Multiperiod Design and Planning of Multiproduct Batch Plants with Mixed-Product Campaigns

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This work presents a multiperiod optimization model for multiproduct batch plants operating during several time periods with different characteristics because of seasonal and market fluctuations. This model simultaneously considers decisions about the design, operation, scheduling, and planning of the plant and the corresponding trade-offs among them. Thus, decomposition mechanisms, which have been frequently used in previous approaches, are avoided through a formulation that takes into account the main elements of these problems. Besides, decisions are affected by different context conditions arisen by the multiperiod effect. Through a mixed integer nonlinear program, different alternatives of mixed production campaign are considered, handled by means of a novel set of scheduling constraints. This approach is posed for a fermentors network with high detail level in the description of the unit operations in a plant that produces yeast and ethanol. © 2009 American Institute of Chemical Engineers *AIChE J,* 55: 2356–2369, 2009

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Introduction

Multiperiod models are posed when costs, demands, and resources typically vary from period to period because of market or seasonal changes. Usually, models are formulated without taking into account that variations in the context conditions. Thus, models should consider them to achieve an actual description.

Models for optimal design and planning of multiproduct batch plants with several time periods have in general an objective, for example, maximize total profit or minimize cost, which is subject to constraints that represent mass balances, process performance equations, and design equations. Some constraints can be valid for all periods or for an individual period. These models typically involve both continuous and discrete variables, and consequently most mathematical formulations for this problem result in a mixed integer nonlinear programming (MINLP) model.

Multiperiod optimization models for design and planning in the chemical industry have received considerable attention in the last years.[1–15] Often, taking into account the model complexity, these problems have been solved through decomposition approaches.

Birewar and Grossmann[1,2] addressed the problem of sizing and scheduling of a multiproduct batch plant for the unlimited intermediate storage and zero wait (ZW) transfer policies with mixed-product campaigns (MPCs). These are the first works that considered both problems simultaneously. Very simple process models are used, employing the fixed time and size factors approach and only one time period.

Paules and Floudas[3] presented a two-stage stochastic programming approach for the synthesis problem of heat-integrated distillation sequences for a finite number of periods of operation. They propose a synthesis strategy that combines a superstructure with a partitioning of the design variables into two classes: structural and periodic. The resulting mathematical

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formulation is a large-scale MINLP problem. The structure of the problem can be exploited by viewing the multiperiod design problem as a two-stage stochastic programming problem with the feed composition and flow rate as the uncertain parameters. A nested solution procedure is detailed that combines the generalized benders decomposition and the outer approximation (OA)/equality relaxation algorithms.

Varvarezos et al.[4,5] developed an efficient optimization method for convex nonlinear and mixed-integer nonlinear multiperiod design optimization problems. They propose an outer approximation-based decomposition method for solving these problems. The method is applied to multiperiod multiproduct batch plant problems operating with single-product campaigns (SPCs).

Van den Heever and Grossmann[6] presented a general disjunctive multiperiod nonlinear optimization model, which incorporates design as well as operation and expansion planning, and takes into account the corresponding costs incurred in each time period for the multiproduct batch plant design problem. They use a simple approach with fixed time and size factors, and SPCs. Two algorithms for the resolution of these problems are proposed: logic-basic OA algorithm and a bilevel decomposition algorithm. They conclude that the first method performs best for the smaller problems, whereas the second one is superior for larger problems.

Several valuable works in this area are developed by Bhatia and Biegler.[7–9] In the first work,[7] a more detailed modeling through dynamic optimization is used, which allows including process considerations in the model, working with only one period. Scheduling conditions are also considered using the constraints proposed by Birewar and Grossmann.[1] The optimization model has differential and algebraic equations involving state and control variables. The method of orthogonal collocation over finite elements is proposed to transform the infinite-dimensional problem to a finite-dimensional NLP problem. Uncertainty in process parameters is taken into account in a later work.[8] The uncertainty is treated like planning scenarios that are addressed through a multiperiod formulation. A closed loop state feedback correlation strategy is proposed. Then, in the last work,[9] an efficient decomposition algorithm for solving multiperiod design problems is proposed using interior point methods within a reduced Hessian successive quadratic programming framework.

Bassett et al.[10] proposed solution approaches for industrially relevant large-scale scheduling problems. They show that the most promising approach uses a reverse rolling window in conjunction with a disaggregation heuristic. Decompositions approaches are also discussed to reduce the problem to manageable proportions.

Dietz et al.[11] presented a multicriteria design of multiproduct batch plants, where the design variables are the size of the equipment items as well as the operating conditions. This formulation takes into account the composition of the production campaigns. Given the important combinatorial aspect of the problem, the proposed approach consists in coupling a stochastic algorithm, indeed a genetic algorithm, with a discrete-event simulator.

Moreno et al.[12–14] presented several works, where design and planning decisions are simultaneously taken into account. All these works use a very simple formulation considering the posynomial approach with fixed time and size factors for process design and SPCs. A multiperiod scenario is taken into account with different structural options for the plan design,[12,13] where, in the more sophisticated approach, the configuration of the plant can be adjusted in each period. In the last work,[14] a new structural option is introduced using the duplication in series for each operation.

Finally, Corsano et al.[15] presented a work, where synthesis, design, and scheduling are simultaneously considered in a NLP model. They develop a heuristic procedure to decompose the problem, where in the first step the optimal structure of the plant is obtained. Then, several multiproduct production campaigns are proposed using the previous results. Finally, a model is solved for each proposed campaign where the plant is also sized. After considering all the suggested campaigns, the optimal solution is achieved. Seasonal fluctuations are not taken into account since only one time period is considered.

In this work, a general MINLP model is proposed for the simultaneous optimization of the design, operation, scheduling, and planning of a multiproduct batch plant, considering a multiperiod approach. Costs, raw materials, and demands typically vary from period to period because of market or seasonal reasons, so in this model several time periods are considered.

A characteristic of this model is the high detail level in the process units. Batch blending and recycles, integration with other plants that provides raw materials, imported resources, variable processing times and unit sizes, and investment and operative costs are considered in this formulation.

The production planning is also a model decision, which is simultaneously taken in the overall formulation. The model determines the optimal MPC from a set of different campaigns proposed by the designer according to heuristic criteria, where the campaign selection is modeled through binary variables. Therefore, the number of binary variables is relatively small, that is, there are as many integer variables as campaigns in each period. Therefore, the computational effort is relatively small and the solutions are obtained in reasonable times without resorting to decomposition procedures.

The cited elements are affected by the multiperiod formulation. Parameters are different depending on the period. Thus, costs, resource availability, demands, etc. can vary and different operative conditions, production campaigns, etc. must be generated to consider these fluctuations in each period.

The examples presented in this work are concerned with an integral optimization of fermentation processes. The behavior of the fermentors is described by a set of algebraic and differential equations written as finite difference equations in an equation-oriented environment. Nonconventional constraints related to connections among batch items, detailed kinetic models, and the operation costs corresponding to inoculums and different available substrates are included in the model. This level of detail has been posed by few authors. Some exceptions that can be mentioned are Bhatia and Biegler,[7] even though with a different formulation since neither batch blending and recycles nor MPCs are considered.

Comparing with previous approaches, this work simultaneously considers the decisions involved in the operation,

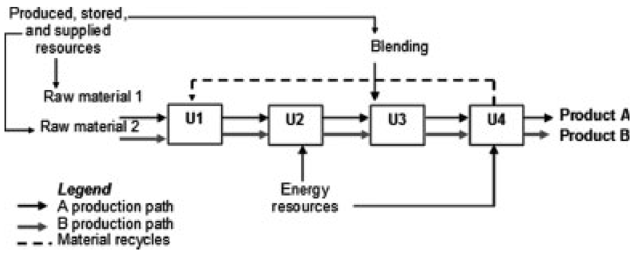

Figure 1. Flowsheet of a multiproduct batch plant.

design, and scheduling of multiproduct plants without resorting to decomposition approaches. Besides, a detailed process model is included, involving multiperiod effects and MPCs. This last subject has not been usually addressed in previous formulations and SPCs have been used, excluding solutions with potential improvements in the total cost.

The remainder of this article is organized as follows: in the next section, the problem and the modeling strategy are discussed. A general mathematical model for a multiproduct batch plant in a multiperiod context is presented in Model Formulation section. Two numeric examples for the optimal design, operation, and planning of batch fermentation networks and an instance for production planning in designed plants are presented in the Examples section. Finally, conclusions of this work are outlined.

Problem Statement

In a multiproduct batch plant, two or more products are processed following the same production path. Each unit can be characterized by a processing time and no simultaneous removal and input material is performed. In this work, special considerations as the batch blending and batch recycling are taken into account, decisions considered by few authors in previous published works. In the first case, an extra feeding is added to the batch unit before beginning the operation. In the second one, a partial or total amount of the production obtained in some unit is recycled to another unit. Figure 1 shows a general multiproduct batch plant, where several operative decisions have been taken into account.

Suppose that in the batch plant N_p products are processed in N_j batch units during T time periods ($t = 1, \ldots, T$). In this model, the plant structure is fixed for all periods and the unit sizes will be determined. The number of periods, the total time horizon HT, and the length for each time period, H_t, are problem data.

For each time period t, there are raw materials r ($r = 1, \ldots, N_r$) available, which can be used from different sources. They can be produced by other plants in the neighborhood of the multiproduct plant. These resources are considered as ‘‘produced resources’’ and are included so as to consider multiplant complexes, where different resources are shared by several plants or processes. They present different conditions than elements provided by external suppliers. The multiproduct plant can also receive resources in each period from other not nearby plants, which are called ‘‘supplied resources’’. Finally, some unused available resources can be stored from one period to another. These resources are denominated ‘‘stored resources’’. Therefore, the model considers three kinds of resources: produced, supplied, and stored. The former has cost lower than the second ones, whereas an extra cost is added for the stored resources.

The transfer policy adopted between batch stages is ZW, which means that a batch of product i ($i = 1, \ldots, N_p$) after been processed in unit j ($j = 1, \ldots, N_j$) is immediately transferred to unit $j + 1$, which must be available. In this model, only one unit per stage is considered.

Another characteristic of the multiproduct plant model is the sequence adopted to process the products, that is, the production campaign. There are two production modes: the SPC configuration and the MPC configuration. In the first one, a product is produced until the production requirement is reached and then the production is changed to the next product. This type of campaign assumption greatly simplifies the design problem.[1] For MPCs, a production sequence of different products is planned for each time period and then the sequence is cyclically repeated over the period. Aside from providing a more steady supply of products from the commercial point of view, the sequencing of batches of different products in MPCs can reduce idle times to increase the utilization of the equipment. Figure 2 schematizes the SPC and the MPC configurations for the production of three

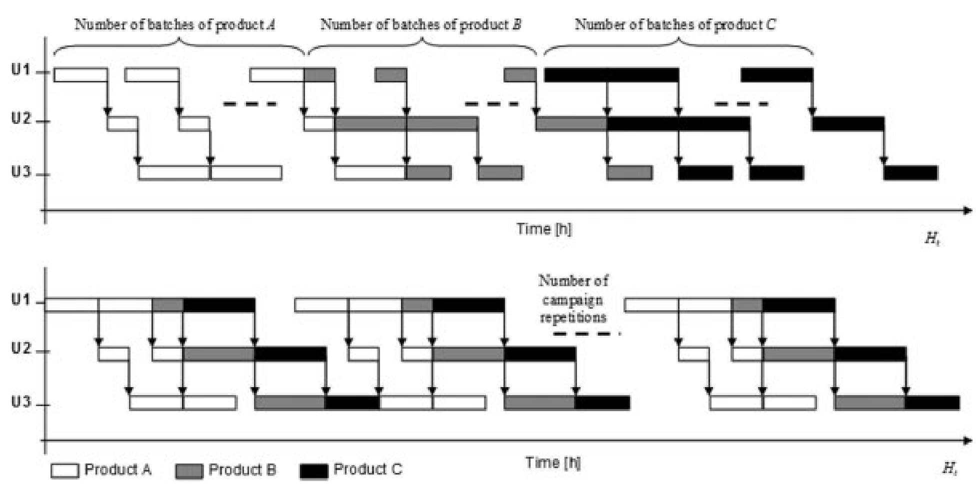

Figure 2. SPC and MPC Gantt charts for production of A, B, and C.

products (A, B, and C) in a time period t. For MPC, the campaign chosen is A-A-B-C.

The model presented in this work considers a set of possible campaign configurations selected by the designer according to some criteria (commercial policy, stock availability, product demand, etc.). The different alternative campaigns are modeled with binary variables, so that if the variable associated to a campaign configuration takes value "1," that configuration is selected. Only one campaign configuration is selected for each time period t.

Overall, the multiperiod design and planning model for multiproduct batch plant considering MPC can be stated as follows:

Given

- N_p products to be produced in N_j batch units
- T time periods during the time horizon HT
- A set of N_s external sources of raw material
- A set of possible production campaigns
- The product prices
- The cost coefficient for investment cost, raw material procurement cost, and operating cost for each time period

determine

- The plant design (unit sizes); and for each time period
- Production of each product
- Batch blending and recycles for each batch unit
- Component concentrations
- Processing times
- Resource consumptions: produced, supplied, and stored resources
- The production campaign to maximize the net present value of the profit along the global time horizon, taking into account incomes from product sales and expenditures from investment and operating costs.

Model Formulation

In this section, a mathematical model for the optimal design, operation, and scheduling of a multiproduct batch plant over a multiperiod scenario is presented. A detailed list of the parameters and variables can be found in Nomenclature section.

The objective function to be maximized is as follows:

$$\text{Max} \sum_{t=1}^{T}\sum_{i=1}^{N_p} p_{it}Q_{it} - \left(\sum_{j=1}^{N_j} \alpha_j V_j^{\beta_j} + \sum_{t=1}^{T}\sum_{r=1}^{N_r} \left(c_{rt}^{\text{prod}} F_{rt}^{\text{prod}} + \sum_{s=1}^{N_s} c_{rt}^{s} F_{rt}^{s} + c_{rt}^{\text{stor}} F_{rt}^{\text{stor}} \right) + \sum_{t=1}^{T} OC_t \right), \quad (1)$$

where p_{it} represents the sale prices for product i in period t, Q_{it} is the total production of i in time period t, α_j, β_j, and c_{rt} are cost coefficients, V_j is the size of unit j, and F_{rt}^{prod}, F_{rt}^{S}, and F_{rt}^{stor} are the produced, supplied, and stored resource r in time period t, respectively. The first term in the main parenthesis represents the investment cost corresponding to a power law expression on the units capacity.[16] The terms c_{rt}^{prod} F_{rt}^{prod} represent the production cost of each resource r produced in the nearby plants. The terms c_{rt}^{s} F_{rt}^{s} correspond to the cost of each resource r supplied by the external supplier s, with $s = 1, \ldots, N_s$, and the terms c_{rt}^{stor} F_{rt}^{stor} are stock costs. OC_t represents additional operative costs in each period t, like water and inoculums costs.

Thus, the objective function takes into account costs of different kinds of raw materials, with different procurement policies. They depend on specific resources conditions, availability, capacity, etc. The cost coefficients should be adjusted to consider available options and to avoid "forbidden" alternatives (e.g., stored material that is degraded in short time).

Let F_{rt} be the total volume of r consumed in period t, Nb_{it} the number of batches of product i produced in time period t, and F_{rijt} the volume of r consumed for producing a batch of i in unit j at period t, then the following equations represent the balances among consumed, stored, and supplied resources

$$F_{rt} = \sum_{j=1}^{N_j}\sum_{i=1}^{N_p} F_{rijt}Nb_{it} \quad \text{for } r = 1, \ldots, N_r, \quad t = 1, .., T \quad (2)$$

$$\sum_{s=1}^{N_s} F_{rt}^{s} + F_{rt}^{\text{prod}} + F_{r,t-1}^{\text{stor}} - F_{rt} = F_{rt}^{\text{stor}} \quad \text{for } r = 1, .., N_r, \quad t = 1, .., T. \quad (3)$$

Equation 2 describes the total volume of resource r consumed at time period t, which is the sum of the volumes of r consumed at each unit j for the production of each product i.

Equation 3 represents the stock balance for the resource r in period t. This means that the total amount of r stored in period t is equal to the total amount of r supplied by all the sources s in period t, plus the total available resource r provided by the nearby plants in period t, plus the amount of r stored in the previous period, minus the total amount of resource r consumed at period t. When t =1, $F_{r,0}^{\text{stor}}$ represents the initial stock, generally taken equal to zero. This term is most useful when the model only considers operative decisions in a planning context with a given plant.

In many cases, a subproduct obtained in some stage in a production process can be partially or totally recycled to another stage of the process of the same or another product. Because batch processes are considered in this work, the recycled amount is added to a unit before beginning the processing, together with the blend of raw materials. Also, the not recycled amount of the subproduct can be discarded. Let e be a subproduct, then the following constraint establishes that the total amount of e consumed in all the stages of the production processes of all the products plus the discarded amount of e is equal to the total produced amount of e

$$\sum_{i \in IP_e}\sum_{j \in EP_e} F_{ijt}^{e} Nb_{it} = \sum_{i' \in IC_e}\sum_{j' \in EC_e} f_{i'j't}^{e} Nb_{i't} + \text{D}_\text{e} \quad \forall e, t = 1, \ldots, T \quad (4)$$

where F_{ijt}^{e} is the amount of e produced at stage j in the process production of product i in period t and $f_{i'j't}^{e}$ the amount of e consumed at stage j' in the process production of product i' in period t. IP_e and EP_e represent the set of products and stages that produce e, respectively, whereas IC_e and EC_e represent the set of products and stages that consume e, respectively, and D_e is the total amount of discarded e. For model simplification purposes, the inventory cost of raw materials and effluents is not considered in this work.

Let V_{ijt}^{ini} and V_{ijt}^{fin} be the batch volume of product i in unit j at period t before beginning the processing in that unit and after processing respectively. Let σ_{ijt} be the process conversion factor for producing i in unit j at period t, which can be a process variable or a process parameter. In a simpler formulation, σ_{ijt} is taken as a constant value, whereas in more detailed models σ_{ijt} is written as a function of processing variables. Then, the following volume balances are stated:

$$V_{ijt}^{\text{ini}} = \sum_{r=1}^{N_r} F_{rijt} + \sum_{e \in R_e} f_{ijt}^{e} + V_{i,j-1,t}^{\text{fin}} \quad i = 1, ..., N_p, \quad j = 2, ..., N_j, \quad t = 1, ..., T, \quad (5)$$

$$V_{ijt}^{\text{ini}} = \sum_{r=1}^{N_r} F_{rijt} + \sum_{e \in R_e} f_{ijt}^{e} \quad i = 1, ..., N_p, \quad j = 1, \quad t = 1, ..., T, \quad (6)$$

$$V_{ijt}^{\text{fin}} = \sigma_{ijt} V_{ijt}^{\text{ini}} \quad i = 1, ..., N_p, \quad j = 1, ..., N_j, \quad t = 1, ..., T. \quad (7)$$

Then, the design constraint for each unit j is

$$V_j \geq S_{ijt} V_{ijt}^{\text{ini}} \quad i = 1, ..., N_p, \quad j = 1, ..., N_j, \quad t = 1, .., T, \quad (8)$$

where S_{ijt} represents the size factor of unit j for product i in period t and corresponds to the relationship between the unit size and the material to be processed. These sizes factors can be constants if the production recipe is known or can be a process variable. Then, Eq. 8 means that the size of a unit has to be large enough to accommodate all the products produced in the plant over all the time periods.

In case of handling several components x, $x = 1, \ldots, N_x$, connection and balance constraints should be settled for each of them. In such way, the mass balances for each component are given by

$$V_{ijt}^{\text{ini}} C_{xijt}^{\text{ini}} = \sum_{r=1}^{N_r} C_{xijt}^{r} F_{rijt} + C_{xi,j-1,t}^{\text{fin}} V_{i,j-1,t}^{\text{fin}} \quad i = 1, ..., N_p, \quad j = 1, ..., N_j, \quad t = 1, .., T, \quad x = 1, ..., N_x \quad (9)$$

$$C_{xijt}^{\text{fin}} = v_{ijt}^{x} C_{xijt}^{\text{ini}} \quad i = 1, ..., N_p, \quad j = 1, ..., N_j, \quad t = 1, .., T, \quad x = 1, ..., N_x, \quad (10)$$

where C_x represents the concentration of component x, so that Eq. 9 expresses that the initial total amount of component x for the production of i at unit j in time period t is equal to the total final amount of the same component in the previous unit plus the total amount of this component that the extra feedings can contribute. This extra feeding represents a blend of the raw materials (batch blending) which is added at each unit.

v_{ijt}^{x} is a component conversion factor so that Eq. 10 describes the behavior of component x for product i processed in j at period t. Generally, this factor is considered a fixed parameter. In this work, the conversion of components is obtained from differential equations. The growth or decrease of each component in each unit along the processing time is described by differential equations that are embedded in the overall model as algebraic equations using some numerical method. According to the stability of the differential equations, simpler or complex methods have to been used. In this work, the Trapezoidal method is used, which is an implicit one-step method that possesses a special stability property.[17]

Suppose that the concentration of a component x is described by the differential equation

$$\frac{dC_x}{d\tau} = f(\tau, C_x), \quad (11)$$

where τ is the integral variable (processing time). Let h be the step size of the discretization so that the m grid points of the Trapezoidal method are defined by $\tau_{\text{final}} = \tau_{\text{initial}} + hm$ and $h = \tau_{l+1} - \tau_l$. Then, the algebraic equation corresponding to the Trapezoidal method is

$$C_x^{l+1} = C_x^{l} + \frac{h}{2}\left(f(\tau_l, C_x^{l}) + f(\tau_{l+1}, C_x^{l+1})\right). \quad (12)$$

In this way, Eqs. 12 for each component x are embedded in the overall model as algebraic equations. A more detailed description of this modeling strategy is presented in the work by Corsano et al.[18]

For each product i, there are lower and upper bounds for the demand that can vary from one period to other depending on market and seasonal factors. The total production of product i is obtained from the last process unit. Let λ_{it} be the product conversion factor for product i in period t and P_{ix} the product specification for component x, then:

$$B_{it} = \lambda_{it} V_{ijt}^{\text{fin}} \quad i = 1, ..., N_p, \quad t = 1, .., T, \quad j = N_j \quad (13)$$

$$Q_{it} = B_{it} Nb_{it} \quad i = 1, ..., N_p, \quad t = 1, ..., T \quad (14)$$

$$P_{ix} \leq C_{xijt}^{\text{fin}} B_{it} \quad i = 1, ..., N_p, \quad t = 1, ..., T, \quad x = 1, ..., N_x, \quad j = N_j \quad (15)$$

$$Q_{it}^{\min} \leq Q_{it} \leq Q_{it}^{\max} \quad i = 1, ..., N_p, \quad t = 1, .., T. \quad (16)$$

Again, λ_{it} can be a process variable or parameter if the recipe is known.

In this formulation, the designer proposes a set of possible MPCs according to some criteria. Corsano et al.[15] presented a heuristic procedure for determining possible campaigns, and then for each campaign a NLP problem was modeled and solved. Unlike that procedure, in this work, all the possible campaigns are simultaneously considered. Let CT_{jt} be the cycle time of a unit j during time period t, that is, the total time that the unit j is occupied for processing a campaign. Let SL_{bjt} be the idle time at unit j after processing the batch in position b in the sequence ($b = 1, \ldots, n_{bt}$) and before processing the next batch in the campaign sequence of period t, and $T_{b(i)jt}$ be the processing time of a batch of product i that is in the $b(i)$ position in the campaign sequence at unit j in period t. Then the cycle time is defined by:

$$\text{CT}_{jt} = \sum_{b=1}^{n_{bt}} \left(T_{b(i)jt} + \text{SL}_{bjt}\right), \quad j = 1, ..., N_j, \quad t = 1, .., T \quad (17)$$

where n_{bt} is the number of batches in the campaign in period t.

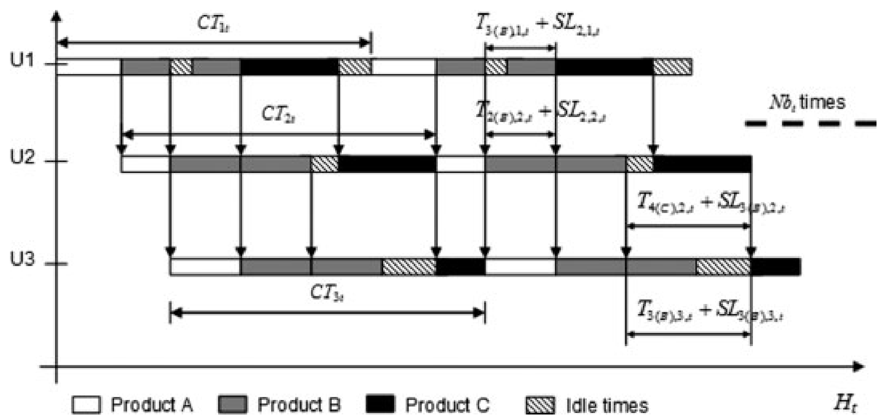

Figure 3. Gantt chart for campaign *A-B-B-C*.

For example, if three products A, B, and C are processed in the plant in a given period t, and the designer decides to produce one batch of A, two batches of B, and one batch of C, the cycle time for each unit j is given by:

$$\mathrm{CT}_{jt} = T_{1(A)jt} + \mathrm{SL}_{1jt} + T_{2(B)jt} + \mathrm{SL}_{2jt} + T_{3(B)jt} + \mathrm{SL}_{3jt} + T_{4(C)jt} + \mathrm{SL}_{4jt}. \quad (18)$$

Figure 3 shows the Gantt chart for a batch plant whit three units and three products with the production sequence equal to *A-B-B-C*. For each time period t, the campaign is repeated Nb_t times.

To avoid the task superposition at each unit, the following constraints are stated for each unit j in each period t:

$$T_{b+1(i),jt} + \mathrm{SL}_{bjt} = T_{b(i),j+1,t} + \mathrm{SL}_{b,j+1,t} \quad i = 1,...,N_p, \quad j = 1,...,N_j - 1, \quad b = 1,...,n_{bt} - 1, \quad t = 1,...,T \quad (19)$$

$$CT_{jt}Nb_t \leq H_t \quad j = 1,...,N_j, t = 1,...,T. \quad (20)$$

Equations 17 and 19 are illustrated in Figure 3. Equation 19 is posed for each couple of consecutive batches. Note that two successive batches can be of the same or different products, as it is shown in the figure with the second and third batches and with the first and second ones, respectively.

The total number of batches of product i over the period t, Nb_{it}, is equal to the number of times that a product i is produced in a campaign multiplied by the number of campaign repetitions:

$$Nb_{it} = n_{it}Nb_t \quad i = 1,...,N_p, \quad t = 1,...,T, \quad (21)$$

where n_{it} is the number of batches of product i in the campaign of period t.

The set of campaign configurations can vary from one period to other, so Eqs. 17 and 19–21 are stated for each time period t according to the set of campaign configurations proposed by the designer. In this work, different campaign configurations are proposed for each time period and they are handled in terms of binary variables. So, these equations are rewritten to incorporate the binary variables which select the optimal alternative.

The configured campaigns differ in the number of batches of each product or in the production sequence. Let K_t be the set of different available campaigns in time period t and k_t denotes each campaign configuration in K_t. Let $y_{k_t t}$ be the binary variable that takes value 1 if the campaign configuration k_t is selected in time period t or 0 otherwise. Then, Eqs. 17 and 19–21 must be posed for each proposed campaign. Moreover, the cycle time CT_{jt}, the number of batches Nb_{it} and n_{it}, and the campaign repetition Nb_t depend on the campaign configuration k_t. Therefore, CT_{jt}, Nb_{it}, n_{it}, and Nb_t in Eqs. 17 and 19–21 are replaced by CT_{jtk_t}, Nb_{itk_t}, n_{itk_t}, and Nb_{tk_t}, respectively.

In every period, the solution must only consider the campaign that optimizes the objective function. To formulate these equations so as to fulfill these conditions only for the selected campaign in each time period, a "Big-M" formulation is used. For a general constraint $h(x) \geq 0$, with binary variables $y_{k_t t}$, the following constraints must be posed:

$$h(x) \geq -M(1 - y_{k_t t}) \quad \forall k_t \in K_t, \quad t = 1,...,T$$
$$\sum_{k_t \in K_t} y_{k_t t} = 1 \quad \text{for each } t = 1,...,T, \quad (22)$$

where M is a large valid upper bound. In this way, the general constraint $h(x) \geq 0$ is trivially satisfied when $y_{k_t t}$ is zero and must be fulfilled with the alternative selected through the only binary variable $y_{k_t t}$ with value one.

The Eqs. 17 and 19–21 have to be first written as inequality constraints, and then the big-M formulation must be applied to them. Therefore, Eqs. 17 and 19–21 are transformed in

$$\mathrm{CT}_{jtk_t} - \left(\sum_{bk_t=1}^{n_{bt}} \left(T_{b(i)k_t,jt} + \mathrm{SL}_{bk_t,jt} \right) \right) \geq -M_1(1 - y_{k_t t}) \quad j = 1,...,N_j, \quad t = 1,..,T, \quad k_t \in K_t \quad (23)$$

$$-\mathrm{CT}_{jtk_t} + \sum_{bk_t=1}^{n_{bt}} \left(T_{b(i)k_t,jt} + \mathrm{SL}_{bk_t,jt} \right) \geq -M_1(1 - y_{k_t t}) \quad j = 1,...,N_j, \quad t = 1,..,T, \quad k_t \in K_t \quad (24)$$

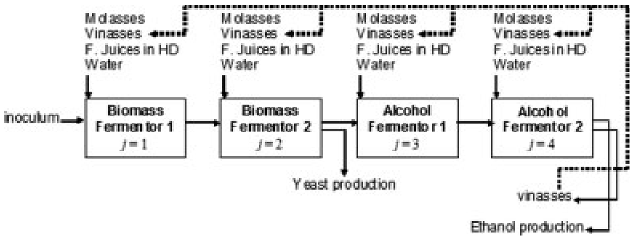

Figure 4. Fermentation process scheme.

$$T_{b+1(i)k_t,jt} + \mathrm{SL}_{bk_t,jt} - \left(T_{b(i)k_t,j+1,t} + \mathrm{SL}_{b(i)k_t,j+1,t}\right) \geq -M_2(1 - y_{k_t t}) \quad \begin{array}{l} i = 1,\dots,N_p - 1, \quad j = 1,\dots,N_j - 1 \\ p = 1,\dots,n_{it}, \quad t = 1,\dots,T, k_t \in K_t \end{array} \tag{25}$$

$$-\left(T_{b+1(i)k_t,jt} + \mathrm{SL}_{bk_t,jt}\right) + T_{b(i)k_t,j+1,t} + \mathrm{SL}_{bk_t,j+1,t} \geq -M_2(1 - y_{k_t t}) \quad \begin{array}{l} i = 1,\dots,N_p - 1, \quad j = 1,\dots,N_j - 1 \\ p = 1,\dots,n_{it}, \quad t = 1,\dots,T, \quad k_t \in K_t \end{array} \tag{26}$$

$$H_t - \mathrm{CT}_{jtk_t} Nb_{tk_t} \geq -M_3(1 - y_{k_t t}) \quad j = 1,\dots,N_j, \quad t = 1,\dots,T, \quad k_t \in K_t \tag{27}$$

$$Nb_{itk_t} - n_{itk_t} Nb_{tk_t} \geq -M_4(1 - y_{k_t t}) \quad i = 1,\dots,N_p, \quad t = 1,\dots,T, \quad k_t \in K_t \tag{28}$$

$$-Nb_{itk_t} + n_{itk_t} Nb_{tk_t} \geq -M_4(1 - y_{k_t t}) \quad i = 1,\dots,N_p, \quad t = 1,\dots,T, \quad k_t \in K_t. \tag{29}$$

Then, Eqs. 2–16 and 22–29 with the objective function (Eq. 1) constitute the mathematical model for the optimal design, operation, and scheduling of a multiproduct batch plant with several time periods considering a set of different campaigns settled by the designer.

Examples

Problem description

In this section, the optimization model previously presented is applied to a batch fermentation plant to produce two products: yeast for cattle feed and ethanol that in the model are called A and B. There are two types of fermentors: biomass and alcohol fermentors, and the plant configuration consists in two fermentors of each type connected in series as it is shown in Figure 4. These fermentors can be fed with molasses diluted with water, filters juice, and/or distillery vinasses, which contribute to the total substrate concentration. For each fermentor, the total volume blending and their concentration are optimization variables. The vinasses are a residue of the ethanol process, which has variable substrate concentration, so there is a relationship between the vinasses used in fermentor feedings and the size and final substrate concentration of the second fermentor in ethanol production. In Figure 4, the vinasses recycles are denoted by the dot line.

There is a sugar plant close to the multiproduct batch plant. For the sugar plant, two seasons are distinguished: harvest (HD) and no-harvest date (NHD). During the harvest date, the sugar plant can provide molasses and filter juices to the multiproduct plant. In addition, molasses can be imported from other plants when they are required, with operative costs bigger than the cost of molasses obtained from the nearby sugar plant. Molasses that are not consumed during the harvest date can be stored, whereas filter juices cannot, because they are degraded in a short time.

During the NHD, filter juices are not available and the fermentors can be fed with water, vinasses, and stored and supplied molasses. The vinasses can be used according to ethanol production and the vinasses store is not allowed because of their degradation.

For yeast production, only biomass fermentors are used, whereas for ethanol production the two fermentations are used. Both yeast and ethanol production processes receive a variable amount of inoculum at the first biomass fermentor, which has an operative cost.

For units' performance, detailed models are adopted. A description of the development of these models is given by Corsano et al.[18] Mass balances are obtained from the following differential equations, which are written as algebraic equations using the trapezoidal method and included in the overall model.

$$\frac{dC_{\mathrm{X}ijt}}{dt} = \mu_{ijt} C_{\mathrm{X}ijt} - \upsilon_{ijt} C_{\mathrm{X}ijt}, \quad i = A, B;\ j = 1,\dots,4;\ t = \mathrm{HD}, \mathrm{NHD} \tag{30}$$

$$\frac{dC_{\mathrm{S}ijt}}{dt} = -\frac{\mu_{ijt} C_{\mathrm{X}ijt}}{Y_{x/s}}, i = A, B;\ j = 1,\dots,4;\ t = \mathrm{HD}, \mathrm{NHD} \tag{31}$$

$$\frac{dC_{\mathrm{D}ijt}}{dt} = \upsilon_{ijt} C_{\mathrm{X}ijt}, \quad i = A, B; \quad j = 1,\dots,4; \quad t = \mathrm{HD}, \mathrm{NHD} \tag{32}$$

$$\frac{dC_{\mathrm{E}ijt}}{dt} = \frac{\mu_{ijt} C_{\mathrm{X}ijt}}{Y_{p/x}}, \quad i = B; \quad j = 3,4; \quad t = \mathrm{HD}, \mathrm{NHD} \tag{33}$$

$$\mu_{ijt} = \mu_{ijt}^{\max} \frac{C_{\mathrm{S}ijt}}{k_s + C_{\mathrm{S}ijt}}, \quad i = A, B;\ j = 1,\dots,4; \quad t = \mathrm{HD}, \mathrm{NHD}, \tag{34}$$

where C_{X}, C_{S}, C_{D}, and C_{E} are the biomass, substrate, nonactive biomass, and ethanol concentration (kg m^{-3}), respectively, $Y_{x/s}$ is the biomass yield coefficient, $Y_{x/p}$ product yield coefficient, k_s is the substrate saturation constant (kg m^{-3}), μ is the specific growth rate of biomass (h^{-1}), and $\mu^{\max}$ the maximum specific growth rate of biomass (h^{-1}). Equation 33 is only used for third and fourth fermentors in ethanol production. For this example, t = HD, NHD.

The objective function adopted is the maximization of the total net benefit given by the expression:

$$\mathrm{Max} \sum_{t=\mathrm{HD,NHD}} \sum_{i=1}^{N_p} p_{it} Q_{it} - \left(\sum_{j=1}^{N_j} \alpha_j V_j^{\beta_j} + \sum_{t=\mathrm{HD,NHD}} \sum_{r \in R} \left(c_{rt}^{\mathrm{prod}} F_{rt}^{\mathrm{prod}} + c_{rt}^{s} F_{rt}^{s} + c_{rt}^{\mathrm{stor}} F_{rt}^{\mathrm{stor}} \right) + \sum_{t=\mathrm{HD,HND}} c_t^{\mathrm{inoc}} \mathrm{Inoc}_t, \right)$$

where Q_{it} is the period production of product i and p_{it} its selling price, and V_j represents the fermentors size. The

Table 1. Model Decision

Design	Operation	Planning and Scheduling
Unit sizes: V_j	Fermentation feedings flow rates: F_{rijt}	Production Q_{it}
	Raw materials: F_{rt}^{prod}, F_{rt}^{s}, F_{rt}^{stor}	Idle times: SL_{bjt}
	Batch recycling flow rates: $F_{DV,ijt}$	Cycle times: CT_{jt}
	Batch sizes: V_{ijt}^{ini}, V_{ijt}^{fin}	Batch numbers: n_{it}, n_{bt}, Nb_{it}, Nb_t
	Component concentrations C_{xijt}	Campaign configuration: $y_{k,t}$
	Unit processing times for each product T_{ijt}	
	Conversion factors: ν, σ, λ	
	Sizes factors: S_{ijt}	

resources r are molasses (M), filter juice (FJ), distillery vinasses (DV), and fresh water (FW), where $c_{\text{DV},t}^{\text{prod}} = 0$ as the cost of producing vinasses is considered in the ethanol processing and investment costs. $F_{\text{DV},t}^{s}= 0$ and $F_{\text{DV},t}^{\text{stor}} = 0$ as vinasses are neither supplied by extra plants nor stored. $F_{\text{FJ,NHD}}^{\text{prod}} = F_{\text{FJ,NHD}}^{s} = F_{\text{FJ,NHD}}^{\text{stor}} = 0$, because neither produced nor supplied filter juice are in NHD and the store of filter juice is not possible for their degradation. The nearby sugar plant does not produce water, so all the water used in fermentation come from supplied sources, that is $F_{\text{FW},t}^{\text{prod}} = F_{\text{FW},t}^{\text{stor}} = 0$. The inoculums cost is considered in the last term.

As no substances are added during the course of the fermentations, in this model $V_{ijt}^{\text{ini}} = V_{ijt}^{\text{fin}}$.

The yeast and ethanol period demands are bounded to known values for each period. Different solution schemes are found according to these bound values.

For both periods, eight campaign configurations are proposed. Let A be a batch of yeast production, B a batch of ethanol production, and the set of different campaign configurations K equal to {A, B, A-B, A-A-B, A-A-A-B, A-A-A-A-B, A-A-B-B, A-B-B}. This set of campaign configurations may be changed according to the designer knowledge or criteria.

A constraint for vinasses recycle is that total consumed vinasses in period t have to be lower than the 70% of total batch size of ethanol production at the last unit ($j = 4$), and vinasses substrate concentration is estimated as the final substrate concentration in the last alcohol fermentor. The constraints for these requirements are as follows:

$$\sum_{i=1}^{N_p}\sum_{j=1}^{N_j} F_{\text{DV},ijt} n_{it} \leq 0.7 V_{B4t}^{\text{fin}}, \quad \forall t = \text{HD}, \text{NHD}$$

$$C_{St}^{\text{DV}} = C_{SB4t}^{\text{fin}}, \quad \forall t = \text{HD}, \text{NHD},$$

where V_{B4t}^{fin} denotes the final batch size of the last ethanol fermentor, C_{S} is the substrate concentration, and $j = 4$ represents the last ethanol fermentor.

As the substrate is consumed in the fermentors, the processing time of the last ethanol fermentor will influence on vinasses substrate concentration. When molasses cost is high, the use of vinasses instead molasses results attractive because the vinasses have no costs as they are a residue of ethanol process. Therefore, the model presents different trade-offs between operative and design variables that are worth exploring. On the other hand, when vinasses substrate concentration is a low value, the use of vinasses reduces the fresh water consumption, as vinasses are used instead of water in the blends.

Table 2. Model Parameters for Example 1

	Yeast Production	Ethanol Production	
	Biomass Fermentors	Biomass Fermentors	Ethanol Fermentors
$\mu^{\max}$	0.5 h^{-1}	0.5 h^{-1}	0.1 h^{-1}
k_s	20 g l^{-3}	20 g l^{-3}	20 g l^{-3}
α	40,020	40,020	24,200
β	0.6	0.6	0.45
$Y_{x/s}$	0.4	0.4	0.124
$Y_{x/p}$			0.23
υ	0.02 h^{-1}	0.02 h^{-1}	0.02 h^{-1}
HD demand	$\leq$6000 t	$\leq$8000 t	
NHD demand	$\leq$3500 t	$\leq$3500 t	
Inoculum cost	0.5$ t^{-1}	0.5$ t^{-1}	
HD selling prices	80$ t^{-1}	50$ t^{-1}	
NHD selling prices	100$ t^{-1}	60$ t^{-1}	

The decisions that are simultaneously made in this model are shown in Table 1, where some optimization variables are stated.

In the following sections, two different scenarios are proposed and the optimal solution is compared with solutions obtained through other approaches.

Example 1

Table 2 shows the parameter values considered in this first example.

The sugar plant produces 40,000 t of molasses and 67,480 t of filter juices during HD. The multiproduct plant can import up to 20,000 t of molasses during HD from other sugar plants and 17,500 t of molasses during NHD. Also, the molasses not consumed in HD can be stored and used in NHD. The inventory cost is equal to 3.5$/t, whereas the supplied molasses cost is equal to 5$/t. The molasses and filter juices produced in the sugar plant have a cost of 0.1$/t.

The optimal solution obtained for the multiperiod plant with MPCs is shown in Table 3. The optimal MPC is A-A-B

Table 3. Example 1 Economical Results

	Mixed-Product Campaign Model Solution	Single-Product Campaign Model Solution
Product sales	1,173,700	1,141,400
Produced molasses and filter juice cost	10,748	10,748
Supplied molasses cost	112,500	112,500
Stored molasses cost	51,310	48,118
Biomass fermentor units cost	320,510	318,420
Alcohol fermentor units cost	165,460	179,790
Fresh water cost	7018	7588
Inoculum cost	41,298	35,397
Total annual benefit	464,856	428,839

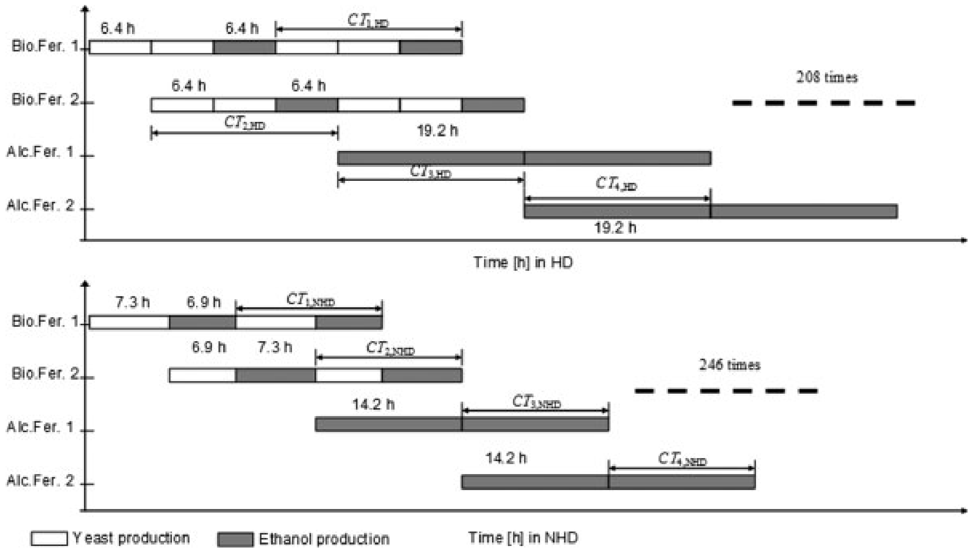

Figure 5. Gantt chart for mixed-product campaigns of Example 1.

in HD an *A-B* in NHD. Figure 5 shows the Gantt chart for both periods. The campaign is repeated 208 times in HD and 246 times in NHD. Unit cycle time is equal to 19.2 h in HD and 14.2 h in NHD. There is no idle time in both periods. This interesting result has been obtained taking into account that this model has been solved using operation times as optimization variables. Then, the optimal solution has adjusted the operation times so as to avoid idle times.

The produced filter juices are totally consumed in HD, and Table 4 shows the molasses and inoculum consumptions in each period. As can be observed, 12,000 t (upper bound) are supplied in HD and 14,660 t are stored in that period because the available amount of molasses is not enough for NHD production. In NHD the amount of supplied molasses is equal to the given upper bound for this resource.

The optimal values for the ethanol production reach their upper bounds in both periods, while for yeast are equal to 4038 t in HD and 2406.5 for NHD. This occurs because ethanol is the most profitable production. Then, according to the available raw material, the production of yeast is carried out. It is worth mentioning that some units are sub-occupied. For example, the second biomass fermentor for ethanol production is 16% occupied in HD and 22.3% in NHD taking into account that yeast is the limiting product. Figure 6 shows the optimal design and the optimal batch sizes for each product in each time period. Note that in HD no feeding is added to the second biomass fermentor of ethanol production. The biomass fermentation process for this production in HD can be considered as an only biomass fermentation stage of 12.8 h.

The total produced vinasses in HD are recycled to the first and second biomass fermentor of yeast production and to the first biomass fermentor, and to the last alcohol fermentor of ethanol production. The vinasses substrate concentration is equal to 2.6 g/l. In NHD, the total produced vinasses are recycled to both biomass fermentor of yeast production and to the first biomass fermentor of ethanol production, and the vinasses substrate concentration is equal to 4.3 g/l.

A novel result that can be observed of this multiproduct plant model with mixed campaigns is that because of some units are not used by all products, the operating times of such stages are longer, decreasing the costs. This means that a better equipment employment is obtained, as occurs at the alcohol fermentation stages. This would not occur if the tasks scheduling constraints were not taken into account in the plant design model or if a SPCs were adopted, as will be shown in the following example.

The example here presented was also modeled and solved for a SPC configuration to compare its optimal solution with the MPC optimal solution. Figure 7 shows the optimal Gantt chart for each period, and in the second column of Table 3 the economical results are presented. For this case, the total annual benefit is 7.7% lower than the MPC optimal solution. Although the difference between the objective function values of both production schemes is small, the design, operation, and planning are significantly different in both cases.

Table 4. Consumed Molasses and Inoculum

	Mixed-Product Campaign Model Solution		Single-Product Campaign Model Solution	
	HD	NHD	HD	NHD
Consumed molasses in biomass fermentors (t)	18,216	13,655	13,715	17,136
Consumed molasses in alcohol fermentors (t)	19,124	11,505	24,537	7112
Supplied molasses (t)	12,000	10,500	12,000	10,500
Stored molasses (t)	14,660	–	13,748	–
Inoculum (t)	58.6	23.5	39.8	30.6

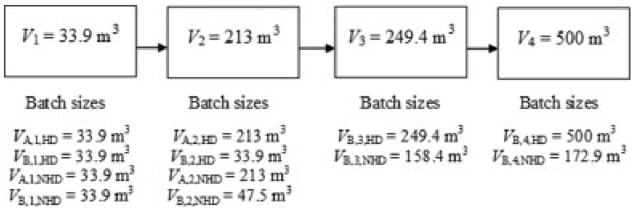

Figure 6. Optimal design for Example 1.

The production of yeast is equal to 3430.8 t in HD and 3500 t (the upper bound) in NHD, whereas the ethanol production reaches its upper bound in HD (8000 t) and 1949 t in NHD. The total sales incomes are 3% reduced. The reduction of ethanol production in NHD is due to two factors: a lower amount of available molasses, and a longer time cycle in alcohol fermentation process. As the amount available molasses is lower, the fermentation feeds of ethanol production are more diluted, and so more processing times are needed to reach proper product concentrations. As the production processes of both products are not overlapped, the ethanol time cycle is longer and therefore fewer batches are produced. A reduction in the time cycle implies a bigger unit size, because the product concentration increases along the time and the final product is reached as the product concentration per volume.

The produced vinasses cannot be stored for long time periods because of their degradation and because the inventory units should be extremely large. Although SPC is easier to formulate and solve than the MPC from the point of view of mathematical programming, SPC is impracticable from the operative point of view when recycles are considered. Here, SPC has been solved only with academic aims to assess the performance of the optimal solution found. In this case, the total produced vinasses in HD are recycled to the second biomass fermentors for yeast production and to the first and second alcohol fermentor for ethanol production, whereas in NHD the vinasses are recycled to the first and second biomass fermentor for yeast production and to the first biomass fermentor for ethanol production. The 6% of the total produced vinasses are discarded. In this work, treatment cost was not taken into account, but in a more detailed integration scheme it must be also considered.[19] The vinasses substrate concentration is 4.5 g/l in HD and 2.4 g/l in NHD.

Processing time for alcohol fermentation in HD is smaller than in MPC model solution. Therefore, to keep the unit sizes not too large, the molasses consumption in these stages is increased and the blending is more concentrated. This effect can be observed in Table 4. Anyway, the unit size of the first alcohol fermentor is bigger than the MPC model solution and the investment cost of this stage is 8.6% increased. On the contrary, the molasses consumption in NHD is lower, because the availability of molasses is smaller, and therefore the processing time of alcohol fermentation stage must be longer to reach proper levels of product concentration.

Figure 7 shows that in NHD the ethanol production has idle time in the first biomass fermentor. This occurs because no blending is added to the second biomass fermentor and so, to keep high the substrate concentration, the processing time is short. This two biomass fermentation stages can be considered as only one operation with processing time equal to 22.6 h.

Comparing with MPC model solution, the inoculum consumption is reduced in HD because more concentrated blending is used and less yeast is produced. But in NHD the inoculum consumption is increased because the blending is more diluted. Anyway the total inoculum cost in SPC is 14% smaller than the MPC model. It must be considered that in SPC the total production is smaller than in MPC model solution.

Figure 8 shows the plant design and the batch sizes of each product.

This example and the following ones were implemented and solved in GAMS[20] in a Pentium IV, 1.60 Ghz. The code DICOPT was used for solving the MINLP problems. The number of continuous variables and constraints is about

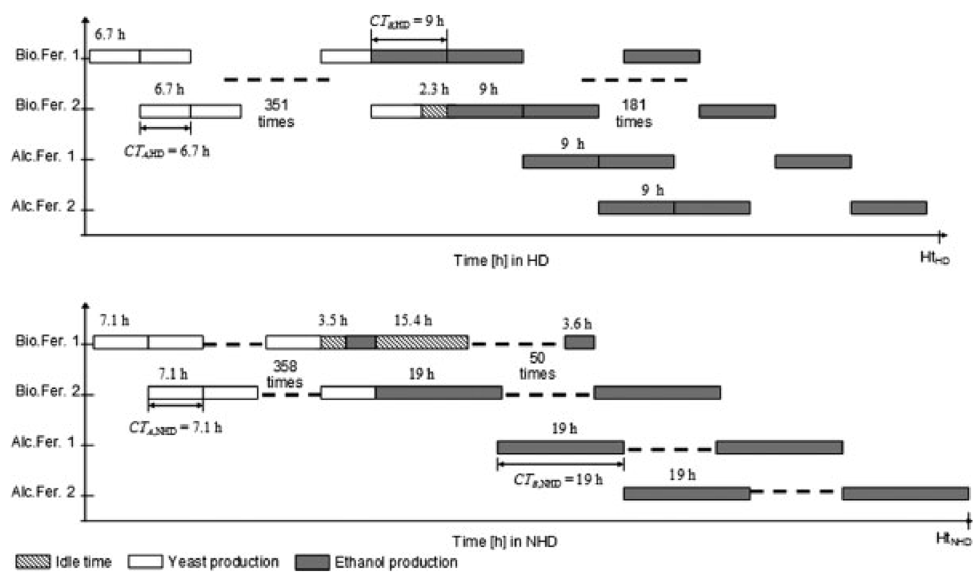

Figure 7. Gantt chart for single-product campaign in HD and NHD of Example 1.

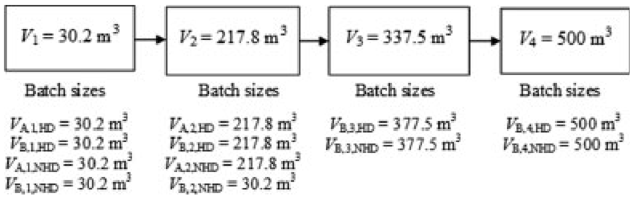

Figure 8. Optimal design for SPC model of Example 1.

1500 and 1700, respectively, the number of binary variables is equal to the number of different campaign configurations, in this case are 16 (eight in each period), and the CPU processing times varied between 25 and 50 s.

Example 2

This example considers a multiproduct plant that produces yeast and ethanol in three harvest date periods with different characteristics and one nonharvest date period: HD1, HD2, HD3, and NHD. The process parameters are the same as those shown in Table 2, and the HD and NHD sugar plant productions, maximum demand, and prices are shown in Table 5. The differences between the harvest periods are due to variations on sugar cane production amounts and quality. The plant configuration is two biomass fermentors and two ethanol fermentors. For this example, the number of continuous variables and constraints is about 3000 and 3500, respectively, the number of binary variables is equal to the number of different campaign configuration, in this case are 32 (eight in each period), and the CPU processing 180 s.

The optimal solution adopts the campaign *A-A-B* for HD1 and HD2, *A-B* for HD3, and only *B* for NHD. The most outstanding result of this example is that the first biomass fermentor is not used. In the optimal solution, the blending volume and the inoculum amount for the first biomass fermentor in all periods are equal to zero. Therefore, the first biomass fermentor size is zero and this unit is not used. Table 6 shows the batch size and unit processing times for each production in each period. In NHD there is idle time (2 h) at biomass fermentor for ethanol production. The campaign is repeated 86 times in HD1, 61 times in HD2, 106 times in HD3, and 72 times in NHD.

The production of yeast is equal to 2017.8 t in HD1, 1485 t in HD2, and 1275 t in HD3. The ethanol production is equal to its upper bound in each period.

No molasses are supplied in HD1, but in the remaining periods, the maximum amount of molasses is supplied. The stored molasses are 1212.5 t in HD1, 6482.4 t in HD2, and 6700.4 t in HD3. All the produced filter juices are consumed in HD1, HD2, and HD3, and all the produced vinasses are

Table 5. Harvest and Nonharvest Date Resources and Prices

	HD1	HD2	HD3	NHD
Produced molasses (t)	20,000	10,000	6000	0
Produced filter juices (t)	33,740	20,000	10,000	0
Yeast price (\$ t^{-1})	80	90	90	100
Ethanol prices (\$ t^{-1})	50	60	80	100
Max yeast demand (t)	3000	2000	4000	3000
Max ethanol demand (t)	4000	2000	2000	3000
Max supplied molasses (t)	6000	6000	6000	4500
Time horizon (h)	2000	2000	2000	1500

recycled in each period. The vinasses substrate concentration is equal to 0.01, 0.4, 4.3, and 0.5 g l^{-1} in each period, respectively.

The total annual net benefit is equal to 470,407$, as the incomes for sales are 1,189,800$ and the total annual costs are as follows: 294,850$ the biomass fermentor cost, 179,750 alcohol fermentors cost, 50,383$ stored molasses cost, 82,500$ supplied molasses cost, 9974$ produced molasses and filter juices costs, 96,590$ inoculum costs, and 5346$ fresh water costs. No comparison is done with the previous example because the selling prices and the availability of raw materials are different, as well as the maximum product demands. But it is worth noting that the investment cost for biomass fermentor is lower in this case, where only one unit is used, whereas the inoculum cost is higher than the previous case because the use of units in series reduces the amount of consumed inoculum.

Example 3: planning approach

The proposed model can be also applied to plan the production in a given multiproduct plant, that is, solve the model for fixed unit sizes and determine the optimal planning and processing conditions. In this case, the analysis is focused on the different operating variables like raw material distribution, amount of inoculum, processing times, recycle allocation, and on scheduling decisions: the campaign configuration.

Suppose that a plant with two biomass fermentors and two alcohol fermentors is given. The unit sizes are 33.9, 213, 249.4, and 500 m^3, respectively, and the adopted model parameters are the same of the previous model. The unit sizes are the values obtained in the optimal solution of MPC model previously presented.

The optimal solution in this case is the same as that presented for MPC model. Table 7 shows the inoculums consumption and the raw material distribution in each period.

If selling prices of yeast vary to 30$ t^{-1} in HD, 50$ t in NHD, and for ethanol to 80$ t^{-1} in HD and 100$ t^{-1} in

Table 6. Batch Sizes and Processing Times of Example 2 Optimal Solution

	$V_{i,1,t}$ (m^3)	$T_{i,1,t}$ (h)	$V_{i,2,t}$ (m^3)	$T_{i,2,t}$ (h)	$V_{i,3,t}$ (m^3)	$T_{i,3,t}$ (h)	$V_{i,4,t}$ (m^3)	$T_{i,4,t}$ (h)
Yeast HD1	0	0	298.9	7.75	–	–	–	–
Ethanol HD1	0	0	108.3	7.75	377.1	23.25	500	23.25
Yeast HD2	0	0	298.9	10.78	–	–	–	–
Ethanol HD2	0	0	14.9	10.78	254.9	32.34	460	32.34
Yeast HD3	0	0	298.9	14.27	–	–	–	–
Ethanol HD3	0	0	33.5	4.53	197.4	18.8	249.6	18.8
Ethanol NHD	0	0	68.9	18.8	377.1	20.8	500	20.8

Table 7. Inoculum and Raw Material Consumption

	Inoculum (t)	Molasses (t)	Filter Juices (t)	Vinasses (t)	Fresh Water (t)
Yeast HD	55.7	17,110	11,495	65,684	–
Ethanol HD	2.9	20,230	55,985	13,162	24,273
Yeast NHD	22.8	11,388	–	24,896	19,744
Ethanol NHD	0.7	13,770	–	7370	26,178

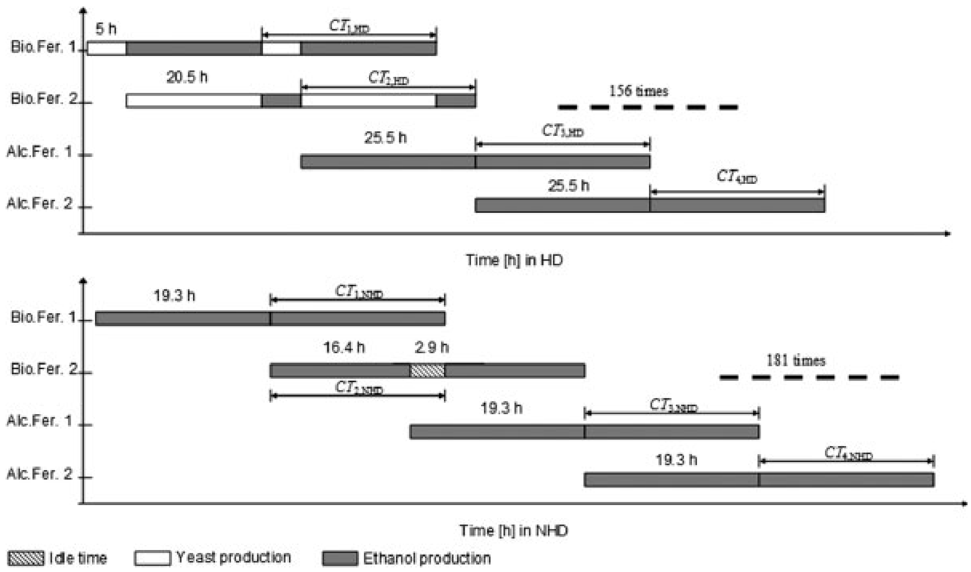

Figure 9. Gantt chart for mixed-product campaigns of planning approach.

NHD, the inoculum cost is duplicated, that is 1$ t^{-1}, and the maximum demand is 6000 t for each product in HD and 3500 t for yeast and 7000 t for ethanol in NHD, the optimal solution adopts the campaign *A-B* in HD and only *B* in NHD. The yeast production in HD is equal to 1323.6 t, whereas the ethanol production is equal to 6000 t in HD and 7000 t in NHD (their upper bounds). The yeast production consumes large amount of inoculum, and in this case the inoculum cost is duplicated. On the other hand, the amount of available molasses is lower than in the previous case, and, taking into account that ethanol production is more profitable, yeast is not produced in NHD.

The Gantt chart for this solution is displayed in Figure 9. It can be noted that in NHD the ethanol production has idle time in the second biomass fermentor. This occurs because the amount of available molasses is not enough and the blending is more diluted, so to keep high the substrate concentration for the following unit, the processing time is shorter.

Table 8 shows the inoculum and raw material consumption. It is worth noting that the amount of inoculum is decreased in this case, because the inoculum cost is higher than in the previous case and because yeast is not produced in NHD.

Furthermore, total available supplied molasses are imported to produce ethanol and only the 2% of the produced molasses in the sugar plant is used for yeast production. All the produced filter juices are consumed in HD and vinasses are used instead of water in that period. The vinasses substrate concentration is equal to 0.5 g l^{-1} and 32% of the total produced vinasses are discarded. In HD, 15,926 t of molasses are stored to reach the ethanol production. In NHD, all the produced vinasses, all the available molasses and fresh water are used in ethanol production.

In this example, a new scenario has been solved. The main difference with the previous one lies in the fact that the plant is given. Only operation, planning, and scheduling decisions have to be considered. However, this scenario is very usual because selling prices and operating costs will change with market fluctuations. Then, new operating conditions must be adjusted to satisfy these new requirements. This example shows how the proposed model can be used as

Table 8. Inoculum and Raw Material Consumption for Example with Different Selling Prices

	Inoculum (t)	Molasses (t)	Filter Juices (t)	Vinasses (t)	Fresh Water (t)
Yeast HD	0.2	477	32,198	2118	–
Ethanol HD	0.05	15,597	35,282	34,648	–
Ethanol NHD	2^{-3}	26,426	–	68,743	5191

a tool for planning the production of a given plant and to analyze different production schemes.

Conclusions

In this work, a detailed model for the optimal design, operation, scheduling, and planning for a multiproduct batch plant taking into account a multiperiod context was presented. Most of the previous works focused on a unique problem considering stable conditions to solve a simpler mathematical program. However, this approach hinders the correct assessment of the strong trade-offs among the variables involved. These effects are more significant if seasonal and market fluctuations are severe. Then, for a rigorous analysis, all these decisions must be contemplated simultaneously and the variations during the time must be also included.

The formulation presented in this work take into account the elements previously cited, including a high level of detail in the processing unit description. As was shown in the examples, new operation alternatives, such as batch blending and recycles, are allowed.

Through the solved examples, the capability of the model could be assessed. All the links and trade-offs between variables were explored. Also, the variations through the time periods were analyzed resulting in different production campaigns in each period. All the remaining elements of the problem were also adjusted to satisfy the different requirements being addressed. This flexible model can be also adapted to solve different scenarios, as was shown for instance, for the case of a given plant where operation and planning conditions had to be determined. Small solution times were required, even when highly nonlinear formulations and noncontinuous variables were used.

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Notation

Sets

EC_e = set of stages that consume subproduct e
EP_e = set of stages that produce subproduct e
IC_e = set of products that consume subproduct e
IP_e = set of products that produce subproduct e
K_t = set of mixed product campaigns configuration of period t
R_e = set of subproduct to be recycled

Indices

b = position of a batch in the production sequence $b = 1, \ldots, n_{bt}$
e = subproduct
fin = final
i = products, $i = 1, \ldots, N_p$
ini = initial
j = units, $j = 1, \ldots, N_j$
k_t = mixed-product campaign configuration in set K_t
min = minimum
max = maximum
prod = produced resource
r = raw material $r = 1, \ldots, N_r$
s = external supplied resource $s = 1, \ldots, N_s$
stor = stored resource
t = time period, $t = 1, \ldots, T$
x = component $x = 1, \ldots, N_x$

Parameters

α_j = investment cost coefficient of unit j
β_j = investment cost exponent of unit j
c_{rt}^{prod} = production cost for raw material r in time period t
c_{rt}^{s} = supply cost for raw material r in time period t
c_{rt}^{stor} = inventory cost for raw material r in time period t
H_t = time horizon of period t
HT = total production horizon time
M = large valid upper bound
n_{bt} = number of batches in the campaign of period t
n_{it} = number of batches of the product i in the campaign of period t
p_{it} = sale prices of product i in period t
P_{ix} = product specification for component x

Binary variables

$y_{k_t t}$ = selection of mixed product campaign in period t

Continuous variables

B_{it} = batch size of final product i in time period t
C_x = concentration of component x
C_x^r = concentration of component x in raw material r
CT_{jt} = cycle time for unit j in period t
D_e = total amount of e discarded
f_{ijt}^e = amount of e consumed at stage j in the process production of product i in period t
F_{ijt}^e = amount of e produced at stage j in the process production of product i in period t
F_{rt} = total volume of resource r in period t
Nb_{it} = number of batches of product i produced in time period t
Nb_t = number of campaigns in time period t
OC_t = operative cost in period t
Q_{it} = total production of product i in period t
S_{ijt} = size factor of unit j for product i in period t
SL_{bjt} = idle time at unit j after processing the batch in position b and before processing the next batch in the campaign sequence of period t
$T_{b(i)jt}$ = processing time of a batch of product i in position b at unit j in period t
V_j = size of unit j
V_{ijt} = batch size for product i of unit j in period t
λ_{it} = product conversion factor for producing i in period t
σ_{ijt} = process conversion factor for producing i at unit j in period t
τ = integral variable
v_{ijt}^x = component conversion factor of component x for product i produced at j in period t

Literature Cited

1. Birewar DB, Grossmann IE. Incorporating scheduling in the optimal design of multiproduct batch plant. *Comput Chem Eng*. 1989;13:141–161.
2. Birewar DB, Grossmann IE. Simultaneous synthesis, sizing, and scheduling of multiproduct batch plants. *Ind Eng Chem Res*. 1990;29:2242–2251.
3. Paules GE, Floudas CA. Stochastic programming in process synthesis: a two-stage model with MINLP recourse for multiperiod heat-integrated distillation sequences. *Comput Chem Eng*. 1992;16:189–210.
4. Varvarezos DK, Grossmann IE, Biegler LT. An outer-approximation method for multiperiod design optimization. *Ind Eng Chem Res*. 1992;31:1466–1477.
5. Varvarezos DK, Biegler LT, Grossmann IE. Multiperiod design optimization with SQP decomposition. *Comput Chem Eng*. 1994;18:579–595.
6. Van den Heever SA, Grossmann IE. Disjunctive multiperiod optimization methods for design and planning of chemical process systems. *Comput Chem Eng*. 1999;23:1075–1095.
7. Bhatia TK, Biegler LT. Dynamic optimization in the design and scheduling of multiproduct batch plants. *Ind Eng Chem Res*. 1996;35:2234–2246.
8. Bhatia TK, Biegler LT. Dynamic optimization for batch design and scheduling with process model uncertainty. *Ind Eng Chem Res*. 1997;36:3708–3717.

9. Bhatia TK, Biegler LT. Multiperiod design and planning with interior point methods. *Comput Chem Eng.* 1999;23:919–932.
10. Bassett MH, Pekny JF, Reklaitis GV. Decompositon techniques for the solution of large-scale scheduling problems. *AIChE J.* 1996;42:3373–3387.
11. Dietz A, Azzaro-Pantel C, Pibouleau L, Doménech S. A framework for multiproduct batch plant design with environmental consideration: application to protein production. *Ind Eng Chem Res.* 2005;44:2191–2206.
12. Moreno MS, Montagna JM, Iribarren OA. Multiperiod optimization for the design and planning of multiproduct batch plants. *Comput Chem Eng.* 2007;31:1159–1173.
13. Moreno MS, Montagna JM. New alternatives in the design and planning of multiproduct batch plants in a multiperiod scenario. *Ind Eng Chem Res.* 2007;46:5645–5658.
14. Moreno MS, Montagna JM. Optimal simultaneous design and operational planning of vegetable extraction processes. *Trans IChemE Part C: Food Bioproducts Process.* 2007;85:360–371.
15. Corsano G, Montagna JM, Iribarren OA, Aguirre PA. A heuristic method for the optimal synthesis and design of batch plants considering mixed product campaigns. *Ind Eng Chem Res.* 2007;46:2769–2780.
16. Ravemark DE, Rippin DWT. Optimal design of multiproduct batch plants. *Comput Chem Eng.* 1998;22:177–183.
17. Atkinson KE. *An Introduction to Numerical Analysis*, 2nd ed. New York: Wiley, 1989.
18. Corsano G, Aguirre PA, Iribarren OA, Montagna JM. Batch fermentation networks model for optimal synthesis, design, and operation. *Ind Eng Chem Res.* 2004;43:4211–4219.
19. Corsano G, Montagna JM, Aguirre PA, Iribarren OA. Processes integration: multiplant complex vs. multipurpose plant assessment. *Ind Eng Chem Res.* 2006;45:4256–4269.
20. Brooke A, Kendrick D, Meeraus A, Raman R. *GAMS, A User Guide*. California: Scientific Press, 1998.

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