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TECHNICAL NOTE
HOLDUP MODELS FROM EXPERIMENTAL CONDUCTANCE DATA IN A BUBBLE COLUMN*

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ABSTRACT

Mathematical models for the calculation of gas holdup in liquid–gas systems from experimental measures of the electrical conductance at several frother levels are presented in this paper.

Experimental data were taken by a computer connected to conductance electrodes fixed in an acrylic transparent pilot flotation column.

In the first stage of data analysis a relationship between the relative conductance and gas holdup on constant frother concentration was established. In the second stage, variable frother concentration was introduced. Finally, a relationship between the relative conductance, the gas holdup and the interaction between gas holdup and the square root of frother concentration was determined.

Alternative models are statistically analyzed. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords
Column flotation; flotation bubbles; modelling; process control

INTRODUCTION

The conductivity of a liquid decreases when non–conducting gas bubbles are introduced. Frothers reduce the coalescence of bubbles causing an increment of gas holdup.

Tests have been carried out in an acrylic transparent flotation column, 3 m high and 9.2 cm inner diameter, provided with a porous sparger, conductivity sensors for the automatic control of froth level and a conductimetric cell of parallel planes, connected to a computer to record conductance at increasing superficial air rates (Mattenella *et al.*, 1994).

The experimental data were related by means of the ratio between the conductance at a superficial air velocity (C) and the conductance of the liquid phase in the absence of air (C_0), under a fixed frother concentration.

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The following variables were initially considered in the regression analysis:

x_1: air holdup, measured by the rising of the collection zone in the column (ΔH) in relation to the initial liquid height (H_0)

x_2: frother concentration in ppm

y: dependent variable (C/C_0)

Table 1 shows the experimental data collected as described above.

TABLE 1 Experimental data

0 ppm		10 ppm		20 ppm		30 ppm	
y exp.	gas holdup	y exp.	gas holdup	y exp.	gas holdup	y exp.	gas holdup
1	0	1	0	1	0	1	0
0.998	0.006	0.976	0.015	0.975	0.016	0.97	0.016
0.961	0.021	0.923	0.051	0.924	0.052	0.923	0.052
0.929	0.043	0.873	0.089	0.879	0.085	0.871	0.094
0.888	0.072	0.843	0.112	0.819	0.138	0.837	0.122
0.85	0.095	0.81	0.138			0.783	0.17
0.816	0.118	0.78	0.163				

CONSTRUCTION OF MODELS

Many authors have estimated gas holdup in flotation columns from different points of view (Banisi *et al.*, 1994; Bergh *et al.*, 1995). In this paper the constructed models from experimental data are statistically evaluated.

Data shown in Table 1 have initially been analyzed by considering the changes of conductance due to changes in gas holdup on constant frother concentrations. That is, an appropriate model of regression to assess $y = f(x_1)$ under a fixed frother concentration ($x_2 = 0$, 10, 20 and 30 ppm) was studied.

A linear relationship that fits every set of experimental data is:

$$\hat{y} = a + b\,x_1 \tag{1}$$

where the coefficient **a** is very close to 1 and **b** is always negative.

Assuming the simplification **a** = 1, Eq. (1) becomes:

$$\hat{y} = 1 + b\,x_1 \tag{2}$$

From (1) and (2) it follows that:

$$1 - \hat{y} = -\,b\,x_1 \tag{3}$$

and then:

$$1 - y = 1 - C/C_0 = (C_0 - C) / C_0 \quad (4)$$

Equation (4) also measures the decrease of bed conductance in relation to the initial conductance.

We call this magnitude z. Its estimated values are:

$$\hat{z} = 1 - \hat{y} = -b\, x_1 = m\, x_1 \quad (5)$$

It means that the bed conductance decreases proportionally to the increase of gas holdup, expressed in Eq. (5) through a positive constant **m**, which depends on the frother concentration.

The second stage of data analysis shows that **m** increases linearly with the square root of frother concentration:

$$m = 1.5302 - 0.05 \,.\, \sqrt{x_2} \quad (6)$$

Substituting (6) in (5):

$$\hat{z} = (1.5302 - 0.05 \,.\, \sqrt{x_2}) \,.\, x_1 \quad (7)$$

From now on the frother concentration variable is represented through its square root:

$$x_2 = \sqrt{\text{Frother concentration}} \quad (8)$$

GENERALIZED STATISTICAL MODEL

A statistical model that fits the transformations mentioned above is:

$$z = \beta_1\, x_1 + \beta_{12}\, x_1\, x_2 + \varepsilon \quad (9)$$

where ε is a random variable, normally distributed, with mean 0 and constant variance σ^2 (Peña Sánchez de Rivera, 1995). It has been proved that the quadratic terms of a general model of second order are not significant, and neither are the linear terms x_2.

Two other similar models transformed by powers have been proposed:

$$z = \beta_1\, x_1^p + \beta_{12}\, (x_1\, x_2)^q + \varepsilon \quad (10)$$

Model 1

This has been formulated from Eq. (7) by addition of the effects of the two variables:

$$\hat{z} = 1.5302\, x_1 - 0.05\, x_1\, x_2 \quad (11)$$

Model 2

Formulated from Model 1, fitting the coefficients by least squares.

$$\hat{z} = 1.5494\, x_1 - 0.044\, x_1\, x_2 \quad (12)$$

Model 3

This was formulated transforming the two factors through the powers p and q, respectively. Values of the exponents p = 0.9077 and q = 1.017 have been obtained by direct searching methods with the aim of improving the determination coefficient and reducing the residual variance.

$\hat{\beta}_1 = 1.2495$ and $\hat{\beta}_{12} = -0.0363$ are least squares estimates

$$\hat{z} = 1.2495\, x_1^{0.9077} - 0.0363\, (x_1\, x_2)^{1.017} \tag{13}$$

Model 4

This model has been built under the criterion of maximizing the correlation coefficient between the ordered residuals $(z - \hat{z})_{(h)}$ and the vector of percentiles $F_Z^{-1}\left(\frac{h}{n+1}\right)$ characteristic of the standard normal $Z \sim N(0, 1)$. We have called it the **normality coefficient of residuals**.

This model-building procedure does not use the least squares estimation method.

$$\hat{z} = 1.267\, x_1^{0.9} - 0.0435\, (x_1\, x_2) \tag{14}$$

SUMMARY OF THE MODELS

The coefficient of determination (R^2), the value of the statistic F, the normality coefficient (R_N^2) and the difference between the residual deviation (S_R) and the regression slope ($\hat{\beta}_R$) have been used to evaluate the behavior of the models (Table 2).

TABLE 2 Efficiency measurements of the models

Models	Fitting degree		Normality measurements of residuals			
	R^2	F	R_N^2	Regression Slope $\hat{\beta}_R$	Residuals Deviation S_R	Difference $S_R - \hat{\beta}_R$
1. $\hat{z}=1.5302x_1 - 0.05x_1\, x_2$	0.9970	3,157	0.9702	$4.036\ 10^{-3}$	$4.06\ .\ 10^{-3}$	$2.82\ 10^{-5}$
2. $\hat{z}=1.5494x_1 - 0.044x_1\, x_2$	0.9944	1,687	0.9364	$4.875\ 10^{-3}$	$4.997\ 10^{-3}$	$1.22\ 10^{-4}$
3. $\hat{z}= 1.2495x_1^{0.9077} - 0.0363(x_1\, x_2)^{1.017}$	0.9990	9,490	0.9692	$2.210\ 10^{-3}$	$2.226\ 10^{-3}$	$1.66\ 10^{-5}$
4. $\hat{z}=1.267x_1^{0.9} - 0.0435(x_1\, x_2)$	0.9988	7,907	0.9823	$2.581\ 10^{-3}$	$2.583\ 10^{-3}$	$1.92\ 10^{-6}$

Furthermore, the residuals have been graphically analyzed by means of histograms and dot diagrams (Figure 1).

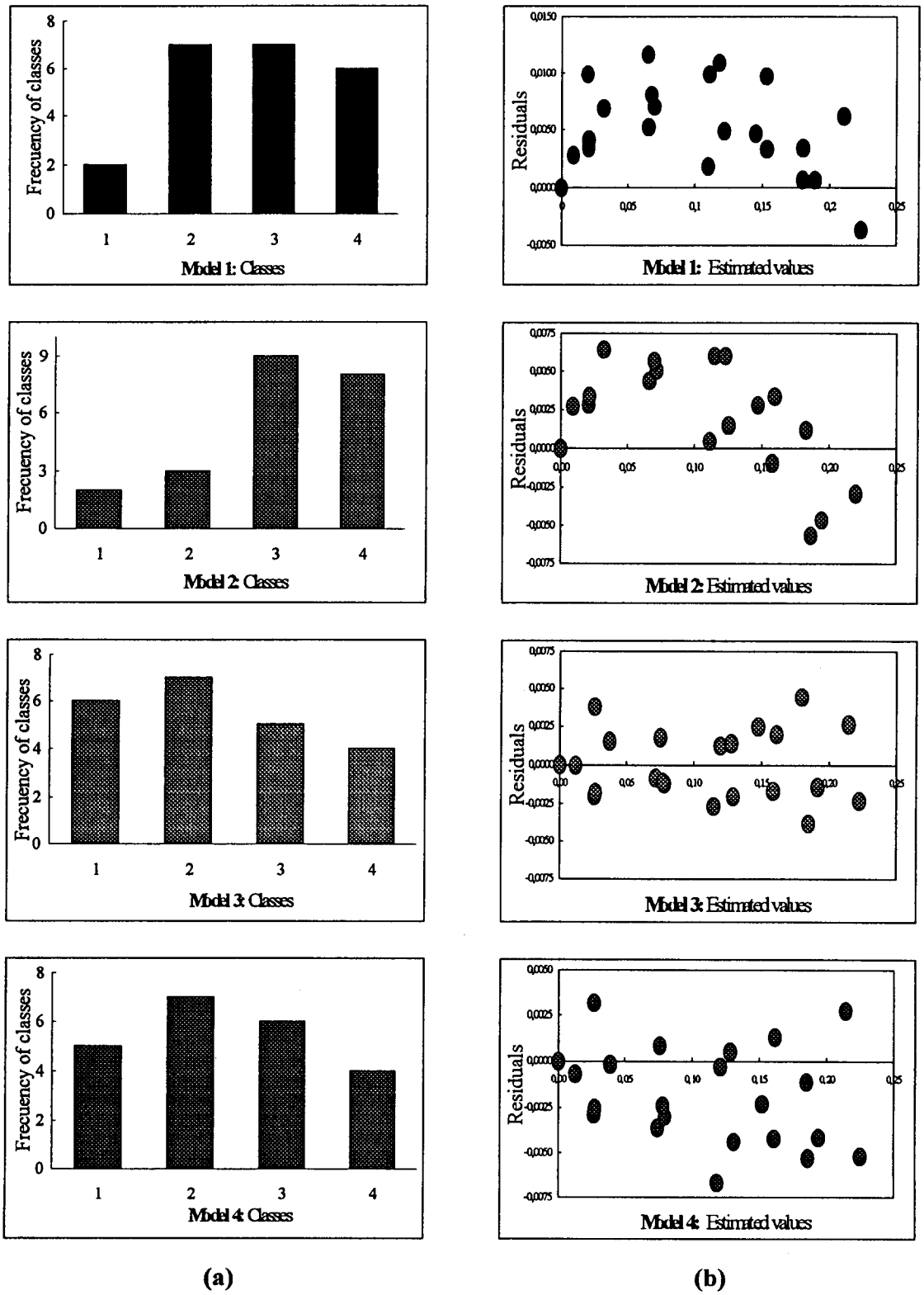

Fig.1 Analysis of residuals: (a) Frequency histograms, (b) Dot diagrams.

CONCLUSIONS

In general, the four formulated models offer a reasonable fitting to the experimental data. However, a more precise analysis by means of numerical measurements and a plot of residuals showed that Model 4 is the one that better satisfies the requirements imposed by hypothesis to a regression model. Its coefficients have not been obtained by standard regression techniques, thus avoiding the classical restriction of the least squares method, that is the summation of residuals must be zero.

Although the solution to the problem suggested formulations for $(C - C_0)/C$ as a function of gas holdup (x_1) and the interaction between gas holdup and square root of frother concentration (x_1x_2), the actual problem is in obtaining the gas holdup. The proposed models allow the estimation of the gas holdup for any values of relative conductance and frother concentration in the experimental range.

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