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A Comparison Study on Criteria to Select the Most Adequate Weighting Matrix

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Abstract

The practice of spatial econometrics revolves around a weighting matrix, which is often supplied by the user on previous knowledge. This is the so called **W** issue and, probably, the aprioristic approach is not the best solution. However, we have to concur that, nowadays, there few alternatives for the user. Under these circumstances, our contribution focuses on the problem of selecting a **W** matrix from among a finite set of matrices, all of them deemed appropriate for the case. We develop a new and simple method based on the Entropy corresponding to the distribution probability estimated for the data. Other alternatives to ours, which are common in current applied work, are also reviewed. The main part of the paper consists of a large Monte Carlo resolved in order to calibrate the effectiveness of our approach compared to the others. A case study is also included.

JEL Classification: C4, C5, R1.

Keywords: Weights matrix, Model Selection, Entropy, Monte Carlo.

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1 Introduction

Let us begin with a mantra: the weighting matrix is the most characteristic element in a spatial model. This commonplace is probably true and we have no problem in asserting once again; in fact, it is the reason for our work. As known, spatial models deal primarily with phenomena such as spillovers, trans-boundary competition or cooperation, flows of trade, migration, knowledge, etc. in very complex networks. Rarely does the user know about how these events operate in practice. Indeed, they are mostly unobservable phenomena which are, however, required to build the model. Traditionally the gap has been solved by providing externally this information, in the form of a weighting matrix. By the way, we must recognize that the **W** alternative is not the unique solution to deal with such kind of unobservables (Oud and Folmer, 2008, for example, develop a latent variables approach that does not need of **W**), but is the most simple.

Hays et al. (2010) give a fairly sensible explanation of the generalization of a predefined **W**. Network analysts are more interested in the formation of networks, taking units attributes and behaviors as given. This is spatial dependence due to selection, where relations of homophily and heterophily are crucial. The spatial econometricians are more interested in what they call *'computing the effects of alters actions on ego's actions through the network*'; in this case, the patterns of connectivity are taken as given. This form of spatial dependence is due to influence and the notions of contagion and interdependence are crucial. So, if the network is predefined in our work, why not supplying it externally?.

However, popularity does not mean absence of dispute; in fact, the **W** issue has been often cause of dispute. In the early stages of the discipline, terms like 'join' or 'link' were very common (for instance, in Moran, 1948, or Whittle, 1954). The focus at that time was mainly on testing for the presence of spatial effects, for which is not so important the specification of a highly detailed weighting matrix; contiguity, nearness, rough measures of separation can be appropriate notions for that purpose. The work of Ord (1975) is crucial in the evolution of this debate because of its strong emphasis on the task of modelling spatial relationships. It is evident that the weights matrix needs more attention if we want to avoid estimation biases and wrong inference. Anselin (1988, 2002) puts the **W** issue in the center of the debate about specification of spatial models. However, after decades of practicing, the **W** issue is still rather obscure. Our purpose is to offer some help to the user in the crucial decision of choosing a weighting matrix for her equation.

The objective is clear: we have to '*determine which ... units in the spatial system have an influence on the particular unit under consideration ... expressed in notions of neighborhood and nearest neighbor*' relations, in words of Anselin (1988, p.16) or '*to define for any set of points or area objects the spatial relationships that exist between them*' as stated by Haining (2003, p. 74). The problem is how should it be done. Roughly speaking, we may distinguish two approaches to the building of **W**: (i) specifying

W exogenously; (ii) estimating **W** from data. The exogenous approach is by far the most common and includes, for example, use of a binary contiguity criterion, k-nearest neighbours, kernel functions based on distance, etc. The second approach uses the topology of the space and the nature of the data, and takes many forms.We find ad-hoc procedures in which a predefined objective guides the search such as the maximization of Moran's I in Kooijman (1976) or the local statistical model of Getis and Aldstadt (2004). Benjanuvatra and Burridge (2015) develop a quasi maximum-likelihood, QML, algorithm to estimate the weights in **W** assuming partial knowledge about the form of the weights. More flexible approaches are possible if we have panel information such as in Bhattacharjee and Jensen-Butler (2013) or Beenstock and Felsenstein (2012). Endogeneity of the weight matrix is another topic introduced recently in the field (i.e., Qu and f. Lee, 2015), which connects with the concept of *coevolution* put forward by Snijders et al. (2007) whose basis is difficult to object: in the long run network connectivity must evolve with the model, that is, with the endogenous variable. The recent literature on spatial econometrics is developing according to the second approach, but our contribution pertains to the first one where still remains most part of the applied research.

Before continue, we may wonder if the **W** issue, even in our context of pure exogeneity, is really a problem to worry for or it is the *biggest myth* of the discipline as claimed by LeSage and Pace (2014). Their argument is that only dramatic different choices for **W** would lead to significant differences in the estimates or in the inference from the model. We partly agree with them in the sense that is a bit silly to argue whether it is better the 5 or the 6 nearest-neighbor matrix; surely there will be almost no difference between the two. However, there is consistent evidence, obtained mainly by Monte Carlo (Florax and Rey, 1995; Franzese Jr and Hays, 2007; Lee and Yu, 2012; Debarsy and Ertur, 2016) showing that the misspecification of **W** (mistakes a little bigger than marginal mismatches) tends to attenuate, or inflate, the estimates of the coefficients of spatial dependence with an inverse impact on the slope coefficients. Moreover, the magnitude of the bias increases for the estimates of the marginal direct/indirect effects. So, we are not pretty sure that '*far too much effort has gone into fine-tunning spatial weight matrices*' as stated by LeSage and Pace (2014). Our impression is that any useful result is welcomed in this field but, overall, we need practical, clear guides to solve the problem.

Another question of concern are the criticisms of Gibbons and Overman (2012). As said, it is common in spatial econometrics to assume that the weighting matrix is known (although things are changing), which is the cause of (weak) identification problems in the models; this flaw extends to the instruments, moment conditions, etc. There is little to say in relation to this point. In fact, there is a kind of shift-share move where the interdependence strength coefficients (i.e., the ρ parameter) and the pattern in which the interaction happens (the matrix **W**) virtually merge, becoming not separable; they are jointly identified (we do estimate $\rho\mathbf{W}$). Hays et al. (2010) and Bhattacharjee and Jensen-Butler (2013) agree in this point.

Bavaud (1998, p. 153), given this state of confusion, was very skeptic, '*there is no such thing as "true", "universal" spatial weights, optimal in all situations*' and continues by stating that the weighting matrix '*must reflect the properties of the particular phenomena, properties which are bound to differ from field to field*'. We share his skepticism about the concept of truth. Perhaps it would suffice with a 'reasonable' weighting matrix, the best among those that we consider. In practical terms, this means that the problem of selecting a weighting matrix can be interpreted as a problem of model selection. In fact, different weighting matrices result in different spatial lags of the endogenous or the exogenous variables included in the model. Finally, different equations with different regressors, or different structures, amounts to a model selection problem.

We believe that this is the dominant approach in the current literature devoted to discussing the **W** issue, that we want to explore and extend further in the present paper. Section 2 introduces four selection criteria that fit well to the problem of selecting a weighting matrix from among a finite set of them. Section 3 presents the main features of the Monte Carlo solved in the fourth Section. Section 5 includes two well known case studies which are revised in the light of our findings. Sixth Section concludes.

2 Criteria to select a W matrix from among a finite set of them

As said, the **W** problem has been present in the literature since very early. However the case of choosing a one of them from among a finite set of such matrices is relatively recent. Anselin (1984) is among the first to pose formally. He suggested a *Cox* statistic derived in an analytical framework of non-nested models. Leenders (2002), on this basis, elaborates a more simple J-test using the classical augmented regressions in this literature. Later on, Kelejian (2008) extends the approach of Leenders to a SAC model, with spatial lags of the endogenous variable and in the error term, using GMM estimates. Piras and Lozano (2012) confirm the adequacy of the J-test to compare different weighting matrices stressing that we should make use of a full set of instrument to increase GMM accuracy. Burridge and Fingleton (2010) show that the Chi-square asymptotic approximations for the J-tests produces irregular results, excessively liberal or conservative in a series of leading cases; they suggest a simple bootstrap resampling method. Burridge (2012) focuses on the propensity of the spatial GMM algorithm to deliver spatial parameter estimates lying outside the invertibility region of the model which, in turn, affects the bootstrap; he suggest the use of a QML algorithm to remove the problem. Kelejian and Piras (2011) generalized and modify the original version of Kelejian to account for all the available information, according to the findings of Piras and Lozano. Finally, Kelejian and Piras (2016) adapt the J test to a panel data setting with unobserved fixed effects and additional endogenous variables which reinforces the adequacy of the GMM framework.

Another milestone in the J test literature is Hagemann (2012), who copes with the reversion problem originated by the lack of a definite null hypothesis in the test. He introduces the minimum J test, MJ. His approach is based on the idea that if there is a finite set of competing models, only the model with the smallest J statistic can be the correct one. In this case, the J statistic will converge to the Chi-square distribution but will diverge if any of the models is correct. The author proposes a wild bootstrap to test if the model with the minimum J is correct. This approach has been applied by Debarsy and Ertur (2016) to a spatial setting with good results.

Our intention is to use only the first part of the procedure of Hagemann, given that we know that there is a correct model in the Monte Carlo that follows. So assuming that we have a collection of m competing weighting matrices, such as: $\mathcal{W} = \{\mathbf{W}_1; \mathbf{W}_2; ...; \mathbf{W}_m\}$ for the same model, then:

1. We are going to estimate the m models; each one corresponds to a different weighting matrix belonging to $\mathcal{W}$. Following Burridge (2012) and given that our interest lies on the small sample case, the models are estimated by QML.

2. For each model, we obtain the battery of J statistic as usual, after estimating, also by QML, the corresponding extended equations.

3. The chosen matrix is the one associated to the minimum J statistic. We do not test if this matrix is really the correct matrix.

Another popular method for choosing between models deals with the so-called *Information Criteria*. Most are developed around a loss function, such as the *Kullback-Leibler*, KL, quantity of information which measures the closeness of two density functions. One of them corresponds to the true probability distribution that generated the data, obviously not known, the other is the distribution estimated from the data. The criteria differ in the role assigned to the aprioris and in way of solving the approximation to the unknown true density function (Hansen, 2005). The two most common procedures are the AIC (Akaike, 1973) and the Bayesian BIC criteria (Schwarz et al., 1978). The first compares the models on equal basis whereas the second incorporates the notion of model of the null. Both criteria are characterized by their lack of specificity in the sense that the selected model is the closest to the true model in terms of KL without any other consideration (i.e., a good global fit does not mean that the model is the best alternative to estimate the parameters of interest; Pötscher, 1991). AIC and BIC lead to single expressions that depend on the accuracy of the ML estimation of the models (in this sense, they are parametric methods) plus a penalty term related to the number of parameters entering the model, such as the following:

$$\left.\begin{array}{ll} AIC(k): & -2l(\tilde{\gamma}) + 2k, \\ BIC(k): & -2l(\tilde{\gamma}) + k\log(n), \end{array}\right\} \quad (1)$$

where $l(\widetilde{\gamma})$ means the estimated log-likelihood at the ML estimates, $\widetilde{\gamma}$, k is the number of non-zero parameters in the model and n the number of observations. For the case that we are considering the models only differ in the weighting matrix, so k and n are the same for every case. This means that the decision depends on the estimated log-likelihood or, what is the same, on the balance between the estimated variance and the Jacobian them given that, for a standard spatial model of SAR or SEM type we can write: $l(\widetilde{\gamma}) \propto \log \left[\frac{1}{\widetilde{\sigma}^n} \left| I - \widetilde{\rho} \mathbf{W} \right| \right]$, being σ the standard deviation and ρ the corresponding spatial dependence coefficient. To minimize any of the two statistics in (1) the objective is to maximize the concentrated estimated log-likelihood, $l(\widetilde{\gamma})$. The same as before, the *Information Criteria* approach implies:

1. We have to estimate by QML each one of the m models corresponding to each weighting matrix in $\mathcal{W}$.

2. For each model, we obtain the corresponding AIC statistic (BIC produces the same results).

3. The matrix in the model with minimum AIC statistic should be chosen.

An important part of the recent literature on spatial econometrics has a Bayesian foundation, which includes the topic of choosing a weighting matrix from among a finite set of alternatives. The point of departure, once again, is to recognize that differences in the weighting matrix, everything else constant, amounts to different models. The Bayesians are well equipped to cope with these type of problems; and extended solution is to calculate posterior the model probabilities associated with each weight matrix as the basis for taking a decision. There are excellent reviews that can be consulted such as Hepple (1995a,b, 2004), Besag and Higdon (1999) and especially, LeSage and Pace (2009). For the sake of completeness, let us highlight the main elements in this approach.

The analysis is made conditional to a model, which is not under discussion. Thus, we have a collection of m weighting matrices in $\mathcal{W}$, a set of k parameter in γ, some of which are of dispersion, σ, others of position, β, and others of spatial dependence, ρ and θ, and a panel data set with nT observations in y. The point of departure is the joint probability of all these elements represented as:

$$p\left(\mathbf{W}_i;\gamma;y\right) = \pi\left(\mathbf{W}_i\right)\pi\left(\gamma \mid \mathbf{W}_i\right)L\left(y \mid \gamma;\mathbf{W}_i\right), \quad (2)$$

where $\pi(\cdot)$ are the prior distributions and $L\left(y \mid \gamma;\mathbf{W}_i\right)$ the likelihood for y conditional on the parameters and the matrix. Bayes' rule leads to the posterior joint probability for matrices and parameters:

$$p\left(\mathbf{W}_i;\gamma \mid y\right) = \frac{\pi\left(\mathbf{W}_i\right)\pi\left(\gamma \mid \mathbf{W}_i\right)L\left(y \mid \gamma;\mathbf{W}_i\right)}{L\left(y\right)}, \quad (3)$$

whose integration over the space of parameters, $\gamma \in \Upsilon$, produces the posterior probability for matrix $\mathbf{W}_i$:

$$p\left(\mathbf{W}_i \mid y\right) = \int\limits_{\Upsilon} p\left(\mathbf{W}_i; \gamma \mid y\right) d\gamma. \quad (4)$$

The presence of spatial structures in the model complicates the resolution of (4) which usually requires of numerical integration. The priors are always a point of concern in this approach and, usually, practitioners prefer the use of diffuse priors. For example, LeSage and Pace (2009, Section 6.3) suggest $\pi\left(\mathbf{W}_i\right) = \frac{1}{m}\ \forall i$, a NIG conjugate prior for β and σ where $\pi_\beta\left(\beta \mid \sigma\right) \sim N\left(\beta_0; \sigma^2\left(\kappa X'X\right)^{-1}\right)$, being X the matrix of the exogenous variables in the model, and $\pi\left(\sigma\right)$ a inverse gamma, $IG(a, b)$. For the parameter of spatial dependence they suggest a $Beta(d, d)$ distribution, being d the amplitude of the sampling space of ρ. For example, the defaults in the MATLAB codes of LeSage (2007) are $\beta_0 = 0$, $\kappa = 10^{-12}$ and $a = b = 0$. In sum, the use of the *Bayesian* approach implies the following:

1. Fix the priors for all the terms appearing in the equation. In this point, we have followed the suggestions of LeSage and Pace.

2. For each matrix, obtain the corresponding posterior probability of (4) for which we need (i) solve the ML estimation of the corresponding model and (ii) solve the numerical integration of (4).

3. The matrix chosen will be that associated with the highest posterior probability.

Our own proposal to deal with the selection problem is based on the notion of *Entropy*, who dates back to Shannon and Weaver (1949). In our statistical framework, *Entropy* is viewed as a measure of the information carried by a distribution of probability. Let us assume an univariate continuous variable, y, whose probability density function is $p(y)$; then, *Entropy* is defined as:

$$h(p) = -\int_I p(y) \log p(y) dy, \quad (5)$$

being I the domain of the random variable y. Higher *Entropy* means less information or, what is the same, more uncertainty about y. Our case fits well with Shannon's framework: we observe a random variable, y, and there are a finite set of rival distribution functions potentially capable of having generated such data. The features of our decision problem are also well defined because each distribution function differs from the others only in the weighting matrix; the other elements are the same. We are not interested in the Laplacian principle of indifference (select the density with maximum *Entropy*, or less informative, to avoid the use of unwarranted information). Quite the opposite: given that in our case there is no unwarranted information, we are looking for the more informative probability distribution so our objective is to minimize *Entropy*.

As with the other three cases, the application of this principle requires the complete specification of the distribution function, which means that the user knows the form of the model (equations 7 or

8 below, except the $\mathbf{W}$ matrix); additionally we add a Gaussian distribution. Next, we should remind that for the case of a $(n \times 1)$ multivariate normal random variable, $y \sim N(\mu; \Sigma)$, the entropy is: $h(y) = \frac{1}{2}\left(n + \log((2\pi)^n |\Sigma|)\right)$. This measure does not depend, directly, on first order moments (parameters of position of the model) but on second order moments (dependence and dispersion parameters). For example, in the case of the *SDM* of (7) below, the entropy is:

$$h(y)_{SDM} = \frac{1}{2}\left(nT + \log((2\pi\sigma^2)^{nT}\left|(B'B)^{-1}\right|)\right), \tag{6}$$

where $B = (I - \rho\mathbf{W})$. Note that the covariance matrix for y in the *SDM* is $V(y) = B^{-1}V(u)B^{'-1}$. If u is indeed a white noise random term with variance σ^2, the covariance matrix of y will match will that which appears in (), $V(y) = \sigma^2 (B'B)^{-1}$. Let us also note that the covariance matrix of y in the *SDEM* of (8) coincides with that of the SDM case, so the expression of the *Entropy* is formally the same as that in (6).

Finally, in order to apply the *Entropy* criterion we must must go through the following steeps:

1. Estimate each one of the m versions of the model that we are considering. As said, each models differs from the other only in the weighting matrix considered. We maintain the *QML* estimation algorithm for reasons given above.

2. For each model, we obtain the corresponding value of the *Entropy*, in the $h_i;\ i = 1, 2, ..., m$ statistic.

3. The decision criterion consists in choosing the weighting matrix corresponding to the model with minimum value of the *Entropy*.

3 Description of the Monte Carlo

This part of the paper is devoted to the design of the Monte Carlo conducted in the next Section in order to to calibrate the performance of the four criteria for selecting $\mathbf{W}$ presented so far: the *MJ* test, the *Bayesian* approach, the *AIC* criterion and the *Entropy* measure. The objective of the analysis is to identify and select the true matrix, which intervened in the generation of the data. Moreover, our focus is on small sample sizes. As will be clear in the next Section, the four criteria reach good standards very quickly; so it is not necessary to simulate very large sample sizes

We are going to simulate a panel setting, with two of the most common *DGPs* in the current literature on applied spatial econometrics; namely, the spatial Durbin Model, *SDM* in expression (7) below, and the spatial Durbin error model, *SDEM* in expression (8):[1]

[1]Main conclusions can be extended to other processes like the spatial lag model or the spatial error model, which are not replicated here (details on request from the authors).

$$y_{it} = \rho \sum_{j=i}^{n} \omega_{ij} y_{jt} + \beta_0 + x_{it}\beta_1 + \theta \sum_{j=i}^{n} \omega_{ij} x_{jt} + \varepsilon_{it}, \tag{7}$$

$$y_{it} = \beta_0 + x_{it}\beta_1 + \theta \sum_{j=i}^{n} \omega_{ij} x_{jt} + u_{it},\ u_{it} = \rho \sum_{j=i}^{n} \omega_{ij} u_{jt} + \varepsilon_{it}. \tag{8}$$

Only one exogenous regressor, x variable, appears in the right hand side of the equations whose observations are obtained from a normal distribution, $x_{it} \sim i.i.d.N\left(0;\sigma_x^2\right)$, where $\sigma_x^2 = 1$; the same applies with respect to the error terms: $\varepsilon_{it} \sim i.i.d.N\left(0;\sigma_\varepsilon^2\right)$, where $\sigma_\varepsilon^2 = 1$. The two variables are not related, $E\left(x_{it}\varepsilon_{it}\right) = 0$. Our space is made of hexagonal pieces which are arranged regularly, one next to the others without discontinuities nor empty spaces.

The two equations are characterized by the presence of, at least, one weighting matrix which plays a central role in the functioning of the model. As said before, the weighting matrix is not observable and the user must take actions to resolve the uncertainty. The decision problem, in a typical situation, consists in choosing one matrix from among a finite set of alternatives which in our simulation are composed by only three candidates: $\mathbf{W}_1$ is built using the traditional contiguity criterion between spatial units; the weights in $\mathbf{W}_2$ are the inverse of the distance between the centroids of the spatial units, $\mathbf{W}_2 = \left\{\omega_{ij} = \frac{1}{d_{ij}}; i \neq j\right\}$; whereas $\mathbf{W}_3$ incorporates a cut-point in the networks of connections of $\mathbf{W}_2$, so that $\mathbf{W}_3 = \left\{\omega_{ij} = \frac{1}{d_{ij}}; i \neq j\ if\ j \in N_8(i);\ 0\ otherwise\right\}$ being $N_8(i)$ the set of 8 nearest neighbors to i. Following usual practice, every matrix has been row-standardized. To keep things simple, the same weighting matrix intervenes with the endogenous and exogenous variables in (7) and with the exogenous and error terms in (8). Due to the row-standardization, the three matrices are non nested in the sense that all the weights are different among them.

Only three different, small cross-sectional sample sizes, n, have been used $n \in \{25, 49, 100\}$; that is enough because, as shown later, higher values of this parameter does not improve the information about the $\mathbf{W}$ dilemma. For the same reason, the number of cross-sections in the panel, T, are limited to only three, $T \in \{1, 5, 10\}$. The values for the coefficient of spatial dependence, ρ, ranges from negatives to positives, $\rho = \{-0.8, -0.5, -0.2, 0.2, 0.5, 0.8\}$. Other global parameters are those associated with the constant term, $\beta_0 = 1$, the x variable, $\beta_1 \in \{1, 5\}$, and its spatial lag, $\theta \in \{1, 5\}$.

In sum, each case consists in:

- Generate the data using a given weighting matrix, $\mathbf{W}_k$, $k = 1, 2, 3$ and a spatial equation, *SDM* or *SDEM*. There are 216 cases of interest in the results of each equation (6 values in ρ, 3 values in n, 3 values in T, 2 values in β_1 and 2 values in θ).

- The spatial equation is assumed to be known so the model can be estimated by maximum likelihood, ML, once the user supplies a $\mathbf{W}$ matrix.
- Compute the four selection criteria, MJ, *Posterior probability*, *Entropy* and AIC for the three alternative weighting matrices for each draw.
- Select the corresponding matrix according to each criterion and compare the result with the *true* matrix in the DGP.
- The process has been replicated $1,000$ times.

Note that the selection of the matrix is made conditional on a correct specification of the equation. We are perfectly aware that this dichotomy is artificial, both decisions are intimately related (in fact, the same matrix but in different equations plays different roles and bears different information). However, this point is not further developed in the present paper. In order to give some intuition, we include the results corresponding to the case of a wrong selection of the spatial equation (i.e, estimate a SDM model whereas the true model in the DGP is a $SDEM$).

4 Results of the Monte Carlo

The Monte Carlo has provided us with a lot of information in relation to the four aforementioned criteria. The aim of this Section is to summarize this volume of information. Let us advance an little spicy: in strictly quantitative terms, the *Entropy* measure is the best criterion. What is more surprising, the *Bayesian* approach is only marginally better than the simple AIC in a setting of small samples, practically indistinguishable for large samples, but much more complicated to obtain; moreover, the MJ test is, by large, the worse alternative among the four criteria. The last two observations are really striking for us given the strong support that both criteria, the *Bayesian* and the MJ test, have received up to now in the specialized literature.

Tables 1 to 3 summarize the main results of our Monte Carlo. These Tables show the percentage of correct selections attained by each criterion, ordered according to the value of the spatial dependence coefficient (rho in the Tables). The averages are obtained accumulating, for each n or T, the cases corresponding to the different values of β's or θ in the simulation. The detail of the results for the 216 cases corresponding to the three DGP/Estimated-equation combinations appear in Tables A1 to A3 in the Appendix. A cell in bold indicates that the respective criterion reaches the maximum rate of correct selections. A quick look at the Tables reveals that gray backgrounds are concentrated in the *Entropy* criterion columns.

The predominance of the *Entropy* criterion for selecting the weighting matrix extends regularly for all the cases, without exception: for correctly specified models, as in Tables 1 and 2, and misspecified equations, as in Table 3, for negative as well as positive values of the spatial autocorrelation coefficient, for small and large cross-sections and for simple to large panels. Especially remarkable are the good results attained by this criterion for the case of small sample sizes. The first panel in the three Tables shows that the averages rates of correct selections are well above 80% for only 25 spatial units. The misspecification in the estimated equation has only a marginal impact on the effectiveness of the three criteria. As can be seen in Table 3, the percentages of right selections are similar to those obtained for correctly specified models, well above 80% in most of the cases. Moreover, the ordering among the criteria continues to be the same: *Entropy*, *Bayes*, *AIC* and *MJ* test.

Table 1: Average percentage of correct selections. DGP: SDM. Equation estimated: SDM.

Aggregated by cross-section, sample size (n)						**Aggregated by time series, sample size (T)**					
	ρ	$h(y)$	*Bayes*	*MJ*	*AIC*		ρ	$h(y)$	*Bayes*	*MJ*	*AIC*
$n = 25$	−0.8	**84.4**	77.8	52.4	79.6	$T = 1$	−0.8	**77.1**	70.5	42.4	73.4
	−0.5	**77.7**	62.5	52.0	61.8		−0.5	**70.6**	58.3	41.4	61.8
	−0.2	**66.5**	48.7	53.1	50.2		−0.2	**63.2**	42.3	43.8	45.7
	0.2	**68.2**	59.8	65.0	61.2		0.2	**59.2**	39.8	53.1	44.8
	0.5	**77.5**	72.6	73.2	72.1		0.5	**65.6**	54.9	60.4	58.7
	0.8	**85.4**	81.7	74.5	75.5		0.8	**78.8**	73.7	69.2	69.6
$n = 49$	−0.8	**91.3**	88.7	57.6	90.1	$T = 5$	−0.8	**95.0**	93.0	62.3	93.5
	−0.5	**85.6**	77.5	58.6	78.7		−0.5	**88.4**	80.2	63.7	79.5
	−0.2	**76.4**	55.5	58.6	58.8		−0.2	**77.6**	57.3	62.4	60.1
	0.2	**77.2**	67.9	73.1	70.0		0.2	**81.3**	76.6	78.1	76.1
	0.5	**86.3**	81.7	81.6	82.0		0.5	**93.2**	92.5	87.1	89.8
	0.8	**94.6**	93.8	88.1	87.4		0.8	**98.5**	98.3	88.2	89.9
$n = 100$	−0.8	**94.7**	94.3	63.9	95.2	$T = 10$	−0.8	**98.2**	97.4	69.2	97.9
	−0.5	**90.2**	87.2	66.6	88.7		−0.5	**94.5**	88.7	72.1	87.9
	−0.2	**81.4**	61.9	62.8	66.6		−0.2	**83.5**	66.5	68.2	69.8
	0.2	**84.5**	76.4	79.4	77.1		0.2	**89.5**	87.7	86.4	87.4
	0.5	**92.4**	90.5	85.6	89.7		0.5	**97.5**	**97.5**	92.9	95.4
	0.8	**97.2**	96.3	89.5	92.4		0.8	**99.8**	**99.8**	94.6	95.8

As indicated in the Tables 1-3, these averages are obtained accumulating panels of different length, from 1 to 10 cross-sections in the left of the Tables, or samples with different number of spatial units (from 25 to 100) in the right hand. The case of a single cross-section with 25 spatial units appears at the top of Tables A1 to A3, in the Appendix, yielding rates of correct selection around 60% for high values of ρ; this percentage is well above 90% at the bottom of the Tables, where the number of cross-sections in the panel is 10. In a similar vein, the increase in the cross-sectional size, n, maintaining the number of cross-sections, T, also has highly beneficial effects for the four criteria. The rate of correct selections for the case of a hundred of spatial units tends to be higher than 75% on average for the case of a single cross-section, but this percentage fluctuates around 60% using 49 spatial unit and hardly reaches 50% for the case of 25 spatial cells. These percentages improve quickly if the time dimension of the panel increases. In general, the rate of correct selections is nearly 100%, using 5 to 10 cross-sections.

Table 2: Average percentage of correct selections. DGP: SDEM. Equation estimated: SDEM.

Aggregated by cross-section, sample size (n)						**Aggregated by time series, sample size (T)**					
	ρ	$h(y)$	*Bayes*	MJ	AIC		ρ	$h(y)$	*Bayes*	MJ	AIC
$n = 25$	−0.8	**86.4**	77.3	56.7	82.5	$T = 1$	−0.8	**80.2**	68.8	46.3	75.9
	−0.5	**81.1**	65.2	57.5	69.6		−0.5	**74.5**	58.6	47.7	67.1
	−0.2	**70.1**	52.5	56.5	58.2		−0.2	**67.1**	45.7	47.7	53.7
	0.2	**63.8**	52.4	56.9	57.7		0.2	**59.2**	39.0	49.6	47.1
	0.5	**69.6**	62.6	55.7	63.7		0.5	**59.5**	48.4	50.1	51.9
	0.8	**77.2**	73.8	54.0	67.0		0.8	**64.7**	61.8	52.9	56.6
$n = 49$	−0.8	**92.5**	88.5	64.5	91.0	$T = 5$	−0.8	**96.2**	94.7	71.1	95.4
	−0.5	**87.7**	78.8	65.6	81.9		−0.5	**91.5**	84.8	72.5	84.9
	−0.2	**79.7**	62.6	65.2	66.3		−0.2	**81.7**	67.1	71.6	69.6
	0.2	**72.9**	61.4	65.2	65.9		0.2	**76.2**	69.3	70.3	73.1
	0.5	**80.6**	75.7	64.5	75.1		0.5	**87.1**	85.5	67.8	83.7
	0.8	**87.3**	87.1	64.0	79.6		0.8	**95.9**	95.4	64.6	86.8
$n = 100$	−0.8	96.3	95.8	75.1	**96.4**	$T = 10$	−0.8	**98.7**	98.2	78.9	98.6
	−0.5	**93.0**	91.3	76.4	92.1		−0.5	**95.8**	91.6	79.3	91.7
	−0.2	**86.6**	75.1	76.4	77.2		−0.2	**87.6**	77.4	78.8	78.3
	0.2	**83.0**	74.9	75.1	78.3		0.2	**84.2**	80.3	77.3	81.7
	0.5	**90.5**	89.1	72.9	88.1		0.5	**94.1**	93.5	75.2	91.3
	0.8	95.4	**95.6**	69.4	90.7		0.8	99.3	**99.4**	70.0	93.9

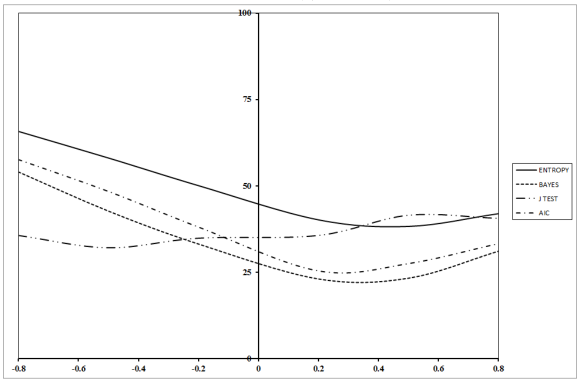
Table 3: Average percentage of correct selections. DGP: SDEM. Equation estimated: SDM.

Aggregated by cross-section, sample size (n)						**Aggregated by time series, sample size (T)**					
	ρ	$h(y)$	*Bayes*	MJ	AIC		ρ	$h(y)$	*Bayes*	MJ	AIC
$n = 25$	−0.8	**85.2**	77.2	51.4	77.6	$T = 1$	−0.8	**79.6**	71.3	41.8	74.5
	−0.5	**79.5**	64.1	55.1	62.3		−0.5	**73.4**	60.1	44.7	63.0
	−0.2	**68.8**	52.4	57.4	52.7		−0.2	**64.9**	44.4	46.4	47.0
	0.2	**63.0**	54.0	59.1	55.4		0.2	**55.9**	35.9	49.2	41.5
	0.5	**67.6**	62.4	55.9	59.4		0.5	**56.1**	45.3	49.6	46.7
	0.8	**74.7**	72.2	42.5	59.4		0.8	**60.3**	57.4	46.5	50.4
$n = 49$	−0.8	**92.1**	88.6	60.4	89.9	$T = 5$	−0.8	**95.2**	92.3	64.3	91.7
	−0.5	**87.8**	78.9	62.7	79.8		−0.5	**90.4**	81.6	67.4	80.0
	−0.2	**79.5**	61.3	64.7	63.1		−0.2	**81.0**	63.4	69.5	64.5
	0.2	**72.0**	60.7	65.5	64.1		0.2	**75.0**	68.5	69.4	70.9
	0.5	**77.9**	73.5	63.2	70.2		0.5	**84.3**	83.9	65.7	78.2
	0.8	**84.4**	84.3	47.1	72.1		0.8	93.1	**93.7**	44.3	77.9
$n = 100$	−0.8	**96.0**	95.0	67.0	95.8	$T = 10$	−0.8	**98.6**	97.2	72.6	96.9
	−0.5	**92.3**	88.6	70.4	89.8		−0.5	**95.8**	89.9	76.1	88.8
	−0.2	**85.2**	68.9	71.5	71.3		−0.2	**87.7**	74.9	77.7	75.6
	0.2	**79.3**	70.0	71.1	73.4		0.2	**83.3**	80.2	77.0	80.5
	0.5	**87.6**	85.9	68.2	83.3		0.5	**92.7**	92.5	72.0	87.9
	0.8	93.0	**93.3**	39.6	84.4		0.8	98.5	**98.8**	38.3	87.6

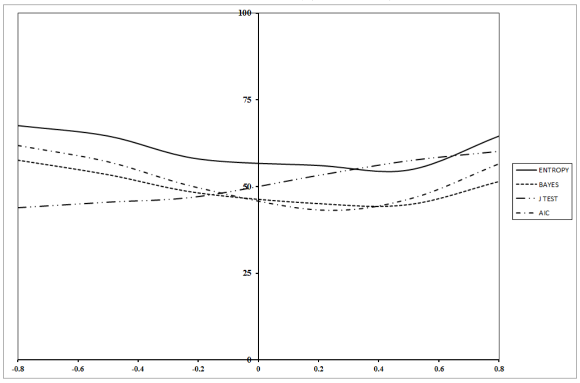
Figures 1 to 3 depict the case of a single cross-section for the three DGP/Estimated-equations combinations and different values of the spatial autocorrelation coefficient and β and θ parameters. The conclusions of these Figures can be extended to the other cases. Overall, as the number of cross-sections, T, increases the curves turn into a straight line stuck to the 100% mark (see Figures A1 to A3 in the Appendix for the same cases as in Figures 1 to 3 but with $T = 10$). It is clear that the probability of choosing the correct weighting matrix increases with the value of the autocorrelation coefficient and also with the signal of spatial spillovers in the model coming from θ.

Figure 1: Percentages of correct seletions. DGP: SDM. Equation estimated: SDM.

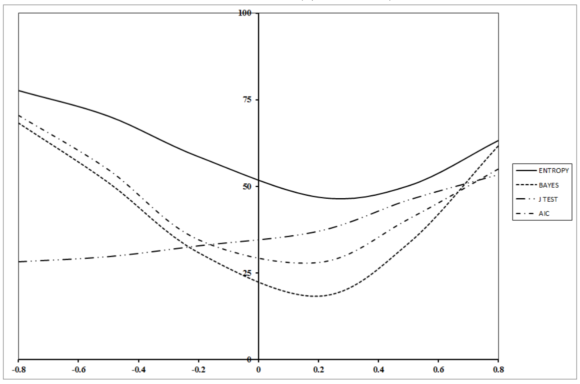
$CASE: n = 25; \beta_1 = 1; \theta = 1$



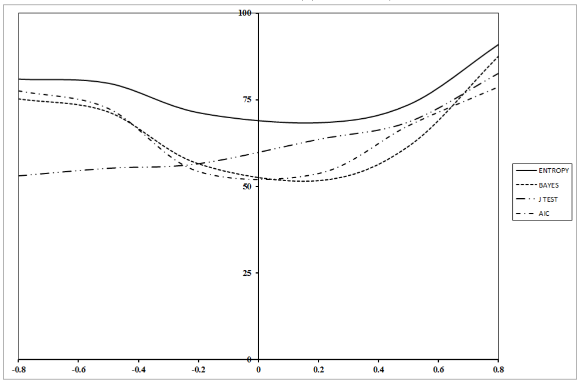
$CASE: n = 25; \beta_1 = 1; \theta = 5$



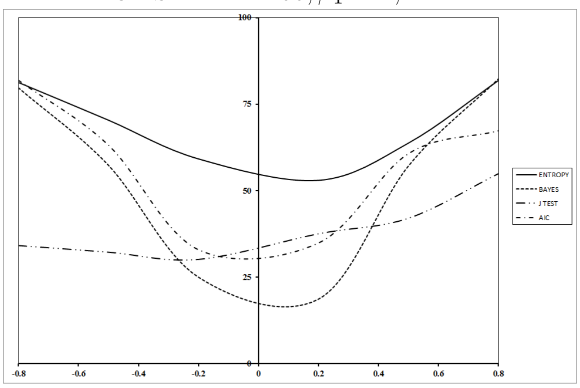
$CASE: n = 49; \beta_1 = 1; \theta = 1$



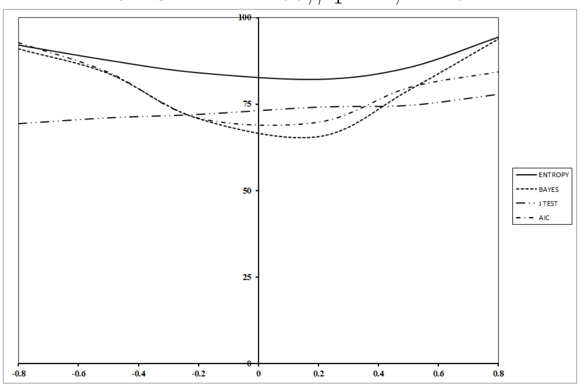
$CASE: n = 49; \beta_1 = 1; \theta = 5$



$CASE: n = 100; \beta_1 = 1; \theta = 1$



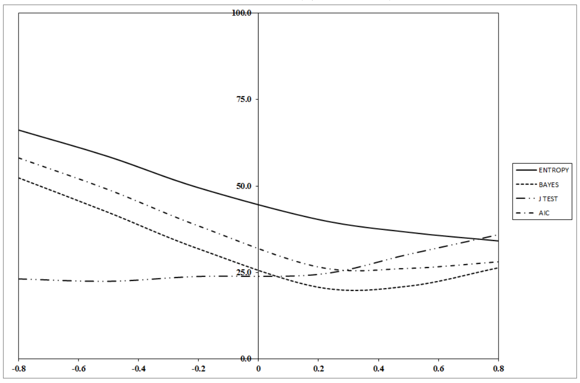
$CASE: n = 100; \beta_1 = 1; \theta = 5$



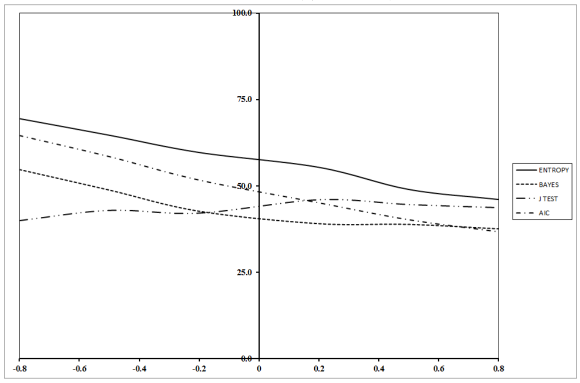
The value of parameter β_1, as expected, has no impact in any criteria. Another interesting feature is the asymmetry of the selection curves. Negative spatial dependence, scarcely considered in the literature, is beneficial for the detection of the correct weighting matrix, especially when the signals from spatial spillover are weak (low value in the parameter θ) and the number of panel cross-sections is also low. The asymmetry appears in all criteria, except in the MJ test whose behavior, contrary to the others, worsens in case of negative values in parameter ρ. The impact of the asymmetry disappears with higher values in T as well as in parameter θ.

Figure 2: Percentages of correct seletions. DGP: SDEM. Equation estimated: SDEM.

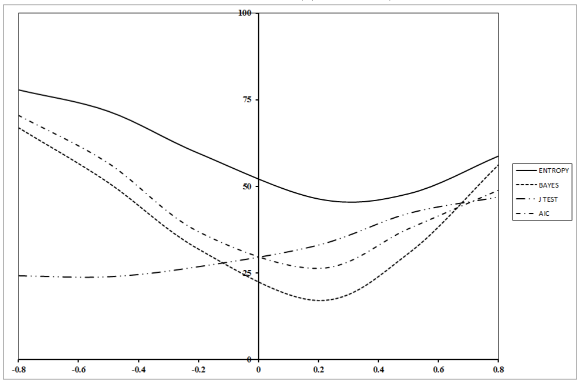
$CASE : n = 25; \beta_1 = 1; \theta = 1$



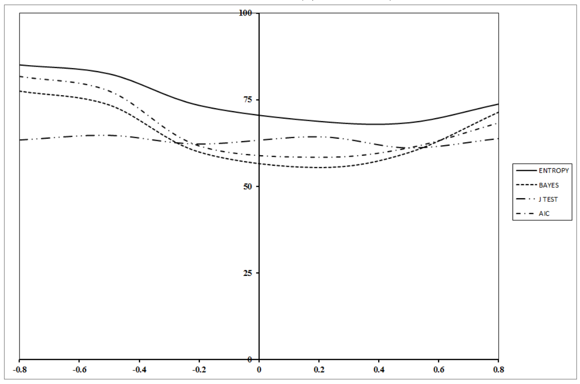
$CASE : n = 25; \beta_1 = 1; \theta = 5$



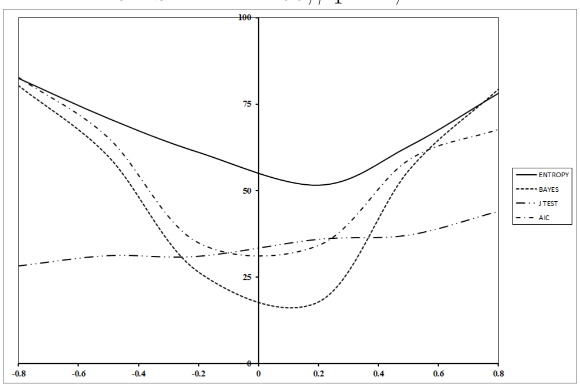
$CASE : n = 49; \beta_1 = 1; \theta = 1$



$CASE : n = 49; \beta_1 = 1; \theta = 5$



$CASE : n = 100; \beta_1 = 1; \theta = 1$



$CASE : n = 100; \beta_1 = 1; \theta = 5$

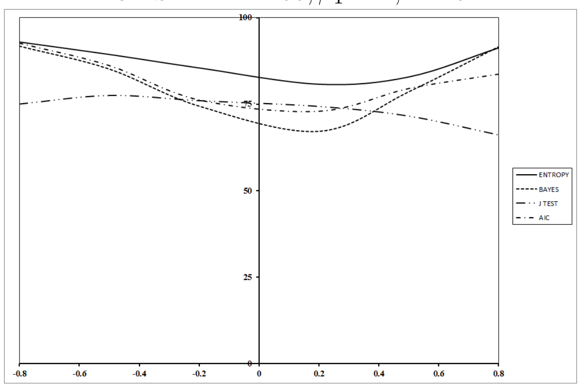
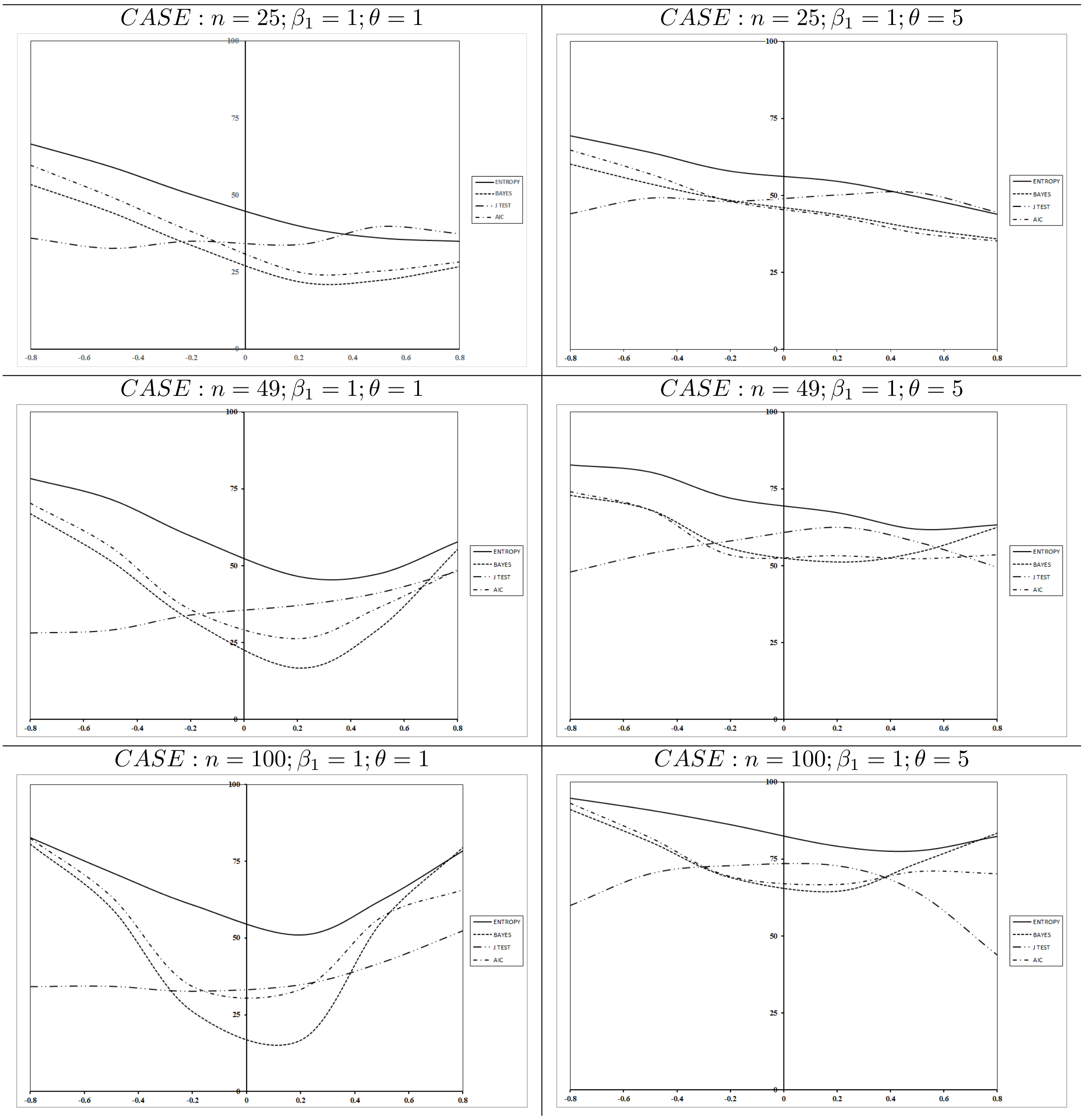


Figure 3: Percentages of correct seletions. DGP: SDM. Equation estimated: SDM.



To complete the picture, we are going to estimate, for each DGP/Estimated-equation combination, a *response-surface* to model the empirical probability of choosing the correct weighting matrix using the corresponding criterion, p_i. As usual, a logit transformation of the empirical probabilities is carried out, so the estimated equation is:

$$\log\left(\frac{p_i + (2r)^{-1}}{1 - p_i + (2r)^{-1}}\right) = p_i^* = \eta + z_i\varphi + \epsilon_i, \tag{9}$$

where p_i^* is the logit transformation, often known as the *logit*, r the number of replications of each experiment (1000 in all the cases; $(2r)^{-1}$ appears in 9 to assure the the *logit* is defined even when the probability of correct selection is 0 or 1; Maddala, 1983); η is an intercept term, z_i the design matrix whose columns reflect the conditions of each experiment, φ is a vector of parameters and ϵ_i the error term assumed to be independent and identically distributed (this assumption is reasonable if all experiments come from the same study, as ours, and are obtained under identical circumstances; Florax and De Graaff, 2004) Let us remind that the number of observations for each *response-surface* equation is 216 (so $i = 1, 2, ..., 216$). Table 4 shows the results for the three DGP/Estimated-equation combination combination.

Table 4: Estimated response surfaces.

SDEM case	constant	n	T	β_1	θ	$\lvert\rho\rvert$	R^2	F_{AV}
Entropy	-5.4864 (-131.7)	0.0029 (8.66)	0.0378 (13.06)	0.0035 (0.66)	0.0482 (9.03)	0.2898 (6.65)	0.64	74.34 $pval = 0$
Bayes	-6.2233 (-84.48)	0.0051 (8.52)	0.0660 (12.87)	-0.0017 (-0.18)	0.0904 (9.58)	0.6813 (8.83)	0.66	81.57 $pval = 0$
MJ test	-6.1300 (-125.4)	0.0044 (10.95)	0.0520 (15.27)	0.0106 (1.70)	0.1569 (25.05)	-0.0378 (-0.74)	0.82	196.74 $pval = 0$
AIC	-5.9177 (-105.4)	0.0042 (9.25)	0.0506 (12.93)	0.0044 (0.61)	0.0795 (11.04)	0.4590 (7.81)	0.67	87.21 $pval = 0$
SDM case	constant	n	T	β_1	θ	$\lvert\rho\rvert$	R^2	F_{AV}
Entropy	-5.4386 (-154.9)	0.0023 (8.01)	0.0351 (14.35)	0.0070 (1.58)	0.0423 (9.40)	0.3478 (9.46)	0.68	133.46 $pval = 0$
Bayes	-6.1117 (-80.28)	0.0033 (5.31)	0.0548 (10.33)	0.0052 (0.52)	0.0861 (8.82)	0.8117 (10.18)	0.60	63.33 $pval = 0$
MJ test	-5.8998 (-74.41)	0.0024 (3.60)	0.0476 (8.27)	0.0185 (1.75)	0.1036 (9.77)	0.1668 (1.93)	0.47	36.74 $pval = 0$
AIC	-5.9340 (-105.2)	0.0034 (7-37)	0.0478 (12.19)	0.0091 (1.21)	0.0721 (9.98)	0.6301 (10.67)	0.67	83.61 $pval = 0$
MIX case	constant	n	T	β_1	θ	$\lvert\rho\rvert$	R^2	F_{AV}
Entropy	-5.5026 (-133.9)	0.0028 (8.43)	0.0411 (14.36)	0.0002 (0.04)	0.0466 (8.86)	0.2905 (6.75)	0.66	80.25 $pval = 0$
Bayes	-6.1882 (-88.69)	0.0040 (6.95)	0.0648 (13.34)	-0.0001 (-0.01)	0.0916 (10.24)	0.7103 (9.73)	0.67	85.13 $pval = 0$
MJ test	-5.6677 (-79.01)	0.0020 (3.35)	0.0379 (7.58)	0.0007 (0.07)	0.1162 (12.63)	-0.3854 (-5.13)	0.54	50.92 $pval = 0$
AIC	-5.9741 (-105.4)	0.0043 (9.23)	0.0558 (14.13)	0.0000 (0.00)	0.0696 (9.58)	0.4728 (7.97)	0.68	88.38 $pval = 0$

Note: t-ratios appear between brackets. F_{AV} means F test of the null that all coefficients are zero except the constant.

Overall, the estimates confirm our suppositions. The main factor influencing the empirical probability of choosing the correct weighting matrix is the parameter of spatial dependence, absolute value of ρ in Table 4. Its contribution is crucial in the case of the *Bayesian* criteria and, to a lesser extend, also in the case of the AIC approach. On the contrary, the MJ test is very peculiar: this parameter is not significant for $SDEM$ processes, hardly significant for SDM models and becomes a handicap for misspecified equations.The second more influential factor is the signal of the spatial spillovers, parameter θ. Clearly, its impact is beneficial for all the cases though it is of greater importance for the MJ test; *Bayes* and AIC are a bit less sensitive whereas its effect on *Entropy* is the lowest. Sample size is also relevant for the four approaches and T seems to be more important than n, both in term of p-values and estimated impact. Finally, as said before, parameter β_1 is not significant in any circumstance which means that the *signal-to-noise* ratio is not a crucial factor to consider when choosing the best weighting matrix for our spatial equation.

Table 5 completes the *response-surface* analysis with the F tests of equality in the coefficients of the *response-surface* estimates. According to the sequence of F tests, the most dissimilar method is the MJ approach, and second is the *Entropy* criterion. In the opposite sense, *Bayes* and AIC are the most similar approaches, even indistinguishable in the case of SDM equations.

Table 5: F test for the equality of coefficients in the response-surface estimates

SDEM case	*Entropy*	*Bayes*	*MJ test*	*AIC*
Entropy	–	25.863 (0.00)	218.37 (0.00)	16.771 (0.00)
Bayes	–	–	34.782 (0.00)	4.5580 (0.00)
MJ test	–	–	–	61.772 (0.00)
AIC	–	–	–	–
SDM case	*Entropy*	*Bayes*	*MJ test*	*AIC*
Entropy	–	21.831 (0.00)	178.56 (0.00)	56.876 (0.00)
Bayes	–	–	17.417 (0.00)	1.811 (0.10)
MJ test	–	–	–	28.556 (0.00)
AIC	–	–	–	–
MIX case	*Entropy*	*Bayes*	*MJ test*	*AIC*
Entropy	–	74.819 (0.00)	266.55 (0.00)	46.445 (0.00)
Bayes	–	–	69.142 (0.00)	5.1722 (0.00)
MJ test	–	–	–	85.234 (0.00)
AIC	–	–	–	–

Note: p-value appear between brackets.

To finish this Section let us to move to Table 6 which summarizes the overall standings of the four criteria in our simulation. This Table shows the percentages that each criterion has reached the maximum rate of correct selections in each pair DGP/Estimated-equation. We believe that the conclusion is rather clear: for small sampling sizes, the *Entropy* criterion is the best alternative to choose the weighting matrix. If the equation is correctly specified (the first two panels), the *Entropy* criterion appears in leading place approximately 90% of times, next is *Bayes* and then AIC; MJ is always in the last position. The percentage for the *Entropy* criterion is smaller in the case of misspecified model on the third panel (70.4% on average) but it is still the best alternative to choose the weighting matrix. Overall, mixing all the cases in the fourth panel, three out of four times, *Entropy* is the best alternative whose probability of making the correct selection going quickly to 1 with sample size. *Bayes* and AIC appear to be also consistent criteria, given that their probability of choosing the correct $\mathbf{W}$ matrix according tends to one when the sample size increases (at a lower speed than the case of *Entropy*). The MJ criterion has many problems, including its apparent inconsistency. We can not recommend its use for the case we are studying.

Table 6: Number of times that each criterion represents the maximum value of correct decision for each study case, in percentage.

Aggregated by cross-section, sample size (n)					**Aggregated by time series, sample size (T)**				
			DGP: SDM	*Equation estimated: SDM*					
n	$h(y)$	*Bayes*	MJ	AIC	**T**	$h(y)$	*Bayes*	MJ	AIC
25	90.3	11.1	6.9	0.0	**1**	87.5	1.4	5.6	5.6
49	79.2	23.6	6.9	16.7	**5**	75.0	27.8	11.1	19.4
100	66.7	38.9	20.8	38.9	**10**	73.6	44.4	18.1	30.6
Total	78.7	24.5	11.6	18.5	**Total**	78.7	24.5	11.6	18.5
			DGP: SDEM	*Equation estimated: SDEM*					
n	$h(y)$	*Bayes*	MJ	AIC	**T**	$h(y)$	*Bayes*	MJ	AIC
25	93.1	4.2	1.4	1.4	**1**	90.3	6.9	1.4	1.4
49	79.2	29.2	4.2	13.9	**5**	76.4	27.8	5.6	16.7
100	72.2	50.0	18.1	34.7	**10**	77.8	48.6	16.7	31.9
Total	81.5	27.8	7.9	16.7	**Total**	81.5	27.8	7.9	16.7
			DGP: SDEM	*Equation estimated: SDM*					
n	$h(y)$	*Bayes*	MJ	AIC	**T**	$h(y)$	*Bayes*	MJ	AIC
25	73.6	13.9	12.5	0.0	**1**	83.3	5.6	11.1	0.0
49	72.2	27.8	5.6	9.7	**5**	61.1	30.6	4.2	15.3
100	65.3	44.4	11.1	25.0	**10**	66.7	50.0	13.9	19.4
Total	70.4	28.7	9.7	11.6	**Total**	70.4	28.7	9.7	11.6
			All DGPs	*All estimated equations*					
n	$h(y)$	*Bayes*	MJ	AIC	**T**	$h(y)$	*Bayes*	MJ	AIC
25	85.7	9.7	6.9	0.5	**1**	87.0	4.6	6.0	2.3
49	76.9	26.9	5.6	13.4	**5**	70.8	28.7	7.0	17.1
100	68.1	44.4	16.7	32.9	**10**	72.7	47.7	16.2	27.3
Total	76.9	27.0	9.7	15.6	**Total**	76.9	27.0	9.7	15.6

5 Empirical applications

The empirical applications in this section are based on two well-known economic models. The first one is a model of growth estimated by Ertur and Koch (2007) using a cross-section sample of 91 countries over the period 1960–1995. The other example is an economic model of productivity estimated by Munnell (1990) using panel data on 48 states in US over the period 1970–1979. Our aim in this section is to test the hypothesis about the appropriate selection of spatial weight matrices in each case.

5.1 Study case 1: Ertur and Koch (2007)

Ertur and Koch (2007) build a growth equation to model technological interdependence between countries using spatial externalities. The main hypotheses of interaction is that the stock of knowledge in one country produces externalities that may cross national borders and spill over into neighboring countries, with an intensity which decreases with distance. The authors use the criterion of pure geographical distance.

The benchmark model assumes an aggregate Cobb-Douglas production function, for the country i in time period t, with constant returns to scale in labour and physical capital:

$$Y_i(t) = A_i(t) K_i^{\alpha}(t) L_i^{1-\alpha}(t), \tag{10}$$

where $Y_i(t)$ is output, $K_i(t)$ is the level of reproducible physical capital, $L_i(t)$ is the level of labour, and $A_i(t)$ is the aggregate level of technology specified as:

$$A_i(t) = \Omega(t) k_i^{\phi}(t) \prod_{j \neq i}^{n} A_i^{\delta \omega_{ij}}(t). \tag{11}$$

The aggregate level of technology $A_i(t)$ of any country i depends on three elements. First, a certain proportion of technological progress is exogenous and identical in all countries: $\Omega(t) = \Omega(0)e^{\mu t}$, where is a constant rate of technological growth. Second, each country's aggregate level of technology increases with the aggregate level of physical capital per worker $k_i^{\phi}(t) = (K_i(t)/L_i(t))^{\phi}$ with the parameter $\phi \in [0;1]$ capturing the strength of home externalities by physical capital accumulation. Finally, the third term captures the external effects of knowledge embodied in capital located in a different country whose impact crosses national borders at a diminishing intensity, $\delta \in [0;1]$. The terms ω_{ij} represent the connectivity between country i and its neighbours; this weight is assumed to be exogenous, non-negative and finite.

Following Solow, the authors assume that, in every country i, a constant fraction of output s_i is saved and that labour grows exogenously at the rate n_i. Also, they assume a constant and identical annual rate of depreciation of physical capital for all countries, denoted τ. The evolution of output per worker in country i is governed by the usual fundamental dynamics of the Solow equation which, after some manipulations, lead to the steady-state real income per worker in that country (Ertur and Koch, 2007, p. 1038, eq. 9):

$$y = \Omega + (\alpha + \phi) k - \alpha \delta \mathbf{W} k + \delta \mathbf{W} y. \tag{12}$$

This is a spatially augmented Solow model and coincides with the predictor obtained by Solow adding spillover effects. In terms of spatial econometrics, it is immediate to recognize a *Spatial Durbin Model*, SDM, in the equation which can be expressed as:

$$y = x\beta + \rho \mathbf{W} y + \mathbf{W} x \theta + \varepsilon. \tag{13}$$

Equation (13) is estimated using information on real income, investment and population growth for a sample of 91 countries over the period $1960-1995$. Regarding the spatial weighting matrix, Ertur and Koch consider two geographical distance functions: the inverse of squared distance (which is the main hypothesis) and the negative exponential of squared distance (to check robustness in the specification). We also consider a third matrix using the inverse of the distance.

Let us call the three weighting matrices as $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_3$ which are row-standardized: $\omega_{hij} = \omega^*_{hij}/\sum_{j=1}^{n}\omega^*_{hij};\ h = 1,2,3$ where:

$$\omega^*_{1ij} = \begin{cases} 0 & if\ i=j \\ d_{ij}^{-2} & otherwise \end{cases} \quad ; \quad \omega^*_{2ij} = \begin{cases} 0 & if\ i=j \\ e^{-d_{ij}} & otherwise \end{cases} \quad ; \quad \omega^*_{3ij} = \begin{cases} 0 & if\ i=j \\ d_{ij}^{-1} & otherwise \end{cases} \quad , \qquad (14)$$

with d_{ij} as the great-distance between the capitals of countries i and j.

The authors analyze several specifications checking for different theoretical restrictions and alternatives spatial equations. We concentrate our revision in the non-restricted equation of Ertur and Koch (in the sense that it includes more coefficients than advised by theory). Table 7 presents the SDM version of this equation using the three alternative weighting matrices specified before (the first two columns coincides with those in Table I, columns 3-4, pp. 1047, of Ertur and Koch, 2007). The the last four rows in the Table shows the value of the selection criteria corresponding to each case.

Table 7: Ertur & Koch case. Unrestricted SDM estimates

Model/Weight matrix	***SDM / W1***	***SDM / W2***	***SDM / W3***
constant	1.178 (0.62)	0.678 (0.36)	5.045 (0.96)
$\log(s)$	0.829 (8.24)	0.795 (7.60)	0.908 (8.50)
$\log(n+0.05)$	−1.500 (−2.62)	−1.452 (−2.61)	−1.711 (−2.68)
$\mathbf{W}\times\log(s)$	−0.283 (−1.51)	−0.345 (−2.06)	0.468 (1.19)
$\mathbf{W}\times\log(n+0.05)$	0.528 (0.62)	0.118 (0.15)	2.177 (1.02)
$\mathbf{W}\times\log(y)$	0.716 (9.61)	0.643 (8.43)	0.899 (13.67)
Selection Criteria			
Entropy	28.021	29.631	34.616
Bayesian	0.871	0.127	0.002
MJ test	11.158	9.388	10.208
AIC	95.885	99.100	109.132

Note: t-ratios appear between brackets.

The preferred model by Ertur and Koch corresponds to the pair $SDM/\mathbf{W}_1$ which coincides with the selection corresponding to the criterion of minimum *Entropy,* the *Bayesian* posterior probability and AIC. The selection of the minimum J test is $\mathbf{W}_2$.

Other results in Ertur and Koch concern to the Spatial Error Model version of the steady-state equation of (12), or SEM model. The intention of the authors is to visualize the presence of spatial correlation in the traditional non spatial Solow equations; our intention is just to add another example of selection of weighting matrices in misspecified models. The main results appear in Table 8 (which can be compared with columns 2-3 of Table II, in Ertur and Koch, 2007, p. 1048).

Table 8: Ertur & Koch case. Unrestricted SEM estimates

Model/Weight matrix	***SEM / W1***	***SEM / W2***	***SEM / W3***
constant	6.457 (4.22)	6.706 (4.62)	5.892 (3.02)
$\log(s_i)$	0.828 (8.36)	0.804 (7.87)	0.992 (8.94)
$\log(n_i + 0.05)$	−1.702 (−3.03)	−1.552 (−2.85)	−2.269 (−3.65)
$\mathbf{W} \times \varepsilon_i$	0.823 (15.69)	0.737 (12.19)	0.937 (22.08)
Selection Criteria			
Entropy	30.973	31.734	42.049
Bayesian	0.655	0.345	0.000
MJ test	0.171e^{-12}	0.043e^{-12}	0.085e^{-12}
AIC	97.870	99.391	120.021

Note: t-ratios appear between brackets.

In spite of the misspecified model, the selection of most adequate **W** matrix does not change. Using the values of *Entropy* criterion we select the first model, in which intervenes the matrix $\mathbf{W}_1$, the same as with the Bayesian approach and the *AIC* criterion; *MJ* continues selecting $\mathbf{W}_2$.

5.2 Study case 2: Munnell (1990)

Munnell et al. (1990) suggests a Cobb-Douglas production function in each state of the US (excluding "islands" Alaska and Hawaii and the district of Columbia, for a total of 48 states) observed over the years between 1970 and 1986. The dependent variable, output of the production function, is the gross state product, log(*gsp*), and the explanatory variables considered are the endowment of public capital, log(*pcap*) (roads, water facilities and other utilities), the private capital, log(*pc*), employment, log(*emp*), and the unemployment rate, *unemp*, in order to proxy for the effects of the business cycle. The model can be expressed formally as follows:

$$\log(gsp_{it}) = \alpha + \beta_1 \log(pcap_{it}) + \beta_2 \log(pc_{it}) + \beta_3 \log(emp_{it}) + \beta_4 unemp_{it} + \mu_i + \varepsilon_{it}, \quad (15)$$

with $i = 1, \ldots, 48$, and $t = 1970, ..., 1979$. This dataset had been used by Millo et al. (2012) and Álvarez et al. (2017) among others. For this example, we consider subset of the original data including only the observations between 1970 and 1979. This selection is to keep the number of regions and time periods into the panel dimensions used in the Monte Carlo simulation.

We estimate a fixed effect model with the most complex spatial models used in Monte Carlo, *SDEM* and *SDM*. Following previous studies, our main hypothesis of spatial spill-over is based on contiguity criterion ($\mathbf{W}_1$ in our case). This matrix presents an average of connectivity of 4.5 neighbours and the median of neighbours is 4. Using this information, we propose two very near alternative spatial weighting matrices using k-nearest neighbours criterion: $\mathbf{W}_2$ is constructed using 4 nearest neighbors and $\mathbf{W}_3$ with 5 nearest neighbors.

Let us call the three weighting matrices as $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{W}_3$ which are row-standardized: $\omega_{hij} = \omega^*_{hij} / \sum_{j=1}^{n} \omega^*_{hij};\ h = 1, 2, 3$ where:

$$\omega^*_{1ij} = \begin{cases} 1 & if\ (i,j)\ have\ common\ frontier \\ 0 & otherwise \end{cases};$$

$$\omega^*_{2ij} = \begin{cases} 1 & if\ d_{ij} \leq d_{i(k=4)} \\ 0 & otherwise \end{cases}; \tag{16}$$

$$\omega^*_{3ij} = \begin{cases} 1 & if\ d_{ij} \leq d_{i(k=5)} \\ 0 & otherwise \end{cases},$$

with d_{ij} as the Euclidean-distance between the centroids of counties i and j, and $d_{i(k)}$ indicates the distance of $k - th$ neighbor.

Previously to applied the set of criteria, we check the significance of spatial effects using LM tests obtaining as final specification a $SDEM$ model with two spatial lagged explanatory variables, $\mathbf{W} \times \log(pc)$ and $\mathbf{W} \times \log(emp)$. This is the preferred model and, also, we present the results of SDM model as a misspecified model. All results are presented in the Tables 9 and 10.

Table 9: Munnell case. SDEM estimates

Model/Weight matrix	***SDEM / W1***	***SDEM / W2***	***SDEM / W3***
$\log(pcap)$	0.103 (3.07)	0.111 (3.32)	0.106 (3.15)
$\log(pc)$	0.289 (8.50)	0.289 (8.31)	0.288 (8.30)
$\log(emp)$	0.629 (18.02)	0.637 (17.76)	0.606 (17.37)
$unemp$	−0.003 (−2.40)	−0.003 (−2.34)	−0.002 (−1.79)
$\mathbf{W} \times \log(pc)$	−0.201 (−3.81)	−0.232 (−4.71)	−0.250 (−4.43)
$\mathbf{W} \times \log(emp)$	0.191 (4.13)	0.224 (4.66)	0.283 (5.14)
$\mathbf{W} \times \varepsilon$	0.420 (8.28)	0.357 (6.56)	0.432 (7.81)
Selection Criteria			
Entropy	−1.131	−1.117	−1.128
Bayesian	0.763	0.000	0.237
MJ test	15.890	5.958	0.292
AIC	−2.134	−2.115	−2.132

Note: t-ratios appear between brackets.

The criterion of minimum Entropy, the Bayesian posterior probability and AIC are coincident to select as preferred model the pair $SDEM/\mathbf{W}1$. The selection of the minimum J test is $SDEM/\mathbf{W}3$.

For the misspecified model, the Spatial Durbin Model, the selection for each criterion of the weighting matrix is similar to the previous one model. In Table 10, Entropy, the Bayesian posterior probability and AIC select the pair $SDM/\mathbf{W}1$ and minimum J test chooses $SDM/\mathbf{W}3$.

Table 10: Munnell case. SDM estimates

Model/Weight matrix	***SDM / W1***	***SDM / W2***	***SDM / W3***
$\log(pcap)$	0.108 (3.34)	0.126 (3.92)	0.114 (3.58)
$\log(pc)$	0.311 (8.60)	0.306 (8.42)	0.306 (8.45)
$\log(emp)$	0.589 (16.04)	0.614 (16.45)	0.578 (15.85)
$unemp$	−0.003 (−2.72)	−0.003 (−2.71)	−0.002 (−2.21)
$\mathbf{W} \times \log(pc)$	−0.293 (−5.94)	−0.299 (−6.35)	−0.318 (−6.37)
$\mathbf{W} \times \log(emp)$	−0.118 (−2.05)	−0.067 (−1.09)	−0.070 (−1.07)
$\mathbf{W} \times \log(y)$	0.430 (8.77)	0.359 (6.80)	0.430 (7.98)
Selection Criteria			
Entropy	−1.137	−1.121	−1.131
Bayesian	0.961	0.000	0.039
MJ test	8.693	6.246	1.122
AIC	−2.144	−2.121	−2.139

Note: t-ratios appear between brackets.

6 Conclusions

Much of the applied literature on spatial econometrics and regional data sets an exogenous approximation to the **W** matrix. It is assumed that the user has some previous knowledge with respect to the network of interaction among the sampling units, which allows him to build a weighting matrix on purely apriori grounds. In recent years, new literature has been published advocating for a more data driven approach to the **W** issue. We strongly support this tendency, which should be dominant in the future, but now our focus is on the exogenous approach.

The problem that we pose is relatively frequent in applied work: the user has a finite collection of weighting matrices, all of them consistent with the case of study, and she/he needs to select one of them. Which is the best **W**?. This question is far from being new in the literature where we can find different proposal: the *Bayesian* posterior probability, the J tests in all its variants or the simple model selection criteria such as AIC or BIC, very common in mainstream econometrics (probably there are other alternatives). We add a fourth criterion such as the *Entropy* of the estimated distribution function. *Entropy* is a measure of uncertainty, popular in applied statistic, which fits pretty well in our problem. It depends on the estimated covariance matrix offering a more complete picture of the goodness of the distribution function (let us remind that every distribution function is linked to a particular **W**).

The conclusions of our Monte Carlo are very enlightening. First, the different criteria do a good work in the sense that they select, almost with security, the correct matrix for the case of large samples. Let us add, however, that the J test, even using the refined approach of Hagemann (2012), is the worst alternative.The length of the time series appear to be more beneficial than the number of cross-sectional units in the sample. In second place, it is important to highlight that the four criteria are not indifferent, especially in samples of small size (in n or in T). The ordering is clear: *Entropy* in first place, *Bayesian* posterior probability in continuation with *AIC* slightly worse and then J in the fourth position.

Our recommendation for applied researchers is check for the adequacy of the weighting matrix and, in case of various candidates, take a decision using well-defined criteria such as the *Entropy*. The case studies presented in Section 5 illustrates this procedure.

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Table A1: Percentage of correct selections. Details: DGP: SDM Equation estimated: SDM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
1	-0.8	1	1	65.8	54.0	35.7	57.7	77.7	68.3	28.3	70.6	81.3	79.8	34.2	81.9
1	-0.5	1	1	58.1	42.7	32.1	48.5	70.3	51.1	29.8	54.9	70.4	57.4	32.3	63.2
1	-0.2	1	1	50.1	33.2	34.9	38.2	58.6	30.9	32.9	34.7	59.2	25.0	30.2	33.0
1	0.2	1	1	40.2	23.1	35.7	25.5	46.9	18.3	37.1	28.1	53.0	18.6	37.6	34.9
1	0.5	1	1	38.3	23.3	41.6	27.6	50.2	33.5	46.1	40.6	63.8	57.1	42.1	60.7
1	0.8	1	1	42.0	31.1	40.7	33.4	63.3	61.7	53.4	55.1	81.9	82.3	55.0	67.3
1	-0.8	1	5	67.6	57.7	43.9	61.9	81.0	75.3	53.1	77.7	92.1	91.0	69.4	92.9
1	-0.5	1	5	64.6	53.5	45.5	57.2	79.8	71.5	55.3	72.6	87.7	83.9	71.1	84.3
1	-0.2	1	5	58.0	48.2	47.1	49.7	71.3	56.5	56.6	54.4	84.1	70.9	72.1	71.0
1	0.2	1	5	56.1	45.1	53.2	43.3	68.4	51.6	63.6	53.8	82.2	65.7	74.2	69.8
1	0.5	1	5	54.8	44.8	57.4	46.4	73.6	61.5	68.6	67.6	85.6	79.0	74.7	79.8
1	0.8	1	5	64.6	51.5	60.1	56.6	91.0	87.6	82.6	78.8	94.4	93.8	77.9	84.4
1	-0.8	5	1	66.9	57.7	42.6	62.2	81.8	74.8	46.4	76.6	87.0	85.7	55.3	88.2
1	-0.5	5	1	59.4	46.1	34.2	50.4	69.5	55.6	33.7	58.7	74.7	63.5	38.5	68.1
1	-0.2	5	1	49.5	31.0	31.9	36.2	59.0	28.6	29.1	33.0	60.5	22.3	26.9	32.5
1	0.2	5	1	44.2	27.4	41.3	30.1	50.0	23.4	44.8	30.9	56.8	26.9	45.8	39.4
1	0.5	5	1	47.1	35.7	49.3	37.5	63.6	48.7	58.1	55.1	76.9	70.3	59.2	69.8
1	0.8	5	1	58.2	49.1	55.2	50.6	89.3	85.7	78.2	76.0	92.1	92.0	73.9	81.3
1	-0.8	5	5	64.5	51.1	36.1	55.1	78.9	69.8	30.1	73.6	80.9	80.2	33.7	82.0
1	-0.5	5	5	60.5	47.1	34.9	51.1	73.6	56.4	39.6	59.0	78.8	70.2	49.5	73.6
1	-0.2	5	5	58.8	46.8	46.4	48.6	70.9	53.0	54.0	53.7	78.1	60.8	64.0	63.6
1	0.2	5	5	57.8	47.1	56.5	48.4	71.5	55.5	67.5	58.9	83.4	74.5	79.5	74.6
1	0.5	5	5	60.2	48.5	62.5	52.1	80.5	67.6	77.4	78.2	92.0	88.4	87.9	88.7
1	0.8	5	5	74.9	67.1	70.0	69.9	96.3	94.6	91.8	90.4	98.0	87.6	92.0	91.7

Table A1 (continue): Percentage of correct selections. Detail. DGP: SDM Equation estimated: SDM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
5	-0.8	1	1	86.5	77.0	30.7	78.3	92.6	92.6	35.2	93.9	98.2	98.2	33.0	99.0
5	-0.5	1	1	72.9	46.5	33.6	46.7	82.5	69.7	35.7	72.4	88.4	88.0	34.6	89.7
5	-0.2	1	1	51.8	22.6	34.1	26.9	65.3	30.4	40.1	37.7	72.8	39.7	45.8	48.5
5	0.2	1	1	46.9	33.8	39.6	41.1	60.0	49.5	51.7	55.8	72.2	67.1	63.0	48.5
5	0.5	1	1	70.7	66.9	48.2	64.6	82.7	82.0	64.1	74.9	92.8	93.7	76.7	88.6
5	0.8	1	1	91.0	89.1	52.4	72.9	95.6	96.5	72.2	79.6	99.4	99.5	81.8	91.1
5	-0.8	1	5	95.4	92.6	79.8	91.8	98.8	98.9	93.9	99.3	99.9	99.9	98.3	99.9
5	-0.5	1	5	92.6	85.8	82.2	79.5	97.8	96.7	94.7	97.0	99.6	99.6	99.3	99.8
5	-0.2	1	5	90.5	80.8	85.0	71.8	98.0	95.1	95.1	93.3	99.8	99.6	99.4	99.4
5	0.2	1	5	90.6	85.9	89.7	85.7	97.6	97.3	97.5	95.4	100.0	100.0	100.0	99.5
5	0.5	1	5	93.4	92.7	90.5	90.4	99.6	99.6	99.3	96.4	100.0	100.0	100.0	99.6
5	0.8	1	5	98.2	97.4	84.1	83.0	99.9	99.9	99.4	93.5	100.0	100.0	100.0	98.5
5	-0.8	5	1	93.7	89.1	70.3	89.5	96.6	96.4	79.6	98.0	99.4	99.4	91.3	99.8
5	-0.5	5	1	76.6	55.1	44.4	51.7	84.9	76.3	52.0	77.0	93.5	93.8	68.6	94.2
5	-0.2	5	1	50.9	17.6	27.0	24.5	59.8	18.3	28.9	31.8	68.1	25.7	29.3	39.3
5	0.2	5	1	56.5	45.6	53.1	51.3	73.9	66.6	68.8	69.1	86.0	85.5	84.1	82.6
5	0.5	5	1	87.7	83.3	79.2	83.5	95.5	95.8	92.9	89.0	98.6	98.8	97.8	95.8
5	0.8	5	1	98.6	97.8	78.5	81.7	99.7	99.7	98.5	89.5	100.0	100.0	100.0	97.9
5	-0.8	5	5	87.6	79.7	38.7	80.4	92.8	93.8	44.0	94.1	98.3	98.4	52.6	98.5
5	-0.5	5	5	83.3	66.6	61.8	60.8	91.8	87.2	72.4	87.6	96.7	96.7	85.3	97.1
5	-0.2	5	5	81.7	70.3	76.4	65.6	94.6	89.0	90.6	85.3	98.0	98.0	97.3	97.4
5	0.2	5	5	92.2	88.8	90.9	86.9	99.1	99.3	99.1	97.6	100.0	100.0	100.0	100.0
5	0.5	5	5	97.5	96.8	96.3	96.1	100.0	100.0	99.9	98.5	100.0	100.0	100.0	99.7
5	0.8	5	5	99.8	99.8	91.3	92.9	99.9	99.9	100.0	98.6	100.0	100.0	100.0	100.0

Table A1 (continue): Percentage of correct selections. Detail. DGP: SDM Equation estimated: SDM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
10	-0.8	1	1	93.7	89.9	32.8	92.2	97.4	97.2	34.0	98.6	99.6	99.6	35.3	100.0
10	-0.5	1	1	85.0	61.8	32.9	61.3	88.6	81.5	37.7	82.9	95.4	95.6	41.2	96.2
10	-0.2	1	1	60.9	29.1	38.5	36.7	75.2	41.0	48.5	47.4	81.8	59.2	60.3	61.2
10	0.2	1	1	60.3	52.9	53.2	54.6	73.6	69.1	64.5	68.9	86.7	84.7	77.1	84.3
10	0.5	1	1	85.0	84.1	61.6	80.0	91.8	92.7	75.1	88.4	98.5	99.0	89.5	94.4
10	0.8	1	1	97.9	98.0	67.0	84.1	99.8	99.7	81.3	91.3	100.0	100.0	92.9	97.2
10	-0.8	1	5	99.0	98.8	93.5	98.4	99.9	99.9	98.1	99.9	100.0	100.0	99.9	100.0
10	-0.5	1	5	98.8	96.2	94.5	93.7	99.8	99.8	99.2	99.9	99.9	99.9	99.9	100.0
10	-0.2	1	5	97.0	94.2	95.0	90.8	99.2	99.4	99.0	99.1	100.0	100.0	100.0	100.0
10	0.2	1	5	99.0	98.1	98.3	97.7	99.5	99.3	99.4	98.8	100.0	100.0	100.0	100.0
10	0.5	1	5	99.6	99.7	99.4	97.8	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.9
10	0.8	1	5	100.0	100.0	98.5	92.8	100.0	100.0	99.9	98.6	100.0	100.0	100.0	99.9
10	-0.8	5	1	98.4	97.1	82.9	97.1	99.5	99.5	92.6	99.8	100.0	100.0	98.3	100.0
10	-0.5	5	1	88.3	68.9	53.5	66.5	91.6	87.5	66.0	86.4	97.5	97.8	81.7	98.0
10	-0.2	5	1	53.9	21.0	28.0	29.7	65.6	25.4	31.0	38.0	74.9	41.4	28.3	53.7
10	0.2	5	1	75.4	70.7	69.8	71.3	86.5	85.2	83.4	83.3	93.5	93.9	92.0	91.0
10	0.5	5	1	95.9	95.7	91.9	90.8	98.6	98.8	97.8	95.8	100.0	100.0	99.7	99.0
10	0.8	5	1	100.0	100.0	96.7	90.6	100.0	100.0	99.8	98.0	100.0	100.0	100.0	99.5
10	-0.8	5	5	93.4	88.9	42.3	90.4	98.2	98.3	55.5	98.5	99.7	99.7	65.6	99.9
10	-0.5	5	5	92.2	80.0	74.4	74.2	97.1	96.1	87.2	95.4	99.7	99.8	97.0	99.8
10	-0.2	5	5	94.9	89.5	92.3	84.0	98.7	98.1	97.7	97.1	99.9	99.9	99.8	100.0
10	0.2	5	5	99.1	99.2	99.1	98.5	99.8	99.8	99.8	99.8	100.0	100.0	100.0	100.0
10	0.5	5	5	100.0	100.0	100.0	98.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
10	0.8	5	5	100.0	100.0	99.5	97.6	100.0	100.0	100.0	99.9	100.0	100.0	100.0	100.0

Table A2: Percentage of correct selections. Details: DGP: SDEM Equation estimated: SDEM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
1	-0.8	1	1	66.2	52.3	23.1	58.1	77.9	67.0	24.3	70.5	82.5	80.5	28.2	82.8
1	-0.5	1	1	58.5	42.3	22.4	48.9	71.7	51.1	24.0	56.7	70.9	60.0	31.2	65.3
1	-0.2	1	1	49.5	31.8	23.8	38.4	59.6	31.9	26.9	37.0	61.1	26.6	31.0	34.9
1	0.2	1	1	40.3	20.7	24.4	26.5	46.4	17.0	33.1	26.4	51.6	17.9	35.9	34.1
1	0.5	1	1	36.6	21.0	30.2	26.1	47.9	30.6	42.2	37.8	62.8	55.8	37.1	58.8
1	0.8	1	1	34.1	26.3	35.9	28.0	58.8	56.2	46.9	48.9	78.2	79.4	44.0	67.7
1	-0.8	1	5	69.5	54.7	40.0	64.7	85.1	77.5	63.4	81.8	93.0	91.8	75.1	92.7
1	-0.5	1	5	64.7	48.8	43.0	58.6	82.5	73.5	64.7	77.6	89.4	85.2	77.6	86.2
1	-0.2	1	5	59.7	42.7	42.2	51.8	73.4	59.9	62.3	61.8	85.5	74.5	76.1	76.2
1	0.2	1	5	55.4	39.1	46.1	45.2	68.8	55.4	64.3	58.5	80.8	67.2	74.4	72.9
1	0.5	1	5	49.0	38.9	44.7	40.3	68.4	59.7	61.2	61.2	82.9	78.6	71.6	79.5
1	0.8	1	5	46.1	37.6	43.8	36.8	73.8	71.4	63.8	68.4	91.3	91.6	66.1	83.7
1	-0.8	5	1	64.9	40.7	22.6	56.2	77.5	62.8	23.6	70.2	93.0	91.8	75.1	92.7
1	-0.5	5	1	59.6	31.1	23.4	49.5	70.6	41.8	26.5	55.8	89.4	85.2	77.6	86.2
1	-0.2	5	1	50.6	19.4	21.5	38.1	59.7	22.9	26.7	35.5	85.5	74.5	76.1	76.2
1	0.2	5	1	40.1	13.5	27.7	27.5	46.6	15.3	32.3	27.3	80.8	67.2	74.4	72.9
1	0.5	5	1	34.2	14.9	28.5	24.2	50.4	30.9	41.8	36.3	82.9	78.6	71.6	79.5
1	0.8	5	1	37.2	25.0	36.3	26.6	58.1	59.5	51.9	48.8	91.3	91.6	66.1	83.7
1	-0.8	5	5	73.6	42.0	46.0	66.2	88.6	75.0	62.1	83.9	90.8	88.9	72.4	90.6
1	-0.5	5	5	67.7	36.0	44.9	60.0	80.6	64.9	62.7	73.3	88.5	83.2	74.7	86.9
1	-0.2	5	5	60.5	34.5	46.7	53.1	76.8	56.5	64.3	66.1	83.3	72.8	74.6	75.4
1	0.2	5	5	54.0	34.8	46.3	44.0	67.7	52.8	62.9	58.7	77.6	67.2	73.4	71.1
1	0.5	5	5	49.2	35.5	42.8	39.3	66.7	57.8	59.3	59.1	82.6	78.5	70.2	81.1
1	0.8	5	5	49.1	44.2	47.5	40.3	71.6	70.8	64.2	64.7	87.1	87.6	67.7	81.1

Table A2 (continue): Percentage of correct selections. Detail. DGP: SDEM Equation estimated: SDEM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
5	-0.8	1	1	87.9	78.5	36.6	81.0	93.0	93.3	44.3	94.9	98.4	98.5	53.9	99.2
5	-0.5	1	1	76.4	50.2	39.7	50.6	84.6	73.5	47.5	74.5	91.8	90.9	56.6	92.8
5	-0.2	1	1	55.3	26.7	35.8	30.6	68.3	38.8	46.0	43.9	75.2	52.2	56.8	57.8
5	0.2	1	1	42.4	31.7	37.1	37.9	57.1	42.3	45.3	52.6	68.1	58.8	52.4	64.0
5	0.5	1	1	66.4	60.2	37.3	62.7	78.6	77.3	41.5	72.2	91.3	92.7	50.9	86.5
5	0.8	1	1	89.0	86.5	35.8	71.6	93.4	94.3	45.6	78.8	99.0	98.9	48.5	90.6
5	-0.8	1	5	99.1	98.0	94.4	97.9	99.8	99.8	98.8	99.7	100.0	100.0	99.8	100.0
5	-0.5	1	5	97.5	94.6	93.9	93.0	98.9	99.0	98.6	98.9	99.9	99.9	100.0	99.9
5	-0.2	1	5	94.4	89.9	92.9	88.7	98.6	98.5	98.4	98.2	100.0	100.0	100.0	100.0
5	0.2	1	5	91.1	87.1	90.0	89.0	96.9	96.8	96.8	96.4	99.7	99.7	99.7	99.5
5	0.5	1	5	89.4	87.8	84.2	87.5	98.4	97.1	93.2	96.9	99.9	99.9	99.2	99.7
5	0.8	1	5	95.3	93.3	75.9	86.5	99.4	99.4	88.0	95.0	100.0	100.0	96.1	98.6
5	-0.8	5	1	87.3	81.0	32.1	81.9	92.6	91.5	45.7	93.8	98.0	98.1	54.7	98.6
5	-0.5	5	1	77.1	51.9	36.5	49.6	82.6	70.9	47.2	73.6	92.6	92.9	57.7	93.5
5	-0.2	5	1	53.2	26.9	36.0	32.2	66.5	35.0	45.3	42.0	78.8	51.8	59.4	57.7
5	0.2	5	1	43.7	30.2	37.0	37.7	56.4	41.3	44.6	49.2	72.7	62.1	56.1	68.1
5	0.5	5	1	64.4	57.2	37.2	56.6	79.9	77.8	45.4	73.4	90.3	91.1	49.3	85.9
5	0.8	5	1	87.9	86.3	37.6	71.1	94.0	95.4	43.3	77.8	98.4	98.6	46.8	90.7
5	-0.8	5	5	99.0	98.0	94.3	98.1	99.4	99.5	98.6	99.6	100.0	100.0	100.0	100.0
5	-0.5	5	5	97.4	94.8	94.0	93.0	99.2	99.3	98.3	98.9	99.9	99.9	99.9	100.0
5	-0.2	5	5	91.9	87.7	90.7	86.5	98.4	98.4	98.1	98.2	99.8	99.8	99.5	99.8
5	0.2	5	5	90.1	85.5	88.7	88.0	96.6	96.3	96.4	95.5	99.8	99.9	99.7	99.8
5	0.5	5	5	89.7	87.9	84.0	87.2	97.7	97.2	93.0	96.4	99.7	99.7	98.9	99.4
5	0.8	5	5	94.6	93.1	75.5	87.0	99.5	99.4	86.5	95.4	100.0	100.0	95.1	99.0

Table A2 (continue): Percentage of correct selections. Detail. DGP: SDEM Equation estimated: SDEM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
10	-0.8	1	1	95.4	92.7	47.4	94.2	97.7	97.8	56.7	98.9	99.9	99.9	70.8	100.0
10	-0.5	1	1	88.4	68.8	48.2	68.6	90.4	85.7	58.3	86.7	97.2	97.7	71.1	97.6
10	-0.2	1	1	65.2	36.9	43.8	41.5	78.5	54.2	56.6	56.3	84.9	74.1	71.7	73.3
10	0.2	1	1	55.8	45.3	45.6	49.8	69.0	60.8	54.2	63.7	83.0	80.9	69.3	80.8
10	0.5	1	1	80.1	76.3	45.6	73.3	89.9	89.8	49.6	85.3	96.2	96.8	62.1	93.0
10	0.8	1	1	96.9	97.2	43.1	83.0	99.6	99.7	43.6	90.5	100.0	100.0	51.9	97.1
10	-0.8	1	5	99.8	99.7	99.0	99.9	100.0	100.0	99.8	100.0	100.0	100.0	100.0	100.0
10	-0.5	1	5	99.7	99.6	99.1	99.3	99.9	99.9	99.9	99.9	100.0	100.0	100.0	100.0
10	-0.2	1	5	98.5	98.3	98.2	98.0	99.7	99.7	99.6	99.7	100.0	100.0	100.0	100.0
10	0.2	1	5	98.3	98.0	97.9	98.1	99.2	99.1	99.0	98.8	100.0	100.0	100.0	100.0
10	0.5	1	5	98.6	98.1	96.1	97.2	99.7	99.7	98.8	99.6	100.0	100.0	100.0	100.0
10	0.8	1	5	99.7	99.4	88.9	97.0	100.0	100.0	95.1	99.2	100.0	100.0	99.8	100.0
10	-0.8	5	1	94.4	90.5	46.0	92.4	97.8	97.9	57.0	98.3	99.7	99.8	70.6	99.8
10	-0.5	5	1	86.1	63.9	45.4	64.8	91.2	86.5	59.3	87.2	96.8	96.8	70.7	96.8
10	-0.2	5	1	63.1	36.9	47.9	40.8	77.6	55.6	57.7	56.9	85.3	74.6	71.3	74.7
10	0.2	5	1	57.1	46.1	46.0	52.0	70.1	59.4	54.0	64.6	81.4	78.4	66.1	76.8
10	0.5	5	1	79.6	76.4	43.3	73.2	90.1	90.7	49.3	83.9	97.0	97.2	64.2	93.7
10	0.8	5	1	97.4	97.7	40.5	80.8	99.4	99.5	43.5	89.0	99.8	99.8	51.5	95.9
10	-0.8	5	5	100.0	99.8	99.0	99.8	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
10	-0.5	5	5	99.9	100.0	99.7	99.5	100.0	100.0	99.9	100.0	100.0	100.0	100.0	100.0
10	-0.2	5	5	99.0	98.5	98.6	98.5	99.7	99.8	99.9	99.7	100.0	100.0	100.0	100.0
10	0.2	5	5	97.2	96.2	96.2	96.2	99.8	99.9	99.7	99.5	100.0	100.0	100.0	100.0
10	0.5	5	5	98.5	97.4	94.1	96.9	99.5	99.5	99.1	99.2	100.0	100.0	99.9	100.0
10	0.8	5	5	99.1	99.0	87.1	95.2	100.0	100.0	95.7	98.9	100.0	100.0	99.6	100.0

Table A3: Percentage of correct selections. Details: DGP: SDEM Equation estimated: SDM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
1	-0.8	1	1	66.6	53.5	36.1	59.8	78.4	67.0	28.2	70.4	82.7	80.6	34.2	82.5
1	-0.5	1	1	59.2	44.5	32.8	49.5	71.7	51.7	29.1	56.3	71.4	59.8	34.3	63.6
1	-0.2	1	1	50.2	33.7	35.1	38.2	59.7	32.4	34.0	35.8	60.8	26.1	32.7	34.3
1	0.2	1	1	40.0	21.9	34.0	25.0	46.6	16.7	37.1	26.3	51.0	16.7	34.8	33.3
1	0.5	1	1	36.1	22.3	39.8	25.3	47.3	29.1	41.1	36.2	62.4	55.3	42.0	56.9
1	0.8	1	1	35.0	26.8	37.5	28.3	57.8	55.3	48.1	48.7	78.4	79.4	52.3	65.7
1	-0.8	1	5	69.4	60.2	44.0	64.8	82.8	72.9	47.9	74.1	94.8	91.2	59.9	93.1
1	-0.5	1	5	64.0	53.8	49.2	57.0	80.5	68.2	54.0	68.1	90.9	80.7	70.3	82.0
1	-0.2	1	5	57.9	48.4	48.1	48.1	72.0	55.7	58.1	53.5	86.2	69.1	72.9	69.4
1	0.2	1	5	54.6	43.8	50.2	43.1	67.3	51.3	62.6	53.3	79.2	64.6	72.9	66.8
1	0.5	1	5	49.6	39.3	51.0	37.8	61.9	54.4	57.8	52.3	77.7	73.7	64.2	71.0
1	0.8	1	5	43.9	35.9	44.5	35.2	63.3	62.5	49.5	53.6	82.4	83.5	43.8	70.3
1	-0.8	5	1	65.7	52.2	35.8	57.7	77.4	67.7	31.4	71.5	85.7	83.4	33.1	85.4
1	-0.5	5	1	59.7	44.9	32.7	50.5	71.1	50.4	30.8	55.6	74.0	63.4	33.3	66.4
1	-0.2	5	1	50.0	33.3	35.3	37.2	60.1	30.1	31.0	34.2	61.3	25.8	31.6	34.5
1	0.2	5	1	40.5	22.4	38.5	26.0	45.8	16.6	38.2	26.3	46.8	15.1	35.7	30.9
1	0.5	5	1	34.6	21.1	36.7	22.7	48.8	29.1	44.6	35.5	65.1	56.4	41.5	60.4
1	0.8	5	1	37.1	26.6	40.6	27.4	58.9	58.1	48.9	49.2	80.0	81.1	49.8	69.4
1	-0.8	5	5	73.2	64.1	45.5	67.8	84.9	74.9	47.7	76.7	93.0	87.9	58.3	90.4
1	-0.5	5	5	67.3	55.9	46.8	59.0	78.8	66.9	54.0	64.5	92.0	81.5	69.0	83.3
1	-0.2	5	5	61.1	51.2	49.1	51.6	74.5	57.5	58.5	57.1	85.2	69.8	70.2	69.6
1	0.2	5	5	54.2	44.1	52.2	45.4	67.8	52.2	61.8	53.7	76.5	65.8	72.3	68.0
1	0.5	5	5	49.2	36.2	50.4	37.2	62.3	52.6	60.2	52.8	78.5	73.9	65.4	72.7
1	0.8	5	5	46.7	40.0	48.7	37.3	60.5	59.5	51.4	50.7	79.9	80.2	43.2	69.1

Table A3 (continue): Percentage of correct selections. Detail. DGP: SDEM Equation estimated: SDM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
5	-0.8	1	1	87.6	76.6	36.1	77.8	94.0	94.2	41.7	94.9	98.6	98.7	48.3	99.3
5	-0.5	1	1	76.6	49.0	38.9	48.1	84.5	71.9	42.9	74.3	91.8	90.3	50.0	92.3
5	-0.2	1	1	54.0	25.0	36.7	28.9	68.5	35.3	46.0	42.3	77.0	48.6	53.8	53.4
5	0.2	1	1	42.7	30.0	36.5	36.7	56.0	41.3	43.8	51.2	66.9	58.7	51.2	63.7
5	0.5	1	1	65.1	59.7	36.6	60.9	77.6	76.8	43.3	69.7	90.7	91.8	48.5	85.9
5	0.8	1	1	88.1	86.7	43.8	69.7	92.8	94.4	47.2	77.7	98.9	98.9	48.9	88.7
5	-0.8	1	5	92.1	84.4	66.0	77.1	99.9	99.6	93.6	99.5	100.0	100.0	99.3	100.0
5	-0.5	1	5	88.8	78.3	75.7	67.2	99.7	98.1	96.8	97.4	100.0	100.0	99.7	99.9
5	-0.2	1	5	89.2	78.4	83.2	70.2	98.4	95.8	96.8	93.7	100.0	99.8	99.8	99.5
5	0.2	1	5	89.1	86.3	88.7	83.0	94.9	95.0	95.8	92.6	99.1	99.2	99.3	98.3
5	0.5	1	5	84.0	84.4	78.1	77.4	92.8	93.6	88.6	84.4	97.4	97.9	95.2	93.4
5	0.8	1	5	85.0	87.5	43.4	66.4	93.7	94.2	49.6	75.0	99.1	99.6	32.2	88.5
5	-0.8	5	1	87.4	78.3	36.4	79.0	92.9	91.7	42.5	94.2	98.0	98.1	47.9	98.9
5	-0.5	5	1	76.3	49.7	37.4	48.9	84.8	71.8	43.1	73.7	92.7	92.6	53.2	93.6
5	-0.2	5	1	53.3	25.4	36.6	29.4	66.8	32.8	45.3	39.8	79.6	47.8	57.7	53.9
5	0.2	5	1	42.2	29.7	37.6	36.7	55.5	40.7	44.0	49.5	71.7	62.5	54.6	66.3
5	0.5	5	1	63.0	57.9	39.9	56.7	77.9	77.7	43.6	71.0	89.2	90.2	51.9	85.0
5	0.8	5	1	87.7	87.1	42.5	69.2	94.0	95.0	48.9	76.6	98.3	98.5	49.4	90.5
5	-0.8	5	5	92.8	86.5	65.6	80.4	99.5	99.5	95.0	99.8	100.0	100.0	99.3	100.0
5	-0.5	5	5	89.6	79.3	75.2	68.1	99.8	98.0	95.8	96.9	100.0	100.0	99.7	100.0
5	-0.2	5	5	86.3	76.5	81.5	69.0	98.6	95.6	96.6	93.8	99.8	99.8	99.7	99.9
5	0.2	5	5	88.3	84.9	86.7	81.6	94.5	94.6	94.7	92.4	99.4	99.6	99.4	98.8
5	0.5	5	5	85.0	86.2	79.6	77.0	91.2	92.7	88.5	83.3	98.0	98.3	94.9	93.7
5	0.8	5	5	87.5	89.0	43.7	67.9	93.8	94.7	47.4	75.2	98.8	98.7	35.1	88.9

Table A3 (continue): Percentage of correct selections. Detail. DGP: SDEM Equation estimated: SDM

Other parameters				CASE n=25				CASE n=49				CASE n=100			
T	ρ	β_1	θ	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC	Entropy	Bayes	MJ	AIC
10	-0.8	1	1	95.2	91.2	42.1	92.6	97.9	97.9	50.5	99.1	99.9	99.9	61.1	100.0
10	-0.5	1	1	88.4	64.0	44.7	63.7	90.8	85.1	52.8	85.9	97.9	97.9	67.8	98.3
10	-0.2	1	1	64.2	33.6	45.5	39.1	78.2	49.9	54.8	53.7	86.5	69.2	69.8	69.6
10	0.2	1	1	55.0	45.9	45.8	48.2	68.7	62.2	55.3	63.9	81.6	79.6	67.4	79.4
10	0.5	1	1	79.9	76.8	44.2	73.0	89.4	89.6	48.4	84.1	96.0	96.5	60.8	91.6
10	0.8	1	1	96.9	97.3	47.0	81.9	99.6	99.6	48.3	89.8	100.0	100.0	47.6	96.1
10	-0.8	1	5	99.2	95.4	82.8	92.7	100.0	100.0	98.6	100.0	100.0	100.0	100.0	100.0
10	-0.5	1	5	98.7	94.4	93.0	88.4	100.0	100.0	99.9	99.9	100.0	100.0	100.0	100.0
10	-0.2	1	5	98.1	94.2	95.1	90.1	99.7	99.7	99.7	99.6	100.0	100.0	100.0	100.0
10	0.2	1	5	97.4	97.3	97.1	95.2	98.4	98.2	98.5	97.9	99.9	100.0	99.9	99.8
10	0.5	1	5	93.6	94.7	86.5	85.6	98.3	98.4	95.0	95.2	99.9	99.9	98.9	99.2
10	0.8	1	5	96.3	97.0	38.7	73.8	99.6	99.6	39.0	90.0	100.0	100.0	13.2	95.8
10	-0.8	5	1	93.8	89.4	42.2	90.5	97.7	97.7	48.4	98.2	99.6	99.6	62.7	99.8
10	-0.5	5	1	86.1	61.4	43.6	61.0	91.6	84.2	53.0	85.5	97.1	97.1	67.4	97.8
10	-0.2	5	1	63.3	35.2	45.8	39.7	77.5	51.4	55.7	54.3	86.4	71.2	69.7	71.6
10	0.2	5	1	56.1	46.2	45.1	51.2	68.6	59.9	54.5	63.7	79.0	78.2	65.3	75.3
10	0.5	5	1	78.4	75.7	43.1	71.4	89.3	89.8	51.2	82.7	96.6	96.8	56.8	91.7
10	0.8	5	1	97.1	97.5	42.2	81.1	99.5	99.7	47.7	89.0	99.7	99.8	47.9	95.1
10	-0.8	5	5	99.3	95.1	83.7	90.4	100.0	100.0	98.8	100.0	100.0	100.0	100.0	100.0
10	-0.5	5	5	99.1	94.5	91.6	85.7	100.0	100.0	99.7	99.9	100.0	100.0	100.0	100.0
10	-0.2	5	5	98.4	94.4	96.8	90.6	99.9	99.7	99.5	99.1	100.0	100.0	100.0	100.0
10	0.2	5	5	95.4	95.7	96.4	93.2	99.4	99.5	99.1	98.3	100.0	100.0	100.0	99.9
10	0.5	5	5	93.0	94.1	85.2	87.5	97.8	98.0	95.6	94.6	99.9	99.9	97.9	97.8
10	0.8	5	5	94.8	95.5	37.2	74.3	99.0	99.0	38.7	89.5	100.0	100.0	12.1	94.8

Figure A1. Percentages of correct selection.

DGP: SDM Equation estimated: SDM. T=10

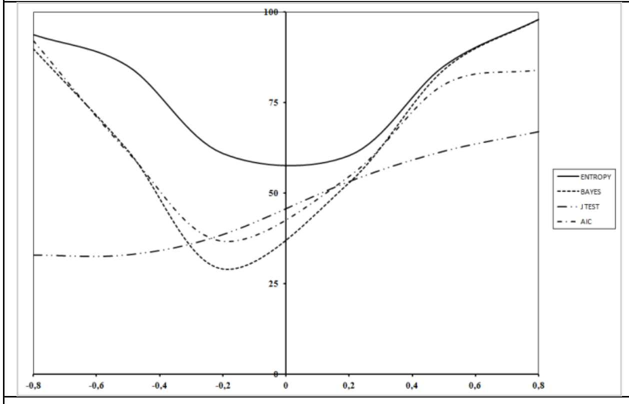
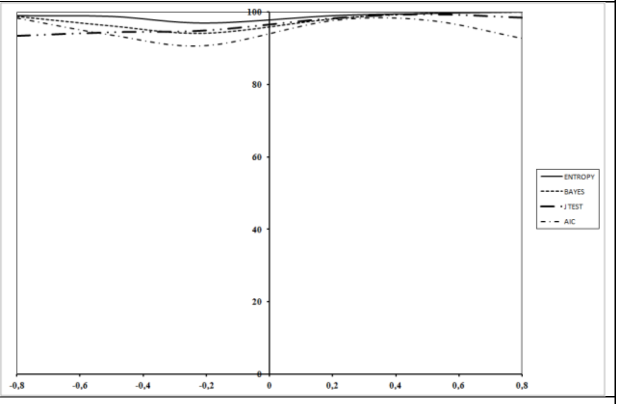
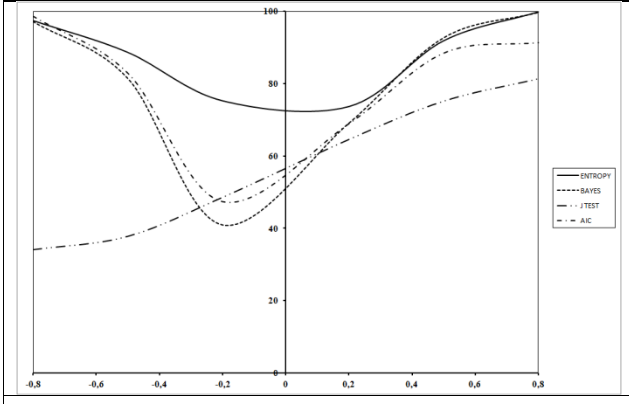
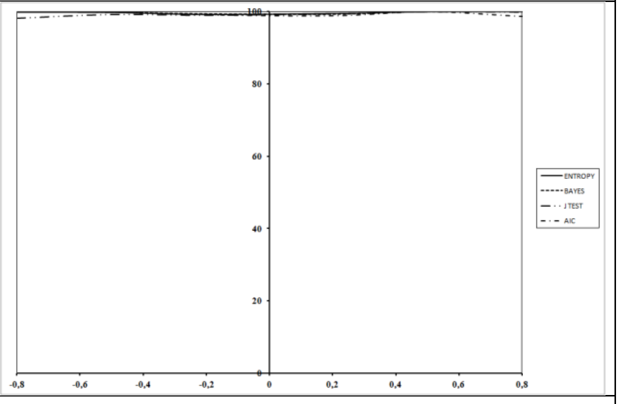
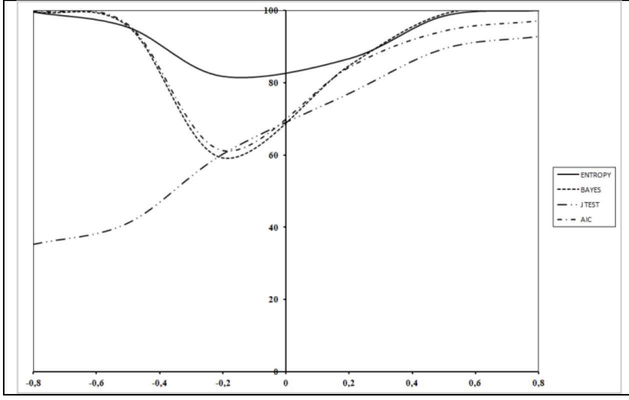
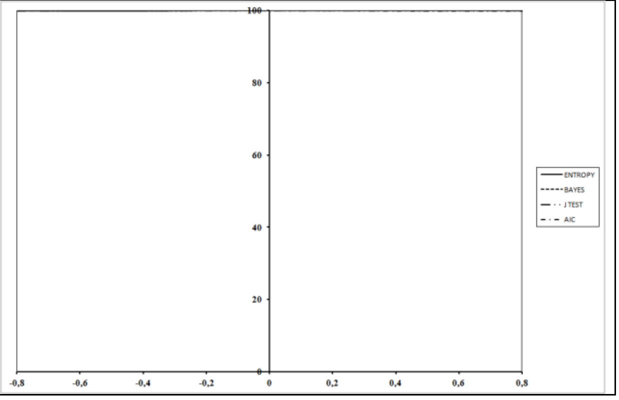
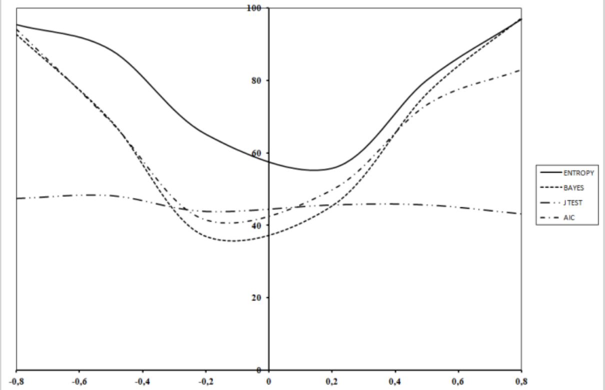
CASE: n=25; β_1=1; θ=1	CASE: n=25; β_1=1; θ=5
	
CASE: n=49; β_1=1; θ=1	CASE: n=49; β_1=1; θ=5
	
CASE: n=100; β_1=1; θ=1	CASE: n=100; β_1=1; θ=5
	

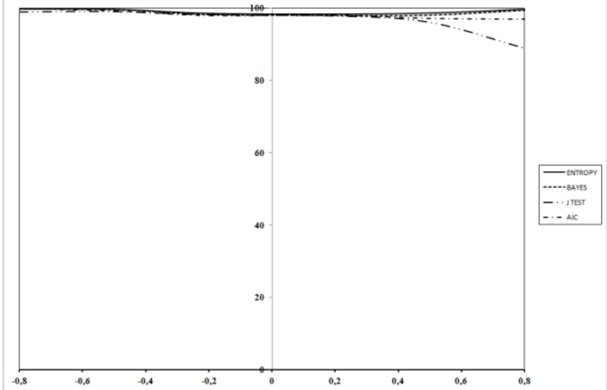
Figure A2. Percentages of correct selection.

DGP: SDEM Equation estimated: SDEM. T=10

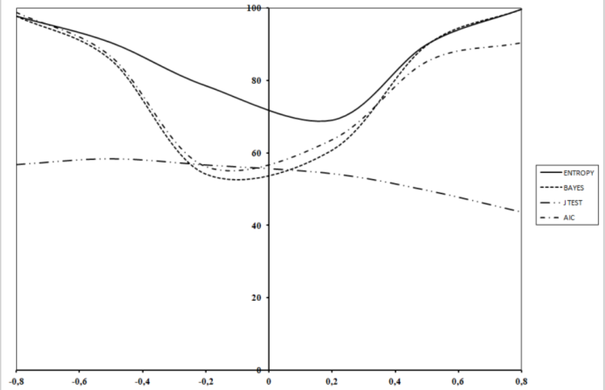
CASE: n=25; β_1=1; θ=1



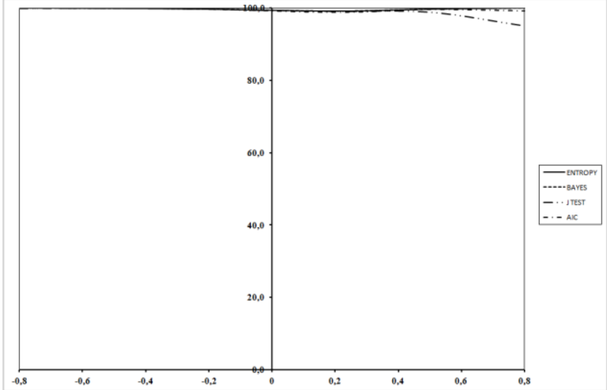
CASE: n=25; β_1=1; θ=5



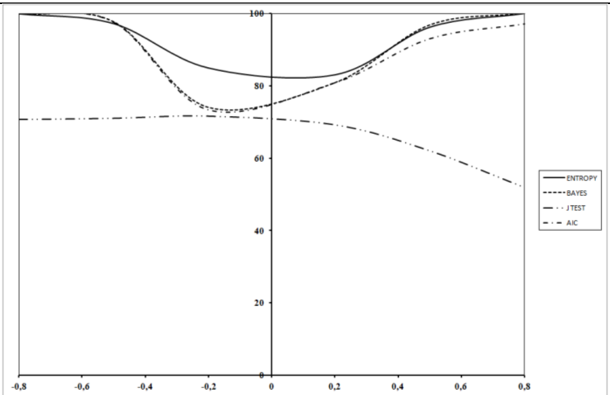
CASE: n=49; β_1=1; θ=1



CASE: n=49; β_1=1; θ=5



CASE: n=100; β_1=1; θ=1



CASE: n=100; β_1=1; θ=5

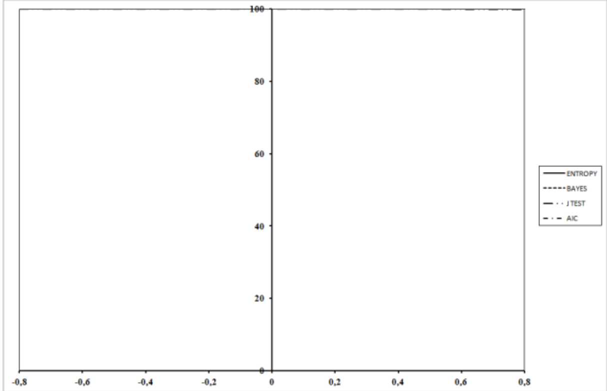
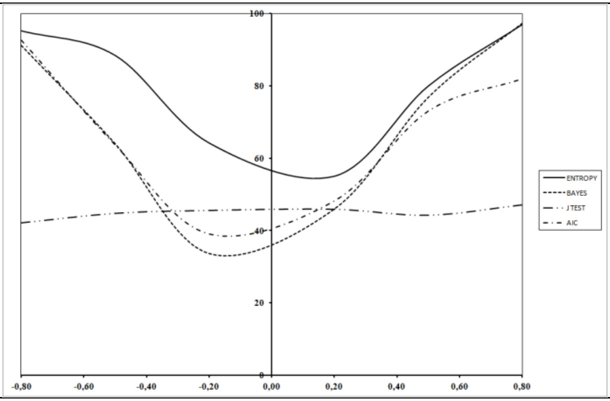


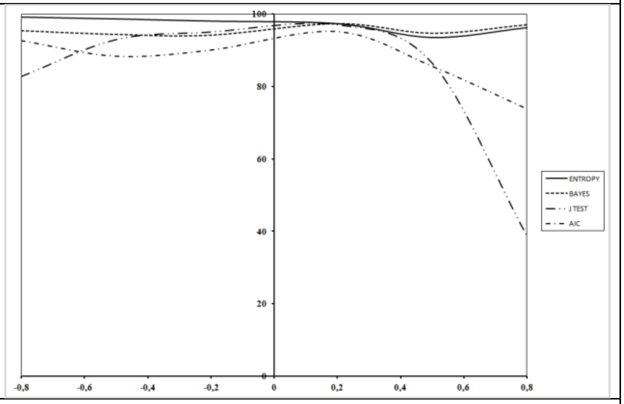
Figure A3. Percentages of correct selection.

DGP: SDEM Equation estimated: SDM. T=10

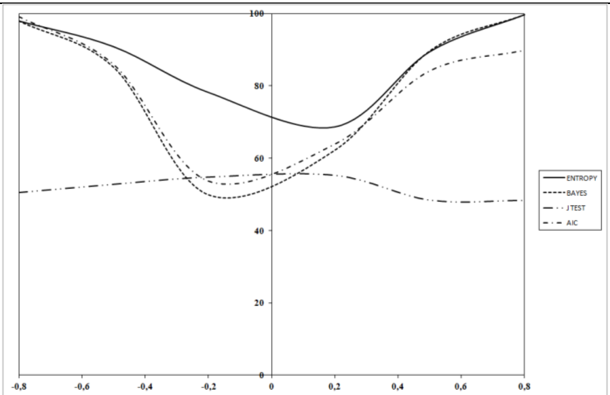
CASE: n=25; β_1=1; θ=1



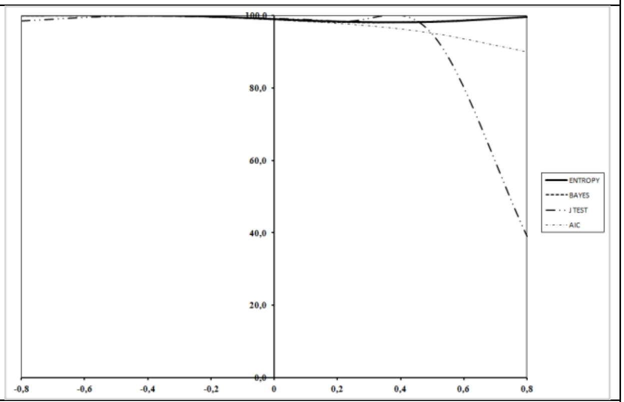
CASE: n=25; β_1=1; θ=5



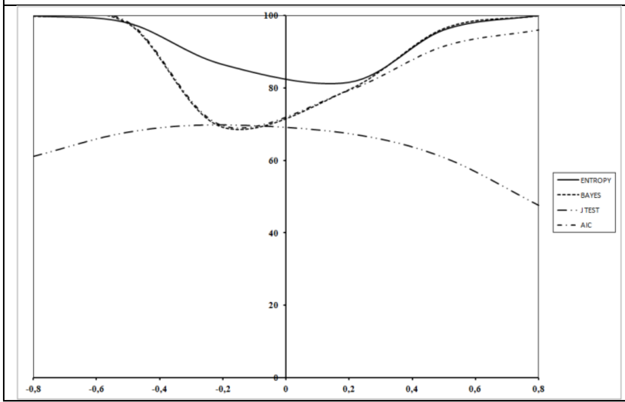
CASE: n=49; β_1=1; θ=1



CASE: n=49; β_1=1; θ=5



CASE: n=100; β_1=1; θ=1



CASE: n=100; β_1=1; θ=5