Logical and Generalized Disjunctive Programming for Supplier and Contract Selection under Provision Uncertainty

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Supply-chain optimization is a key issue in gainnig competitiveness in today's economies where buyers and suppliers develop long-term relationships. Final agreements are formalized by signing contracts involving the purchasing of large amounts of materials, taking advantage of economies of scale. Although establishing agreements with suppliers is definitely an important step in reducing uncertainty, uncertainty never completely disappears. For that reason, the problem addressed in this article is the delivery and purchase optimization in a supply chain under provision uncertainty. Several decisions are presented in this problem that can naturally be posed in terms of of discrete decision models and generalized disjunctive programming. Uncertainty is modeled through a new approach that avoids the drawbacks of traditional methods. The goal pursued in this article is to contribute to better raw-material usage and tactical decisions to face an uncertain provision process in the supply chain.

1. Introduction

In recent years, supply-chain optimization has gained increasing interest in both research and business environments. This situation has been motivated by a tough competitive market where the efficiency of the entire supply chain is essential for companies' survival. In fact, there has been constant interest in the integration process between partners. In this challenge, decisions related to suppliers are especially relevant; business relations with suppliers influence customer satisfaction and represent most of a company's costs. For that reason, business partners are involved in a constant negotiation process, playing a dynamic role in the supply chain. As a consequence, permanent changes have occurred in the production and distribution network.[1] In this work, a dynamic representation of the supply chain is introduced in which, given a maximum number of suppliers, the provision channel is reconfigured in each time period by selecting a subset of them.

As supported by Ojala and Hallikas,[2] the provision process represents the core of supply-chain management. Indeed, many works have focused on this problem, pointing up different facets of it. Some have stressed the importance of supplier selection and designed graphical, heuristic, and optimization methods for that purpose. Others have evaluated the most important supplier characteristics through qualitative methods. For instance, Chen et al.[3] develop a supplier capability and price analysis chart for evaluating supplier performance. Mohebbi[4] presented an analytical model for computing the stationary distribution of the inventory system considering that a supplier can take two possible states: available and unavailable. It was shown that one of the main factors that greatly influence the efficiency of every production and inventory control operation is the reliability of its supply process. Random supply interruptions were modeled to diminish their negative impacts on the customer service level and the operating costs of the supply chain. In addition, Ruiz-Torres and Mahmoodi[5] argued that supplier's decision making is a critical component given the recent increased reliance of many organizations on their suppliers. In their article, they presented a decision model that optimizes the demand allocation across a set of suppliers. Highlighting the importance of supply network interruptions, they used a decision-tree approach to determine the optimal number of suppliers in the presence of risks. In the present work, a disjunctive optimization approach is developed to determine the number of suppliers, quantities to order, and material selection to minimize the actualized expected costs in the time horizon considered.

Much work has been directed toward proving that contractual policies are effective tools against risks in supply-chain management.[6,7] Berger et al.[8] argued that the interdependency between partners motivates an integration process where buyers and suppliers develop long-term relationships characterized by stability, cooperation, and mutual benefit. Park et al.[9] studied the purchasing process considering a disjunctive model formulation for the optimal selection of suppliers and purchase contracts. They showed that signing contracts is a business practice that contributes to reducing uncertainty and taking advantage of economies of scale. Contract selection has received increasing attention in the literature as was pointed out by Naraharisetti et al.[10] Laínez et al.[11] also supported the idea that the strategic use of option contracts with suppliers could be a hedge against supply-chain risks. Aligned with these ideas, the aim of this article is to encourage supply-chain integration by signing purchase contract with suppliers, resulting in lower costs and decreasing negative effects of speculation and uncertainty.

The dynamic characteristics of supply chains underline the importance of taking into account uncertainty as a major challenge. Although establishing agreements with suppliers is definitely an important step in reducing uncertainty, uncertainty never completely disappears. In fact, risks still exist in every link of supply chains, ultimately affecting costs and demand satisfaction. Subrahmanyam et al.[12] identified various levels of uncertainty such as sales, material purchase, equipment purchase, equipment reliability, and manufacturing. Hwarng and Xie[13] presented a simulation approach of the supply-chain dynamics influenced by various factors, including demand pattern, ordering policy, demand-information sharing, and lead time. Although many risk sources have been recognized, in general, much research deals with demand uncertainty. However, because supply interruptions could also strongly affect demand satisfaction and enterprise revenue, uncertainty in the provision process is modeled in this work.

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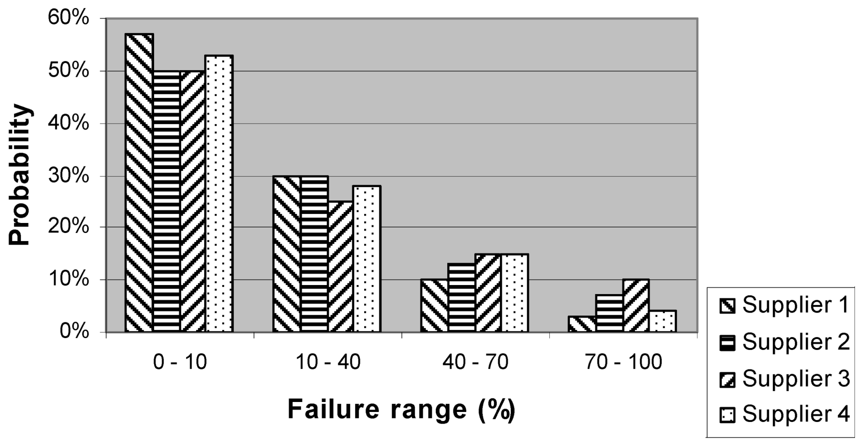

Figure 1. Discrete probability distribution of delivery failure.

Zimmermann[14] provides a very detailed explanation and taxonomy of the causes of uncertainty. According to this author, instead of deciding whether probabilistic, fuzzy set, or evidence theory is the best method for modeling uncertainty, the choice of the appropriate approach depends on the context in which it is applied. In general, there are two main strategies for treating uncertainty: probabilistic and scenario-based methods.[1,15−17] Probabilistic models formulate certain parameters as random variables with a known distribution. Alternatively, scenario-based formulations, also called stochastic models, include uncertainty representing a set of probable scenarios. In this work, uncertainty is modeled using a discrete probabilistic distribution of the supply process, taking advantage of the linear scenario-based formulation and considering managers' knowledge of supplier behavior.

Finally, in problems addressing supply-chain planning, the emphasis is generally placed on physical parameters such as stock level, demand satisfaction, and so on. However, it is relevant to consider financial issues because of their impact on enterprise performance.[11,18] Aside from the increase in international prices, financial considerations are particularly important in Latin American countries where inflationary processes evolve, decreasing the value of the unit of currency even more. Cost functions are actualized with an appropriate discount rate in the objective function, and because of the economic context mentioned before, cost data include the forecasted inflation rate.

This article presents a general formulation for purchase contract selection in the supply chain under provision uncertainty. Uncertainty is handled through a new approach that avoids the drawbacks of the traditional methods. Some discrete decisions presented in this problem are modeled by generalized disjunctive programming, thus strengthening the expressiveness of the formulation. The main purpose of this work is to encourage a better decision process whereby companies can select their materials, purchase conditions, and suppliers on the basis of lower costs and higher supplier reliability.

This work is outlined as follows: In the next section, the problem is introduced and described; then, the model formulation is developed. In section 4, two case studies and their results are presented considering different representations of probabilistic distributions of provision failure. A sensitivity analysis is shown in section 5 where uncertainty parameters are modified to capture some important features of the modeled context. Finally, conclusions and some discussion are presented in section 6.

2. Problem Description

The problem addressed in the present article is material delivery and purchase optimization in the supply chain. A medium-term production plan divided into several time periods is considered. A multilevel decision structure is presented in the problem. Diverse materials are purchased from various manufacturers through the signing of different contract types.

The purchase problem was already presented by Rodríguez and Vecchietti[19] for the paper supply chain. With the aim of a broader application context, purchase, supplier, and contract selection is considered here for any supply chain handling a set of raw material families. The use of material families is a common practice in many industries such as the production of paper, furniture, textile, and food, among others. Each supplier might offer sets of material families with different types and qualities. Materials with similar properties are grouped into families, giving flexibility to purchase decisions and allowing a set of possible material formulations for the same final product. In addition to this combinatory problem, raw material purchase decisions have a great influence on company profits, emphasizing the importance of purchase process optimization.

A discrete probability distribution for the performance of each supplier is proposed to model two possible uncertain situations: In the first case (Figure 1), supplier failures determine the probability that a certain amount will not be delivered; for instance, given a quantity ordered from supplier 1, there is a 57% probability that the supplier will fail to provide between 0% and 10% of the quantity ordered. In the second case (Figure 2), suppliers fail to provide the total amount ordered; under this assumption, supplier 1 presents a 25% probability of failing to provide the total quantity ordered and a 75% probability of delivering the complete order.

Several decision levels are presented in this problem. In the first level, none, one, or more suppliers can be chosen to provide the material families required. Note that a many-to-many relation is considered, as one supplier could provide all families and all suppliers could provide a given family. The second decision level is related to the material type selected from a particular family set. In general, it is assumed that all suppliers could provide any material from the family. However, by introducing a slight change in the formulation, it is also possible to constrain the material supplied by each provider. Once the material type is decided, the third and last step is to determine which contract type will be signed with the selected supplier(s). The decision process presented herein provides a hierarchical structure that must be satisfied when formulating the problem.

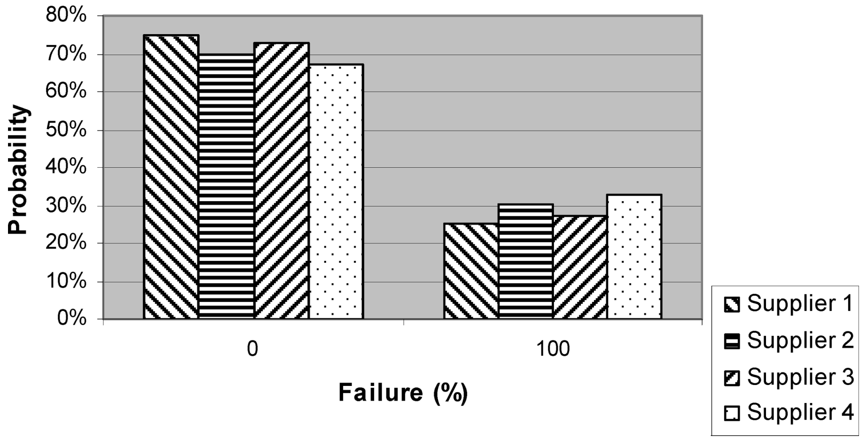

Figure 2. Discrete probability distribution of total failure.

According to Laínez et al.,[11] there are several contract types that can be signed with suppliers. The quantity-flexibility contract is defined in this work as a contract in which the buyer commits to purchase a certain amount of material at a fixed price. The second contract type considered is the revenue-sharing contract, in which the buyer pays an upfront fee to the supplier for the right to sell the materials and commits to sharing revenue obtained from these sales. Finally, the buyback contract is presented that basically is a put option whereby the buyer is able to sell excess material back to the supplier. Given these descriptions, the contracts considered in this work correspond to the quantity-flexibility contract type. Nevertheless, some special issues are included in the contract policies to capture the true nature of the commitments tackled. In the first, a minimum quantity is established, and the price presents a minimum discount over the total quantity purchased. The second can be selected only in the case that the same material was ordered from the supplier in the previous period, encouraging longer relationships between companies. Moreover, it considers a minimum quantity larger than in the first contract with a special discount. In the last contract type, the price includes the corresponding interest rate due to longer payment terms and the largest minimum quantity to order. This last contract could be an attractive alternative when a lack of financial liquidity and/or inflationary processes are presented.

The objective function to optimize is not a trivial decision. In developing countries, it is particularly relevant to take into account the inflationary process and its consequences on purchase contract decisions because of the time horizon involved. Therefore, it is important to evaluate the actualized costs, defining the objective function as the minimization of the present value of purchase costs, financial costs due to payment terms, inventory costs, and expected costs due to unsatisfied demand.

3. Model Formulation

Because of the multilevel decision structure, Boolean variables are used for selecting suppliers, material family, materials supplied, and contract types in each time period, subject to demand, stock, and capacity constraints. To capture a representative formulation, discrete decisions and constraints are posed using nested disjunctions, by generalized disjunctive programming (GDP),[20] and logic propositions.

3.1. Algebraic Constraints. In the first place, certain algebraic restrictions related to demand and stock must be satisfied in the time horizon planned. These constraints are given by eqs 1−6 below.

Equation 1 is the material family demand constraint. It determines that the quantity ordered for all materials k of family f in period t from all suppliers j, q_{jkt}, plus the quantity of family f in stock in period t, s_{ft}, must be greater than or equal to the family demand in period t, FD_{ft}. Without losing generality, it is considered that the time at which the order is made is the same as the delivery time for every supplier. It is also assumed that stock, demand, and material quantities are already transformed into comparative units.

$$\sum_{k\in \mathrm{FK}_{fk}} \sum_{j} q_{jkt} + s_{ft} \geq \mathrm{FD}_{ft} \quad \forall t \in T, \forall f \in F \tag{1}$$

Equation 2 establishes an excess of material upper bound, μ, that provides flexibility to the purchase plan. The quantity ordered plus the quantity in stock, s_{ft}, for family f in period t must be lower than the family demand plus the upper bound constrained by the parameter $\mu > 0$:

$$\sum_{k\in \mathrm{FK}_{fk}} \sum_{j} q_{jkt} + s_{ft} \leq \mathrm{FD}_{ft}(1 + \mu) \quad \forall t \in T, \forall f \in F \tag{2}$$

The initial stock expected of family f in period t, s_{ft}, is calculated by eq 3 as the initial stock of family f in period $t - 1$ plus the expected quantity of family f provided by all suppliers selected in period $t - 1$, $\mathrm{eq}_{f(t-1)}$, minus the family sales in the previous period, $d_{f(t-1)}$. Note that the variable $\mathrm{eq}_{f(t-1)}$ is calculated using eqs 30−45 (see section 3.3), depending on the suppliers selected, the amount of materials k from family f ordered from each of them, and the expected failure distribution associated with the suppliers.

$$s_{ft} = s_{f(t-1)} + \mathrm{eq}_{f(t-1)} - d_{f(t-1)} \quad \forall f \in F, \forall t \geq t_2 \in T \tag{3}$$

Equation 4 determines that sales of family f in period t must be lower than or equal to the demand forecast for family f in period t:

$$d_{ft} \leq \mathrm{FD}_{ft} \quad \forall f \in F, \forall t \in T \tag{4}$$

Equation 5 constrains the expected quantity in stock according to the stock capacity parameter, SC:

$$\sum_{f} s_{ft} \leq \mathrm{SC} \quad \forall t \in T \tag{5}$$

Equation 6 defines the initial stock of family f in period t_1 as being equal to the parameter IS_f, which is input data in the model:

$$s_{ft_1} = \mathrm{IS}_f \quad \forall f \in F \tag{6}$$

3.2. Modeling Contract Discrete Decisions and Logic Restrictions. Disjunction 7 selects the contract type for each material k and supplier j in period t. Each term corresponds to the contract types modeled. The first constraint in each case restricts the minimum amount of material that must be ordered to the supplier. The second constraint calculates the purchase costs, w_{jckt}, according to the amount ordered, q_{jkt}; the price, PC_{jkt}; and the corresponding contract discount or interest rate, δ_{jc}. In the third constraint, the positive variable $m_{jckt'}$ determines the amount paid for material k to supplier j in period t' according to the payment policy of contract c. In this case, the set $\mathrm{TP}_{ctt'}$ establishes that the purchase ordered in period t must be paid in period t' according to contract c. Note that the gap between t and t' gives the financial benefit offered. In the first two contracts, period t is equal to t'; however, the third contract has a longer payment term. Although variable $m_{jckt'}$ could be calculated directly in the second constraint, thereby avoiding the use of w_{jckt}, this formulation is considered worthy to distinguish the cost concept given by the second equation and the effective money outflows calculated in the last equation.

$$\begin{bmatrix} y3_{jc_1kt} \\ q_{jkt} \geq \mathrm{Qmin}_{c_1j} \\ w_{jc_1kt} = q_{jkt}\mathrm{PC}_{jkt}(1 - \delta_{jc_1}) \\ w_{jc_1kt} = m_{jc_1kt'} \end{bmatrix} \vee \begin{bmatrix} y3_{jc_2kt} \\ q_{jkt} \geq \mathrm{Qmin}_{c_2j} \\ w_{jc_2kt} = q_{jkt}\mathrm{PC}_{jkt}(1 - \delta_{jc_2}) \\ w_{jc_2kt} = m_{jc_2kt'} \end{bmatrix} \vee$$
$$\begin{bmatrix} y3_{jc_3kt} \\ q_{jkt} \geq \mathrm{Qmin}_{c_3j} \\ w_{jc_3kt} = q_{jkt}\mathrm{PC}_{jkt}(1 + \delta_{jc_3}) \\ w_{jc_3kt} = m_{jc_3kt'} \end{bmatrix} \quad \forall j \in J, \forall k \in K, \forall (c,t,t') \in \mathrm{TP}_{ctt'} \tag{7}$$

The first contract type establishes a minimum quantity to order, given by Qmin_{c_1j}. It also has a discount applied to the total amount ordered. The second contract type has a larger minimum quantity to order, Qmin_{c_2j}, and a bigger discount. Encouraging longer business relations, proposition 8 determines that this contract type can be selected only in the case that material k was already ordered from supplier j in period $t - 1$. In the last contract type, the minimum quantity to order, Qmin_{c_3j}, is the highest, and instead of a discount, the second equation considers an interest rate due to the payment term. In the rest of the contract types, the payment must be made within the same period in which the order was made.

In propposition 8, according to the corresponding contract rules, the second contarct type, c_2, can be selected for material k of supplier j only if one of the contract types has been selected in the previous period for material k of supplier j. If not, contract type c_2 cannot be selected for this case. Because of this same rule, in proposition 9, contract type c_2 cannot be chosen for the initial period t_1 because it is the first period in the planning horizon.

$$[y3_{jc_2kt} \wedge (\underset{c}{\veebar} y3_{jckt-1})] \vee \neg\, y3_{jc_2kt} \quad \forall j \in J, \forall k \in K, \forall t \in T \tag{8}$$

$$\neg y3_{jc_2kt_1} \quad \forall j \in J, \forall k \in K \tag{9}$$

Considering the corresponding values of the parameters in each contract type, disjunction 7 can be simplified to

$$\underset{c \in C}{\veebar} \begin{bmatrix} y3_{jckt} \\ q_{jkt} \geq \mathrm{Qmin}_{cj} \\ w_{jckt} = q_{jkt}\mathrm{PC}_{jkt}(1 - \delta_{jc}) \\ w_{jckt} = m_{jckt'} \end{bmatrix} \quad \forall j \in J, \forall k \in K, \forall (c,t,t') \in \mathrm{TP}_{ctt'} \tag{10}$$

where $\delta_{jc_1} \geq 0$, $\delta_{jc_2} \geq 0$, and $\delta_{jc_3} \leq 0$.

As was mentioned in the Problem Description, two decision levels are needed before the contract types can be selected, which is posed in nested disjunction 11, including disjunction 10 in the deepest term.

$$\begin{bmatrix} y1_{jft} \\ \sum_{k \in \mathrm{FK}_{jk}} q_{jkt} \leq \sum_{k \in \mathrm{FK}_{jk}} \mathrm{Qmax}_{jkt} \\ \underset{k \in \mathrm{FK}_{fk}}{\veebar} \begin{bmatrix} y2_{jfkt} \\ q_{jkt} \leq \mathrm{Qmax}_{jkt} \\ \underset{\substack{c \in C \\ (c,t,t') \in \mathrm{TP}_{ctt'}}}{\veebar} \begin{bmatrix} y3_{jckt} \\ q_{jkt} \geq \mathrm{Qmin}_{cj} \\ w_{jckt} = q_{jkt}\mathrm{PC}_{jkt}(1 - \delta_{jc}) \\ w_{jckt} = m_{jckt'} \end{bmatrix} \end{bmatrix} \end{bmatrix} \vee \begin{bmatrix} \neg y1_{jft} \\ \sum_{k \in \mathrm{FK}_{fk}} q_{jkt} = 0 \end{bmatrix} \quad \forall j \in J, \forall f \in F, \forall t \in T \tag{11}$$

In the first level of eq 11, Boolean variable $y1_{jft}$ selects which families f are bought from each supplier j in each period t. In the negative case, no material is ordered related to family f from supplier j in period t. In the affirmative case, the total quantity ordered must be lower than the maximum capacity of supplier j. In the next level, variable $y2_{jfkt}$ indicates that one material k must be selected from family f, according to FK_{fk}, in period t. The set FK_{fk} defines which materials correspond to each family f. In this term, the quantity ordered of material k in period t cannot be greater than the supplier capacity for that material and period. In the third term, disjunction 10 is introduced, where variable $y3_{jckt}$ indicates that contract type c must be chosen to order material k from supplier j in period t.

To present a broader formulation, the use of material families is considered in this approach. However, it is important to notice that, if materials are considered instead of families, this formulation is still valid, just with the first level in the decision hierarchy ignored.

An additional logical constraint is also required in the problem formulation by

$$\neg\left(\underset{k \notin \mathrm{FK}_{fk}}{\vee} y2_{jfkt}\right) \quad \forall j \in J, \forall f \in F, \forall t \in T \tag{12}$$

where proposition 12 determines that material k cannot be selected for family f if this material does not belong to family f of set FK_{fk}.

Although the expressiveness of the hierarchical decisions by means of nested disjunctions, they cannot be implemented directly. These disjunctions must be transformed into GDP form.[21,22] For that purpose, the disjunctions in 11 must be rewritten as single disjunctions, and some additional constraints must also be included in the model.

Disjunction 11 must be replaced by expressions 13−15 and 10:

$$\begin{bmatrix} y1_{jft} \\ \sum_{k \in \mathrm{FK}_{fk}} q_{jkt} \leq \sum_{k \in \mathrm{FK}_{fk}} \mathrm{Qmax}_{jkt} \end{bmatrix} \vee \begin{bmatrix} \neg y1_{jft} \\ \sum_{k \in \mathrm{FK}_{fk}} q_{jkt} = 0 \end{bmatrix} \quad \forall j \in J, \forall f \in F, \forall t \in T \tag{13}$$

Table 1. Example of Probability Failure Distribution for Supplier j_1

p_{j_1r}	r
0.5	0.15
0.3	0.35
0.2	0.5

Table 2. Example of Probability Failure Distribution for Supplier j_2

p_{j_2r}	r
0.45	0.15
0.35	0.35
0.1	0.5

$$\underset{k\in \mathrm{FK}_{kf}}{\vee}\begin{bmatrix} y2_{jfkt} \\ q_{jkt} \le \mathrm{Qmax}_{jkt}\end{bmatrix} \quad \forall j\in J, \forall f\in F, \forall t\in T \quad (14)$$

$$\left[y1_{jft} \wedge \left(\underset{k\in \mathrm{FK}_{fk}}{\underline{\vee}} y2_{jfkt}\right)\right] \vee \neg \left[y1_{jft} \vee \left(\underset{k\in \mathrm{FK}_{fk}}{\underline{\vee}} y2_{jfkt}\right)\right] \quad \forall j\in J, \forall f\in F, \forall t\in T \quad (15)$$

In proposition 15, if family f is selected for supplier j in period t, then only one material k related to family f in FK_{fk} must be selected for supplier j. If, instead, family f is not selected for supplier j in period t, then material k related to family f in FK_{fk} must not be selected for supplier j.

Proposition 16 determines that, if a material k of any family is selected for supplier j in period t, then one contract type c must also be selected for that material, supplier, and period. On the contrary, if material k is not selected from any family for supplier j in period t, then no contract c must be chosen for that material, supplier, and period:

$$\left[\left(\underset{f}{\vee} y2_{jfkt}\right) \wedge \left(\underset{c}{\underline{\vee}} y3_{jckt}\right)\right] \vee \neg \left[\left(\underset{f}{\vee} y2_{jfkt}\right) \vee \left(\underset{c}{\underline{\vee}} y3_{jckt}\right)\right] \quad \forall j\in J, \forall k\in K, \forall t\in T \quad (16)$$

3.3. Modeling Provision Uncertainty. As was mentioned in the Introduction, uncertainty in the provision process can occur as a result of various factors such as machinery breakdowns, material shortages, capacity constraints, or political crisis. Because this affects the delivery performance, uncertainty should be included in modeling of the supply decision process. While the probabilistic model introduces some nonlinear terms in the formulation, the stochastic representation considers a set of scenarios, thereby increasing the model size. Instead of using those traditional uncertainty techniques, a discrete random variable is used to represent the amount delivered by the suppliers. This novel mixed integer linear programming formulation avoids the disadvantages of the traditional methods in modeling uncertainty by using the expected value of the variable under analysis. Some examples of discrete probabilistic distributions are presented for the supplier failures in Figures 1 and 2. These distributions must be obtained as a result of analyzing statistical data about supplier historical delivery performances. The variable considered is the amount of material expected from the total quantity of material family f ordered in period t. Note that, if a given supplier presents a perfect historical delivery performance, even under adverse conditions, then its failure probability would be 0%, and the expected delivery quantity will be exactly the quantity ordered, so this procedure remains valid. It is worth mentioning that the number of suppliers selected in each time period will restrict the formula used for the expected quantity. For that reason, a decision-tree approach is used to show the scenarios handled in the problem

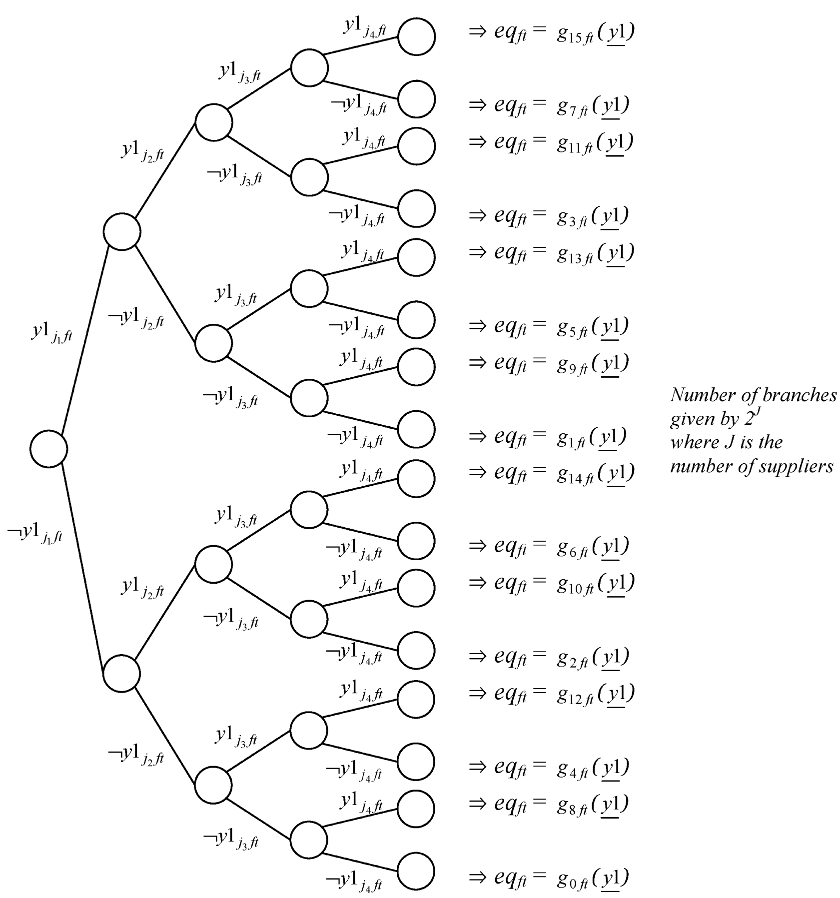

Figure 3. Decision-tree approach for the selection of four potential suppliers.

that are then jointly considered using the expected value formula. For illustrative purposes, the tree presented in Figure 3 represents the possible formulations when a total of four suppliers could be selected in each time period.

The diagram of Figure 3 shows that, if all suppliers are selected, then the expected quantity of family f in period t, eq_{ft}, is given by $g_{15ft}(\underline{\mathrm{y1}})$, where $g_{15ft}(\underline{\mathrm{y1}})$ is a function of the four suppliers selected, their failure probability distributions, and the quantity of family f ordered. Similarly, each branch is related to a different value of eq_{ft} according to the variables and parameters mentioned. In the last branch, for instance, no supplier is selected for family f in period t; as a consequence, eq_{ft} is equal to zero.

Note that, if, for example, only supplier j_1 is selected with the failure probability distributions given by Table 1, then the expected quantity is constrained by the probability p_{j_1r} of supplier j_1 in each possible failure r. In this case, supplier j_1 presents a 50% probability of failure for 15% of the amount ordered, a 30% probability to fail for 35% of the quantity ordered, and a 20% probability to fail for 50% of the amount ordered.

The variable qf_{jft} is defined as the quantity of material family f ordered from supplier j in period t, where

$$\mathrm{qf}_{jft} = \sum_{k \in \mathrm{FK}_{ft}} q_{jkt} \quad \forall j \in J, \forall f \in F, \forall t \in T \qquad (17)$$

Then, the quantity delivered that corresponds to each expected failure is defined by the equation

$$\mathrm{qr}_{jrft} = \mathrm{qf}_{jft}(1 - r) \quad \forall j \in J, \forall f \in F, \forall t \in T, \forall r \in R \qquad (18)$$

In this case, suppose that 100 units of material family f_1 have been ordered from supplier j_1 in period t_1. Then, the expected quantity of this material, eq_{ft}, is defined by the equation

$$\mathrm{eq}_{f_1t_1} = 0.5 \times (1 - 0.15) \times 100 + 0.3 \times (1 - 0.35) \times 100 + 0.2 \times (1 - 0.5) \times 100 = 72 \text{ units} \qquad (19)$$

In general, the expected quantity if only one supplier, j_1, has been chosen can be represented as

$$\mathrm{eq}_{ft} = p_{j_1r_1}\mathrm{qr}_{j_1r_1ft} + p_{j_1r_2}\mathrm{qr}_{j_1r_2ft} + p_{j_1r_3}\mathrm{qr}_{j_1r_3ft} = \sum_r p_{j_1r}\mathrm{qr}_{j_1rft} \quad \forall f \in F, \forall t \in T \qquad (20)$$

Note that the variable qr_{jrft} could be replaced by $\mathrm{qf}_{jft}(1 - r)$ as shown in eq 18. In that case, the number of scenarios modeled given by the size of set r does not constrain the number of variables, showing one of the most important advantages of this formulation. For simplicity, qr_{jrft} is used in the following discussion; however, the results presented are modeled using the corresponding term given by eq 18.

Considering another possible situation, if two suppliers have been selected for family f_1 in period t_1, then the expected quantity will be constrained by the supplier failure distributions and the quantity of family f_1 ordered from each of them. Given the failure probability distribution for supplier j_2 in Table 2, if suppliers j_1 (Table 1) and j_2 are selected and 100 units are requested from each of them, then the expected quantity expected for family f_1 is given by eq 21. Note that, in this case, scenarios are generated by considering the probability distributions of both suppliers at the same time. Assuming event independence, the probability of each scenario is given by the product of the probabilities of the two suppliers.

$$\begin{aligned}\mathrm{eq}_{f_1t_1} = {} & 0.5 \times 0.45 \times [(1 - 0.15) \times 100 + (1 - 0.15) \times 100] + 0.5 \times 0.35 \times [(1 - 0.15) \times 100 + (1 - 0.35) \times 100] + 0.5 \times 0.1 \times [(1 - 0.15) \times 100 + (1 - 0.5) \times 100] + 0.3 \times 0.45 \times [(1 - 0.35) \times 100 + (1 - 0.15) \times 100] + 0.3 \times 0.35 \times [(1 - 0.35) \times 100 + (1 - 0.35) \times 100] + 0.3 \times 0.1 \times [(1 - 0.35) \times 100 + (1 - 0.5) \times 100] + 0.2 \times 0.45 \times [(1 - 0.5) \times 100 + (1 - 0.15) \times 100] + 0.2 \times 0.35 \times [(1 - 0.5) \times 100 + (1 - 0.35) \times 100] + 0.2 \times 0.1 \times [(1 - 0.5) \times 100 + (1 - 0.5) \times 100] = 130.8 \text{ units}\end{aligned} \qquad (21)$$

In general, for two suppliers j_1 and j_2, the expected quantity, eq_{ft}, is given by

$$\begin{aligned}\mathrm{eq}_{ft} = {} & p_{j_1r_1}p_{j_2r_1}(\mathrm{qr}_{j_1r_1ft} + \mathrm{qr}_{j_2r_1ft}) + p_{j_1r_1}p_{j_2r_2}(\mathrm{qr}_{j_1r_1ft} + \mathrm{qr}_{j_2r_2ft}) + p_{j_1r_1}p_{j_2r_3}(\mathrm{qr}_{j_1r_1ft} + \mathrm{qr}_{j_2r_3ft}) + p_{j_1r_2}p_{j_2r_1}(\mathrm{qr}_{j_1r_2ft} + \mathrm{qr}_{j_2r_1ft}) + p_{j_1r_2}p_{j_2r_2}(\mathrm{qr}_{j_1r_2ft} + \mathrm{qr}_{j_2r_2ft}) + p_{j_1r_2}p_{j_2r_3}(\mathrm{qr}_{j_1r_2ft} + \mathrm{qr}_{j_2r_3ft}) + p_{j_1r_3}p_{j_2r_1}(\mathrm{qr}_{j_1r_3ft} + \mathrm{qr}_{j_2r_1ft}) + p_{j_1r_3}p_{j_2r_2}(\mathrm{qr}_{j_1r_3ft} + \mathrm{qr}_{j_2r_2ft}) + p_{j_1r_3}p_{j_2r_3}(\mathrm{qr}_{j_1r_3ft} + \mathrm{qr}_{j_2r_3ft})\end{aligned} \quad \forall f \in F, \forall t \in T \qquad (22)$$

By applying the distributive property, eq 22 can be transformed into the equation

$$\begin{aligned}\mathrm{eq}_{ft} = {} & p_{j_1r_1}p_{j_2r_1}\mathrm{qr}_{j_1r_1ft} + p_{j_1r_1}p_{j_2r_1}\mathrm{qr}_{j_2r_1ft} + p_{j_1r_1}p_{j_2r_2}\mathrm{qr}_{j_1r_1ft} + p_{j_1r_1}p_{j_2r_2}\mathrm{qr}_{j_2r_2ft} + p_{j_1r_1}p_{j_2r_3}\mathrm{qr}_{j_1r_1ft} + p_{j_1r_1}p_{j_2r_3}\mathrm{qr}_{j_2r_3ft} + p_{j_1r_2}p_{j_2r_1}\mathrm{qr}_{j_1r_2ft} + p_{j_1r_2}p_{j_2r_1}\mathrm{qr}_{j_2r_1ft} + p_{j_1r_2}p_{j_2r_2}\mathrm{qr}_{j_1r_2ft} + p_{j_1r_2}p_{j_2r_2}\mathrm{qr}_{j_2r_2ft} + p_{j_1r_2}p_{j_2r_3}\mathrm{qr}_{j_1r_2ft} + p_{j_1r_2}p_{j_2r_3}\mathrm{qr}_{j_2r_3ft} + p_{j_1r_3}p_{j_2r_1}\mathrm{qr}_{j_1r_3ft} + p_{j_1r_3}p_{j_2r_1}\mathrm{qr}_{j_2r_1ft} + p_{j_1r_3}p_{j_2r_2}\mathrm{qr}_{j_1r_3ft} + p_{j_1r_3}p_{j_2r_2}\mathrm{qr}_{j_2r_2ft} + p_{j_1r_3}p_{j_2r_3}\mathrm{qr}_{j_1r_3ft} + p_{j_1r_3}p_{j_2r_3}\mathrm{qr}_{j_2r_3ft}\end{aligned} \quad \forall f \in F, \forall t \in T \qquad (23)$$

Grouping the corresponding terms in eq 23gives

$$\mathrm{eq}_{ft} = p_{j_2r_1}\sum_r p_{j_1r}\mathrm{qr}_{j_1rft} + p_{j_2r_2}\sum_r p_{j_1r}\mathrm{qr}_{j_1rft} + p_{j_2r_3}\sum_r p_{j_1r}\mathrm{qr}_{j_1rft} + p_{j_1r_1}\sum_r p_{j_2r}\mathrm{qr}_{j_2rft} + p_{j_1r_2}\sum_r p_{j_2r}\mathrm{qr}_{j_2rft} + p_{j_1r_3}\sum_r p_{j_2r}\mathrm{qr}_{j_2rft} \quad \forall f \in F, \forall t \in T \qquad (24)$$

To represent the general case, the variable eq_{ft} can be generalized as in eq 25. In this case, two suppliers, j and j', are presented, where $j, j' \in \{j_1, j_2\}$. Note also that failure range r is redefined as r' for supplier j', but $r, r' \in \{r_1,..., r_n\}$.

$$\mathrm{eq}_{ft} = \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{r \in R} \sum_{r' \in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \quad \forall f \in F, \forall t \in T \qquad (25)$$

Extending this procedure to three suppliers, the expected quantity, eq_{ft}, is given by

$$\mathrm{eq}_{ft} = \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{\substack{j'' \in J \\ j'' < j' \\ j'' \neq j}} \sum_{r \in R} \sum_{r' \in R} \sum_{r'' \in R} p_{jr}p_{j'r'}p_{j''r''}\mathrm{qr}_{jrft} \quad \forall f \in F, \forall t \in T \qquad (26)$$

Similarly, this approach can be applied for any number of suppliers selected. If four suppliers have been chosen, the expected quantity eq_{ft} is given by

$$\mathrm{eq}_{ft} = \sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{\substack{j''\in J\\ j''<j'\\ j''\neq j}}\sum_{\substack{j'''\in J\\ j'''<j''\\ j'''\neq j}}\sum_{r\in R}\sum_{r'\in R}\sum_{r''\in R}\sum_{r'''\in R} p_{jr}p_{j'r'}p_{j''r''}p_{j'''r'''}\mathrm{qr}_{jrft} \qquad \forall f\in F, \forall t\in T \tag{27}$$

As shown in eqs 20 and 25−27, the number of suppliers selected constrains the expression used to calculate the expected quantity of family f in period t (eq_{ft}) given by the function $g_{ift}(\underline{y1})$. The supplier selection presented in the decision-tree approach can be rewritten as proposition 28. This hierarchical disjunctive representation is used to determine the supplier selection when there are at most four potential suppliers. A detailed definition of $g_{ift}(\underline{y1})$ is given by eqs 29−44.

$$\left[\begin{matrix} y1_{j_1ft} \\ \left[\begin{matrix} y1_{j_2ft} \\ \left[\begin{matrix} y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{15ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{7ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \vee \left[\begin{matrix} \neg y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{11ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{3ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \end{matrix}\right] \vee \left[\begin{matrix} \neg y1_{j_2ft} \\ \left[\begin{matrix} y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{13ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{5ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \vee \left[\begin{matrix} \neg y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{9ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{1ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] \vee$$

$$\left[\begin{matrix} \neg y1_{j_1ft} \\ \left[\begin{matrix} y1_{j_2ft} \\ \left[\begin{matrix} y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{14ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{6ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \vee \left[\begin{matrix} \neg y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{10ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{2ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \end{matrix}\right] \vee \left[\begin{matrix} \neg y1_{j_2ft} \\ \left[\begin{matrix} y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{12ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{4ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \vee \left[\begin{matrix} \neg y1_{j_3ft} \\ \begin{bmatrix} y1_{j_4ft} \\ g_{8ft}(\underline{y1}) \end{bmatrix} \vee \begin{bmatrix} \neg y1_{j_4ft} \\ g_{0ft}(\underline{y1}) \end{bmatrix} \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] \qquad \forall f\in F, \forall t\in T \tag{28}$$

According to the procedure presented in eqs 20 and 25−27, the functions $g_{ift}(\underline{y1})$ are defined through eqs 29−44.

$$g_{15ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,\ldots,j_4\}}\sum_{\substack{j'\in\{j_1,\ldots,j_4\}\\ j'\neq j}}\sum_{\substack{j''\in\{j_1,\ldots,j_4\}\\ j''<j'\\ j''\neq j}}\sum_{\substack{j'''\in\{j_1,\ldots,j_4\}\\ j'''<j''\\ j'''\neq j}}\sum_{r\in R}\sum_{r'\in R}\sum_{r''\in R}\sum_{r'''\in R} p_{jr}p_{j'r'}p_{j''r''}p_{j'''r'''}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \tag{29}$$

$$g_{7ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,\ldots,j_3\}}\sum_{\substack{j'\in\{j_1,\ldots,j_3\}\\ j'\neq j}}\sum_{\substack{j''\in\{j_1,\ldots,j_3\}\\ j''<j'\\ j''\neq j}}\sum_{r\in R}\sum_{r'\in R}\sum_{r''\in R} p_{jr}p_{j'r'}p_{j''r''}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \tag{30}$$

$$g_{11ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,j_2,j_4\}}\sum_{\substack{j'\in\{j_1,j_2,j_4\}\\ j'\neq j}}\sum_{\substack{j''\in\{j_1,j_2,j_4\}\\ j''<j'\\ j''\neq j}}\sum_{r\in R}\sum_{r'\in R}\sum_{r''\in R} p_{jr}p_{j'r'}p_{j''r''}\mathrm{qr}_{jrft} \qquad \forall f\in F, \forall t\in T \tag{31}$$

$$g_{3ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,j_2\}}\sum_{\substack{j'\in\{j_1,j_2\}\\ j'\neq j}}\sum_{r\in R}\sum_{r'\in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \qquad \forall f\in F, \forall t\in T \tag{32}$$

$$g_{13ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,j_3,j_4\}}\sum_{\substack{j'\in\{j_1,j_3,j_4\}\\ j'\neq j}}\sum_{\substack{j''\in\{j_1,j_3,j_4\}\\ j''<j'\\ j''\neq j}}\sum_{r\in R}\sum_{r'\in R}\sum_{r''\in R} p_{jr}p_{j'r'}p_{j''r''}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \tag{33}$$

$$g_{5ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,j_3\}}\sum_{\substack{j'\in\{j_1,j_3\}\\ j'\neq j}}\sum_{r\in R}\sum_{r'\in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \tag{34}$$

$$g_{9ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_1,j_4\}} \sum_{\substack{j'\in\{j_1,j_4\}\\ j'\neq j}} \sum_{r\in R} \sum_{r'\in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \forall f\in F, \forall t\in T \quad (35)$$

$$g_{1ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{r\in R} p_{j_1r}\mathrm{qr}_{j_1rft} \quad \forall f\in F, \forall t\in T \quad (36)$$

$$g_{14ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_2,\dots,j_4\}} \sum_{\substack{j'\in\{j_2,\dots,j_4\}\\ j'\neq j}} \sum_{\substack{j''\in\{j_2,\dots,j_4\}\\ j''<j'\\ j''\neq j}} \sum_{r\in R} \sum_{r'\in R} \sum_{r''\in R} p_{jr}p_{j'r'}p_{j''r''}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \quad (37)$$

$$g_{6ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_2,j_3\}} \sum_{\substack{j'\in\{j_2,j_3\}\\ j'\neq j}} \sum_{r\in R} \sum_{r'\in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \quad (38)$$

$$g_{10ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_2,j_4\}} \sum_{\substack{j'\in\{j_2,j_4\}\\ j'\neq j}} \sum_{r\in R} \sum_{r'\in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \quad (39)$$

$$g_{2ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{r\in R} p_{j_2r}\mathrm{qr}_{j_2rft} \quad \forall f\in F, \forall t\in T \quad (40)$$

$$g_{12ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{j\in\{j_3,j_4\}} \sum_{\substack{j'\in\{j_3,j_4\}\\ j'\neq j}} \sum_{r\in R} \sum_{r'\in R} p_{jr}p_{j'r'}\mathrm{qr}_{jrft} \quad \forall f\in F, \forall t\in T \quad (41)$$

$$g_{4ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{r\in R} p_{j_3r}\mathrm{qr}_{j_3rft} \quad \forall f\in F, \forall t\in T \quad (42)$$

$$g_{8ft}(\underline{y1}) = \mathrm{eq}_{ft} = \sum_{r\in R} p_{j_4r}\mathrm{qr}_{j_4rft} \quad \forall f\in F, \forall t\in T \quad (43)$$

$$g_{0ft}(\underline{y1}) = \mathrm{eq}_{ft} = 0 \quad \forall f\in F, \forall t\in T \quad (44)$$

Although the nested disjunctions presented in proposition 28 help in explaining the decision and the implications associated with the selection of suppliers, three main problems appear in this procedure. First, the formulation depends on the number of potential suppliers available, leading to a nongeneral formulation. In addition, as already explained for disjunction 11, the implementation of hierarchical disjunctions is not straightforward. Finally, it would be important to find a relation between the selection of suppliers by variables $\mathrm{y1}_{jft}$ and the subindex i used in $g_{ift}(\underline{y1})$ to calculate eq_{ft}. Note that these nested disjunctions must satisfy only one constraint corresponding to the deepest term. This characteristic facilitates a variable transformation that groups the variables from different levels into a unique variable. This transformation is done by introducing new Boolean variables and transforming the disjunctions using a binary representation as follows:

Table 3. Examples of Binary Representations According to Supplier Selection

supplier selection for family f in period t	binary representation of that choice
$\mathrm{y1}_{j_4ft} \wedge\neg\, \mathrm{y1}_{j_3ft} \wedge \mathrm{y1}_{j_2ft} \wedge\neg\, \mathrm{y1}_{j_1ft}$	1010
$\neg\mathrm{y1}_{j_4ft} \wedge\neg\, \mathrm{y1}_{j_3ft} \wedge \mathrm{y1}_{j_2ft} \wedge\neg\, \mathrm{y1}_{j_1ft}$	0010
$\neg\mathrm{y1}_{j_4ft} \wedge \mathrm{y1}_{j_3ft} \wedge \mathrm{y1}_{j_2ft} \wedge\neg\, \mathrm{y1}_{j_1ft}$	0110

Table 4. Material Family Compositions (FK_{fk})

family	materials
f_1	k_1, k_2, k_3
f_2	k_4, k_5, k_6, k_7
f_3	k_8, k_9, k_{10}
f_4	k_{11}, k_{12}, k_{13}
f_5	k_{14}, k_{15}, k_{16}, k_{17}
f_6	k_{18}, k_{19}, k_{20}
f_7	k_{21}, k_{22}
f_8	k_{23}, k_{24}, k_{25}
f_9	k_{26}, k_{27}, k_{28}, k_{29}
f_{10}	k_{30}, k_{31}, k_{32}

Table 5. Payment Terms Given by $\mathrm{TP}_{ctt'}$, $t = t'$ for All Contracts Except c_3

	time period	
contract	t	t'
c_1	t	t
c_2	t	t
c_3	t	$t + 2$

First, suppose that the selection of suppliers defines a binary number. Then, if supplier j_n is selected for family f in period t, variable $\mathrm{y1}_{j_nft}$ is true, and the corresponding binary number is 1. On the contrary, if supplier j_n is not selected, the Boolean variable is false, and its binary number is 0. Defining a combination of binary numbers for the whole solution, then, a 0−1 representation is obtained. If, for instance, suppliers j_4 and j_3 have been selected whereas suppliers j_1 and j_2 have not, the corresponding representation of this choice would be 1100, meaning that the first binary number refers to supplier j_4, the second to j_3, and so on. Table 3 shows some other possible examples presenting the value of the Boolean variables and the equivalent representation using binary numbers.

This approach allows one to determine a relation between the selection of suppliers and the corresponding value of

Table 6. Family Demand (FD_{ft})

family	time period		
	t_1	t_2	t_3
f_1	105	170	160
f_2	182	175	180
f_3	50	55	60
f_4	110	120	130
f_5	100	105	110
f_6	200	150	180
f_7	200	200	200
f_8	150	145	210
f_9	100	250	220
f_{10}	120	145	200

Table 7. Initial Stock of Family *f* (IS_f)

family	initial units in stock
f_1	40
f_2	45
f_3	40
f_4	40
f_5	40
f_6	30
f_7	32
f_8	40
f_9	30
f_{10}	45

Table 8. Minimum Quantity to Order According to Supplier *j* and Contract *c* ($Qmin_{cj}$)

supplier	contract		
	c_1	c_2	c_3
j_1	30	55	70
j_2	40	60	75
j_3	32	52	77
j_4	38	55	80

subindex i used in the definition of $g_{ift}(y1)$, translating the binary number into its decimal representation. The conversion procedure to transform a number from binary to decimal base consists of multiplying each binary number per 2^{ord}, where ord represents the order of the number in the binary representation. If the number 1010 given in base 2 is converted into a base-10 number, the procedure is

$$1010_2 \leftrightarrow 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 8 + 0 + 2 + 0 = 10_{10} \quad (45)$$

giving the equivalent number 10 in base 10.

This algorithm establishes a relation between the set of suppliers selected (represented as a binary number) and the corresponding value of subindex i, as shown in eq 46:

$$i_{ft} = \sum_{j=1}^{J} 2^{j-1} Y1_{jft} \quad \forall f \in F, \forall t \in T \quad (46)$$

where $i_{ft} \in I_{ft} = \{0,..., 2^J - 1\}$ and $Y1_{jft} \in \{0, 1\}$.

The value of subindex i will be the corresponding decimal representation of the binary number that symbolizes the suppliers selected. Considering all possible solutions of supplier selection leads to the complete representation of the set I_{ft} for each family f and period t.

In addition to the generality of the procedure presented and the clear relation between the supply conformation and the corresponding constraint $g_{ift}(\underline{y1})$ to be satisfied, this approach also makes it possible to simplify disjunction 28. To this end, a new Boolean variable, v_{ift}, is introduced equivalent to the conformation of suppliers selected according to the value of subindex i. Then, function $g_{ift}(\underline{y1})$ must be satisfied if and only if v_{ift} is true; that is

$$v_{ift} \Leftrightarrow g_{ift}(\underline{y1}) \quad \forall i \in I, \forall f \in F, \forall t \in T \quad (47)$$

If, for instance, the solution for family f_1 in period t_1 is given by $\neg y1_{j_4f_1t_1} \wedge \neg\, y1_{j_3f_1t_1} \wedge y1_{j_2f_1t_1} \wedge \neg\, y1_{j_1f_1t_1}$, then the procedure applies as follows

$$\neg y1_{j_4f_1t_1} \wedge \neg\, y1_{j_3f_1t_1} \wedge y1_{j_2f_1t_1} \wedge \neg\, y1_{j_1f_1t_1} \rightarrow \text{binary representation } 0010$$

$$i_{f_1t_1} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 0 + 0 + 2 + 0$$
$$= 2$$

Then, variable $v_{2f_1t_1}$ is true, and function $g_{2f_1t_1}(\underline{y1})$ must be satisfied.

By this general formulation, each combination of selected suppliers leads to a different value of the expected quantity (eq_{ft}) of family f in period t given by $g_{ift}(\underline{y1})$. It is worth mentioning that the form of the expected quantity, eq_{ft}, is defined by the procedure shown in eqs 20 and 25−27.The closer the eq_{ft} variable is to the demand forecasted, the better, given that a deficit in the supplied quantity implies less real revenues and could also affect customer satisfaction. As a consequence, the decision regarding suppliers will also consider the expected costs due to supplier delivery failures. It is extremely important to consider this feature when formulating the objective function.

Finally, by applying the developed approach, disjunction 28 can be replaced by the general disjunction

$$\begin{bmatrix} v_{0ft} \\ g_{0ft} \end{bmatrix} \vee \begin{bmatrix} v_{1ft} \\ g_{1ft} \end{bmatrix} \vee \begin{bmatrix} v_{2ft} \\ g_{2ft} \end{bmatrix} \vee \cdots \vee \begin{bmatrix} v_{(2^J-1)ft} \\ g_{(2^J-1)ft} \end{bmatrix} \quad \forall f \in F, \forall t \in T \quad (48)$$

3.4. Modeling Additional Stock Constraints. The following constraints are modeled to calculate the average stock, $savg_{ft}$, of family f in period t, needed in the objective function. This variable can be calculated as

$$savg_{ft} = \frac{\sum_e sd_{fet}}{e_h} \quad \forall f \in F, \forall t \in T, e_h \in E \quad (49)$$

Table 9. Regular Price of Material *k* Sold by Supplier *j* in Period t_1 from k_1 to k_{16} (PC_{jkt_1})

supplier	material															
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	k_{16}
j_1	0.50	0.47	0.80	0.71	0.75	0.51	0.49	0.78	0.72	0.75	0.45	0.46	0.78	0.70	0.80	0.51
j_2	0.52	0.49	0.81	0.78	0.72	0.52	0.45	0.68	0.81	0.74	0.55	0.48	0.81	0.72	0.75	0.54
j_3	0.51	0.48	0.79	0.69	0.73	0.53	0.48	0.72	0.71	0.72	0.52	0.47	0.88	0.71	0.74	0.50
j_4	0.55	0.45	0.82	0.70	0.77	0.54	0.47	0.70	0.70	0.79	0.51	0.48	0.81	0.75	0.72	0.51

Table 10. Regular Price of Material k Sold by Supplier j in Period t_1 from k_{17} to k_{32} (PC_{jkt_1})

	material															
supplier	k_{17}	k_{18}	k_{19}	k_{20}	k_{21}	k_{22}	k_{23}	k_{24}	k_{25}	k_{26}	k_{27}	k_{28}	k_{29}	k_{30}	k_{31}	k_{32}
j_1	0.47	0.80	0.71	0.75	0.51	0.49	0.79	0.71	0.73	0.51	0.47	0.78	0.81	0.77	0.80	0.81
j_2	0.57	0.78	0.73	0.70	0.52	0.48	0.78	0.70	0.71	0.52	0.45	0.81	0.71	0.74	0.82	0.80
j_3	0.48	0.79	0.71	0.75	0.50	0.45	0.78	0.72	0.73	0.53	0.48	0.82	0.75	0.75	0.88	0.83
j_4	0.49	0.77	0.75	0.72	0.50	0.50	0.81	0.73	0.72	0.50	0.49	0.81	0.72	0.75	0.85	0.84

Table 11. Contract Discount (or Interest Rate if $\delta_{jc} < 0$)

	contract		
supplier	c_1	c_2	c_3
j_1	0.15	0.15	−0.13
j_2	0.06	0.12	−0.09
j_3	0.095	0.15	−0.125
j_4	0.055	0.121	−0.078

where

$$\mathrm{sd}_{fet} = \mathrm{sd}_{f(e-1)t} - \frac{s_{ft}}{e_h} \quad \forall f \in F, \forall t \in T, \forall e \in E = \{e_2, ..., e_h\} \tag{50}$$

In eq 50, the variable sd_{fet} indicates the family stock in each subperiod e of period t, $\mathrm{sd}_{f(e-1)t}$ is the family stock in the previous period, and s_{ft}/e_h represents a constant material output rate from the stock. In eq 51, the initial stock of family f in the first subperiod e_1 of period t is constrained to be equal to the initial stock of family f in period t.

$$\mathrm{sd}_{fe_1t} = s_{ft} \quad \forall f \in F, \forall t \in T \tag{51}$$

3.5. Objective Function. The objective function defines the minimization of the actualized expected costs over the time horizon. The costs considered are the corresponding material purchase costs, the inventory costs, and the expected costs due to the unsatisfied demand calculated as lost sales. The objective function is presented as

$$\min \sum_t \frac{\sum_j \sum_c \sum_k m_{jckt} + \sum_f \mathrm{savg}_{ft}\mathrm{COSTavg}_{ft}\mathrm{MS} + \sum_f \mathrm{Ap}_{ft}(\mathrm{FD}_{ft} - d_{ft})}{(1 + \mathrm{RR})^t} \tag{52}$$

The positive variable m_{jckt} determines the money outflows due to material k purchased from supplier j to be paid in period t according to the payment policy of contract c.

In the second term, the positive variable savg_{ft} represents the average stock expected of family f in period t, calculated in eq 49. The parameter $\mathrm{COSTavg}_f$ corresponds to the average cost of family f, and parameter MS indicates a percentage of the raw material average costs. This percentage is used to estimate the average stock costs and includes the following major components: capital costs, storage costs, obsolescence costs, and quality costs. In general, all of these costs are rolled together into a single inventory cost rate, expressed as a percentage of the product or material value per unit time.[23]

The last term corresponds to the expected loses due to the unsatisfied demand. It is considered that the difference between the family demand and the expected sales of family f represents the amount of product that could have been sold. These units are quantified by multiplying them by the average sale price of family f in period t, Ap_{ft}.

The return rate, RR, corresponds to the capital cost, which is calculated with the capital asset pricing model.[24] The formula for the capital cost RR is

$$\mathrm{RR} = \mathrm{RR}^{\mathrm{free}} + \mathrm{RR}^{\mathrm{risk}} \tag{53}$$

where $\mathrm{RR}^{\mathrm{free}}$ corresponds to the expected return of a theoretical risk-free asset and is the minimum return an investor expects for any investment. Traditionally, for long-term investments, $\mathrm{RR}^{\mathrm{free}}$ is equal to the country Treasury bill rate. In this case, the appropriate $\mathrm{RR}^{\mathrm{free}}$ value considered is a certificate of deposit rate (from a safe bank) because the time horizon is short and the investment corresponds to working capital instead of fixed assets. The $\mathrm{RR}^{\mathrm{risk}}$ element can be rewritten as

$$\mathrm{RR}^{\mathrm{risk}} = \beta(\mathrm{RR}^{\mathrm{market}} - \mathrm{RR}^{\mathrm{free}}) \tag{54}$$

where $\mathrm{RR}^{\mathrm{market}}$ represents the expected return of the market and β indicates the sensitivity of returns from asset a to market returns. This parameter can be calculated as

$$\beta = \sigma^{a\,\mathrm{market}} / (\sigma^{\mathrm{market}})^2 \tag{55}$$

where $\sigma^{a\,\mathrm{market}}$ in eq 55 represents the covariance between the returns from asset a and the market returns and $(\sigma^{\mathrm{market}})^2$ defines the market variance. All of these coefficients are model data.

The final mathematical model for the case of four potential suppliers is given by expressions 1−6, 8−10, 12−16, 29−44, 46, and 48−52.

4. Model Results

Two case studies considering four potential suppliers are presented here to illustrate formulation performance. The models were posed in the GAMS system and executed over a PC having an Intel Pentium D 2.8 GHz processor. Disjunctions were modeled using GDP, and the disjunctive program was solved with LogMIP.[25]

4.1. Case Study 1. In this example, supplier uncertainty is modeled by considering two possible extreme situations: raw material order delivery completely fails or is completely successful, meaning that, in the first case, no raw material is delivered by the supplier, whereas in the second case, the whole

Table 12. Maximum Quantity of Material k to Order from Supplier j in Period t from k_1 to k_{16} (Qmax_{jkt})[a]

	material															
supplier	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	k_{16}
j_1	150	50	50	145	60	50	60	158	60	65	45	148	50	160	50	60
j_2	160	70	65	180	80	80	50	160	45	60	80	150	50	150	45	50
j_3	150	55	45	165	65	40	53	159	50	66	42	140	60	162	20	49
j_4	160	50	64	175	60	77	52	142	40	70	70	160	22	180	48	60

[a] Suppliers' capacity is the same for all periods t.

Table 13. Maximum Quantity of Material k to Order from Supplier j in Period t from k_{17} to k_{32} ($Qmax_{jkt}$)

supplier	material															
	k_{17}	k_{18}	k_{19}	k_{20}	k_{21}	k_{22}	k_{23}	k_{24}	k_{25}	k_{26}	k_{27}	k_{28}	k_{29}	k_{30}	k_{31}	k_{32}
j_1	70	170	60	70	180	70	165	60	85	170	85	110	95	125	110	80
j_2	66	150	65	75	185	75	152	65	82	175	80	100	110	120	100	82
j_3	65	175	50	70	150	60	145	70	80	180	90	125	104	130	100	80
j_4	73	175	63	72	180	70	165	70	84	185	80	105	100	120	105	75

Table 14. Probability of Failure of Supplier j in the Range or Proportion r of the Ordered Quantity (p_{jr}): Case Study 1

supplier	failure (r)	
	0%	100%
j_1	75%	25%
j_2	70%	30%
j_3	73%	27%
j_4	67%	33%

Table 15. Average Price of Family f in Period t_1 (Ap_{ft_1})

family	Ap_{ft_1}
f_1	0.90
f_2	0.92
f_3	1.10
f_4	0.90
f_5	0.93
f_6	1.12
f_7	0.74
f_8	1.11
f_9	0.95
f_{10}	1.21

order is sent. The supplier failure probability distributions are presented in Figure 2. Data for this example are provided in the next two subsections.

4.1.1. Model Sets. The sizes of the model sets are as follows: number of families (F) = 10, number of materials (K) = 32, number of contract types (C) = 3, and number of potential suppliers (J) = 4.

Tables 4 and 5 provide data on the material family compositions and payment terms, respectively.

4.1.2. Model Parameters. Model parameters related to the family demands, initial stocks of family f, minimum quantities to order, regular prices, contract discounts, maximum quantities to order, supplier failure probabilities, and average prices are provided in Tables 6−15.

The demand upper bound is given by $\mu = 0.50$, and the stock capacity is SC = 5000.

The regular price of material k sold by supplier j in period t (PC_{jkt}) is calculated using the inflation rate ir (ir = 0.10) as

$$PC_{jkt} = PC_{jk(t-1)}(1 + \mathrm{ir})$$

The average cost of family f in period t ($COSTavg_{ft}$) is determined by calculating the average of PC_{jkt} for all suppliers and for all materials belonging to f given by FK_{kf}.

Table 17. Expected Quantity of Family f in Period t (eq_{ft}): Case Study 1

f	time period		
	t_1	t_2	t_3
f_1	65.0	170.0	160.0
f_2	137.0	175.0	180.0
f_3	22.50	42.50	60.0
f_4	70.0	120.0	130.0
f_5	60.0	105.0	110.0
f_6	170.0	150.0	180.0
f_7	168.0	200.0	200.0
f_8	110.0	145.0	210.0
f_9	70.0	250.0	220.0
f_{10}	75.0	145.0	200.0

Table 18. Expected Sales for Family f in Period t (d_{ft}): Case Study 1

f	time period		
	t_1	t_2	t_3
f_1	105	170	160
f_2	182	175	180
f_3	50	55	60
f_4	110	120	130
f_5	100	105	110
f_6	200	150	180
f_7	200	200	200
f_8	150	145	210
f_9	100	250	220
f_{10}	120	145	200

The average price of family f in period t is calculated using the interest rate ir as

$$Ap_{ft} = Ap_{f(t-1)}(1 + \mathrm{ir})$$

The percentage of the raw material average costs used to calculate stock costs is MS = 0.25, and the discount rate is RR = 0.15.

4.1.3. Model Solution. In Table 16, several decisions are presented. The first two columns show which materials k are selected for each family f in period t for supplier j; this information is given by the variable $y2_{jfkt}$. In the intersection of rows and columns are presented the quantities of material k ordered from supplier j in period t (variable q_{ijkt}) and the contract selected c (variable $y3_{jckt}$). Note that, when the quantity ordered is not null, this also means that the supplier j has been selected for family f ($y1_{fkt}$).

Table 16. Materials Purchased, Quantities Ordered, and Contracts and Suppliers Selected in Each Time Period: Case Study 1

f	k	j_1			j_2			j_3		
		t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
f_1	k_2	86.7 − c_3	151.7 − c_2	162.7 − c_2	–	–	–	–	77 − c_3	52 − c_2
f_2	k_7	–	70 − c_3	55 − c_2	195.7 − c_3	175 − c_3	198.2 − c_3	–	–	–
f_3	k_8	–	–	–	–	–	85.7 − c_3	–	–	–
f_3	k_9	30 − c_1	56.7 − c_1	–	–	–	–	–	–	–
f_4	k_{11}	93.3 − c_3	160 − c_2	173.3 − c_2	–	–	–	–	–	–
f_5	k_{17}	80 − c_3	140 − c_2	146.7 − c_2	–	–	–	–	–	–
f_6	k_{19}	156.7 − c_3	200 − c_2	170 − c_2	–	–	–	–	–	–
f_6	k_{20}	–	–	–	75 − c_3	–	75 − c_3	–	–	–
f_7	k_{22}	30 − c_1	72 − c_3	72 − c_2	–	–	–	199.3 − c_3	200 − c_2	200 − c_2
f_8	k_{24}	146.7 − c_3	193.3 − c_2	200 − c_2	–	–	85.7 − c_3	–	–	–
f_9	k_{27}	–	146.7 − c_3	106.7 − c_2	100 − c_3	200 − c_3	200 − c_3	–	–	–
f_{10}	k_{30}	–	–	–	107.1 − c_3	126.8 − c_3	200 − c_3	–	77 − c_3	82.2 − c_2

Table 19. Supplier Selection for Family$f$ in Period t ($y1_{jft}$) and Its Relationship with the Variable v_{ift}: Case Study 1

f	t	supplier selection for family f in period t	binary representation and Boolean variable v_{ift}
f_1	t_1	$\neg y1_{j_4f_1t_1} \wedge\neg\ y1_{j_3f_1t_1} \wedge\neg\ y1_{j_2f_1t_1} \wedge\ y1_{j_1f_1t_1}$	$0001 \Rightarrow i_{f_1t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_1t_1}$
	t_2	$\neg y1_{j_4f_1t_2} \wedge\ y1_{j_3f_1t_2} \wedge\neg\ y1_{j_2f_1t_2} \wedge\ y1_{j_1f_1t_2}$	$0101 \Rightarrow i_{f_1t_2} = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5 \Rightarrow v_{5f_1t_2}$
	t_3	$\neg y1_{j_4f_1t_3} \wedge\ y1_{j_3f_1t_3} \wedge\neg\ y1_{j_2f_1t_3} \wedge\ y1_{j_1f_1t_3}$	$0101 \Rightarrow i_{f_1t_3} = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5 \Rightarrow v_{5f_1t_3}$
f_2	t_1	$\neg y1_{j_4f_2t_1} \wedge\neg\ y1_{j_3f_2t_1} \wedge\ y1_{j_2f_2t_1} \wedge\neg\ y1_{j_1f_2t_1}$	$0010 \Rightarrow i_{f_2t_1} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2 \Rightarrow v_{2f_2t_1}$
	t_2	$\neg y1_{j_4f_2t_2} \wedge\neg\ y1_{j_3f_2t_2} \wedge\ y1_{j_2f_2t_2} \wedge\ y1_{j_1f_2t_2}$	$0011 \Rightarrow i_{f_6t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_6t_3}$
	t_3	$\neg y1_{j_4f_2t_3} \wedge\neg\ y1_{j_3f_2t_3} \wedge\ y1_{j_2f_2t_3} \wedge\ y1_{j_1f_2t_3}$	$0011 \Rightarrow i_{f_2t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_2t_3}$
f_3	t_1	$\neg y1_{j_4f_3t_1} \wedge\neg\ y1_{j_3f_3t_1} \wedge\neg\ y1_{j_2f_3t_1} \wedge\ y1_{j_1f_3t_1}$	$0001 \Rightarrow i_{f_3t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_3t_1}$
	t_2	$\neg y1_{j_4f_3t_2} \wedge\neg\ y1_{j_3f_3t_2} \wedge\neg\ y1_{j_2f_3t_2} \wedge\ y1_{j_1f_3t_2}$	$0001 \Rightarrow i_{f_3t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_3t_2}$
	t_3	$\neg y1_{j_4f_3t_3} \wedge\neg\ y1_{j_3f_3t_3} \wedge\ y1_{j_2f_3t_3} \wedge\neg\ y1_{j_1f_3t_3}$	$0010 \Rightarrow i_{f_3t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2 \Rightarrow v_{2f_3t_3}$
f_4	t_1	$\neg y1_{j_4f_4t_1} \wedge\neg\ y1_{j_3f_4t_1} \wedge\neg\ y1_{j_2f_4t_1} \wedge\ y1_{j_1f_4t_1}$	$0001 \Rightarrow i_{f_4t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_4t_1}$
	t_2	$\neg y1_{j_4f_4t_2} \wedge\neg\ y1_{j_3f_4t_2} \wedge\neg\ y1_{j_2f_4t_2} \wedge\ y1_{j_1f_4t_2}$	$0001 \Rightarrow i_{f_4t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_4t_2}$
	t_3	$\neg y1_{j_4f_4t_3} \wedge\neg\ y1_{j_3f_4t_3} \wedge\neg\ y1_{j_2f_4t_3} \wedge\ y1_{j_1f_4t_3}$	$0001 \Rightarrow i_{f_4t_3} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_4t_3}$
f_5	t_1	$\neg y1_{j_4f_5t_1} \wedge\neg\ y1_{j_3f_5t_1} \wedge\neg\ y1_{j_2f_5t_1} \wedge\ y1_{j_1f_5t_1}$	$0001 \Rightarrow i_{f_5t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_5t_1}$
	t_2	$\neg y1_{j_4f_5t_2} \wedge\neg\ y1_{j_3f_5t_2} \wedge\neg\ y1_{j_2f_5t_2} \wedge\ y1_{j_1f_5t_2}$	$0001 \Rightarrow i_{f_5t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_5t_2}$
	t_3	$\neg y1_{j_4f_5t_3} \wedge\neg\ y1_{j_3f_5t_3} \wedge\neg\ y1_{j_2f_5t_3} \wedge\ y1_{j_1f_5t_3}$	$0001 \Rightarrow i_{f_5t_3} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_5t_3}$
f_6	t_1	$\neg y1_{j_4f_6t_1} \wedge\neg\ y1_{j_3f_6t_1} \wedge\ y1_{j_2f_6t_1} \wedge\ y1_{j_1f_6t_1}$	$0011 \Rightarrow i_{f_6t_1} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_6t_1}$
	t_2	$\neg y1_{j_4f_6t_2} \wedge\neg\ y1_{j_3f_6t_2} \wedge\neg\ y1_{j_2f_6t_2} \wedge\ y1_{j_1f_6t_2}$	$0001 \Rightarrow i_{f_6t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_6t_2}$
	t_3	$\neg y1_{j_4f_6t_3} \wedge\neg\ y1_{j_3f_6t_3} \wedge\ y1_{j_2f_6t_3} \wedge\ y1_{j_1f_6t_3}$	$0011 \Rightarrow i_{f_6t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_6t_3}$
f_7	t_1	$\neg y1_{j_4f_7t_1} \wedge\ y1_{j_3f_7t_1} \wedge\neg\ y1_{j_2f_7t_1} \wedge\ y1_{j_1f_7t_1}$	$0101 \Rightarrow i_{f_7t_1} = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5 \Rightarrow v_{5f_7t_1}$
	t_2	$\neg y1_{j_4f_7t_2} \wedge\ y1_{j_3f_7t_2} \wedge\neg\ y1_{j_2f_7t_2} \wedge\ y1_{j_1f_7t_2}$	$0101 \Rightarrow i_{f_7t_2} = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5 \Rightarrow v_{5f_7t_2}$
	t_3	$\neg y1_{j_4f_7t_3} \wedge\ y1_{j_3f_7t_3} \wedge\neg\ y1_{j_2f_7t_3} \wedge\ y1_{j_1f_7t_3}$	$0101 \Rightarrow i_{f_7t_3} = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5 \Rightarrow v_{5f_7t_3}$
f_8	t_1	$\neg y1_{j_4f_8t_1} \wedge\neg\ y1_{j_3f_8t_1} \wedge\neg\ y1_{j_2f_8t_1} \wedge\ y1_{j_1f_8t_1}$	$0001 \Rightarrow i_{f_8t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_8t_1}$
	t_2	$\neg y1_{j_4f_8t_2} \wedge\neg\ y1_{j_3f_8t_2} \wedge\neg\ y1_{j_2f_8t_2} \wedge\ y1_{j_1f_8t_2}$	$0001 \Rightarrow i_{f_8t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_8t_2}$
	t_3	$\neg y1_{j_4f_8t_3} \wedge\neg\ y1_{j_3f_8t_3} \wedge\ y1_{j_2f_8t_3} \wedge\ y1_{j_1f_8t_3}$	$0011 \Rightarrow i_{f_8t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_8t_3}$
f_9	t_1	$\neg y1_{j_4f_9t_1} \wedge\neg\ y1_{j_3f_9t_1} \wedge\ y1_{j_2f_9t_1} \wedge\neg\ y1_{j_1f_9t_1}$	$0010 \Rightarrow i_{f_9t_1} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2 \Rightarrow v_{2f_9t_1}$
	t_2	$\neg y1_{j_4f_9t_2} \wedge\neg\ y1_{j_3f_9t_2} \wedge\ y1_{j_2f_9t_2} \wedge\ y1_{j_1f_9t_2}$	$0011 \Rightarrow i_{f_9t_2} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_9t_2}$
	t_3	$\neg y1_{j_4f_9t_3} \wedge\neg\ y1_{j_3f_9t_3} \wedge\ y1_{j_2f_9t_3} \wedge\ y1_{j_1f_9t_3}$	$0011 \Rightarrow i_{f_9t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_9t_3}$
f_{10}	t_1	$\neg y1_{j_4f_{10}t_1} \wedge\neg\ y1_{j_3f_{10}t_1} \wedge\ y1_{j_2f_{10}t_1} \wedge\neg\ y1_{j_1f_{10}t_1}$	$0010 \Rightarrow i_{f_{10}t_1} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2 \Rightarrow v_{2f_{10}t_1}$
	t_2	$\neg y1_{j_4f_{10}t_2} \wedge\ y1_{j_3f_{10}t_2} \wedge\ y1_{j_2f_{10}t_2} \wedge\neg\ y1_{j_1f_{10}t_2}$	$0110 \Rightarrow i_{f_{10}t_2} = 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6 \Rightarrow v_{6f_{10}t_2}$
	t_3	$\neg y1_{j_4f_{10}t_3} \wedge\ y1_{j_3f_{10}t_3} \wedge\ y1_{j_2f_{10}t_3} \wedge\neg\ y1_{j_1f_{10}t_3}$	$0110 \Rightarrow i_{f_{10}t_3} = 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6 \Rightarrow v_{6f_{10}t_3}$

Table 20. Model Solution Performance: Case Study 1

model statistics	value
number of discrete variables	6104
number of positive variables	5599
number of constraints	7811
OV	2218.27
execution time (s)	54.42

Table 21. Probability of Failure of Supplier j in the Range or Proportion r of the Ordered Quantity (p_{jr}): Case Study 2

	failure (r)			
supplier	10%	40%	70%	100%
j_1	57%	30%	10%	3%
j_2	50%	30%	13%	7%
j_3	50%	25%	15%	10%
j_4	53%	28%	15%	4%

Additional results are reported in Tables 17−20.

4.2. Case Study 2. According to the probabilistic distribution of the undelivered expected quantities in Figure 1, in this second case study, suppliers can fail partially, not delivering certain portions of the quantity ordered.

Table 23. Expected Quantity of Family f in Period t (eq_{ft}): Case Study 2

	time period		
f	t_1	t_2	t_3
f_1	65.0	172.6	157.4
f_2	137.0	175.0	180.0
f_3	11.4	53.6	60.0
f_4	70.0	120.0	130.0
f_5	60.0	105.0	110.0
f_6	170.0	150.0	180.0
f_7	168.0	200.0	200.0
f_8	110.0	145.0	210.0
f_9	70.0	250.0	220.0
f_{10}	75.0	145.0	200.0

4.2.1. Model Parameters. All data and model parameters are the same as in case study 1 except for the failure distribution, which is provided in Table 21.

4.2.2. Model Solution. In Table 22, several decisions are presented. In the first two columns are shown which materials k are selected for each family f in period t for supplier j; this information is given by the variable $y2_{jfkt}$. In the intersection of rows and columns are presented the quantities of material k

Table 22. Materials Purchased, Quantities Ordered, and Contracts and Suppliers Selected in Each Time Period: Case Study 2

		j_1			j_2			j_4		
f	k	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
f_1	k_2	–	$30 - c_1$	–	–	–	–	$82.6- c_3$	$188.2 - c_3$	$200 - c_3$
f_2	k_7	–	–	–	–	$145.7 - c_3$	$152.2 - c_3$	$174.1- c_3$	$80 - c_3$	$80 - c_3$
f_3	k_8	–	–	–	–	–	$78 - c_3$	–	–	–
f_3	k_9	$30 - c_1$	$49.7 - c_1$	–	–	–	–	–	–	–
f_4	k_{11}	$85.8 - c_3$	$147.1 - c_2$	$159.3 - c_2$	–	–	–	–	–	–
f_5	k_{17}	$73.5 - c_3$	$128.7 - c_2$	$134.8 - c_2$	–	–	–	–	–	–
f_8	k_{19}	$131.2 - c_3$	$183.8 - c_2$	$143.4 - c_2$	–	–	–	–	–	–
f_6	k_{20}	–	–	–	–	–	–	$80 - c_3$	–	$80 - c_3$
f_7	k_{22}	$128.7 - c_3$	$174.4 - c_2$	$174.4 - c_2$	$75 - c_3$	–	$75 - c_3$	$80 - c_3$	–	–
f_8	k_{24}	$134.8 - c_3$	$177.7 - c_2$	$180.2 - c_2$	–	–	–	–	–	–
f_8	k_{25}	–	–	–	–	–	–	–	–	$80 - c_3$
f_9	k_{27}	$85.8 - c_3$	$117.9 - c_2$	$81.1 - c_2$	–	$200 - c_3$	$200 - c_3$	–	–	–
f_{10}	k_{30}	–	–	–	–	–	$75 - c_3$	$95.3 - c_3$	$184.2 - c_3$	$180.9 - c_3$

Table 24. Expected Sales of Family f in Period t (d_{ft}): Case Study 2

	time period		
f	t_1	t_2	t_3
f_1	105	170	160
f_2	182	175	180
f_3	50	55	60
f_4	110	120	130
f_5	100	105	110
f_6	200	150	180
f_7	200	200	200
f_8	150	145	210
f_9	100	250	220
f_{10}	120	145	200

ordered from supplier j in period t (variable q_{fjkt}) and the contract selected c (variable $y3_{jckt}$). Note that, when the quantity ordered is not null, this also means that the supplier j has been selected for family f ($y1_{fkt}$).

Additional model results are provided in Tables 23−26.

These two illustrative case studies show some important features about the formulation approach. First, note the short execution times for both cases, in addition to the numbers of equations and variables handled, most of them discrete variables. From another point of view, the case studies also show that a very small change in the uncertainty representation has a significant impact on the solution obtained. This stresses that modeling uncertainty is a remarkable subject regarding the supply process decision making. On the other hand, they illustrate that the accuracy of the data used is also an extremely important point to achieve a reliable solution. In fact, a mistake in the data could give results that are actually far from a "real optimal solution", with the consequence of making wrong decisions. With these ideas in mind, a sensitivity analysis was made in the next sections to evaluate how these parameters influence the optimal solution.

Table 26. Model Solution Performance: Case Study 2

model statistics	value
number of discrete variables	6104
number of positive variables	5599
number of constraints	7811
OV	2029.31
execution time (s)	54.47

Table 27. Probability of Failure of Supplier j in the Range or Proportion r of the Ordered Quantity (p_{jr}): Case Study 2

	failure (r)			
supplier	10%	40%	70%	100%
j_1	45%	35%	15%	5%
j_2	45%	35%	15%	5%
j_3	45%	35%	15%	5%
j_4	45%	35%	15%	5%

5. Sensitivity Analysis

The sensitivity analysis was performed considering that the failure probabilities were the same for all suppliers j in range r, as shown in Table 27. The idea behind this assumption is to disregard the effect of uncertainty on the optimal solution and focus on the economic issues. This kind of analysis will strengthen the solution obtained and suggest tactical strategies for facing negotiation processes with suppliers.

5.1. Model Solution. A comparison of results for supplier selection is provided in Tables 28 and 29. The gray boxes in Table 28 represent decisions made by the model for case study 2 that are no longer optimal for the sensitivity analysis. The gray boxes in Table 29 represent new optimal decisions, and the white boxes represent those decisions that were made in the case study and are still valid.

It can be seen that the uncertainty significantly conditions the final solution. One remarkable conclusion is that supplier j_3 is not chosen in the original model because it is the riskiest supplier, but when the uncertainties are equalized, this supplier

Table 25. Supplier Selection for Family$f$ in Period t ($y1_{jft}$) and Its Relationship with the Variable v_{ift}: Case Study 2

f	t	supplier selection for family f in period t	binary representation and Boolean variable v_{ift}
f_1	t_1	$y1_{j_4f_1t_1} \wedge \neg y1_{j_3f_1t_1} \wedge \neg y1_{j_2f_1t_1} \wedge \neg y1_{j_1f_1t_1}$	$1000 \Rightarrow i_{f_1t_1} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 \Rightarrow v_{8f_1t_1}$
	t_2	$y1_{j_4f_1t_2} \wedge \neg y1_{j_3f_1t_2} \wedge \neg y1_{j_2f_1t_2} \wedge y1_{j_1f_1t_2}$	$1001 \Rightarrow i_{f_1t_2} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9 \Rightarrow v_{9f_1t_2}$
	t_3	$y1_{j_4f_1t_3} \wedge \neg y1_{j_3f_1t_3} \wedge \neg y1_{j_2f_1t_3} \wedge \neg y1_{j_1f_1t_3}$	$1000 \Rightarrow i_{f_1t_3} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 \Rightarrow v_{8f_1t_3}$
f_2	t_1	$y1_{j_4f_2t_1} \wedge \neg y1_{j_3f_2t_1} \wedge \neg y1_{j_2f_2t_1} \wedge \neg y1_{j_1f_2t_1}$	$1000 \Rightarrow i_{f_2t_1} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 \Rightarrow v_{8f_2t_1}$
	t_2	$y1_{j_4f_2t_2} \wedge \neg y1_{j_3f_2t_2} \wedge y1_{j_2f_2t_2} \wedge \neg y1_{j_1f_2t_2}$	$1010 \Rightarrow i_{f_2t_2} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 10 \Rightarrow v_{10f_2t_2}$
	t_3	$y1_{j_4f_2t_3} \wedge \neg y1_{j_3f_2t_3} \wedge y1_{j_2f_2t_3} \wedge \neg y1_{j_1f_2t_3}$	$1010 \Rightarrow i_{f_2t_3} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 10 \Rightarrow v_{10f_2t_3}$
f_3	t_1	$\neg y1_{j_4f_3t_1} \wedge \neg y1_{j_3f_3t_1} \wedge \neg y1_{j_2f_3t_1} \wedge y1_{j_1f_3t_1}$	$0001 \Rightarrow i_{f_3t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_3t_1}$
	t_2	$\neg y1_{j_4f_3t_2} \wedge \neg y1_{j_3f_3t_2} \wedge \neg y1_{j_2f_3t_2} \wedge y1_{j_1f_3t_2}$	$0001 \Rightarrow i_{f_3t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_3t_2}$
	t_3	$\neg y1_{j_4f_3t_3} \wedge \neg y1_{j_3f_3t_3} \wedge y1_{j_2f_3t_3} \wedge \neg y1_{j_1f_3t_3}$	$0010 \Rightarrow i_{f_3t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2 \Rightarrow v_{2f_3t_3}$
f_4	t_1	$\neg y1_{j_4f_4t_1} \wedge \neg y1_{j_3f_4t_1} \wedge \neg y1_{j_2f_4t_1} \wedge y1_{j_1f_4t_1}$	$0001 \Rightarrow i_{f_4t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_4t_1}$
	t_2	$\neg y1_{j_4f_4t_2} \wedge \neg y1_{j_3f_4t_2} \wedge \neg y1_{j_2f_4t_2} \wedge y1_{j_1f_4t_2}$	$0001 \Rightarrow i_{f_4t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_4t_2}$
	t_3	$\neg y1_{j_4f_4t_3} \wedge \neg y1_{j_3f_4t_3} \wedge \neg y1_{j_2f_4t_3} \wedge y1_{j_1f_4t_3}$	$0001 \Rightarrow i_{f_4t_3} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_4t_3}$
f_5	t_1	$\neg y1_{j_4f_5t_1} \wedge \neg y1_{j_3f_5t_1} \wedge \neg y1_{j_2f_5t_1} \wedge y1_{j_1f_5t_1}$	$0001 \Rightarrow i_{f_5t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_5t_1}$
	t_2	$\neg y1_{j_4f_5t_2} \wedge \neg y1_{j_3f_5t_2} \wedge \neg y1_{j_2f_5t_2} \wedge y1_{j_1f_5t_2}$	$0001 \Rightarrow i_{f_5t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_5t_2}$
	t_3	$\neg y1_{j_4f_5t_3} \wedge \neg y1_{j_3f_5t_3} \wedge \neg y1_{j_2f_5t_3} \wedge y1_{j_1f_5t_3}$	$0001 \Rightarrow i_{f_5t_3} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_5t_3}$
f_6	t_1	$y1_{j_4f_6t_1} \wedge \neg y1_{j_3f_6t_1} \wedge \neg y1_{j_2f_6t_1} \wedge y1_{j_1f_6t_1}$	$1001 \Rightarrow i_{f_6t_1} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9 \Rightarrow v_{9f_6t_1}$
	t_2	$\neg y1_{j_4f_6t_2} \wedge \neg y1_{j_3f_6t_2} \wedge \neg y1_{j_2f_6t_2} \wedge y1_{j_1f_6t_2}$	$0001 \Rightarrow i_{f_6t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_6t_2}$
	t_3	$y1_{j_4f_6t_3} \wedge \neg y1_{j_3f_6t_3} \wedge \neg y1_{j_2f_6t_3} \wedge y1_{j_1f_6t_3}$	$1001 \Rightarrow i_{f_6t_3} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9 \Rightarrow v_{9f_6t_3}$
f_7	t_1	$y1_{j_4f_7t_1} \wedge \neg y1_{j_3f_7t_1} \wedge \neg y1_{j_2f_7t_1} \wedge y1_{j_1f_7t_1}$	$1001 \Rightarrow i_{f_7t_1} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9 \Rightarrow v_{9f_7t_1}$
	t_2	$\neg y1_{j_4f_7t_2} \wedge \neg y1_{j_3f_7t_2} \wedge y1_{j_2f_7t_2} \wedge y1_{j_1f_7t_2}$	$0011 \Rightarrow i_{f_7t_2} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_7t_2}$
	t_3	$\neg y1_{j_4f_7t_3} \wedge \neg y1_{j_3f_7t_3} \wedge y1_{j_2f_7t_3} \wedge y1_{j_1f_7t_3}$	$0011 \Rightarrow i_{f_7t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_7t_3}$
f_8	t_1	$\neg y1_{j_4f_8t_1} \wedge \neg y1_{j_3f_8t_1} \wedge \neg y1_{j_2f_8t_1} \wedge y1_{j_1f_8t_1}$	$0001 \Rightarrow i_{f_8t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_8t_1}$
	t_2	$\neg y1_{j_4f_8t_2} \wedge \neg y1_{j_3f_8t_2} \wedge \neg y1_{j_2f_8t_2} \wedge y1_{j_1f_8t_2}$	$0001 \Rightarrow i_{f_8t_2} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_8t_2}$
	t_3	$y1_{j_4f_8t_3} \wedge \neg y1_{j_3f_8t_3} \wedge \neg y1_{j_2f_8t_3} \wedge y1_{j_1f_8t_3}$	$1001 \Rightarrow i_{f_8t_3} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9 \Rightarrow v_{9f_8t_3}$
f_9	t_1	$\neg y1_{j_4f_9t_1} \wedge \neg y1_{j_3f_9t_1} \wedge \neg y1_{j_2f_9t_1} \wedge y1_{j_1f_9t_1}$	$0001 \Rightarrow i_{f_9t_1} = 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \Rightarrow v_{1f_9t_1}$
	t_2	$\neg y1_{j_4f_9t_2} \wedge \neg y1_{j_3f_9t_2} \wedge y1_{j_2f_9t_2} \wedge y1_{j_1f_9t_2}$	$0011 \Rightarrow i_{f_9t_2} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_9t_2}$
	t_3	$\neg y1_{j_4f_9t_3} \wedge \neg y1_{j_3f_9t_3} \wedge y1_{j_2f_9t_3} \wedge y1_{j_1f_9t_3}$	$0011 \Rightarrow i_{f_9t_3} = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3 \Rightarrow v_{3f_9t_3}$
f_{10}	t_1	$y1_{j_4f_{10}t_1} \wedge \neg y1_{j_3f_{10}t_1} \wedge \neg y1_{j_2f_{10}t_1} \wedge \neg y1_{j_1f_{10}t_1}$	$1000 \Rightarrow i_{f_{10}t_1} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 \Rightarrow v_{8f_{10}t_1}$
	t_2	$y1_{j_4f_{10}t_2} \wedge \neg y1_{j_3f_{10}t_2} \wedge \neg y1_{j_2f_{10}t_2} \wedge \neg y1_{j_1f_{10}t_2}$	$1000 \Rightarrow i_{f_{10}t_2} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 \Rightarrow v_{8f_{10}t_2}$
	t_3	$y1_{j_4f_{10}t_3} \wedge \neg y1_{j_3f_{10}t_3} \wedge y1_{j_2f_{10}t_3} \wedge \neg y1_{j_1f_{10}t_3}$	$1010 \Rightarrow i_{f_{10}t_3} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 10 \Rightarrow v_{10f_{10}t_3}$

Table 28. Supplier Selection for Case Study 2

j	f	t_1	t_2	t_3
j_1	f_1		1	
	f_3	1	1	
	f_4	1	1	1
	f_5	1	1	1
	f_6	1	1	1
	f_7	1	1	1
	f_8	1	1	1
	f_9	1	1	1
j_2	f_2		1	1
	f_3			1
	f_7		1	1
	f_9		1	1
	f_{10}			1
j_4	f_1	1	1	1
	f_2	1	1	1
	f_6	1		1
	f_7	1		
	f_8			1
	f_{10}	1	1	1

Table 29. Supplier Selection for Sensitivity Analysis

j	f	t_1	t_2	t_3
j_2	f_2	1	1	1
	f_3		1	1
	f_6	1	1	1
	f_7	1	1	1
	f_8	1	1	1
	f_9	1	1	1
	f_{10}	1	1	1
j_3	f_1		1	
	f_3	1		
	f_7	1	1	1
j_4	f_1	1	1	1
	f_2		1	1
	f_4	1	1	1
	f_5	1	1	1
	f_6	1		1
	f_8			1
	f_9		1	1
	f_{10}			1

is selected for materials provision. This information could help the company to suggest to this supplier to improve his/her delivery performance. Supplier j_2 is also selected for more families and periods, and a similar situation applies to supplier j_4. In contrast, supplier j_1, which was selected on many occasions in case study 2, is not chosen at all in this new scenario. This analysis suggests that this provider actually presents a good package of cost and risk, but from an exclusively economic–financial point of view, it is not very convenient. In the same way, the white boxes in the tables show robust decisions that are not affected by the changes proposed. Those results can be considered the safest ones because they are optimal for both uncertainty scenarios considered.

Other results are presented in Tables 30–33.

Finally, note also that the solution performance is again equivalent to the previous cases considered, demonstrating that the execution of the formulation is not data-dependent.

6. Conclusions

The relevance of the purchase and provision process, as well as its uncertainty in the supply chain, has motivated the present work. From an economic point of view, raw materials significantly impact the product costs, having a great influence on company profits. Furthermore, uncertainty due to machinery breakdowns, material shortages, capacity constraints, or political crises is always a challenging issue affecting the amount provided, prices of raw materials, and also the company performance. To decrease uncertainty and costs, companies establish contracts promoting longer relationships between them. In this work, the provision process is addressed by modeling contracts signed with suppliers and provision uncertainty. No similar work was found in the literature dealing with characteristics similar to those presented here. Managers can use solutions obtained by this approach to improve their negotiation process with suppliers.

The problem is modeled by nested disjunctions and logic propositions to cover the different decision levels considered. This feature facilitates a better understanding of the problem representation and leads to a straightforward formulation. In fact, one of the main advantages of the logic-based formulation is that it can be easily appreciated by company managers. On many occasions, the disjunctive formulation also presents a better solution performance than a traditional mathematical programming approach.

The provision uncertainty is included through the discrete failure probability distribution of each supplier. With this approach, a deterministic representation is obtained. Expected supplier failures are punished in the objective function by calculating the expected revenue losses due to the unsatisfied demand. The strength of this method is addressed by avoiding the typical nonconvexities of traditional probabilistic approaches and the large size of stochastic strategies due to the analysis of a large number of scenarios. A general formulation has been proposed by relating a combination of the selected suppliers with the corresponding value of the expected quantity (eq_{ft}) of family f in period t. This approach is completed by the definition of the Boolean variable v_{ift}, which represents the set of selected suppliers giving a certain expected quantity. This is very useful to save Boolean variables in the formulation and to speed the problem solution.

With the aim of representing the actual context of Latin American economies, real contract types were modeled, and some original considerations were included in the objective function such as the inflationary costs, payment terms, and discount rate. This approach enables a manufacturing company to select suppliers, quantities to order, materials from families, and contract types in each time period considering the effects of provision uncertainty. The objective is to minimize the present purchase, stock costs, and expected losses due to unsatisfied demand. Three examples were solved, showing a robust and rapid performance, as the resolution time was around 1 min for all of the scenarios modeled. The sensitivity analysis shows that uncertainty has a significant impact on the final solution. This analysis helps managers when negotiating with suppliers, encouraging reliable but expensive suppliers to decrease their prices or promoting better performance from less expensive

Table 30. Supplier Selection for Family *f* in Period *t* ($y1_{jft}$) and Its Relationship with the Variable v_{ift}: Anaylsis Study

		j_2			j_3			j_4		
f	k	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
f_1	k_2	–	–	–	–	$32 - c_1$	–	$84.5 - c_3$	$199.4 - c_3$	$200 - c_2$
f_2	k_7	$179.1 - c_3$	$148.8 - c_3$	$155.3 - c_3$	–	–	–	–	$80 - c_3$	$80 - c_3$
f_3	k_8	–	$53 - c_1$	$78.4 - c_3$	–		–	–	–	–
f_3	k_9	–	–	–	$32 - c_1$	–	–	–	–	–
f_4	k_{11}	–	–	–	–	–	–	–	–	–
f_4	k_{12}	–	–	–	–	–	–	$91.5 - c_3$	$156.9 - c_3$	$169.9 - c_3$
f_5	k_{17}	–	–	–	–	–	–	$80 - c_3$	$135.7 - c_3$	$143.8 - c_3$
f_6	k_{19}	–	–	–	–	–	–	–	–	–
f_6	k_{20}	$142.2 - c_3$	$196.1 - c_3$	$155.3 - c_3$	–	–	–	$80 - c_3$	–	$80 - c_3$
f_7	k_{22}	$75 - c_3$	$75 - c_3$	$75 - c_3$	$144.6 - c_3$	$186.4 - c_2$	$186.4 - c_2$	–	–	–
f_8	k_{24}	$143.8 - c_3$	$189.5 - c_3$	$194.5 - c_3$	–	–	–	–	–	–
f_8	k_{25}	–	–	–	–	–	–	–	–	$80 - c_3$
f_9	k_{27}	$91.5 - c_3$	$200 - c_3$	$200 - c_3$	–	–	–	–	$126.8 - c_3$	$87.6 - c_3$
f_{10}	k_{30}	$98 - c_3$	$189.5 - c_3$	$181.4 - c_3$	–	–	–	–	–	$80 - c_2$

Table 31. Expected Quantity of Family *f* in Period *t* (eq_{ft}): Anaylsis Study

	time period		
f	t_1	t_2	t_3
f_1	65	177	153
f_2	137	175	180
f_3	24.8	40.52	60
f_4	70	120	130
f_5	61.2	103.8	110
f_6	170	150	180
f_7	168	200	200
f_8	110	145	210
f_9	70	250	220
f_{10}	75	145	200

Table 32. Expected Sales for Family *f* in Period *t* (d_{ft}): Analysis Study

	time period		
f	t_1	t_2	t_3
f_1	105	170	160
f_2	182	175	180
f_3	50	55	60
f_4	110	120	130
f_5	100	105	110
f_6	200	150	180
f_7	200	200	200
f_8	150	145	210
f_9	100	250	220
f_{10}	120	145	200

Table 33. Model Solution Performance: Analysis Study

model statistics	value
number of discrete variables	6104
number of positive variables	5599
number of constraints	7811
OV	2080.66
execution time (s)	58.83

providers. This study also shows which suppliers offer the best package in terms of cost and risk and, in the same way, which ones represent robust decisions not affected by the changes proposed. All of this additional information enriches the decision-making process and stresses the approach developed.

Notation

Indices

c = contract
e = subperiod time
f = material family
i = supplier subset selected for a provision
j = supplier
k = material
r = supply failure range
t, t' = time period

Sets

C = set of contracts
E = set of time subperiods
F = set of material families
FK_{fk} = set that defines which materials k belong to family f
I = set of supplier subset selected for a provision
J = set of suppliers
K = set of materials
R = set of supply failure ranges
T = set of time periods
$TP_{ctt'}$ = set defining that purchase order in period t must be paid in period t' according to contract c

Parameters

Ap_{ft} = average price of family f in period t
$COSTavg_{ft}$ = average cost of family f in period t
FD_{ft} = demand of family f in period t
IS_f = initial stock of family f
MS = percentage of the raw material average costs to calculate stock costs
p_{jr} = probability of supplier j to fail in the range or proportion r of the quantity ordered
PC_{jkt} = regular price of material k sold by supplier j in period t
$Qmax_{jkt}$ = maximum quantity to order of material k to supplier j in period t
$Qmin_{cj}$ = minimum quantity to order according to supplier j and contract c
RR = discount rate
SC = stock capacity
δ_{jc} = discount (or interest rate) offered by supplier j according to contract c
μ = demand upper bound

Positive Variables

d_{ft} = expected sales for family f in period t
eq_{ft} = expected quantity of family f in period t
m_{jckt} = amount of money to be paid in period t due to the purchase of material k in contract c to supplier j (in the same period or before, according to $TP_{ctt'}$)
q_{jkt} = quantity of material k ordered from supplier j in period t
qf_{jft} = quantity of material family f ordered from supplier j in period t

qr_{jrft} = expected quantity of family f from supplier j in period t according to the failure range r

s_{ft} = expected initial stock of family f in period t

savg_{ft} = expected average stock of family f in period t

sd_{fet} = expected initial stock of family f in subperiod e of period t

w_{jckt} = purchase cost of contract c due to the quantity of material k ordered from supplier j in period t

Boolean Variables

$y1_{jft}$ = true if supplier j is selected to buy family f in period t

$y2_{jfkt}$ = true if material k is selected from family f to be purchased from supplier j in period t

$y3_{jckt}$ = true if contract c is selected to be signed with supplier j to purchase material k in period t

v_{ift} = true is the supplier conformation given by subindex i is selected for family f in period t

Binary Variable

$Y1_{jft}$ = 1 if supplier j is selected to buy family f in period t and 0 otherwise

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