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Multiphoton annihilation of monopolium

H. Fanchiotti and C. A. García Canal

IFLP/CONICET and Departamento de Física,
Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina

V. Vento

Departamento de Física Teórica-IFIC,
Universidad de Valencia-CSIC,
E-46100, Burjassot (Valencia), Spain
vicente.vento@uv.es

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We show that due to the large coupling constant of the monopole–photon interaction the annihilation of monopole–antimonopole and monopolium into many photons must be considered experimentally. For monopole–antimonopole annihilation and lightly bound monopolium, even in the less favorable scenario, multiphoton events (four and more photons in the final state) are dominant, while for strongly bound monopolium, although two photon events are important, four- and six-photon events are also sizable.

Keywords: Magnetic monopole; positronium; monopolium; photon; annihilation.

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Inspired by an old idea of Dirac and Zeldovich[1–3] we proposed that monopolium, a bound state of monopole–antimonopole[3,4] could be easier to detect experimentally than free monopoles.[5,6] We have already studied the annihilation of monopolium into two photons.[7,8] In here we are going to show that monopolium might annihilate most preferably into many low energy photons.[8,9] This result motivates experimental searches of monopolium and monopole–antimonopole by looking into multiphoton decays.

In our description of monopolium decays, we follow closely that of positronium decays with two differences, namely the huge coupling constant in the magnetic case and the dependence on the binding energy. The subtleties associated with the binding potential[6,9] are of no relevance for the present estimation.

The two- and four-photon decay channels of the ground state parapositronium have been studied in QED. In particular, the two-photon channel is known up to $O(\alpha^3 \log^2 \alpha)$ (Refs. 10, 12–14) and the four-photon decay has been studied up to order $O(\alpha)$,[11] where α is the fine structure constant. We show for the ratio of these channels the result to leading order,[15,16]

$$\frac{\Gamma_4}{\Gamma_2} = 0.277\left(\frac{\alpha}{\pi}\right)^2 . \tag{1}$$

The factor in front of the coupling constant, 0.277, contains the contribution of the 4! diagrams of the four-photon amplitude and the 2! diagrams of the two-photon one, to lowest order. The binding energy is very small, a few eV, and has been neglected in the calculation, therefore the energy factors cancel in the ratio.

Let us for the sake of argument increase the photon coupling in Eq. (1) which leads to an increase in the four-particle ratio. This increase in the ratio motivates the present investigation.

Let us assume that the monopole–photon coupling is analogous to the electron–photon coupling except for an effective vertex characterized by the dressed monopole magnetic charge g.[17] Thus we extend the positronium calculation to monopolium just by changing $e \to g$. Recalling the parapositronium calculation in terms of the coupling we get

$$\frac{\Gamma_4}{\Gamma_2} \sim F_{42}\left(\frac{\alpha_g}{\pi}\right)^2 \cdots \frac{\Gamma_{2n}}{\Gamma_2} \sim F_{2n2}\left(\frac{\alpha_g}{\pi}\right)^{2n-2} , \tag{2}$$

where $\alpha_g = \frac{1}{4\alpha} \sim \frac{137}{4} \sim 34.25$ obtained from Dirac's Quantization Condition (DQC).[1,2] The F's represent the contribution of all the Feynman amplitudes to the process shown as subindex after extracting the contribution of the magnetic charge, which is explicitly shown. For example, to leading order, $F_{42} \approx 0.277$ as seen in Eq. (1). We perform the calculations, as is customarily done in monopole physics, to leading order. Due to the large magnetic coupling, the calculations can give at most a qualitative indication of magnitudes, which is what we can pursue at present. At the end of our analysis, we comment on how nonperturbative effects might affect our calculation.

In Fig. 1, we show one of the $2n!$ contributions to the amplitude for a $2n$ photon decay to leading order, and we note that this type of contributions in the above ratios are determined only by vertices and propagators. The calculation for high n with $2n!$ diagrams is out of the scope of any study. Let us discuss first an educated estimation for large n. In the rest frame of the bound system the annihilation into many photons leads to an average momentum for each photon much smaller than the mass of monopolium and therefore much smaller than the mass of the monopole. In order to make an estimation of the above ratios, we consider that in the propagators the monopole mass dominates over the momentum and therefore the calculation of the width, in units of monopole mass, depends exclusively on three factors: the number of diagrams $(2n)!$, the photons' symmetry factor $1/(2n)!$

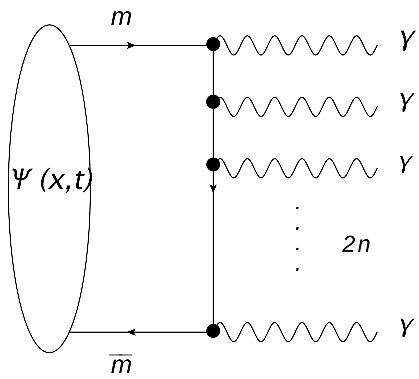

Fig. 1. One of the $2n!$ multiphoton emission diagrams.

and the phase space of the outgoing massless particles, namely

$$(\mathrm{phsp})^{2n} = \frac{1}{2}\frac{1}{(4\pi)^{4n-3}}\frac{M^{4n-4}}{\Gamma(2n)\Gamma(2n-1)}\,, \tag{3}$$

where M is the monopolium mass and $n = 1, 2, 3, \ldots$ $2n$ being the number of photons emitted.

With all these approximations, we obtain the expression

$$\frac{\Gamma_{2n}}{\Gamma_2} = \left(\frac{\alpha_g}{\pi}\right)^{2n-2}\left(\frac{M}{2m}\right)^{4n-4}\frac{2n!}{2!(2n-1)!(2n-2)!}\,. \tag{4}$$

Note that this equation leads to $\Gamma_2/\Gamma_2 = 1$ and for $n = 2$ and $M = 2m$, one recovers the parapositronium case, $\Gamma_4/\Gamma_2 = \left(\frac{\alpha_g}{\pi}\right)^2$, with the interference factor missing (recall Eq. (1)). In order to incorporate this effect we make a second estimate. In the first estimate, we have assumed p^2 to be very small compared with m^2 in the propagator an approximation valid for large n. Let us assume for the second estimate that on the contrary $p^2 \sim m^2$, an approximation which might be adequate for small n. This approximation introduces in Eq. (4) a factor $(\frac{1}{2})^{2n-2}$ leading to

$$\frac{\Gamma_{2n}}{\Gamma_2} = \left(\frac{1}{2}\right)^{2n-2}\left(\frac{\alpha_g}{\pi}\right)^{2n-2}\left(\frac{M}{2m}\right)^{4n-4}\frac{2n!}{2!(2n-1)!(2n-2)!}\,. \tag{5}$$

For $n = 2$, this factor is 0.25 which is very close to true calculation to leading order 0.277. This factor is extremely suppressing for large n, where the approximations discussed initially might be better. We show results with and without this factor to determine a region of confidence. If the leading order calculation were all there, the true result would be between these two limiting expressions. We discuss possible nonperturbative effects at the end of the analysis. In our expressions, we consider the effect of the binding energy not taken into account in the conventional positronium analysis.

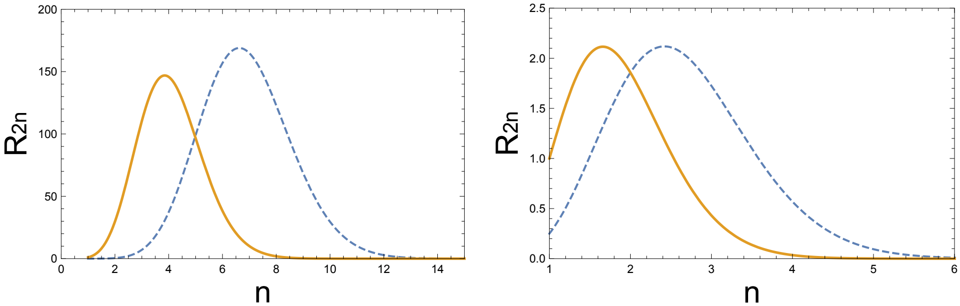

Fig. 2. The Γ_{2n}/Γ_2 ratio as a function of n calculated according to Eqs. (4) and (5). The left figure is for zero binding energy and the right figure for $E_b \sim m$. The solid curves represent the calculation with the interference factor and the dashed curves the one without the factor. In order to have the two curves at the same scale, the no interference ratios had to be divided by 250 left and by 4 right.

In Fig. 2, we plot Eqs. (4) and (5) for two different binding energies and we get bell-shaped distributions. For small binding energies ($M \approx 2m$), the value of n on the average is $\bar{n} \sim 7$ with a deviation of $\Delta n \sim \pm 2$. For large binding energies ($M \sim m$), the average value of n is $\bar{n} \sim 3$ with a deviation of $\Delta n \sim \pm 1$. If the interference factor is included, the multiplicity ($2\bar{n}$) is reduced to $\bar{n} \sim 4$ with $\Delta n \sim \pm 1$ for small binding and $\bar{n} \sim 2$ with $\Delta n \sim \pm 1$ for large binding. Thus, the multiplicity decreases as the binding energy increases. However, even with the strongly suppressing interference factor included, four-photon emission is favored.

Since the above curves do not provide a quantitative estimate of the increase in the ratio, we show in Fig. 3 some ratios as a function of a for a final state of 4, 8 and 12 photons with interference factor (solid) and without factor (dashed) for very small binding energy. This case of small binding energy corresponds very closely to monopole–antimonopole annihilation. The effect is large, even with the interference factor included. Note that the monopole coupling corresponds to $a \sim 11$.

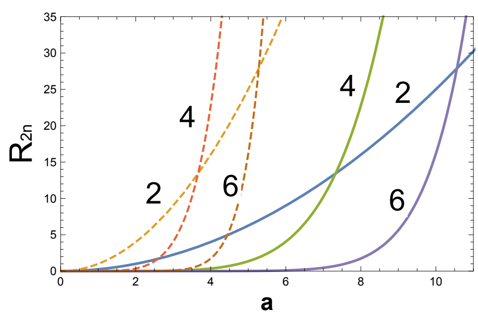

Fig. 3. The Γ_{2n}/Γ_2 ratio as a function of a calculated according to Eqs. (4) and (5) for $n = 2, 4, 6$ and zero binding energy. The solid curves represent the calculation with the interference factor and the dashed curves the one without the factor.

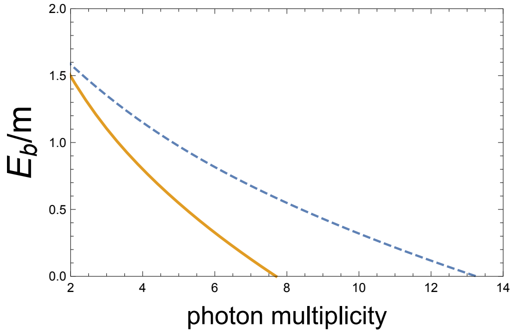

Fig. 4. Binding energy in units of monopole mass as a function of average photon multiplicity in the annihilation. The solid curve represents the calculation with interference factor while the dashed curve is that without that factor.

We now study the dependence of multiplicity with the binding energy. To do so we find the maximum of the ratio in Eqs. (4) and (5) as a function of binding energy. The result is plotted in Fig. 4 where we show the binding energy in units of monopole mass as a function of the average photon multiplicity in the annihilation. The outcome is clear, large average multiplicities, 8–12 photons, arise if the binding energy is small, $M \sim 2m$, while smaller average multiplicities, 4–6 photons, occur for large bindings, $M \sim m$. In the latter case, considerable rates extend up to multiplicities of 8 photons as is shown in Fig. 2 (right).

In order to see quantitatively the increase in the ratios for large binding energy ($M \sim m$), we plot in Fig. 5 as a function of a, the ratios for 4 and 6 photons calculated according to Eqs. (4) and (5). In this case, the rates are smaller and also the multiplicities as seen in Fig. 4. It is important to realize that the binding energy effect is very suppressing in the phase space formula. However, if the mass of the monopole is large (> 1 TeV), binding energies as large as its mass are not to be expected.

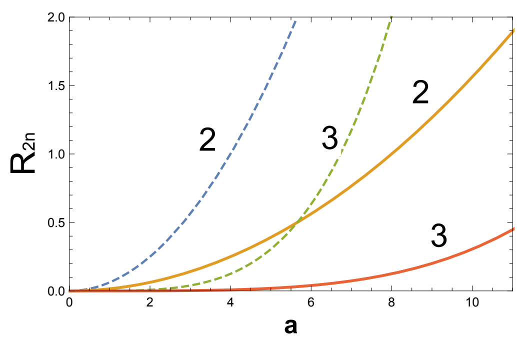

Fig. 5. The Γ_{2n}/Γ_2 ratio as a function of a calculation according to Eqs. (4) and (5) for $n = 2, 3$ and large binding energy ($M \sim m$). The solid curves represent the calculation with the interference factor and the dashed curves the ones without that factor.

We can get an analytic formula for the most probable photon decay channel by calculating the maximum of the logarithm of Eqs. (4) and (5) using Sterling's formula ($k! \sim k^k e^{-k}\sqrt{2\pi k}$). Sterling's formula is quite good even for low k, i.e. for $k = 2$ it gives 1.919, for $k = 3$, 5.836, and $k = 4$, 23.506. Thus, we can consider the equations we are going to derive good approximations for any n. Let us write a generic interference factor $(\delta)^{n-1}$ in front of Eq. (4) for which $\delta = 1$ gives Eq. (4) itself and for $\delta = 0.25$ Eq. (5). The values of n for the maximum decay rates are given by the solutions of the equation

$$n = \frac{\alpha_g\sqrt{\delta}}{2\pi}\left(1 - \frac{E_b}{2m}\right)^2 \exp\left(\frac{1}{4n}\right) + 1\,, \tag{6}$$

for specific values of δ and the binding energy E_b. For large n, which occurs for small binding energy we get the approximate solution

$$n \approx \frac{\alpha_g\sqrt{\delta}}{2\pi} + 1\,, \tag{7}$$

which is very illuminating because it shows explicitly the effect of the coupling constant in increasing the photon multiplicities as seen numerically in Fig. 2.

All the approximations used thus far are valid for Dirac's original formulation. Some coupling schemes lead to small effective couplings close to threshold.[7,8,18] These velocity-dependent schemes proceed by changing $g \to \beta g$, where

$$\beta = \sqrt{1 - \frac{M^2}{s}}\,, \tag{8}$$

where M is the mass of monopolium and s the center of mass energy of the process. In the case of monopole–antimonopole production $M \to 2m$, where m is the mass of the monopole (antimonopole). Thus, in these schemes, all photon widths vanish at threshold. Close to threshold, two photon decays are dominant, since the ratio (Eqs. (4) and (5)) acquire a factor β^{4n-4}. However, by looking at the approximate solution which is now modified to

$$n \approx \frac{\alpha_g\beta^2\sqrt{\delta}}{2\pi} + 1\,, \tag{9}$$

one can understand what happens. Close to threshold the two-photon decay is the dominant process but given the size of the coupling for $\beta > 0.5$, multiphoton decays start to be important. Given that in most processes studied β rises rapidly[7,8] our present analysis holds slightly away from threshold.

Finally, we would like to make some comments about nonperturbative effects. Given the large value of the coupling constant, it is evident that our calculation is merely qualitative and aimed at proposing new signals to discover monopoles at lower photon energies. Let us assume for the following discussion, the worst possible scenario, namely, that nonperturbative effects make the multiphoton channels weaker. We parametrize the nonperturbative effects by an effective δ. How small can δ get to make four- and six-photon decay ratios irrelevant? We use Eq. (6) for $n = 2, 3$ and plot δ as a function of binding energy in Fig. 6.

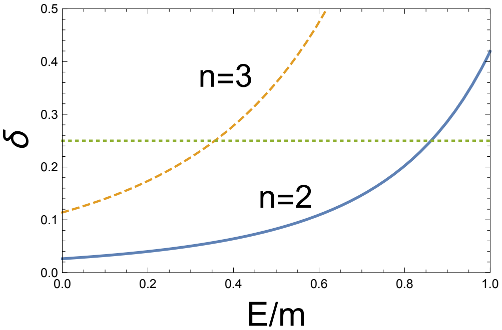

Fig. 6. The interference factor δ as a function of binding energy for four- and six-photon decays according to Eq. (6).

From the figure it is apparent that a small bound monopolium produces preferably multiphoton decays up to very small interference factors. As the binding energy increases the possibility of multiphoton decays decreases. Note however that the four-photon decay is greater or comparable to two-photon decays up to interference factors many times smaller than the one used in this calculation and note that monopole–antimonopole annihilation behaves much like zero binding monopolium.

Our analysis shows that monopole–antimonopole annihilation or lightly bound monopolium decays lead preferably to multiphoton events. In particular, one should look for four or more photons in the final state. If monopolium is strongly bound, the situation changes and although two-photon rates are important, it is also certain that four- and six-photon rates may be sizable. Thus, in any circumstance one should aim at looking at four- and six-photon events with the multiplicity characterizing the dynamics of the binding.

Present searches of low mass monopoles have been carried out by making use of their magnetic properties trapped in matter, by looking for tracks associated with their ionization properties and by studying two-photon decays at colliders.[19,21,22] At colliders like the LHC, the conservation of magnetic charge implies that either monopolium or a pair monopole–antimonopole could be produced. Monopolium, being chargeless, is difficult to detect except for its decay properties. If monopole–antimonopole are produced close to threshold, they might annihilate given their large interaction before reaching the detector. In this experimental scenario, we might not have been able to detect monopolium and/or monopoles because we have been looking into trapped monopoles, ionization remnants or the two-photon channel, all of which according to our present investigation are less probable than four or more photon events. Our study shows that a characteristic feature of monopole–antimonopole annihilation and monopolium decay is to find more than two photons coming from the annihilation vertex. At threshold, these photons have *on average* smaller energy than the energy of a typical collider process and the multiplicity is directly related to the strength of their interaction.

To conclude, we state that in view of the fact that the exact dynamics of monopoles and their properties are not available, large multiplicity of photon events might be the signal for the discovery of these elusive particles. Experiments should be ready to incorporate this feature into their analysis.

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References

1. P. A. M. Dirac, *Proc. R. Soc. London A* **133**, 60 (1931).
2. P. A. M. Dirac, *Phys. Rev.* **74**, 817 (1948).
3. Y. B. Zeldovich and M. Y. Khlopov, *Phys. Lett. B* **79**, 239 (1978).
4. C. T. Hill, *Nucl. Phys. B* **224**, 469 (1983), doi:10.1016/0550-3213(83)90386-3.
5. V. Vento, *Int. J. Mod. Phys. A* **23**, 4023 (2008), doi:10.1142/S0217751X08041669, arXiv:0709.0470 [astro-ph].
6. L. N. Epele, H. Fanchiotti, C. A. García Canal and V. Vento, *Eur. Phys. J. C* **56**, 87 (2008), doi:10.1140/epjc/s10052-008-0628-0, arXiv:hep-ph/0701133.
7. L. N. Epele, H. Fanchiotti, C. A. G. Canal, V. A. Mitsou and V. Vento, *Eur. Phys. J. Plus* **127**, 60 (2012), doi:10.1140/epjp/i2012-12060-8, arXiv:1205.6120 [hep-ph].
8. L. N. Epele, H. Fanchiotti, C. A. G. Canal, V. A. Mitsou and V. Vento, arXiv:1607.05592 [hep-ph].
9. N. D. Barrie, A. Sugamoto and K. Yamashita, *Prog. Theor. Exp. Phys.* **2016**, 113B02 (2016), doi:10.1093/ptep/ptw155, arXiv:1607.03987 [hep-ph].
10. A. A. Penin, *Int. J. Mod. Phys. A* **19**, 3897 (2004), doi:10.1142/S0217751X04020154, arXiv:hep-ph/0308204.
11. G. S. Adkins and E. D. Pfahl, *Phys. Rev. D* **59**, R915 (1998).
12. G. P. Lepage, P. B. Mackenzie, K. H. Streng and P. M. Zerwas, *Phys. Rev. A* **28**, 3090 (1983), doi:10.1103/PhysRevA.28.3090.
13. I. B. Khriplovich and A. S. Yelkhovsky, *Phys. Lett. B* **246**, 520 (1990), doi:10.1016/0370-2693(90)90641-I.
14. A. Czarnecki and S. G. Karshenboim, arXiv:hep-ph/9911410.
15. A. Billoire, R. Lacaze, A. Morel and H. Navelet, *Phys. Lett. B* **78**, 140 (1978), doi:10.1016/0370-2693(78)90367-2.
16. T. Muta and T. Niuya, *Prog. Theor. Phys.* **68**, 1735 (1982), doi:10.1143/PTP.68.1735.
17. D. Zwanziger, *Phys. Rev. D* **3**, 880 (1971), doi:10.1103/PhysRevD.3.880.
18. K. A. Milton, *Rep. Prog. Phys.* **69**, 1637 (2006), doi:10.1088/0034-4885/69/6/R02, arXiv:hep-ex/0602040.
19. L. Patrizii and M. Spurio, *Annu. Rev. Nucl. Part. Sci.* **65**, 279 (2015), doi:10.1146/annurev-nucl-102014-022137, arXiv:1510.07125 [hep-ex].
20. MoEDAL Collab. (B. Acharya *et al.*), *Int. J. Mod. Phys. A* **29**, 1430050 (2014), doi:10.1142/S0217751X14300506, arXiv:1405.7662 [hep-ph].
21. MoEDAL Collab. (B. Acharya *et al.*), *J. High Energy Phys.* **1608**, 067 (2016), doi:10.1007/JHEP08(2016)067, arXiv:1604.06645 [hep-ex].
22. MoEDAL Collab. (B. Acharya *et al.*), *Phys. Rev. Lett.* **118**, 061801 (2017), doi:10.1103/PhysRevLett.118.061801, arXiv:1611.06817 [hep-ex].