a GenAF is imposed by restricting the inner components of $\mathbb{A}\mathtt{rgs}$ to be bounded to some logic $\mathcal{L}^\kappa$, with $\kappa \in \mathbb{N}_0$[1]. Formulae in $\mathcal{L}^\kappa$ are those of FOL that can be built with the help of predicate symbols with arity $\leq \kappa$, including equality and constant symbols, but without function symbols. An example of an $\mathcal{L}^2$-compliant logic is the $\mathcal{ALC}$ DL used to describe basic ontologies. The interested reader is referred to [5,6].

A normal form for a $\mathcal{L}^\kappa$ KB is presented to reorganize the knowledge in the KB through sentences conforming some minimal pattern, which will be interpreted as single arguments in the `GenAF`. Therefore, a `GenAF` may be straightforwardly adapted to deal with dynamics of knowledge as done in [4]: deleting an argument from the framework would mean deleting a statement from the KB. Argumentation frameworks were also related to FOL in [7], however, since there was no intention to cope with dynamics of arguments, no particular structure was provided to manage statements in the KB through single arguments. In this sense, our proposal is more similar to that in [8], although we relate the notions of deduction and conflict to FOL interpretations.

Specifying $\mathbb{A}\mathtt{rgs}$ could bring about some problems: the language for claims may consider conjunctive and/or disjunctive formulae. For the former case, the easiest option is to trigger a different claim for each conjunctive term. For the case of disjunctive formulae for claims, the problem seems to be more complicated. To that matter we introduce the notion of *coalition*, which is a structure capable of grouping several arguments with the intention to support an argument's premise, identify conflictive sources of knowledge, or even to infer new knowledge beyond the one specified through the arguments considered in it. In argumentation theory, an argument's premises are satisfied in order for that argument to reach its claim. This is usually referred as *support relation* [3], handled in this work through coalitions.

Usually, an abstract argument is treated as an indivisible entity that suffices to support a claim; here arguments are also indivisible but they play a smaller role: they are aggregated in structures which can be thought as if they were arguments in the usual sense [7]. However, we will see that they do not always guarantee the achievement of the claim. The idea behind the aggregation of arguments within a structure is similar to that of sub-arguments [9]. Besides, classic argumentation frameworks consider ground arguments, that is, a claim is directly inferred if the set of premises are conformed. In our framework, we consider two different kinds of arguments: *ground* and *schematic*. In this sense, a set of premises might consider free variables, meaning that the claim, and therefore the inference, will depend on them. Thus, when an argument $\mathcal{B}$ counts with free variables in its claim or premises, it will be called schematic; whereas $\mathcal{B}$ is referred as ground, when its variables are instantiated. Instantiation of variables within a schematic argument may occur as a consequence of its premises being supported.

Finally, a basic acceptability semantics is proposed, inspired in the grounded semantics [10]. These semantics ensure the obtention of a consistent set of arguments, from which the accepted knowledge (warranted formulae) can be identified.

2 Foundations for a Generalized AF

For $\mathcal{L}^\kappa$, we use $p, p_1, p_2, \ldots$ and $q, q_1, q_2, \ldots$ to denote monadic predicate letters, $r, r_1, r_2, \ldots$ for dyadic predicate letters, x, y for free variable objects, and a, b, c, d for

[1] Natural numbers are enclosed in the sets $\mathbb{N}_0 = \{0, 1, \ldots\}$ and $\mathbb{N}_1 = \{1, 2, \ldots\}$.

constants (individual names). Besides, the logic $\mathcal{L}_{\mathtt{A}} \subset \mathcal{L}^{\kappa}$ identifies the fragment of $\mathcal{L}^{\kappa}$ describing *assertional formulae* (ground atoms and their negations). Recall that ground atoms are atomic formulae which do not consider variable objects. The logic $\mathcal{L}^{\kappa}$ is interpreted as usual through interpretations $\mathcal{I} = \langle \Delta^{\mathcal{I}}, p^{\mathcal{I}}, p_1^{\mathcal{I}}, \ldots, q^{\mathcal{I}}, q_1^{\mathcal{I}}, \ldots, r^{\mathcal{I}}, r_1^{\mathcal{I}}, \ldots \rangle$, where $\Delta^{\mathcal{I}}$ is the interpretation domain, and $p^{\mathcal{I}}, p_1^{\mathcal{I}}, \ldots, q^{\mathcal{I}}, q_1^{\mathcal{I}}, \ldots, r^{\mathcal{I}}, r_1^{\mathcal{I}}, \ldots$ interpret $p, p_1, \ldots, q, q_1, \ldots, r, r_1, \ldots$, respectively. For an interpretation $\mathcal{I}$, some $a \in \Delta^{\mathcal{I}}$, and a formula $\varphi(x)$, we write $\mathcal{I} \models \varphi(a)$ if $\mathcal{I}, v \models \varphi(x)$, for the assignment v mapping x to a. For simplicity we omit universal quantifiers writing $\varphi(x)$ to refer to $(\forall x)(\varphi(x))$.

As mentioned before, we will rely on a (abstract) language $\mathbb{A}\mathtt{rgs}$ (for arguments) composed by two (unspecified) inner sub-languages: $\mathcal{L}_{\mathtt{pr}}$ (for premises) and $\mathcal{L}_{\mathtt{cl}}$ (claims).

Definition 1 (Argument Language). *Given the logic $\mathcal{L}^{\kappa}$, an **argument language** $\mathbb{A}\mathtt{rgs}$ is defined as $2^{\mathcal{L}_{\mathtt{pr}}} \times \mathcal{L}_{\mathtt{cl}}$, where $\mathcal{L}_{\mathtt{cl}} \subseteq \mathcal{L}^{\kappa}$ and $\mathcal{L}_{\mathtt{pr}} \subseteq \mathcal{L}^{\kappa}$ are recognized as the respective languages for claims and premises in $\mathbb{A}\mathtt{rgs}$.*

Since a premise is supported through the claim of other argument/s, the expressivity of both languages $\mathcal{L}_{\mathtt{pr}}$ and $\mathcal{L}_{\mathtt{cl}}$ should be controlled in order to allow every describable premise to be supported by formulae from the language for claims. Therefore, to handle the language $\mathbb{A}\mathtt{rgs}$ at an abstract level, we will characterize it by relating $\mathcal{L}_{\mathtt{pr}}$ and $\mathcal{L}_{\mathtt{cl}}$.

Definition 2 (Legal Argument Language). *An argument language $2^{\mathcal{L}_{\mathtt{pr}}} \times \mathcal{L}_{\mathtt{cl}}$ is **legal** iff for every $\rho \in \mathcal{L}_{\mathtt{pr}}$ there is a set $\Phi \subseteq \mathcal{L}_{\mathtt{cl}}$ such that $\Phi \models \rho$ **(support)**.*

In the sequel any argument language used will be assumed to be legal. Argumentation frameworks are a tool to reason about potentially inconsistent knowledge bases. Due to complexity matters, it would be interesting to interpret any $\mathcal{L}^{\kappa}$ KB directly as an argumentation framework with no need to transform the KB to a `GenAF`. Intuitively, an argument poses a reason to believe in a claim if it is the case that its premises are supported. This intuition is similar to the notion of material conditionals (implications "$\rightarrow$") in classical logic. Hence, statements from a KB could give rise to a single argument. To this end, we propose a normal form for $\mathcal{L}^{\kappa}$ KBs.

Definition 3 (`pANF`). *Given a knowledge base $\Sigma \subseteq \mathcal{L}^{\kappa}$, and an argument language $\mathbb{A}\mathtt{rgs}$, Σ conforms to the **pre-argumental normal form** (`pANF`) iff every formula $\varphi \in \Sigma$ is an assertion in $\mathcal{L}_{\mathtt{A}}$, or it corresponds to the form $\rho_1 \wedge \ldots \wedge \rho_n \rightarrow \alpha$, where $\alpha \in \mathcal{L}_{\mathtt{cl}}$ and $\rho_i \in \mathcal{L}_{\mathtt{pr}}$ $(1 \leq i \leq n)$. Hence, each formula $\varphi \in \Sigma$ is said to be in `pANF`.*

Example 1. [2] Suppose $\mathcal{L}_{\mathtt{cl}}$ and $\mathcal{L}_{\mathtt{pr}}$ are concretized as follows: $\mathcal{L}_{\mathtt{cl}}$ allows disjunctions but prohibits conjunctions; whereas $\mathcal{L}_{\mathtt{pr}}$ avoids both conjunctions and disjunctions. This would require for a formula like $(p_1(x) \wedge p_2(x)) \vee (p_3(x) \wedge p_4(x)) \rightarrow q_1(x) \wedge (q_2(x) \vee q_3(x))$ to be reformatted into the `pANF` formulae $p_1(x) \wedge p_2(x) \rightarrow q_1(x)$, $p_1(x) \wedge p_2(x) \rightarrow q_2(x) \vee q_3(x)$, $p_3(x) \wedge p_4(x) \rightarrow q_1(x)$ and $p_3(x) \wedge p_4(x) \rightarrow q_2(x) \vee q_3(x)$.

Next we formalize the generalized notion of argument independently from a KB. The relation between premises and claims wrt. a KB could be referred to Remark 1.

Definition 4 (Argument). *An **argument** $\mathcal{B} \in \mathbb{A}\mathtt{rgs}$ is a pair $\langle \Gamma, \alpha \rangle$, where $\Gamma \subseteq \mathcal{L}_{\mathtt{pr}}$ is a finite set of finite premises, $\alpha \in \mathcal{L}_{\mathtt{cl}}$, its finite claim, and $\Gamma \cup \{\alpha\} \not\models \bot$ **(consistency)**.*

[2] For simplicity, examples are enclosed within $\mathcal{L}^2$ to consider only predicates of arity ≤ 2.

Usually, *evidence* is considered a basic irrefutable piece of knowledge. This means that evidence does not need to be supported given that it is self-justified by definition. Thus, two options appear to specify evidence: as a separate entity in the framework, or as arguments with no premises to be satisfied. In this article we assume the latter posture, referring to them as *evidential arguments*.

Definition 5 (Evidence). *Given an argument $\mathcal{B} \in \mathbb{A}\mathtt{rgs}$, $\mathcal{B}$ is referred as* ***evidential argument*** *(or just evidence) iff $\mathcal{B} = \langle \{\}, \alpha \rangle$ with $\alpha \in \mathcal{L}_{\mathtt{A}}$ (assertional formulae).*

Given $\mathcal{B} \in \mathbb{A}\mathtt{rgs}$, its claim and set of premises are identified by the functions $\mathfrak{cl} : \mathbb{A}\mathtt{rgs} \longrightarrow \mathcal{L}_{\mathtt{cl}}$, and $\mathfrak{pr} : \mathbb{A}\mathtt{rgs} \longrightarrow 2^{\mathcal{L}_{\mathtt{pr}}}$, respectively. For instance, given $\mathcal{B} = \langle \{\rho_1, \rho_2\}, \alpha \rangle$, its premises are $\mathfrak{pr}(\mathcal{B}) = \{\rho_1, \rho_2\}$, and its claim, $\mathfrak{cl}(\mathcal{B}) = \alpha$. Arguments will be obtained from pANF formulae through an *argument translation function* $\mathfrak{arg} : \mathcal{L}^{\kappa} \longrightarrow \mathbb{A}\mathtt{rgs}$ such that $\mathfrak{arg}(\varphi) = \langle \{\rho_1, \ldots, \rho_n\}, \alpha \rangle$ *iff* $\varphi \in \mathcal{L}^{\kappa}$ is a pANF formula $\rho_1 \wedge \ldots \wedge \rho_n \rightarrow \alpha$ and $\mathfrak{arg}(\varphi)$ verifies the conditions in Def. 4. Otherwise, $\mathfrak{arg}(\varphi) = \langle \emptyset, \bot \rangle$. An evidential argument $\mathfrak{arg}(\varphi) = \langle \emptyset, \alpha \rangle$ appears if φ is $\rightarrow \alpha$.

Example 2 (Continued from Ex. 1). For the formulae given in Ex. 1, the arguments $\langle \{p_1(x), p_2(x)\}, q_1(x) \rangle$, $\langle \{p_1(x), p_2(x)\}, q_2(x) \vee q_3(x) \rangle$, $\langle \{p_3(x), p_4(x)\}, q_1(x) \rangle$ and $\langle \{p_3(x), p_4(x)\}, q_2(x) \vee q_3(x) \rangle$, are triggered by effect of the function "$\mathfrak{arg}$".

As mentioned before, it is important to recall that the notion of argument adopted in this work differs from its usual usage. This is made clear in the following remark.

Remark 1. Given a pANF KB $\Sigma \subseteq \mathcal{L}^{\kappa}$, a formula $\varphi \in \Sigma$, and its associated argument $\mathfrak{arg}(\varphi) = \langle \Gamma, \alpha \rangle$; it follows $\Sigma \models (\bigwedge \Gamma) \rightarrow \alpha$, but $\Gamma \models \alpha$ does not necessarily hold.

A more restrictive definition of argument could consider conditions like $\Gamma \not\models \alpha$, and/or $\Gamma \setminus \{\rho\} \not\models \rho$, with $\rho \in \Gamma$. However, its appropriate discussion exceeds the scope of this article. For the usual notion of argument see *argumental structures* in Def. 15.

The formalization of the GenAF will rely on *normality conditions*: user defined constraints in behalf of the appropriate construction of the argumentation framework.

Definition 6 (GenAF). *A* ***generalized abstract argumentation framework*** (GenAF) *is a pair $\langle \mathbf{A}, \mathbf{N} \rangle$, where $\mathbf{A} \subseteq \mathbb{A}\mathtt{rgs}$ is a finite set of arguments, and $\mathbf{N} \subseteq \mathbb{N}\mathtt{orm}$, a finite set of normality condition functions $\mathfrak{nc} : 2^{\mathbb{A}\mathtt{rgs}} \longrightarrow \{true, false\}$. The domain of functions $\mathfrak{nc}$ is identified through $\mathbb{N}\mathtt{orm}$, and $\mathbb{G}$ identifies the class of every* GenAF*. The set $\mathbf{E} \subseteq \mathbf{A}$ encloses every evidential argument from $\mathbf{A}$.*

A normality condition required through $\mathbf{N}$, could be to require evidence to be consistent, that is no pair of contradictory evidential arguments should be available in the framework. Other conditions could be to restrict arguments from being non-minimal justifications for the claim, or from including the claim itself as a premise.

(evidence coherency) there is no pair $\langle \{\}, \alpha \rangle \in \mathbf{A}$ and $\langle \{\}, \neg\alpha \rangle \in \mathbf{A}$.
(minimality) there is no pair $\langle \Gamma, \alpha \rangle \in \mathbf{A}$ and $\langle \Gamma', \alpha \rangle \in \mathbf{A}$ such that $\Gamma' \subset \Gamma$.
(relevance) there is no $\langle \Gamma, \alpha \rangle \in \mathbf{A}$ such that $\alpha \in \Gamma$.

Other normality conditions may appear depending on the concretization of the logic for arguments and the environment the framework is set to model. The complete study of these features falls out of the scope of this article. Given a $\mathtt{GenAF}$ $T = \langle \mathbf{A}, \mathbf{N} \rangle$, we will say that T is a *theory iff* for every normality condition $\mathfrak{nc} \in \mathbf{N}$ it follows $\mathfrak{nc}(\mathbf{A}) = true$. From now on we will work only with theories, thus unless the contrary is stated, every framework will be assumed to conform a theory. Moreover, the framework specification is done in such a way that its correctness does not rely on the normality conditions required. Thus, a $\mathtt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle$, with $\mathbf{N} = \emptyset$, will be trivially a theory.

In order to univocally determine a single GenAF from a given KB and a set of normality conditions, it is necessary to assume a comparison criterion among formulae in the KB. Such criterion could be defined for instance, upon entrenchment of knowledge, *i.e.*, levels of importance are related to formulae in the KB. As will be seen later, this criterion will determine the *argument comparison criterion* from which the attack relation is usually specified in the classic argumentation literature. Next, we define a *theory function* to identify the GenAF associated to a KB.

Definition 7 (Theory Function). *Given a* pANF *knowledge base* $\Sigma \subseteq \mathcal{L}^{\kappa}$*, and a set* $\mathbf{N} \subseteq \mathbb{N}\mathtt{orm}$ *of normality condition functions* $\mathfrak{nc}$*, a **theory function*** $\mathfrak{genaf} : 2^{\mathcal{L}^{\kappa}} \times 2^{\mathbb{N}\mathtt{orm}} \longrightarrow \mathbb{G}$ *identifies the* GenAF $\mathfrak{genaf}(\Sigma, \mathbf{N}) = \langle \mathbf{A}, \mathbf{N} \rangle$*, where* $\mathbf{A} \subseteq \{\mathfrak{arg}(\varphi) | \varphi \in \Sigma$ *and* $\mathfrak{arg}(\varphi)$ *is an argument*$\} \cup \{\mathfrak{arg}(\varphi \rightarrow \varphi') | (\neg\varphi' \rightarrow \neg\varphi) \in \Sigma$ *and* $\mathfrak{arg}(\varphi \rightarrow \varphi')$ *is an argument*$\}$ *and* $\mathbf{A}$ *is the maximal set (wrt. set inclusion and the comparison criterion in* Σ*) such that for every* $\mathfrak{nc} \in \mathbf{N}$ *it holds* $\mathfrak{nc}(\mathbf{A}) = true$*.*

The GenAF obtained by the function "$\mathfrak{genaf}$" will consider a maximal subset of the KB Σ such that the resulting set of arguments (triggered by "$\mathfrak{arg}$") is compliant with the normality conditions. Note that also the counterpositive formula of each one considered is assumed to conform an argument in the resulting GenAF. This is natural since counterpositive formulae from the statements in a KB are implicitly considered to reason in classical logic. In a GenAF, this issue is done by considering both explicitly.

3 The GenAF Argumentation Machinery

The purpose of generalizing an abstract argumentation framework comes from the need of managing different argument languages specified through some FOL fragment. Given the specification of $\mathbb{A}\mathtt{rgs}$, different possibilities may arise, for instance, the language for claims may accept disjunction of formulae. Thus, it is possible to infer a formula in $\mathcal{L}_{\mathtt{cl}}$ from several arguments in the GenAF through their claims. Consider for example, two arguments $\langle \{p_1(x)\}, q_1(x) \vee q_2(x) \rangle$ and $\langle \{p_2(x)\}, \neg q_2(x) \rangle$, the claim $q_1(x)$ may be inferred. This kind of constructions are similar to arguments themselves, but are implicitly obtained from the GenAF at issue. To such matter, the notion of *claiming-coalition* is introduced as *a coalition required to infer a new claim*.

In general, a *coalition* might be interpreted as *a minimal and consistent set of arguments guaranteeing certain requirement*. We say that a coalition $\widehat{\mathcal{C}} \subseteq \mathbb{A}\mathtt{rgs}$ is consistent *iff* $\mathfrak{prset}(\widehat{\mathcal{C}}) \cup \mathfrak{clset}(\widehat{\mathcal{C}}) \not\models \bot$, while minimality ensures that $\widehat{\mathcal{C}}$ guarantees a requirement θ *iff* there is no proper subset of $\widehat{\mathcal{C}}$ guaranteeing θ. The functions $\mathfrak{clset} : 2^{\mathbb{A}\mathtt{rgs}} \longrightarrow 2^{\mathcal{L}_{\mathtt{cl}}}$ and $\mathfrak{prset} : 2^{\mathbb{A}\mathtt{rgs}} \longrightarrow 2^{\mathcal{L}_{\mathtt{pr}}}$ are defined as $\mathfrak{clset}(\widehat{\mathcal{C}}) = \{\mathfrak{cl}(\mathcal{B}) | \mathcal{B} \in \widehat{\mathcal{C}}\}$, and $\mathfrak{prset}(\widehat{\mathcal{C}}) = \bigcup_{\mathcal{B} \in \widehat{\mathcal{C}}} \mathfrak{pr}(\mathcal{B})$, to respectively identify the set of claims and premises from $\widehat{\mathcal{C}}$. In this article, three types of coalitions will be considered. Regarding claiming-coalitions, the requirement θ is a new inference in $\mathcal{L}_{\mathtt{cl}}$ from the arguments considered by the coalition.

Definition 8 (Claiming-Coalition). *Given a* GenAF $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, and a formula* $\alpha \in \mathcal{L}_{\mathtt{cl}}$*, a set of arguments* $\widehat{\mathcal{C}} \subseteq \mathbf{A}$ *is a claiming-coalition, or just a* ***claimer****, of* α *iff* $\widehat{\mathcal{C}}$ *is the minimal coalition guaranteeing* $\mathfrak{clset}(\widehat{\mathcal{C}}) \models \alpha$ *and* $\widehat{\mathcal{C}}$ *is consistent.*

Note that a claiming-coalition containing a single argument $\mathcal{B}$ is a primitive coalition for the claim of $\mathcal{B}$. As said before, an argument needs to find its premises supported as a functional part of the reasoning process to reach its claim. In this framework, due to the characterization of $\mathbb{A}\mathtt{rgs}$, sometimes a formula from $\mathcal{L}_{\mathtt{pr}}$ could be satisfied only through several formulae from $\mathcal{L}_{\mathtt{cl}}$. This means that a single argument is not always enough to support a premise of another argument. Thus, we will extend the usual definition of supporter [3] by introducing the notion of *supporting-coalition*.

Definition 9 (Supporting-Coalition). *Given a* GenAF $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, an argument* $\mathcal{B} \in \mathbf{A}$*, and a premise* $\rho \in \mathfrak{pr}(\mathcal{B})$*. A set of arguments* $\widehat{\mathcal{C}} \subseteq \mathbf{A}$ *is a supporting-coalition, or just a* ***supporter****, of* $\mathcal{B}$ *through* ρ *iff* $\widehat{\mathcal{C}}$ *is the minimal coalition guaranteeing* $\mathfrak{clset}(\widehat{\mathcal{C}}) \models \rho$ *and* $\widehat{\mathcal{C}} \cup \{\mathcal{B}\}$ *is consistent.*

Example 3. Assume $\mathbf{A} = \{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4\}$, where $\mathcal{B}_1 = \langle \{p_1(x)\},\ q_1(x) \rangle$, $\mathcal{B}_2 = \langle \{p_1(x)\},\ q_2(x) \rangle$, $\mathcal{B}_3 = \langle \{p_2(x)\}, p_1(x) \vee q_1(x) \rangle$, and $\mathcal{B}_4 = \langle \{p_3(x)\}, \neg q_1(x) \rangle$. The set $\widehat{\mathcal{C}} = \{\mathcal{B}_3, \mathcal{B}_4\}$ is a supporter of $\mathcal{B}_2$. Note that $\widehat{\mathcal{C}}$ cannot be a supporting-coalition of $\mathcal{B}_1$ since it violates (supporter) consistency.

When not every necessary argument to conform the supporting-coalition is present in $\mathbf{A}$, the (unsupported) premise is referred as free.

Definition 10 (Free Premise). *Given a* GenAF $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$ *and an argument* $\mathcal{B} \in \mathbf{A}$*, a premise* $\rho \in \mathfrak{pr}(\mathcal{B})$ *is* ***free*** *wrt.* $\mathbf{A}$ *iff there is no supporter* $\widehat{\mathcal{C}} \subseteq \mathbf{A}$ *of* $\mathcal{B}$ *through* ρ*.*

From Ex. 3, premises $p_2(x) \in \mathfrak{pr}(\mathcal{B}_3)$, $p_3(x) \in \mathfrak{pr}(\mathcal{B}_4)$, and $p_1(x) \in \mathfrak{pr}(\mathcal{B}_1)$ are free wrt. $\mathbf{A}$; whereas $p_1(x) \in \mathfrak{pr}(\mathcal{B}_2)$ is not.

When a schematic argument is fully supported from evidence ($\widehat{\mathcal{C}} \subseteq \mathbf{E}$), its claim is ultimately instantiated ending up as a ground formula. Therefore, an argument $\mathcal{B}$ may be included in a supporting coalition $\widehat{\mathcal{C}}$ of $\mathcal{B}$ itself due to the substitution of variables. This situation is made clearer later and may be referred to Ex. 5. The quest for a supporter $\widehat{\mathcal{C}}$ of some argument $\mathcal{B}$ through a premise ρ in it, describes a recursive supporting process given that each premise in $\widehat{\mathcal{C}}$ needs to be also supported. When this process does ultimately end in a supporter containing only evidential arguments, we will distinguish $\rho \in \mathfrak{pr}(\mathcal{B})$ not only as non-free but also as *closed*.

Definition 11 (Closed Premise). *Given a* GenAF $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, and an argument* $\mathcal{B} \in \mathbf{A}$*, a premise* $\rho \in \mathfrak{pr}(\mathcal{B})$ *is* ***closed*** *wrt.* $\mathbf{A}$ *iff there exists a supporter* $\widehat{\mathcal{C}} \subseteq \mathbf{A}$ *of* $\mathcal{B}$ *through* ρ *such that either* $\mathfrak{prset}(\widehat{\mathcal{C}}) = \emptyset$*, or every premise in* $\mathfrak{prset}(\widehat{\mathcal{C}})$ *is closed.*

The idea behind closing premises is to identify those arguments that effectively state a reason from the GenAF to believe in their claims. Such arguments will be those for which the support of each of its premises does ultimately end in a set of evidential arguments –and therefore no more premises are required to be supported. Thus, every premise in an argument is closed *iff* the claim is *inferrable*. This is natural since inferrable claims can be effectively reached from evidence. Finally, when the claiming-coalition of an inferrable claim passes the acceptability analysis, the claim ends up *warranted*. Acceptability analysis and warranted claims will be detailed later, in Sect. 5.

Definition 12 (Inferrable Formula). *Given a* $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, a formula* $\alpha \in \mathcal{L}_{\texttt{cl}}$ *is* ***inferrable*** *from* $\mathbf{A}$ *iff there exists a claiming-coalition* $\widehat{\mathcal{C}} \subseteq \mathbf{A}$ *for* α *such that either* $\mathfrak{prset}(\widehat{\mathcal{C}}) = \emptyset$*, or every premise in* $\mathfrak{prset}(\widehat{\mathcal{C}})$ *is closed.*

The supporting process closing every premise in a claiming-coalition $\widehat{\mathcal{C}}$ to verify whether the claim is inferrable, clearly conforms a tree rooted in $\widehat{\mathcal{C}}$. We will refer to such tree as *supporting-tree*, and to each branch in it as *supporting-chain*.

Definition 13 (Supporting-Chain). *Given a* $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, a formula* $\alpha \in \mathcal{L}_{\texttt{cl}}$*, and a sequence* $\lambda \in (2^{\mathbf{A}})^n$ *such that* $\lambda = \widehat{\mathcal{C}}_1 \ldots \widehat{\mathcal{C}}_n$*, where* $n \in \mathbb{N}_1$*,* $\widehat{\mathcal{C}}_1$ *is a claiming-coalition for* α*, and for every* $i \in \mathbb{N}_1$ *it follows* $\widehat{\mathcal{C}}_i \subseteq \mathbf{A}$*, and* $\widehat{\mathcal{C}}_{i+1}$ *is a supporting-coalition through some* $\rho_i \in \mathfrak{prset}(\widehat{\mathcal{C}}_i)$*. The notations* $|\lambda| = n$ *and* $\lambda[i]$ *are used to respectively identify the* ***length*** *of* λ *and the* ***node*** $\widehat{\mathcal{C}}_i$ *in it. The last supporting-coalition in* λ *(referred as* ***leaf****) is identified through the function* $\mathfrak{leaf}(\lambda) = \lambda[|\lambda|]$*. The function* $\overleftarrow{\lambda} : (2^{\mathbf{A}})^n \times \mathbb{N}_0 \longrightarrow \mathcal{L}_{\texttt{cl}} \cup \mathcal{L}_{\texttt{pr}} \cup \{\bot\}$ *identifies the* ***link*** $\overleftarrow{\lambda}[0] = \alpha$*; or* $\overleftarrow{\lambda}[i] = \rho_i$ *(*$0 < i < |\lambda|$*), where* $\rho_i \in \mathfrak{prset}(\lambda[i])$ *is supported by* $\lambda[i+1]$*; or* $\overleftarrow{\lambda}[i] = \bot$ *(*$i \geq |\lambda|$*). The set* $\lambda^* = \bigcup_i \lambda[i]$ *(with* $0 < i \leq |\lambda|$*) identifies the set of arguments included in* λ*. Finally,* λ *is a* ***supporting-chain for*** α *wrt.* $\mathbf{A}$ *iff it guarantees:*

(minimality) $\widehat{\mathcal{C}} \subseteq \lambda^*$ *is a supporter (claimer if* $i = 0$*) of* $\overleftarrow{\lambda}[i]$ *iff* $\widehat{\mathcal{C}} = \lambda[i+1]$ *(*$0 \leq i < |\lambda|$*).*
(exhaustivity) *every* $\rho \in \mathfrak{prset}(\mathfrak{leaf}(\lambda))$ *is free wrt.* λ^**.*
(acyclicity) $\overleftarrow{\lambda}[i] = \overleftarrow{\lambda}[j]$ *iff* $i = j$*, with* $\{i, j\} \subseteq \{0, \ldots, |\lambda| - 1\}$*.*
(consistency) $\mathfrak{prset}(\lambda^*) \cup \mathfrak{clset}(\lambda^*) \not\models \bot$*.*

From the definition above, a supporting-chain is a finite sequence of interrelated supporting-coalitions $\widehat{\mathcal{C}}_i$ through a link $\rho_i \in \mathfrak{prset}(\widehat{\mathcal{C}}_i)$ supported by $\widehat{\mathcal{C}}_{i+1}$. It is finite indeed, given that the set $\mathbf{A}$ is also finite, and that no link could be repeated in the chain (acyclicity). The minimality condition (wrt. set inclusion over λ^*) stands to consider as less arguments from $\mathbf{A}$ as it is possible in order to obtain the same chain, whereas the exhaustivity condition (wrt. the length $|\lambda|$) ensures that the chain is as long as it is possible wrt. λ^* (without cycles), that is, λ has all the possible links that can appear from the arguments considered to build it. Note that from minimality no pair of arguments for a same claim could be simultaneously considered in any supporting-chain. Finally, consistency is required given that the intention of the supporting-chain is to provide a tool to close a premise from the claiming-coalition. Next, supporting-trees are formalized upon the definition of supporting-chains.

Definition 14 (Supporting-Tree). *Given a* $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, a formula* $\alpha \in \mathcal{L}_{\texttt{cl}}$*, and a tree* $\mathcal{T}$ *of coalitions* $\widehat{\mathcal{C}} \subseteq \mathbf{A}$ *such that each node* $\widehat{\mathcal{C}}$ *is either:*

- ***the root*** *iff* $\widehat{\mathcal{C}}$ *is a claiming-coalition for* α*; or*
- ***an inner node*** *iff* $\widehat{\mathcal{C}}$ *is a supporting-coalition through* $\rho \in \mathfrak{prset}(\widehat{\mathcal{C}'})$*, where* $\widehat{\mathcal{C}'} \subseteq \mathbf{A}$ *is either an inner node or the root.*

*The membership relation will be overloaded by writing $\lambda \in \mathcal{T}$ and $\widehat{\mathcal{C}} \in \mathcal{T}$ to respectively identify the branch λ and the node $\widehat{\mathcal{C}}$ from $\mathcal{T}$. The set $\mathcal{T}^* = \bigcup_{\widehat{\mathcal{C}} \in \mathcal{T}} \widehat{\mathcal{C}}$ identifies the set of arguments included in $\mathcal{T}$. Hence, $\mathcal{T}$ is a **supporting-tree** iff it guarantees:*

(completeness) *every $\lambda \in \mathcal{T}$ is a supporting-chain of α wrt. $\mathbf{A}$.*
(minimality) *for every $\lambda \in \mathcal{T}$, $\widehat{\mathcal{C}} \subseteq \mathcal{T}^*$ is a supporting-coalition (claimer if $i = 0$) through $\overleftarrow{\lambda}[i]$ iff $\widehat{\mathcal{C}} = \lambda[i+1]$ ($0 \leq i < |\lambda|$).*
(exhaustivity) *for every $\rho \in \mathfrak{prset}(\mathcal{T}^*)$, if there is no $\lambda \in \mathcal{T}$ such that $\overleftarrow{\lambda}[i] = \rho$ ($0 < i < |\lambda|$) then ρ is free wrt. $\mathcal{T}^*$.*
(consistency) $\mathfrak{prset}(\mathcal{T}^*) \cup \mathfrak{clset}(\mathcal{T}^*) \not\models \bot$.

Finally, the notation $\mathfrak{Trees}_{\mathbf{A}}(\alpha)$ identifies the set of all supporting-trees for α from $\mathbf{A}$.

The completeness condition is required in order to restrict the supporting-tree to consider only supporting-chains as their branches. Similar to supporting-chain, minimality is required to avoid considering extra arguments to build the tree, while exhaustivity stands to ensure that every possible supporting-coalition $\widehat{\mathcal{C}} \subseteq \mathcal{T}^*$ through a premise in $\mathfrak{prset}(\mathcal{T}^*)$ is an inner node in the tree. Finally, consistency ensures that the whole supporting process of the premises in the claiming-coalition will end being non-contradictory, even among branches. It is important to note that a supporting-tree for $\alpha \in \mathcal{L}_{\mathtt{cl}}$ determines the set of arguments used in the (possibly inconclusive)[3] supporting process of some claiming-coalition of α. Such set will be referred as *structure*.

Definition 15 (Structure). *Given a* $\mathtt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, and a formula $\alpha \in \mathcal{L}_{\mathtt{cl}}$, a set $\mathbb{S} \subseteq \mathbf{A}$ identifies a **structure** for α iff there is a supporting-tree $\mathcal{T} \in \mathfrak{Trees}_{\mathbf{A}}(\alpha)$ for α such that $\mathbb{S} = \mathcal{T}^*$. The claim and premises of $\mathbb{S}$ can be respectively determined through the functions $\mathfrak{cl} : 2^{\mathbb{A}\mathtt{rgs}} \longrightarrow \mathcal{L}_{\mathtt{cl}}$ and $\mathfrak{pr} : 2^{\mathbb{A}\mathtt{rgs}} \longrightarrow 2^{\mathcal{L}_{\mathtt{pr}}}$, such that $\mathfrak{cl}(\mathbb{S}) = \alpha$ and $\mathfrak{pr}(\mathbb{S}) = \{\rho \in \mathfrak{prset}(\mathbb{S}) \mid \rho$ is a free premise wrt. $\mathbb{S}\}$. Finally, the structure $\mathbb{S}$ is **argumental** iff $\mathfrak{pr}(\mathbb{S}) = \emptyset$, otherwise $\mathbb{S}$ is **schematic**.*

Note that functions "$\mathfrak{pr}$" and "$\mathfrak{cl}$" are overloaded and can be applied both to arguments and structures. This is not going to be problematic since either usage will be rather explicit. Besides, a structure $\mathbb{S}$ formed by a single argument is referred as *primitive iff* $|\mathbb{S}| = 1$. Thus, if $\mathbb{S} = \{\mathcal{B}\}$ then $\mathfrak{pr}(\mathcal{B}) = \mathfrak{pr}(\mathbb{S})$ and $\mathfrak{cl}(\mathcal{B}) = \mathfrak{cl}(\mathbb{S})$. However, not every single argument has an associated primitive structure. For instance, unless relevance would be required as a framework's normality condition, no structure could contain an argument $\langle \{p(x)\}, p(x) \rangle$ given that it would violate (supporting-chain) acyclicity. Finally, when no distinction is needed, we refer to primitive, schematic, or argumental structures, simply as structures.

Example 4. Given two arguments $\mathcal{B}_1 = \langle \{p(x)\}, q(x) \rangle$ and $\mathcal{B}_2 = \langle \{q(x)\}, p(x) \rangle$. The set $\{\mathcal{B}_1, \mathcal{B}_2\}$ cannot be a structure for $q(x)$ since $\{\mathcal{B}_1\}\{\mathcal{B}_2\}\{\mathcal{B}_1\} \ldots$ is a supporting-chain violating acyclicity. Similarly, $\{\mathcal{B}_1, \mathcal{B}_2\}$ could neither be a structure for $p(x)$.

Given two structures $\mathbb{S} \subseteq \mathbb{A}\mathtt{rgs}$ for $\alpha \in \mathcal{L}_{\mathtt{cl}}$, and $\mathbb{S}' \subseteq \mathbb{A}\mathtt{rgs}$ for $\alpha' \in \mathcal{L}_{\mathtt{cl}}$, $\mathbb{S}'$ is a sub-structure of $\mathbb{S}$ (noted as $\mathbb{S}' \sqsubseteq \mathbb{S}$) *iff* $\mathbb{S}' \subseteq \mathbb{S}$. Besides, $\mathbb{S}' \sqsubset \mathbb{S}$ *iff* $\mathbb{S}' \subset \mathbb{S}$.

[3] Inconclusive supporting processes lead to schematic structures with non-free premises wrt. $\mathbf{A}$.

Proposition 1. [4] *Given a* $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, a formula* $\alpha \in \mathcal{L}_{\texttt{cl}}$*, and two structures* $\mathbb{S} \subseteq \mathbf{A}$ *for* α *and* $\mathbb{S}' \subseteq \mathbf{A}$ *for* α*,*

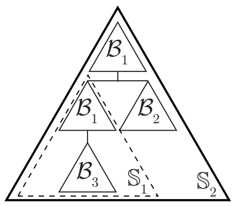
– *if* $\mathbb{S}' \sqsubset \mathbb{S}$ *then* $\mathfrak{pr}(\mathbb{S}) \neq \mathfrak{pr}(\mathbb{S}')$*.*
– *if* $\mathbb{S}$ *is argumental then* $\mathfrak{leaf}(\lambda) \subseteq \mathbf{E}$*, for every* $\lambda \in \mathcal{T}$ *where* $\mathcal{T} \in \mathfrak{Trees}_{\mathbb{S}}(\alpha)$*.*

Lemma 1. *Given a* $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, and a formula* $\alpha \in \mathcal{L}_{\texttt{cl}}$*, a structure* $\mathbb{S} \subseteq \mathbf{A}$ *for* α *is argumental iff* α *is inferrable.*

If a formula $\varphi(x) \in \mathcal{L}_{\texttt{cl}}$ (where x is a free variable) is inferrable then there exists an argumental structure $\mathbb{S}$ for $\varphi(x)$. Note now that since every argumental structure contains an empty set of premises, its supporting-tree $\mathcal{T}$ has only evidential arguments in their leaves. Thus, since the claim of evidential arguments are expressed in the language $\mathcal{L}_{\texttt{A}}$–it considers no free variables– the inner supporting process of $\mathbb{S}$ performed through $\mathcal{T}$ ends up applying a *variable substitution*, for instance mapping x to a, such that $\mathfrak{cl}(\mathbb{S}) = \varphi(a)$. Finally, if a structure states a property about some element of the world through a claim considering only free variables then it is schematic.

Lemma 2. *Given a* $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, and a formula* $\varphi(x) \in \mathcal{L}_{\texttt{cl}}$*, a structure* $\mathbb{S} \subseteq \mathbf{A}$ *for* $\varphi(x)$ *is argumental iff* $\mathfrak{cl}(\mathbb{S}) = \varphi(a)$ *and* $\varphi(a), v \models \varphi(x)$*, where* v *maps* x *to* a*.*

Example 5. Assume the $\texttt{GenAF}$ $\langle \mathbf{A}, \mathbf{N} \rangle$ such that $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\} \subseteq \mathbf{A}$ where $\mathcal{B}_1 = \langle \{p(x)\}, (\exists y)(\neg r(x, y) \vee p(y)) \rangle$, $\mathcal{B}_2 = \langle \{\}, r(a, b) \rangle$, and $\mathcal{B}_3 = \langle \{\}, p(a) \rangle$.



The argumental structure $\mathbb{S}_1 = \{\mathcal{B}_1, \mathcal{B}_3\}$ for $(\exists y)(\neg r(a, y) \vee p(y))$ appears. Moreover, $\widehat{\mathcal{C}}_1 = \{\mathcal{B}_1, \mathcal{B}_2\}$ is a supporter of $\mathcal{B}_1$ through $p(x)$, where the free variables x and y are mapped to a and b, respectively. Note that as a result of such variables substitutions, we have $\mathfrak{pr}(\widehat{\mathcal{C}}_1) = \{p(a)\}$, which in turn will be supported through the primitive coalition $\{\mathcal{B}_3\}$. Hence, the schematic structure $\mathbb{S}_2 = \{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$ for $p(b)$ appears, where $\mathbb{S}_1 \sqsubset \mathbb{S}_2$. Note that, $\mathcal{T} \in \mathfrak{Trees}_{\mathbb{S}_2}(p(b))$ has a unique supporting-chain $\{\mathcal{B}_1\}\{\mathcal{B}_1, \mathcal{B}_2\}\{\mathcal{B}_3\}$.

4 Conflict Identification

As will be formalized in Def. 16, two argumental structures are in conflict whenever their claims cannot be assumed together. Schematic structures may also be conflictive if it is the case that a claim of one of them could support a premise of the other, but a supporting-coalition does not exist given consistency would be violated. A second option of conflict between schematic structures appears when the premises of one of them infer the premises of the other, and either claim is in conflict with some premise from the other, or both claims cannot be assumed together. The intuition for this may be seen as a framework lacking of evidence to close every premise in each structure, but a hypothetical addition of the lacking evidence of one of them would be enough to include in the new framework two different argumental structures containing each original schematic structure. In such a case, the conflict conforms to the first case given.

[4] In this work, proofs were omitted due to space reasons.

This discussion may be made extensive to coalitional sets of structures. Analogous to coalitions of arguments, a *coalition of structures* might be interpreted as *a minimal and consistent set of structures guaranteeing certain requirement*. To go one step further into the formalization of a coalition $\widehat{\mathbb{C}} \subseteq 2^{\mathbb{A}\mathrm{rgs}}$ of structures $\mathbb{S} \subseteq \mathbf{A}$, we will rely on the set $\mathbb{C}^* = \bigcup_{\mathbb{S}\in\widehat{\mathbb{C}}} \mathbb{S}$ of arguments from $\widehat{\mathbb{C}}$. Therefore, we say that a coalition $\widehat{\mathbb{C}}$ of structures $\mathbb{S}$, is consistent *iff* $\mathfrak{prset}(\mathbb{C}^*) \cup \mathfrak{clset}(\mathbb{C}^*) \not\models \bot$, while minimality ensures $\widehat{\mathbb{C}}$ guarantees a requirement θ *iff* there is no proper subset of $\widehat{\mathbb{C}}$ guaranteeing θ, and there is no $\widehat{\mathbb{C}'} \subseteq 2^{\mathbb{A}\mathrm{rgs}}$ guaranteeing θ such that $\mathbb{C}'^* \subset \mathbb{C}^*$. Note that minimality not only looks for the smallest set of structures, but also for the smallest structures.

Coalition of structures are sets grouping structures to guarantee certain requirement θ: *conflict*. For the formalization of the notion of conflict, we will rely on the functions $\mathfrak{clset} : 2^{2^{\mathbf{A}}} \longrightarrow 2^{\mathcal{L}_{\mathrm{cl}}}$ and $\mathfrak{prset} : 2^{2^{\mathbf{A}}} \longrightarrow 2^{\mathcal{L}_{\mathrm{pr}}}$, which are respectively defined as $\mathfrak{clset}(\widehat{\mathbb{C}}) = \{\mathfrak{cl}(\mathbb{S}) | \mathbb{S} \in \widehat{\mathbb{C}}\}$, and $\mathfrak{prset}(\widehat{\mathbb{C}}) = \bigcup_{\mathbb{S}\in\widehat{\mathbb{C}}} \mathfrak{pr}(\mathbb{S})$. Note that functions "$\mathfrak{clset}$" and "$\mathfrak{prset}$" are overloaded and can be applied both to sets of arguments (for instance coalitions $\widehat{\mathcal{C}}$) and to coalitions $\widehat{\mathbb{C}}$ of structures. For this latter case, the functions' outcomes are the claims and premises of the structures included by the coalition $\widehat{\mathbb{C}}$. Next, we specify the notion of conflict between pairs of coalition of structures.

Definition 16 (Conflicting Coalitions). *Given a* `GenAF` $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$*, two coalitions* $\widehat{\mathbb{C}} \subseteq 2^{\mathbf{A}}$ *and* $\widehat{\mathbb{C}'} \subseteq 2^{\mathbf{A}}$ *of structures are in* ***conflict*** *iff it follows:*

- *Both coalitions are related either through dependency or support:*
  (dependency) $\mathfrak{prset}(\widehat{\mathbb{C}}) \models \mathfrak{prset}(\widehat{\mathbb{C}'})$.
  (support) $\mathfrak{clset}(\widehat{\mathbb{C}}) \models \mathfrak{prset}(\widehat{\mathbb{C}'})$.
- *The conflict appears either from claim-clash or premise-clash:*
  (claim-clash) $\mathfrak{clset}(\widehat{\mathbb{C}}) \cup \mathfrak{clset}(\widehat{\mathbb{C}'}) \models \bot$.
  (premise-clash) $\mathfrak{clset}(\widehat{\mathbb{C}}) \cup \mathfrak{prset}(\widehat{\mathbb{C}'}) \models \bot$, *or* $\mathfrak{clset}(\widehat{\mathbb{C}'}) \cup \mathfrak{prset}(\widehat{\mathbb{C}}) \models \bot$.

It is important to note that for any conflicting pair, each involved coalition of structures guarantees minimality and consistency. Later on we will see how acceptability of arguments benefits from these requirements. Next we exemplify the four different types of conflict that may be recognized from a `GenAF` following Def. 16.

Example 6. Let $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5, \mathcal{B}_6, \mathcal{B}_7\} \subseteq \mathbf{A}$ where $\mathcal{B}_1 = \langle\{p_1(x)\}, p_2(x)\rangle$, $\mathcal{B}_2 = \langle\{p_2(x)\}, p_3(x)\rangle$, $\mathcal{B}_3 = \langle\{p_1(x)\}, \neg p_3(x)\rangle$, $\mathcal{B}_4 = \langle\{\neg p_3(x)\}, p_1(x)\rangle$, $\mathcal{B}_5 = \langle\{p_1(x), \neg p_2(x)\}, p_3(x)\rangle$, $\mathcal{B}_6 = \langle\{p_4(x)\}, \neg p_3(x) \vee \neg p_1(x)\rangle$, $\mathcal{B}_7 = \langle\{p_5(x)\}, p_1(x)\rangle$.

(dependency & claim-clash) $\widehat{\mathbb{C}}_1 = \{\{\mathcal{B}_1, \mathcal{B}_2\}\}$ and $\widehat{\mathbb{C}}_2 = \{\{\mathcal{B}_3\}\}$.
(dependency & premise-clash) $\widehat{\mathbb{C}}_3 = \{\{\mathcal{B}_1\}\}$ and $\widehat{\mathbb{C}}_4 = \{\{\mathcal{B}_5\}\}$.
(support & claim-clash) $\widehat{\mathbb{C}}_1 = \{\{\mathcal{B}_1, \mathcal{B}_2\}\}$ and $\widehat{\mathbb{C}}_5 = \{\{\mathcal{B}_6\}, \{\mathcal{B}_7\}\}$.
(support & premise-clash) $\widehat{\mathbb{C}}_1 = \{\{\mathcal{B}_1, \mathcal{B}_2\}\}$ and $\widehat{\mathbb{C}}_6 = \{\{\mathcal{B}_4\}\}$.

In order to decide which coalition of structures succeeds from a conflicting pair, an *argument comparison criterion* "$\succcurlyeq$" is assumed to be determined from the comparison criterion among formulae in the KB (see Sect. 2). Afterwards, two conflicting coalitions of structures $\widehat{\mathbb{C}}_1$ and $\widehat{\mathbb{C}}_2$ are assumed to be ordered by a function "$\mathfrak{pref}$" relying on

"$\succcurlyeq$", where $\mathfrak{pref}(\widehat{\mathbb{C}}_1, \widehat{\mathbb{C}}_2) = (\widehat{\mathbb{C}}_1, \widehat{\mathbb{C}}_2)$ implies the attack relation $\widehat{\mathbb{C}}_1 \mathbf{R}_{\mathbf{A}} \widehat{\mathbb{C}}_2$, *i.e.*, $\widehat{\mathbb{C}}_1$ is a *defeater of* (or it defeats) $\widehat{\mathbb{C}}_2$. In such a case, $\widehat{\mathbb{C}}_2$ is said to be *defeated*. Moreover, if there is no defeater of $\widehat{\mathbb{C}}_1$ then it is said to be *undefeated*. Note that when no pair of arguments is related by "$\succcurlyeq$", both $\widehat{\mathbb{C}}_1 \mathbf{R}_{\mathbf{A}} \widehat{\mathbb{C}}_2$ and $\widehat{\mathbb{C}}_2 \mathbf{R}_{\mathbf{A}} \widehat{\mathbb{C}}_1$ appear from any conflicting pair $\widehat{\mathbb{C}}_1$ and $\widehat{\mathbb{C}}_2$. Finally, the set $\mathbf{R}_{\mathbf{A}} = \{(\widehat{\mathbb{C}}_1, \widehat{\mathbb{C}}_2) \mid \widehat{\mathbb{C}}_1$ and $\widehat{\mathbb{C}}_2$ are in conflict and $\mathfrak{pref}(\widehat{\mathbb{C}}_1, \widehat{\mathbb{C}}_2) = (\widehat{\mathbb{C}}_1, \widehat{\mathbb{C}}_2)\}$ identifies the *attack relations* from a `GenAF` $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$.

Theorem 1. *Given a* `GenAF` $\langle \mathbf{A}, \mathbf{N} \rangle \in \mathbb{G}$, $\mathcal{L}_{\mathtt{cl}} = \mathcal{L}_{\mathtt{pr}} = \mathcal{L}_{\mathtt{A}}$ *iff* $\langle A, \hookrightarrow \rangle$ *is a Dung's AF, where* $A = \{\mathbb{S} \subseteq \mathbf{A} \mid \mathbb{S}$ *is an argumental structure* $\}$ *and* $\hookrightarrow = \{(\mathbb{S}_1, \mathbb{S}_2) \subseteq \mathbf{A} \times \mathbf{A} \mid (\{\mathbb{S}_1\}, \{\mathbb{S}_2\}) \in \mathbf{R}_{\mathbf{A}}\}$.

5 Acceptability Analysis

Assuming a set of normality conditions $\mathbf{N}$, an inconsistent KB Σ leads to conflicting arguments within the associated $\mathfrak{genaf}(\Sigma, \mathbf{N}) = \langle \mathbf{A}, \mathbf{N} \rangle$. Thus, each minimal source of inconsistency within Σ is reflected as an attack in the resulting `GenAF`. Since the objective of a `GenAF` is to reason about a KB under uncertainty, there is a need for a mechanism that allows us to obtain those arguments that prevail over the rest. That is, those arguments that can be consistently assumed together, following some policy. For instance, structures with no defeaters should always prevail, since there is nothing strong enough to be posed against them. The tool we need to resolve inconsistency is the notion of *acceptability of arguments*, which is defined on top of an *argumentation semantics* [10]. There are several well-known argumentation semantics, such as the grounded, the stable, and the preferred semantics [1]. These semantics ensure the obtention of consistent sets of arguments, namely *extensions*. That is, the set of accepted arguments calculated following any of these semantics is such that no pair of conflicting arguments appears in that same extension. Finally, an extension determines a maximal consistent subset of the KB Σ.

It is important to notice that some problems like multiple extensions may arise from semantics like both the *stable* and the *preferred*. This would require to make a choice among them. On the other hand, the outcome of the *grounded semantics* is always a single extension, which could be empty. Finally, since dealing with multiple extensions is a problem that falls outside the scope of this article, we will choose the grounded semantics, which can be implemented with a simple algorithm. Consequently, we define a mapping $\mathfrak{sem} : \mathbb{G} \longrightarrow 2^{\mathbb{A}\mathtt{rgs}}$, that intuitively behaves as follows. The set $X \subseteq \mathbf{A}$ is the minimal set verifying $X \subseteq \bigcup_{(\widehat{\mathbb{C}'}, \widehat{\mathbb{C}}) \in \mathbf{R}_{\mathbf{A}}} \mathbb{C}^*$ for every undefeated $\widehat{\mathbb{C}'}$ defeating $\widehat{\mathbb{C}}$, and for each $\widehat{\mathbb{C}}$ it follows $\mathbb{C}^* \cap X \neq \emptyset$. As a result, other coalition of structures defeated by $\widehat{\mathbb{C}}$ could appear undefeated. Thus, this process is iteratively applied over the set of arguments $\mathbf{A} \setminus X$ until no conflicting pair is identified. Finally, the extension of the `GenAF` is determined.

As stated before, the outcome of a grounded semantics could be an empty extension. Such an issue arises when there is a loop in the structures attack graph, that is $(\widehat{\mathbb{C}'}, \widehat{\mathbb{C}}) \in \mathbf{R}_{\mathbf{A}}$ and $(\widehat{\mathbb{C}}, \widehat{\mathbb{C}'}) \in \mathbf{R}_{\mathbf{A}}$. To overcome this, some argument from either $\widehat{\mathbb{C}}$ or $\widehat{\mathbb{C}'}$ could be included in X, and therefore the loop would be broken, and the process determined by applying "$\mathfrak{sem}$" can be reconsidered.

Given a (potentially inconsistent) pANF knowledge base $\Sigma \subseteq \mathcal{L}^{\kappa}$, and a set of normality conditions $\mathbf{N} \subseteq \mathbb{N}\mathtt{orm}$, it is possible to redefine the notion of entailment "$\models$" from Σ by reasoning about it over its associated GenAF $\mathfrak{genaf}(\Sigma, \mathbf{N})$, such that $\Sigma \models_G \alpha$ *iff* there exists an argumental structure $\mathbb{S}$ for α such that $\mathbb{S} \subseteq \mathfrak{sem}(\mathfrak{genaf}(\Sigma, \mathbf{N}))$. In such a case, the inferrable claim α is said to be *warranted* and therefore, $\Sigma \models_G \alpha$. Note that if Σ is consistent and $\alpha \in \mathcal{L}_{\mathtt{cl}}$, "$\models_G$" equals the classical entailment "$\models$".

Theorem 2. *Given a consistent* pANF *knowledge base* $\Sigma \subseteq \mathcal{L}^{\kappa}$*, a set of normality conditions* $\mathbf{N} \subseteq \mathbb{N}\mathtt{orm}$*, and a formula* $\alpha \in \mathcal{L}_{\mathtt{cl}}$*,* $\Sigma \models \alpha$ *iff* $\Sigma \models_G \alpha$*.*

6 Concluding Remarks

A novel argumentation framework was presented as a generalization of the classical Dung's AF named GenAF. A GenAF aims at providing a straightforward reification tool to reason about inconsistent knowledge bases specified through FOL fragments.

In the last few years, a great effort has been put to the area of ontology change. For instance, ontology evolution intends to restore consistency to inconsistent ontologies. Description logics are probably the most important ontological representation language. Part of our current investigations is done on the research of possible reifications of the here presented GenAF into highly expressible DLs. Consequently, not only ontology evolution could be resolved but also, reasoning about inconsistent ontologies. Some previous work may be referred to [2], where a preliminary investigation on these matters have been done. There, a dynamic version of the GenAF is presented to apply change in a consistent manner to (potentially inconsistent) ontologies.

Finally, since the grounded semantics [1] could return empty extensions, the usage of different semantics [10] is part of the ongoing work to overcome this issue.

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