Computers and Chemical Engineering 34 (2010) 1705–1718

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Inventory and delivery optimization under seasonal demand in the supply chain

Maria Analia Rodriguez, Aldo Vecchietti*

INGAR (CONICET – UTN), Avellaneda 3657, Santa Fe 3000, Argentina

ARTICLE INFO

Article history:
Received 30 April 2009
Received in revised form 8 February 2010
Accepted 11 February 2010
Available online 18 February 2010

Keywords:
Inventory optimization
Purchase decisions
Delivery optimization
Supply chain management

ABSTRACT

This work deals with the inventory, purchase and delivery optimization problem in the supply chain. The formulation of two problems is presented involving several decision levels. The first one optimizes the company inventory and purchase tasks in a medium-term horizon planning, assuming that the total amount purchased is delivered at the beginning of each period. Then, in a more detailed formulation, the purchased amount is distributed among several deliveries giving rise to a non-linear non-convex problem. Some transformation techniques are evaluated to overcome the non-convexities in order to find a global solution in a reasonable execution time. Finally, the results obtained considering some possible scenarios are analyzed and compared.

1. Introduction

Global market conditions promote an increasing interest in integrating strategic, tactical and operational decision-making in order to achieve enterprise-wide optimization. An important number of remarkable research works supports this idea (Grossmann, 2005; Varma, Reklaitis, Blau, & Pekny, 2007). Models and methods have been developed to solve enterprise optimization problems from different points of view (Sarker & Diponegoro, 2009; Sousa, Shah, & Papageorgiou, 2008; You & Grossmann, 2008). Some articles propose the integration of purchase decisions with inventory management and stress the relevance of these issues in the global enterprise optimization (Al-Ameri, Shah, & Papageorgiou, 2008; Mohebbi & Hao, 2006).

The significance of inventory management in the company profits is given by the trade-off between customer satisfaction and capital invested. From an inventory cost perspective, the best condition would be a response-based supply chain where a zero inventory strategy is handled (Bowersox, Closs, & Cooper, 2007). Many companies have realized that inventory cost can be reduced by implementing Just-In-Time (JIT) methodology in their production-distribution channel (Chen & Chang, 2007; Gjerdrum, Shah, & Papageorgiou, 2002). Unfortunately, it is not always possible to achieve that target. Effective inventory management is mostly conditioned by the production system as well as by the demand uncertainties.

From the demand point of view, when products are made-to-stock planning the material inventory strategy depends on the production plan. Even though demands can be predicted with some degree of certainty, the purchase of materials becomes more complex when products are made-to-order. In this case, some other issues must be negotiated with suppliers and formulated in the model, such as the number of deliveries and their deadlines, or order size, to mention just a few of them.

Suppliers could have insufficient capacity to provide the appropriate materials or deliver them in time. This is especially true when the demand pattern is seasonal. Despite the challenging characteristics of inventory planning and optimizing under this seasonal context, the literature on the topic is not so profuse. Chen and Chang (2007) propose two methods for solving a seasonal demand problem with variable lead time and resource constraints. The variable lead time is solved using a linear programming relaxation. Then, a mixed integer program with linearization techniques is constructed for a seasonal demand problem.

As regards production conditions, this work addresses the problem of deciding which materials must be processed so as to satisfy customer orders. In several industries, such as the production of paper, furniture, textiles and food among others, the use of material family is a regular practice (Rodriguez & Vecchietti, 2009). The term "material family" is used in this paper to refer to a set of alternative raw materials to manufacture the same product. Companies use a set of possible raw materials (material family), in order to satisfy product demand. This production policy gives flexibility to purchase decisions, allowing a set of possible formulations for the same final product. In this situation, if there is no stock of the requested material according to the customer order, this is satisfied by using a

* Corresponding author. Tel.: +54 342 453 5568; fax: +54 342 455 3439.
E-mail address: aldovec@santafe-conicet.gov.ar (A. Vecchietti).

0098-1354/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compchemeng.2010.02.009

material of better quality from the family, thus increasing the production cost, commonly called upgrade cost. This opens two main alternatives to analyze. The first one is to anticipate the demand buying the corresponding raw material which increases inventory costs. The second one is to use better quality and expensive materials to satisfy the demand, thus increasing production costs. For these reasons, decisions concerning material purchase and inventory management are really relevant and should not be considered as decoupled subjects.

Many authors stress the importance of considering supply contracts to optimize purchase decisions. Naraharisetti, Adhitya, Karimi, and Srinivasan (2008) highlight the need for rigorous quantitative models that take this issue into account. As explained by Bansal, Karimi, and Srinivasan (2007) these contracts are agreements between the company and a supplier for a fixed period of time, stipulating certain terms, conditions and commitments. Park, Park, Mele, and Grossmann (2006) agree that the signing of provision contracts with suppliers is a common practice in industry and design a disjunctive model to select the best contract type. One motivation for a supply contract is to share risks related to demand, supply, delivery, inventory and price uncertainties (Bansal et al., 2007). Actually, including a supply contract in the purchase optimization model could provide an important background when negotiating the best supply policy with the right suppliers. Though purchase commitments tend to increase order size, the decision of when and how much material to distribute should also be considered as an optimization target. This is motivated by the existence of market competitive conditions forcing the producers to look for new ways of reducing the inventory (Björk & Carlsson, 2007). In this sense, delivery decisions, inventory and purchase planning present different conflicting objectives to achieve. However, it is not possible to optimize all these aspects simultaneously. In fact, any solution must handle a trade-off among them and, consequently, it should be treated as a multi-objective optimization problem. Several articles dealing with the context of multi-objective optimization have been published. Some authors propose different algorithms to solve this type of problems, e.g. the weighting method reported by Zadeh (1963), or the ε-constraint method reported by Haimes (1973). More recently, Jia and Ierapetritou (2007) present a multi-objective optimization framework to solve a short-term scheduling under uncertainty. You and Grossmann (2008) design a responsive supply chain optimizing net present value as the economic criterion and lead time as the responsive one. They also use the ε-constraint technique to treat this problem. Guillén, Mele, Bagajewicz, Espuña, and Puigjaner (2005) formulate the problem of designing the supply chain as a multi-objective stochastic MILP model, which is solved using the ε-constraint method. They propose not only the maximization of the supply chain profits and customer satisfaction but also the financial risk minimization. Gao and Tang (2003) develop a multi-criteria linear programming model to solve the purchasing problem with minimum cost, scrap ratio and delivery tardiness. They use a weighting algorithm to solve the problem. This method has also been selected to be applied in our formulation to optimize the conflicting targets: purchase, inventory and delivery costs.

The model formulation presents non-linearities due to some delivery decisions such as variable order size and number of shipments. A linearization transformation is proposed in order to guarantee an optimal global solution. The results obtained with this procedure are then compared to those obtained from Harjunkoski's method (Harjunkoski, Westerlund, & Pörn, 1999).

In this paper made-to-order products are studied with a very short lead time expected by the customers. The problem deals with the trade-off decision between increasing stock and upgrading production. This context is even more complicated because product demand is influenced by seasonal variations. An optimization approach to solve material purchase; inventory management and delivery problems for the supply chain is presented. Diverse materials are purchased from various manufacturers signing different contract types. The contract models used in this work represent different situations in terms of prices, minimum quantities to order and payment conditions. Materials can be provided by various suppliers with diverse costs. The main target of this work is to optimize the inventory and delivery management integrated with material purchase decisions and handling different contract types with suppliers.

This article is outlined as follows: in the next section the problem is formulated as a non-linear disjunctive model considering several relevant decision levels. A discrete time representation determines a number of periods in the horizon planning while a continuous time representation is used within those periods. In the third section, results show a sensitivity analysis to emphasize the importance of the approach presented in this work. Finally, the conclusions underline the main contributions of this work.

2. Problem formulations

In this section, the formulation of two different problems is presented. The first one consists in the inventory and purchase optimization in a medium-term horizon planning. In the second, new variables and constraints are added in order to include delivery decisions. These changes in the original formulation make this problem non-linear and non-convex.

2.1. Inventory and purchase optimization

The very first problem considered in this work (*PI* problem) is the inventory and material purchase planning over a seasonal demand environment. It deals with several decision levels such as selection of suppliers, materials, purchase contract and inventory levels over a discrete horizon plan. The main constraints restrict demand satisfaction, define inventory levels, calculate the average stock, etc.

2.1.1. Objective function

The objective function defines the maximization of the actualized profits over the time horizon (Eq. (1))

$$Max \sum_t \frac{\sum_p Demand_{pt} \cdot price_{pt} - \left(\sum_j \sum_c \sum_k m_{jckt} + \sum_f savg_{ft} \cdot COSTavg_{ft} \cdot MS \right)}{(1+RR)^t} \quad (1)$$

Revenues are calculated multiplying product demand ($Demand_{pt}$) by the product price ($price_{pt}$). The costs involved are the corresponding material purchase costs and the inventory costs. The money outflow due to the purchase of materials is determined as the positive variable m_{jckt}. It is calculated considering material k purchased from supplier j to be paid in period t according to payment policy of contract c. This variable also includes the fixed cost assumed when selecting the corresponding contract c for supplier j. In the next term, the inventory carrying cost is calculated. For this purpose, $savg_{ft}$ is a positive variable representing the average material stock of family f in period t, calculated in (7). Parameter $COSTavg_f$ corresponds to the average cost of family f and parameter MS indicates a percentage of the raw material average costs. This percentage estimates the average stock costs including the following major components: capital cost, storage cost, obsolescence cost, quality cost. As a general practice, these costs are considered as a single inventory cost rate, represented as a percentage of the material price per time unit (van Ryzin, 2001). The difference between the revenues and the total costs gives the total profits. In order to discount this projected profits, a return rate *RR* is used, corresponding to the capital cost which is calculated with the Capital Asset Pricing Model (Sharpe, 1964).

2.1.2. Model constraints

Material stock constraints are given by Eqs. (2)–(7)

$$s_{ft} = s_{f(t-1)} + \sum_{j} \sum_{k \in FK_{fk}} q_{jk(t-1)} - d_{f(t-1)} \quad \forall f \in F, \;\; \forall t \geq t_2 \in T \tag{2}$$

The initial stock expected of family *f* in period *t*, s_{ft}, is calculated by (2) as the initial stock of family *f* in period $t-1$ plus the amount ordered, $q_{jk(t-1)}$, of materials *k* of family *f* from all suppliers minus the material family consumption in the previous period $d_{f(t-1)}$, calculated by Eq. (3)

$$d_{ft} = \sum_{p \in PL_{plf}} \sum_{l \in PL_{plf}} Demand_{pt} \cdot \alpha_{plf} \tag{3}$$

A product *p* is formed by several components *l* each of which consumes one material family. It is possible that more than one component uses a certain family *f*. Parameter α_{plf} indicates the amount of family *f* corresponding to component *l* for product *p*. The set PL_{plf} relates each component *l* of product *p* with material family *f*. Material consumption d_{ft} is calculated in Eq. (3) taking into account the product demand ($Demand_{pt}$) and the individual consumption of material of each product and component (α_{plf})

$$\sum_{f} s_{ft} \leq SC \quad \forall t \in T \tag{4}$$

Capacity availability for keeping materials in stock is actually finite; Eq. (4) limits the amount in stock according to stock capacity *SC* parameter

$$s_{ft_1} = IS_f \quad \forall f \in F \tag{5}$$

When considering plants that are already operating, material stock in the first period planned is not null. For that reason, Eq. (5) defines the initial stock of family *f* in period t_1 equal to IS_f, a parameter in the model

$$s_{ft} \geq SS \quad \forall f \in F, \;\; \forall t \in T \tag{6}$$

Eq. (6) defines that the material family stored must be bigger than a minimum amount given as a safety stock *SS*.

Inventory handling costs must be calculated considering material average stocks. Constraint (7) is formulated to calculate the average stock $savg_{ft}$ of family *f* in period *t*, used in the objective function. Assuming a constant demand rate and that the whole material order is delivered at the beginning of period *t*, the average stock of family *f* is given by:

$$savg_{ft} = \frac{s_{ft} + \sum_{j} \sum_{k \in FK_{fk}} q_{jkt} + s_{f(t+1)}}{2} \quad \forall f \in F, \;\; \forall t \in T \tag{7}$$

2.1.3. Modeling purchase decisions

The decision process includes several levels. Each level is modeled by disjunctions which are then associated using logic equations and discrete variables

$$\begin{bmatrix} y1_{jft} \\ \sum_{k \in FK_{fk}} q_{jkt} \leq \sum_{k \in FK_{fk}} Qmax_{jkt} \end{bmatrix} \vee \begin{bmatrix} \neg y1_{jft} \\ \sum_{k \in FK_{fk}} q_{jkt} = 0 \end{bmatrix}$$

$$\forall j \in J, \;\; \forall f \in F, \;\; \forall t \in T \tag{8}$$

Disjunction (8) expresses the selection of which material families *f* is bought from each supplier *j* in period *t*. This choice represents the first level in the decision process using Boolean variable $y1_{jft}$. If this variable is false then family *f* is not ordered from supplier *j* in period *t*. If it is true, the total quantity ordered must be lower than the maximum capacity of supplier *j*. In general, this upper limit decreases in high season periods; however, the reduction is not the same for all materials in the family. As presented in the results section, the availability of the least expensive materials tends to decrease more than the most expensive ones

$$\begin{bmatrix} y2_{jkt} \\ q_{jkt} \leq Qmax_{jkt} \end{bmatrix} \vee \begin{bmatrix} \neg y2_{jkt} \\ q_{jkt} = 0 \end{bmatrix} \quad \forall j \in J, \;\; \forall k \in FK_{fk}, \;\; \forall t \in T \tag{9}$$

In the following level given by disjunction (9), variable $y2_{jkt}$ indicates if material *k* is selected from family *f*, according to FK_{fk}, in period *t*. Set FK_{fk} defines which materials correspond to each family *f*. In this term, the quantity ordered of material *k* (q_{jkt}) in period *t* cannot be greater than the supplier capacity for that material and period. If a material *k* is not selected from family *f* then amount to order is zero from supplier *j* in *t*. Note that more than one material could be selected for each family. This situation will occur when the most convenient material presents lower availability ($Qmax_{jkt}$) than required

$$\bigvee_{\substack{c \in C \\ (c,t,t') \in TP_{ctt'}}} \begin{bmatrix} y3_{jckt} \\ q_{jkt} \geq Qmin_{cj} \\ w_{jckt} = q_{jkt} \cdot PC_{jkt} \cdot \left(1 - \delta_{jc}\right) + FC_c \\ w_{jckt} = m_{jckt'} \end{bmatrix}$$

$$\forall j \in J, \;\; \forall k \in K, \;\; \forall t \in T \tag{10}$$

where $c \in \{c_1, c_2, c_3, c_4\}$; $\delta_{jc_1} = 0$; $\delta_{jc_2} > 0$; $\delta_{jc_3} > \delta_{jc_2}$ and $\delta_{jc_4} < 0$.

Disjunction (10) selects contract type *c* using variable $y3_{jckt}$ for each material *k*, supplier *j* in period *t*. $Qmin_{cj}$ is a parameter representing the minimum quantity of material to be ordered to the supplier *j* for contract *c*. Set $TP_{ctt'}$ establishes that the purchase ordered in period *t* must be paid in period t' according to contract *c*.

The second constraint calculates the costs w_{jckt} of purchasing material *k* according to the amount bought q_{jkt}, price PC_{jkt} and corresponding contract discount ($\delta_{jc} > 0$) or interest rate ($\delta_{jc} < 0$). This cost also includes the fixed cost FC_c to be paid whenever contract *c* is signed.

In the third constraint, positive variable $m_{jckt'}$ determines the amount paid for buying material *k* to supplier *j* in period t' according to the payment policy of contract *c*. In this case, according to $TP_{ctt'}$, when $t = t'$ there is no financial benefit. On the contrary, when $t < t'$ the payment term is given by the difference between *t* and t'. In the first three contracts, period *t* is equal to t'; however, the fourth one has a longer payment term. Two different variables are used in disjunction (10), $m_{jckt'}$ and w_{jckt}, in order to distinguish the cost concept given by variable w_{jckt} from the effective money outflows calculated as $m_{jckt'}$.

The first contract models the purchase with no requirements or benefits. For this reason, the cost is calculated using the regular price and the discount is null. The second presents a minimum order size given by $Qmin_{c_2j}$. It also has a discount applied to the total amount ordered. The third one has a bigger amount to order, $Qmin_{c_3j}$, and a bigger discount, too. In order to achieve longer

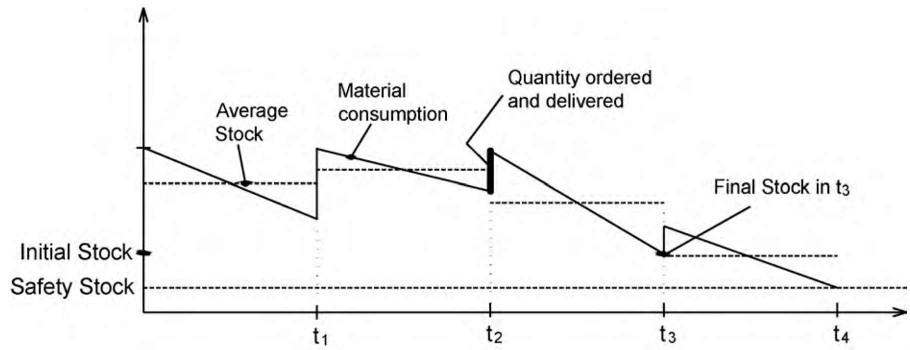

Fig. 1. Inventory evolution when the whole purchase is delivered at the beginning of the period.

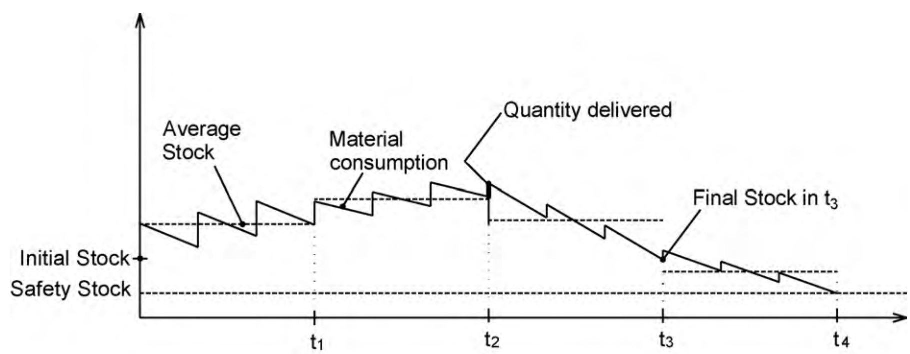

Fig. 2. Inventory evolution considering three deliveries in a time period.

business relationships with the suppliers, Eq. (11) constrains the selection of this contract type only if material k has been already ordered to the same supplier in the previous period. In the latter contract type $Qmin_{c_4j}$ has the highest value and, due to the payment term offered the second equation considers an interest rate.

Decisions in the purchase process must satisfy some contract rules shown in Eqs. (11) and (12). Additionally, decisions presented in disjunctions (8)–(10) are related by a hierarchical structure implemented through Eqs. (13)–(16). So as to express these logical relations by means of mathematical formulation, binary variables are introduced replacing the Boolean variables. For instance, Boolean variable $y3_{jckt}$ is now reformulated as binary variable $Y3_{jckt}$, variable $y2_{jkt}$ is represented as $Y2_{jkt}$ and $y1_{jft}$ is now presented as binary variable $Y1_{jft}$.

Eq. (11) represents the fact that contract c_3 can be chosen for material k of supplier j only if one of the contract types (except c_1) has been selected in the previous period for same material and supplier. On the contrary contract c_3 is not selected

$$\sum_{\substack{c \\ \forall c \neq c_1}} Y3_{jck\,t-1} \geq Y3_{jc_3kt} \quad \forall j \in J, \ \forall k \in K, \ \forall t \in T \tag{11}$$

For the same reason, in (12) contract c_3 cannot be chosen for initial period t_1 since it is the first period in the planning horizon

$$Y3_{jc_3k\,t_1} = 0 \quad \forall j \in J, \ \forall k \in K \tag{12}$$

$$\sum_{k \notin FK_{fk}} Y2_{jkt} = 0 \quad \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{13}$$

Eq. (13) determines that material k cannot be selected for family f if this material k does not belong to family f of set FK_{fk}

$$\sum_{k \in FK_{fk}} Y2_{jkt} \geq Y1_{jft} \quad \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{14}$$

Eq. (14) establishes that if family f is selected, then at least one material k must be selected for this family

$$Y2_{jkt} \leq Y1_{jft} \quad \forall j \in J, \ \forall k \in FK_{fk}, \ \forall t \in T \tag{15}$$

If the corresponding family f is not selected for supplier j in period t, then no material k is selected from that family. This constraint is given by (15)

$$Y2_{jkt} = \sum_{c} Y3_{jckt} \quad \forall j \in J, \ \forall k \in K, \ \forall t \in T \tag{16}$$

In the same direction, constraint (16) determines that if a material k of any family is selected for supplier j in period t then one contract type c must also be selected for that material, supplier and

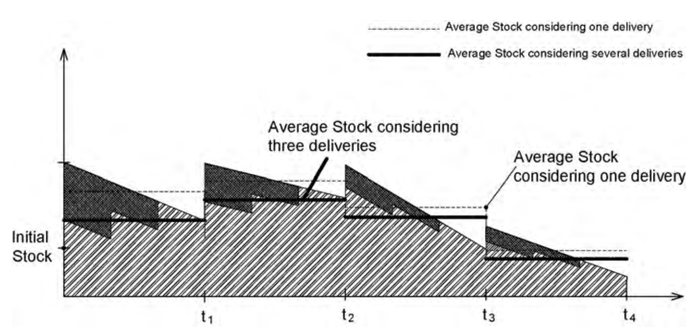

Fig. 3. Inventory differences between multiple deliveries and one delivery in each period.

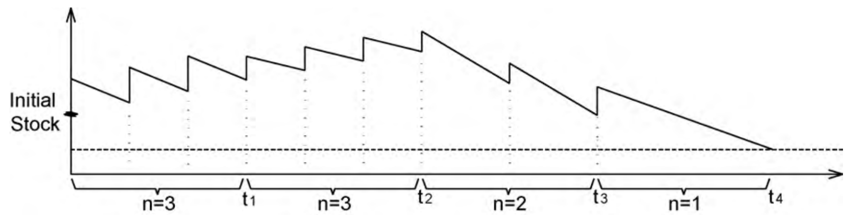

Fig. 4. Variable quantities and number of deliveries in each period.

period. On the contrary, if material k is not selected then no contract c is chosen for that material, supplier and period.

The linear disjunctive model for inventory and purchase problems (PI problem) in a medium-term horizon planning is defined by (1)–(16). The model was implemented in GAMS and the main results obtained are presented in Section 3.

2.2. The delivery problem

Buying materials by means of medium-term commitments promotes the purchasing of large orders. However, the limitations in storage capacity make it difficult to receive the whole purchase at once. In fact, it is usually necessary and convenient to request several partial deliveries in order to complete one purchase order. This gives the opportunity to complement the purchase discounts from the contracts and the optimal delivery quantities according to the two representative costs: delivery and inventory handling costs. This is called the inventory and delivery problem, *PID*.

Fig. 1 shows the evolution of the material stock when the whole order is delivered at the beginning of each period. On the contrary, Fig. 2 shows the inventory fluctuation when several deliveries are planned for each period. In this latter case, the average stock decreases and the delivery cost rises.

The difference in the average material in stock is clarified by overlapping the previous two figures. This is presented in Fig. 3.

Due to seasonal variations in purchase decisions, it is not suitable to define only one delivery size for every period. For this reason, the optimal shipment size must be determined for a certain period, supplier and material family. Then, different configurations in terms of deliveries size (eoq_{jft}) and number (n) can be taken into account in each period for a given material family. This situation is represented in Fig. 4.

The delivery decisions can be then included in the previous formulation as presented in this section. In order to guarantee coherence between the purchase and the delivery problems, Eq. (17) determines that the total amount delivered of family f from supplier j during period t must be equal to the family amount bought from that supplier and period

$$\sum_{k \in FK_{fk}} q_{jkt} = n_{jft} \cdot eoq_{jft} \quad \forall j \in J, \quad \forall f \in F, \quad \forall t \in T \tag{17}$$

Integer variable n_{jft} represents the number of deliveries of material family f from supplier j in period t and eoq_{jft} means the delivery order size. There is a correspondence between the definition of this delivery quantity and the traditional economic order quantity given in inventory management. Similarly, an increase in eoq_{jft} means that average stocks rises and so inventory handling costs. On the contrary, an increase in the number of deliveries diminishes eoq_{jft} size but gives rise to delivery costs.

Fig. 5. Piece wise fixed delivery cost.

Fig. 5 shows that fixed delivery cost (dc_{jft}) depends on both the size and number of shipments. For example, if the quantity ordered is smaller than EOQ_1 the unit cost for each delivery is DC_1 but if the order is larger than EOQ_1 and smaller than EOQ_2, the corresponding cost is DC_2 which is greater than DC_1. A disjunctive representation is used to model this situation, (18)

$$\begin{bmatrix} v1_{jft} \\ eoq_{jft} \le EOQ_1 \\ dc_{jft} = DC_1 \end{bmatrix} \vee \begin{bmatrix} v2_{jft} \\ eoq_{jft} \le EOQ_2 \\ dc_{jft} = DC_2 \end{bmatrix} \vee \begin{bmatrix} v3_{jft} \\ eoq_{jft} \le EOQ_3 \\ dc_{jft} = DC_3 \end{bmatrix}$$
$$\forall j \in J, \quad \forall f \in F, \quad \forall t \in T \tag{18}$$

Disjunction (18) calculates the fixed cost of each shipment according to the amount delivered.

Then, the total delivery costs for a given supplier j, material family f in period t is determined by (19)

$$tdc_{jft} = n_{jft} \cdot dc_{jft} \quad \forall j \in J, \quad \forall f \in F, \quad \forall t \in T \tag{19}$$

As shown in Figs. 1–4, the average stock must be recalculated considering the possibility of several deliveries. In this case, it is not the amount purchased in a given period which directly affects the average material stock but the size of the delivery orders. The average stock variable is then used to estimate the inventory holding costs in the objective function (21)

$$savg_{ft} = \frac{s_{ft} + \sum_j eoq_{jft} + s_{f(t+1)}}{2} \quad \forall j \in J, \quad \forall f \in F, \quad \forall t \in T \tag{20}$$

Eq. (20) determines the average stock of family f in period t as half of the sum of the initial stock s_{ft}, the delivered amount from all suppliers j, and the initial stock in the following period, $s_{f(t+1)}$. See Fig. 6 for more details.

Then, the objective function for the purchase, inventory and delivery problem can be redefined in (21):

$$Max \sum_t \frac{\sum_p Demand_{pt} \cdot price_{pt} - \left(\sum_j \sum_c \sum_k m_{jckt} + \sum_f savg_{ft} \cdot COSTavg_{ft} \cdot MS + \sum_f \sum_j tdc_{jft}\right)}{(1+RR)^t} \tag{21}$$

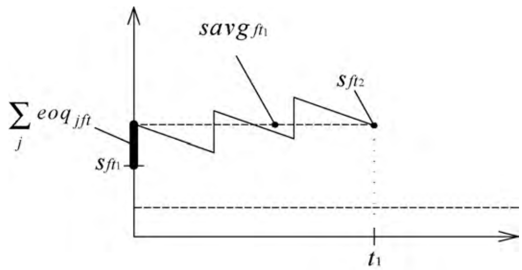

Fig. 6. Determination of the average stock for the *PID* model.

As presented in Eqs. (17) and (19), the delivery problem introduces certain non-linearities in the formulation. These bilinear terms make the problem non-convex preventing the global optimum. Thus, it is necessary to find a suitable transformation procedure in order to overcome this issue and find the global solution in a short execution time.

2.2.1. Model transformation

Both bilinear terms mentioned in the previous paragraph consist of the product of one integer variable by a continuous one. Traditionally, transformation techniques imply the increase of the model size in terms of constraints and equations. One known technique applied by Harjunkoski et al. (1999), Pörn, Harjunkoski, and Westerlund (1999), Rodriguez and Vecchietti (2008) uses a binary formulation for the integer variable. This procedure determines the value of the integer variable n_{jft} as presented in Eq. (22)

$$n_{jft} = \sum_i i \cdot \beta_{ijft} \quad \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{22}$$

where $i \in I = \{1, 2, \ldots, N\}$ and N is the maximum number of deliveries, upper bound of variable n_{jkt}

$$\sum_i \beta_{ijft} \le 1 \quad \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{23}$$

A slack variable is introduced in Eqs. (24)–(26) for the redefinition of the bilinear term $n_{jft} \cdot eoq_{jft}$. Note that when β_{ijft} is equal to 1, meaning that a certain number of periods is selected, then variable $slack1_{ijft}$ is equal to eoq_{jft} but when β_{ijft} is equal to 0, the slack variable is also 0

$$slack1_{ijft} - eoq_{jft} \le 0 \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{24}$$

$$-slack1_{ijft} + eoq_{jft} - SC \cdot \left(1 - \beta_{ijft}\right) \le 0 \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{25}$$

$$slack1_{ijft} - SC \cdot \beta_{ijft} \le 0 \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{26}$$

Then, Eq. (17) can be replaced by (27):

$$\sum_{k \in FK_{fk}} q_{jkt} = \sum_i i \cdot slack1_{ijft} \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{27}$$

Applying the same procedure, it is possible to find a new representation for the bilinear term $n_{jft} \cdot dc_{jft}$.

$$slack2_{ijft} - dc_{jft} \le 0 \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{28}$$

$$-slack2_{ijft} + dc_{jft} - DC_3 \cdot \left(1 - \beta_{ijft}\right) \le 0 \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{29}$$

$$slack2_{ijft} - DC_3 \cdot \beta_{ijft} \le 0 \quad \forall i \in I, \ \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{30}$$

Now, Eq. (19) is rewritten as (31):

$$tdc_{jft} = \sum_i i \cdot slack2_{ijft} \quad \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{31}$$

In general, traditional linearization techniques solve the problem finding a global solution but worsening model performance due to the number of binary and continuous variables and constraints added to the formulation. Note that the number of binary variables introduced by this method is $(i \cdot j \cdot f \cdot t)$ while the total number of slack variables is $2 \cdot (i \cdot j \cdot f \cdot t)$. The number of constraints involved in the transformation is $6 \cdot (i \cdot j \cdot f \cdot t)$.

In order to solve the problem avoiding an excessive number of variables and constraints, a different linearization procedure is introduced. In this case, Boolean variables and disjunctions are used to overcome the bilinear terms (32)

$$\begin{bmatrix} z^1 \\ n_{jft} = 1 \\ \sum_{k \in FK_{fk}} q_{jkt} = eoq_{jft} \\ tdc_{jft} = dc_{jft} \end{bmatrix} \vee \begin{bmatrix} z^2 \\ n_{jft} = 2 \\ \sum_{k \in FK_{fk}} q_{jkt} = 2 \cdot eoq_{jft} \\ tdc_{jft} = 2 \cdot dc_{jft} \end{bmatrix} \vee \cdots \vee \begin{bmatrix} z^N \\ n_{jft} = N \\ \sum_{k \in FK_{fk}} q_{jkt} = N \cdot eoq_{jft} \\ tdc_{jft} = N \cdot dc_{jft} \end{bmatrix} \quad \forall j \in J, \ \forall f \in F, \ \forall t \in T \tag{32}$$

This simple change in the formulation guarantees the global optimization and adds only $(i \cdot j \cdot f \cdot t)$ Boolean variables to the formulation with neither change in the continuous variables nor in the number of constraints. The bilinear constraints in (17) and (19) are now included in the disjunctions avoiding the non-convex terms.

Then, according to the methods proposed, two formulations are presented for the purchase, inventory and delivery problem (*PID* problem). The first one considering Harjunkoski's (Harjunkoski et al., 1999) transformation technique is given by (2)–(16), (18), (20)–(31). The second one, using the disjunctive transformation is represented by (2)–(16), (18), (20)–(21), (32). In order to analyze which linearization is the most convenient, it is important to compare the solution performance under both alternatives.

3. Model results

The formulations were posed in the GAMS system with the aim of showing the main results and model performances. A comparison between the linearization alternatives for the delivery problem was also made. These formulations were executed over a PC having an Intel Pentium D 2.8 GHz processor and 3.5 GB of RAM. Disjunctions were modeled using LogMIP (Grossmann, Meeraus, & Vecchietti, 2005).

3.1. Purchase and inventory (PI) problem results

In this section, the results obtained from *PI* problem are shown and analyzed. The data used to solve it are presented in Appendix A.

3.1.1. Model results

3.1.1.1. Decision variables. The amount purchased of selected materials from suppliers in each time period t is presented in Table 1. In the last column, the total amount is calculated. At

Table 1
Material purchased for the *PI* problem.

q_{jkt}	j_1k_1	j_1k_2	j_1k_3	j_1k_4	j_1k_5	j_1k_8	j_2k_4	Total quantity ordered
t_1	**500**			**400**			261	**1161**
t_2	**200**	**300**	588	**200**	**300**	***85***	**200**	**1873**
t_3			**600**	**150**	**200**	175	**140**	**1265**
t_4			400	185	***110***	120		**815**

this point it is worth mentioning that materials k_1, k_2 and k_3 belong to family f_1, materials k_4, k_5, k_6 and k_7 belong to family f_2 while family f_3 is formed by materials k_8, k_9 and k_{10}. In Table 2, the contract selection is shown for each supplier, material and period.

3.1.1.2. Performance. Table 3 summarizes execution performance for the case study. The objective value was 4353.41, representing total profits.

3.1.2. Analyses of results

Several analyses are carried out in order to highlight the model results. For the sake of clarity, this section is organized in two parts: purchase decisions analysis and financial impact of purchase decisions.

3.1.2.1. Purchase decisions analysis. In this section, it is analyzed the impact of the limited provision capacity and material costs on the raw material purchased. Then, the study is focused on the purchase contracts and the convenience of signing them.

From Table 1 one important point to consider involves the active constraints in the optimal solution. The amounts underlined in bold in Table 1 are those where restriction in the first term of disjunction (9) is active. This limitation is a characteristic of the supplier and independent of the contract signed with him. On the other hand, the amounts in *italic-bold* in Table 1 indicate that the first constraint in disjunction (10) is active where the amount purchased is limited by a minimum ($Qmin_{cj}$). Looking at these results, constraint in disjunction (9) restricts the solution several times while in just two cases the first constraint of disjunction (10) is also active. Stock capacity is not a limitation since the upper bound *SC* is equal to 5000 and the maximum ordered is 1873.

From Table 2, contract c_3 is the most selected one due to its special big discount. So as to choose c_3, another contract must be selected in a previous period for the same material. In the case of this example, it was always contract c_2.

In order to understand the model decisions, some of the data handled in the problem are presented in Figs. 7–10. In Fig. 7 the material cost fluctuation over the time periods is shown. The purchase costs continuously rise from period t_1 to period t_3 for all materials, falling down in period t_4. This situation affects the model solution. On the other hand, Figs. 8–10 present the suppliers' capacity for each material family in the periods analyzed. The availability of the cheapest materials availability goes down from period t_1 to t_3, while the availability of most expensive ones remains constant. These figures clearly show the seasonal behavior of the case analyzed in this work.

With the aim of showing how these costs and the suppliers limitation affect the decision variables, material ordered in each time

Table 2
Contract selection.

y^3_{jckt}	j_1k_1	j_1k_2	j_1k_3	j_1k_4	j_1k_5	j_1k_8	j_2k_4
t_1	c_2			c_2			c_2
t_2		c_4	c_2	c_3	c_2	c_2	c_3
t_3				c_3	c_3	c_3	c_3
t_4				c_3	c_3	c_3	

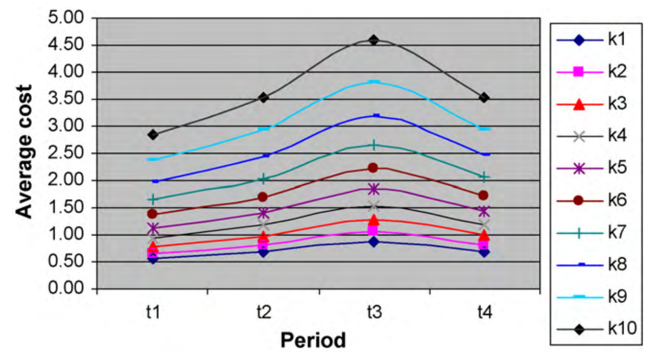

Fig. 7. Material average costs.

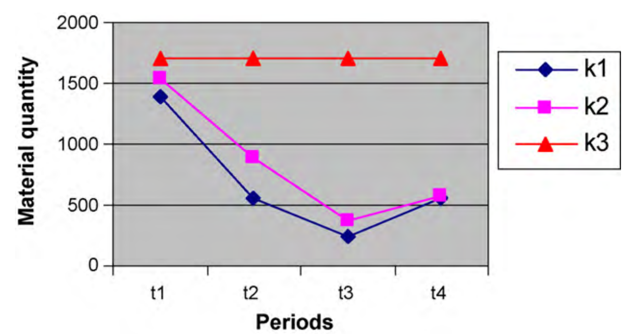

Fig. 8. Suppliers availability for f_1.

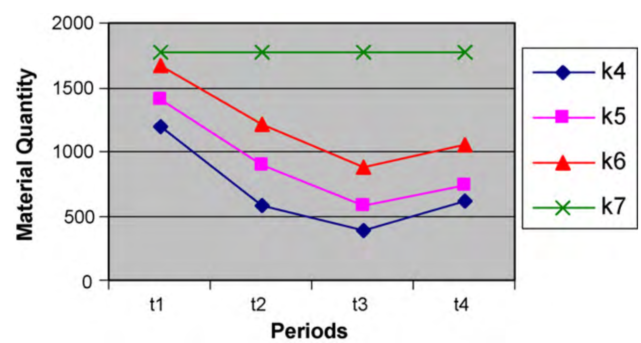

Fig. 9. Suppliers availability for f_2.

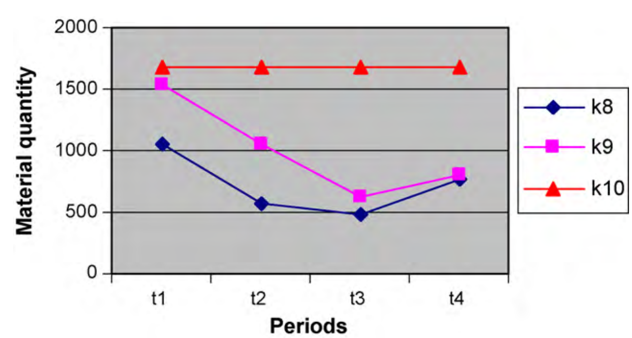

Fig. 10. Suppliers availability for f_3.

Table 3
Solution performance for the purchase and inventory problem (*PI*).

	Number of equations	Number of positive variables	Number of discrete variables	Execution time (s)	Objective value
Solution performance (gap = 0%)	2239	1408	822	11.01	4353.41

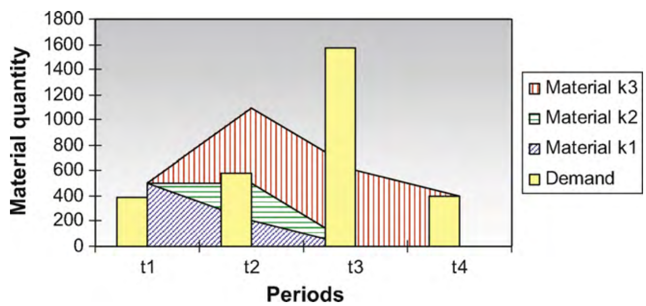

Fig. 11. Demand and material purchase for family f_1.

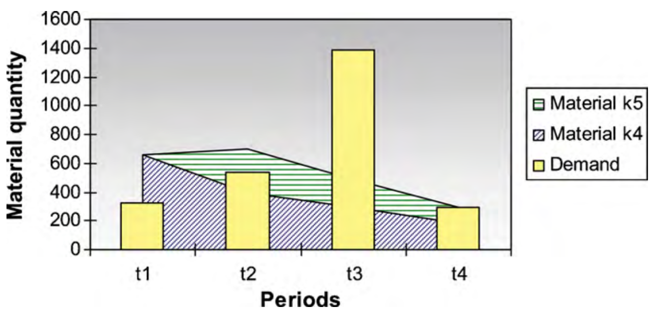

Fig. 12. Demand and material purchase for family f_2.

period is compared to material consumption required to satisfy product demand.

The solution for family f_1 is presented in Fig. 11 showing that the largest quantities ordered are placed in the second period while the highest consumption is actually in the third one. The main reason is that the availability of the least expensive materials (k_1 and k_2) is limited in the third period and purchase costs for all materials are incremented.

The solution for family f_2 is shown in Fig. 12 where a softer curve is observed for materials k_4 and k_5. However, there is still a tendency to buy and store cheaper materials in advance rather than buy expensive ones in the period of peak demand. For this reason, only the least expensive materials are bought, k_4 and k_5, while materials k_6 and k_7 are not ordered at all.

Fig. 13 shows a different situation for family f_3. In this case, the initial stock is sufficient for several periods and, consequently, only a small quantity is required. In fact, only one material is ordered, which is the one with the lowest price (material k_8). In this case, because the inventory handling costs are higher than the purchase

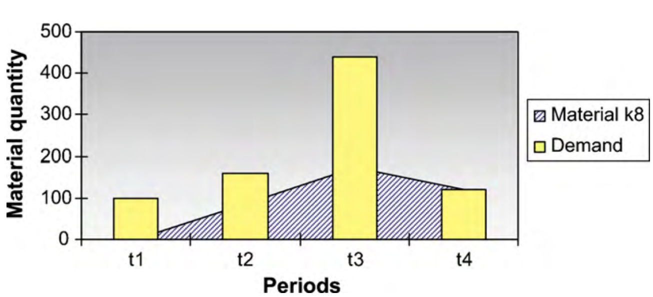

Fig. 13. Demand and material purchase for family f_3.

costs, it is better to buy the material in the periods required.

It must be noted that, in general, the solution prioritizes the use of cheaper materials from each family because of the limited suppliers' capacity and the fluctuation of the purchase costs, even when buying materials in advance increases the financial and stock cost. This fact is analyzed in the following section.

3.1.2.2. Financial impact of purchase decisions. In this section, three different financial analyses are presented. First, it is estimated the financial cost of buying a greater amount of material than the actual needs. Second, the problem is solved without provision restrictions. The results and objective value obtained in this case are compared to the first solution. Finally, in the third analysis, no minimum amount is required for any contract. So, the $Qmin_{cj}$ parameter of the problem is set to 0 and the model is solved again. The results obtained and the comparisons between the objective values are presented at the end of this section.

3.1.2.2.1. Cost of materials bought in advance. The first strategy to analyze the impact of the supplier availability on the company performance is to estimate the financial cost of materials ordered in advance. This is calculated considering the costs of the materials purchased, minus the corresponding cost of the material required by the demand in that period. The total expenditure is determined by the sum of these costs for each period using a discount rate (ir = 8%).

Table 4 shows the demand and the quantity bought from each supplier in the horizon planning for family f_1. It also shows the cost of each material according to the supplier and period. The last column (Cost in advance) of the first row in Table 4 is determined by:

$$(500 + 350 - 383) \times 0.5 = 233.5 \tag{33}$$

The first term in the parentheses in (33) corresponds to material k_1 purchased from supplier j_1, the second term is the initial stock of family f_1 and the third one corresponds to the demand, multiplied by the material cost from supplier j_1 which is 0.5.

In the second period, given by the second row in Table 4, the cost of material bought in advance is estimated by applying a similar procedure given by the following equation:

$$(200 \times 0.6 + 300 \times 0.72 + 588 \times 0.86) - 580 \times 0.6 = 496.0 \tag{34}$$

In this case, more terms are included in (34) compared to (33), since different materials are bought to satisfy family f_1 consumption. The first one represents the amount of material k_1 bought from supplier j_1 multiplied by its cost (0.6), the following term corresponds to material k_2 ordered from supplier j_1 multiplied by its cost (0.72), and the third term in the parenthesis is material k_3 from supplier j_1 and the cost is 0.86. Finally, the cost of material consumption for that period, 580 units, is estimated considering the cheapest cost, which is 0.6.

The actualized total cost for family f_1 is calculated in Eq. (35) where the discount rate is taken into account. All these results are presented in Table 4

$$\frac{233.5}{(1+0.08)^1} + \frac{496.0}{(1+0.08)^2} + 0 + 0 = 641.5 \tag{35}$$

The financial cost of the materials bought in advance for family f_1 is given by the total actualized cost in the last column and row of Table 4.

Table 4
Material purchased in advance and the corresponding actualized costs for family f_1.

Family f_1	Demand	Suppler j_1						Initial stock	Total quantity ordered	Quantity in advance	Cost in advance
		k_1		k_2		k_3					
		Quantity purchased	Cost	Quantity purchased	Cost	Quantity purchased	Cost				
	383	500	0.50		0.6		0.72	350	500	117	233.5
t_2	580	200	0.60	300	0.72	588	0.86		1088	508	496.0
t_3	1575		0.85		1.02	600	1.22		600	0	0
t_4	400		0.60		0.72	400	0.86		400	0	0
Actualized total cost											**641.5**

ir = 0.08.

Table 5
Material purchased in advance and the corresponding actualized costs for family f_2.

Family f_2	Demand	Suppler j_1				Suppler j_2		Initial stock	Total quantity ordered	Quantity in advance	Cost in advance
		k_4				k_4					
		Quantity purchased	Cost	Quantity purchased	Cost	Quantity purchased	Cost				
t_1	326	400	0.86		1.04	261	0.95	400	661	335	657.5
t_2	540	200	1.04	300	1.24	200	1.21		700	160	262.6
t_3	1385	150	1.47	200	1.76	140	1.56		490	0	0
t_4	295	185	1.04	110	1.24		1.31		295	0	0
Actualized total cost											**833.9**

ir = 0.08.

Table 6
Material purchased in advance and the corresponding actualized costs for family f_3.

Family	Demand	Suppler j_1		Inital stock	Total quantity ordered	Quantity in advance	Cost in advance
		k_8					
		Quantity purchased	Cost				
t_1	100			440	0	0	0
t_2	160	85			85	0	0
t_3	440	175			175	0	0
t_4	120	120			120	0	0
Actualized total cost							**0.0**

ir = 0.08.

Similarly to what Table 4 presents for the first family, Table 5 shows that material from family f_2 is ordered in excess in periods t_1 and t_2. As a consequence, an important amount of money (833.9) is invested in stock that cannot be used for other purposes.

Regarding family f_3, shown in Table 6, it is not necessary to buy materials in advance so in this case the financial cost is 0.

3.1.2.2.2. Unrestricted capacity from suppliers. The second approach is to analyze the case where the supplier provision availability is unrestricted for all periods. In this case, a new model is executed with a high value on parameters $Qmax_{jkt}$ ($Qmax_{jkt} \gg M\,\forall j, \forall k, \forall t$). This problem is called UPCP. The model is run and the results for variable q_{jkt} are presented in Table 7.

Table 7 shows that only one material is ordered in the first period, and some in the second one. The largest amount is now presented in the third period. The difference between the objective value in the original case study (4353.41) and the objective value in UPCP (5746.53) gives an approximate idea about the profit lost due to the supplier restricted capacity (Fig. 14). The quantities where constraint *Qmin* is active (first constraint in disjunction (10)) are shown in italics and bold type.

3.1.2.2.3. No minimum order size in the contract selection. In this section, the results obtained solving the problem without a minimum order size in the suppliers' contracts is analyzed. This

Table 7
Material purchased for UPCP problem.

q_{jkt}	j_1k_1	j_1k_4	j_1k_8	j_2k_1	j_2k_4
t_1	***170***				
t_2	618	376			***90***
t_3	700	700	260	700	685
t_4	400	295	120		

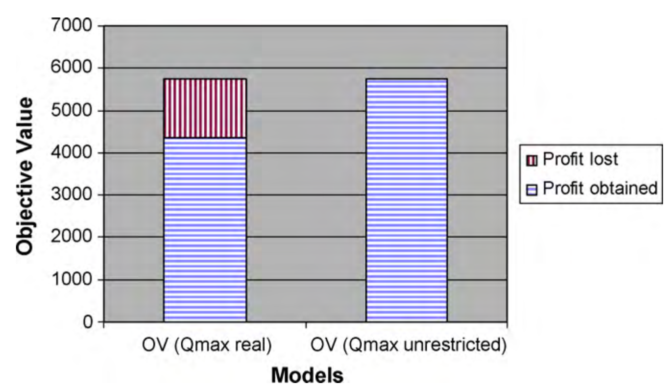

Fig. 14. Objective value comparison.

Table 8
Material purchased for NMQP.

q_{jkt}	j_1k_1	j_1k_2	j_1k_3	j_1k_4	j_1k_5	j_1k_8	j_2k_4	j_2k_5	Total quantity ordered
t_1	500			**51**			**420**		971
t_2	200	300	588	200	300	**80**	200		1868
t_3			600	150	200	**175**	140	**190**	1455
t_4			400	185	**110**	120			815

Table 9
Material purchased for the *PID* problem.

q_{jkt}	j_1k_1	j_1k_2	j_1k_3	j_1k_4	j_1k_5	j_1k_8	j_2k_4	j_2k_5	Quantity ordered
t_1	500			170					**670**
t_2	200	300	588	200	300	85	200	301	**2174**
t_3			600	150	200	175	140	190	**1455**
t_4			400	185	110	120			**815**

Table 10
Contract selection for the *PID* problem.

$y3_{jckt}$	j_1k_1	j_1k_2	j_1k_3	j_1k_4	j_1k_5	j_1k_8	j_2k_4	j_2k_5
t_1	c_2			c_4				
t_2		c_4	c_2	c_3	c_2	c_2	c_2	c_2
t_3			c_3	c_3	c_3	c_3	c_3	c_3
t_4			c_3	c_3	c_3	c_3		

Table 11
Delivery quantities for the *PID* problem.

eoq_{jft}	j_1f_1	j_1f_2	j_1f_3	j_2f_2
t_1	125	85		
t_2	362.67	125	42.5	125.25
t_3	150	116.67	58.33	110
t_4	133.33	147.5	60	

case is modeled by setting the value of the parameter $Qmin_{jc}$ equal to 0. Then, this problem (called NMQP) is solved and results are shown in Table 8. The value of the objective function is 4522.89 having an extra benefit of 169.48 compared to problem PI.

The values in bold in Table 8 are different from those in Table 1 (problem *PI*) showing the influence of parameter $Qmin_{cj}$ in the decision process. For instance, 190 units of material k_5 are now bought from supplier j_2 in the third period, while in the original problem that material was not selected at all.

To sum up, in this section several considerations have been taken into account in order to analyze the solution obtained for the *PI* problem. The information presented helps the managers to understand the behavior of the variables and the interactions among them. So the manager can decide:

- which contracts are the most suitable,
- how suppliers should be ranked,
- which materials are the most requested ones,
- how much capital should be invested in material stock,
- how material purchase fluctuates during the planning horizon,
- which are the limitations to material acquisition, and
- how material stocked changes along the periods.

3.2. Purchase, inventory and delivery (PID) problem results

This section shows the results obtained in the solution of the *PID* problem, where several deliveries are proposed for the purchased material. A comparison is presented between the two linearization strategies proposed for the bilinear terms.

Data used for this problem are those in Appendices A and B.

3.2.1. Model results

3.2.1.1. Decision variables. Table 9 shows the amount of material ordered in each period t from suppliers j; in the last column the total amount is presented. Table 10 shows the contracts selected in the solution. Finally, Table 11 includes the size of the delivery orders. For example, the materials belonging to family f_1 bought from supplier j_1 in period t_1 are delivered in orders of 125 units. This means that 4 deliveries are needed to provide 500 units of material k_1 purchased in the first period from supplier j_1 (Table 9).

3.2.1.2. Performance. The *PID* problem is solved using the two approaches presented in Section 2.2.1. In this section, the model performances are presented and compared. Table 12 shows the results of the *PID* problem using the Harjunkoski's linearization, while Table 13 illustrates the performance of the disjunctive reformulation.

The execution time in Tables 12 and 13 shows a remarkable difference in model performances. Although the number of discrete variables is the same, the size of the first model is bigger due to the number of constraints and positive variables. Addition-

Table 12
Solution performance for the inventory, purchase and delivery problem considering Harjunkoski's reformulation.

	Number of equations	Number of positive variables	Number of discrete variables	Execution time (s)
Solution performance (gap = 1.3%)	3499	1858	1110	10,000

Table 13
Solution performance for the inventory, purchase and delivery problem considering the disjunctive reformulation.

	Number of equations	Number of positive variables	Number of discrete variables	Execution time (s)
Solution performance (gap = 0%)	2816	1534	1110	56

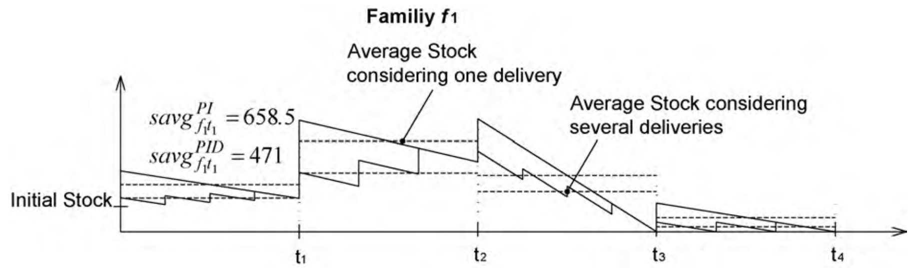

Fig. 15. Inventory fluctuation for family f_1 considering *PI* and *PID* problems.

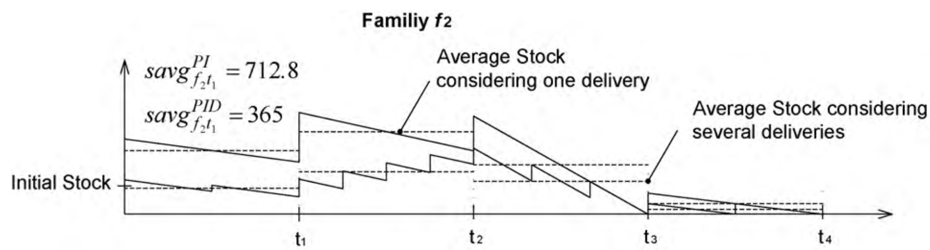

Fig. 16. Inventory fluctuation for family f_2 considering *PI* and *PID* problems.

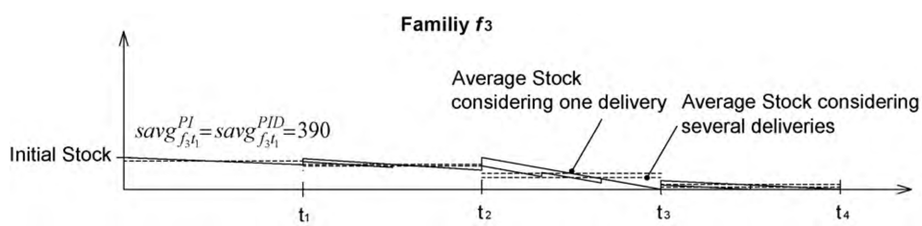

Fig. 17. Inventory fluctuation for family f_3 considering *PI* and *PID* problems.

ally, the second approach presents a much more straightforward representation of the transformed bilinear terms. Indeed, the first formulation obtains a sub-optimal result with a gap of 1.3% in 10,000 s while the second one finds the global optimum in just 56 s. The objective function value is 4475.19, representing the total profits in the horizon modeled.

3.2.2. Analyses of results

Table 9 shows that only two suppliers, j_1 and j_2, are selected. From provider j_1 several materials are bought while from the second one, only two types of material (k_4 and k_5) are chosen.

From Table 10 it can be seen that there is still a tendency to select contract c_3 whenever possible. Similar considerations for the *PI* case are also applicable to this problem.

Table 11 presents the quantities delivered in each period from the providers, represented by variable eoq_{jft}. This information is also depicted in Figs. 15–17 showing the number of deliveries in each case.

Since the difference between *PID* and *PI* problems lies in the shipment, the inventory fluctuation produced by this issue is analyzed below.

Figs. 15–17 compare inventory levels in problem *PI*, with one delivery per period, and problem *PID*, which allows several deliveries.

Fig. 15 shows the solution for material family f_1. The number of deliveries in the *PID* problem varies from three to four according to the period.

Fig. 16 shows the material storage for material family f_2. In this case, the number of deliveries in the *PID* problem varies from two to four.

The storage fluctuation for family f_3 is presented in Fig. 17. As shown in the figure, there is no much difference between both solutions since the size of the purchase orders is small and the number of deliveries fluctuates from one to three.

From Figs. 15–17 it can be seen that the stock level is much lower when several deliveries of smaller size are ordered. In fact, comparing the results of Tables 1 and 9, the values are the same in many cases, but receiving material with several deliveries makes the inventory management much more efficient.

4. Conclusions

The problem discussed in this paper shows how several important decisions can be handled together so as to optimize the company global operation. With the aim of presenting a comprehensive representation, the complete problem is analyzed in two stages. In the first approach, inventory optimization is considered jointly with purchase planning decisions. A new disjunctive model is proposed for solving the integration of inventory management and material purchase also including the selection of providers, specific materials and contract policies. Seasonal demand and material availability make this problem even more interesting since purchase and inventory decisions play an important role in the company profits.

The first formulation is then completed by adding the delivery problem. This consideration makes the model much more real but also more difficult to solve. The non-linearities introduced transform the problem into a non-convex formulation. With the aim of guaranteeing a global solution, two transformation procedures are presented. In the first one, a traditional linearization is used for redefining the corresponding variables and bilinear terms. This transformation adds an important number of constraints as well as binary and continuous variables. A second procedure is developed to overcome the non-linear terms by using disjunctions and Boolean variables. This original alternative is much simpler than the first one and solves the problem efficiently since only Boolean variables are added to the formulation.

With the intention of showing the solution to the problem, results are presented and analyzed in Section 3. Considering the first model, this analysis shows that the amount purchased is restricted by supplier availability on several occasions. In fact, there is a trend towards buying cheaper materials in advance and stock them until they are needed. For that reason, financial costs are estimated using two different techniques showing the impact of these decisions on the company profits. The case study presented and the results obtained strengthen the relevance of the approach proposed.

When including the delivery decisions, results show that the inventory management is greatly improved. Although the total amount of material bought is approximately the same, the purchase order distribution in the horizon planning changes, which highlights the importance of considering all the decisions as a whole. The short execution time gives the possibility of analyzing several scenarios and accomplishing a global optimum from the production perspective.

Notation

Indices

- c contracts
- f material families
- i index used in the transformation for redefining the bilinear terms, $i \in \{1, 2, \ldots, N)$
- j suppliers
- k materials
- l product components
- p products
- t periods

Sets

- FK_{fk} duple for which materials k correspond to family f
- PL_{plf} triplet for which component l of product p correspond to family f
- $TP_{ctt'}$ triplet for which contract c signed in period t must be paid in t'

Variables

- dc_{jft} delivery fixed cost for material family f bought from supplier j in period t
- d_{ft} consumption of material family f in period t
- eoq_{jft} optimal delivery quantity for family f bought from supplier j in period t
- m_{jckt} purchase cost of material k bought from supplier j with contract c in period t
- n_{jft} number of deliveries of family f bought from supplier j in period t
- q_{jkt} quantity purchased of material k from supplier j in period t
- $savg_{ft}$ average stock of family f in period t
- s_{ft} initial stock of family f in period t
- $slack1_{ijft}$ slack variable used in the transformation for redefining the bilinear term $n_{jft} \times eoq_{jft}$
- $slack2_{ijft}$ slack variable used in the transformation for redefining the bilinear term $n_{jft} \times dc_{jft}$
- tdc_{jft} total delivery fixed cost for material family f bought from supplier j in period t
- $v1_{jft}$ Boolean variable indicating the selection of the smallest delivery order size for family f, supplier j and period t
- $v2_{jft}$ Boolean variable indicating the selection of the medium delivery order size for family f, supplier j and period t
- $v3_{jft}$ Boolean variable indicating the selection of the biggest delivery order size for family f, supplier j and period t
- w_{jckt} purchase payment of material k bought from supplier j with contract c in period t
- $y1_{jft}$ Boolean variable indicating the selection of material family f bought from supplier j in period t
- $Y1_{jft}$ binary variable indicating the selection of material family f bought from supplier j in period t
- $y2_{jkt}$ Boolean variable indicating the selection of material k from family f bought from supplier j in period t
- $Y2_{jkt}$ binary variable indicating the selection of material k from family f bought from supplier j in period t
- $y3_{jckt}$ Boolean variable indicating the selection of contract c for purchasing material k from supplier j in period t
- $Y3_{jckt}$ binary variable indicating the selection of contract c for purchasing material k from supplier j in period t
- $z^{i}{}_{jft}$ Boolean variable indicating the selection of the number of deliveries for family f, supplier j and period t
- β_{ijft} binary variable used in the transformation for redefining the integer variable n_{jft}

Parameters

- $COSTavg_{ft}$ average cost of material family f in stock in period t
- DC_1 unit shipment cost for delivery quantities lower than or equal to eoq_1
- DC_2 unit shipment cost for delivery quantities lower than or equal to eoq_2
- DC_3 unit shipment cost for delivery quantities lower than or equal to eoq_3
- $Demand_{pt}$ demand of product p in period t
- EOQ_1 maximum amount for the delivery order which corresponds to unit shipment cost DC_1
- EOQ_2 maximum amount for the delivery order which corresponds to unit shipment cost DC_2
- EOQ_3 maximum amount for the delivery order which corresponds to unit shipment cost DC_3
- FC_c fixed cost of contract c signed
- IS_f initial stock of family f in period t_1
- MS percentage of the raw material average costs to calculate stock costs
- PC_{jkt} unit cost of material k bought from supplier j in period t
- $price_{pt}$ price of product p in period t
- $Qmax_{jkt}$ maximum quantity available of material k of supplier j in period t
- $Qmin_{cj}$ minimum quantity to order signing contract c with supplier j
- RR discount rate used in the objective function to actualize future costs and incomes
- SC stock capacity
- SS safety stock
- α_{plf} consumption of material family f in component l of product p
- δ_{jc} discount or interest rate according to contract c signed with supplier j

Table 14
Material costs in each period t according to suppliers j.

PC_{jkt}	j_1				j_3				j_3			
	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
k_1	0.50	0.60	0.85	0.60	0.55	0.70	0.90	0.76	0.60	0.75	0.92	0.70
k_2	0.60	0.72	1.02	0.72	0.66	0.84	1.08	0.91	0.72	0.90	1.10	0.84
k_3	0.72	0.86	1.22	0.86	0.79	1.01	1.30	1.09	0.86	1.08	1.33	1.01
k_4	0.86	1.04	1.47	1.04	0.95	1.21	1.56	1.31	1.04	1.30	1.59	1.21
k_5	1.04	1.24	1.76	1.24	1.14	1.45	1.87	1.58	1.24	1.56	1.91	1.45
k_6	1.24	1.49	2.12	1.49	1.37	1.74	2.24	1.89	1.49	1.87	2.29	1.74
k_7	1.49	1.79	2.54	1.79	1.64	2.09	2.69	2.27	1.79	2.24	2.75	2.09
k_8	1.79	2.15	3.05	2.15	1.97	2.51	3.23	2.72	2.15	2.69	3.30	2.51
k_9	2.15	2.58	3.66	2.58	2.37	3.01	3.87	3.27	2.58	3.23	3.96	3.01
k_{10}	2.58	3.10	4.39	3.10	2.84	3.61	4.64	3.92	3.10	3.87	4.75	3.61

Table 15
Requirement of material family f to satisfy demand in period t.

d_{ft}	t_1	t_2	t_3	t_4
f_1	383	580	1575	400
f_2	326	540	1385	295
f_3	100	160	440	120

Table 16
Initial stock of family f in period t_1.

	IS_f
f_1	350
f_2	400
f_3	440

Table 17
Fixed cost paid for signing contract c.

	FC_c
c_1	90
c_2	70
c_3	50
c_4	40

Appendix A.

Tables 14–23.

Some other parameters are considered. The percentage of the raw material average costs to calculate stock costs, *MS*, is given as 0.25. The discount rate, *RR*, is 0.08 and the stock capacity, *SC*, to storage all materials in any period is 5000.

Appendix B.

Tables 24 and 25.

Table 19
Demand of product p in period t.

$Demand_{pt}$	t_1	t_2	t_3	t_4
p_1	50	75	200	65
p_2	60	85	255	50
p_3	63	100	225	50
p_4	45	85	220	45
p_5	50	85	240	55

Table 20
Price of product p in period t.

$price_{pt}$	t_1	t_2	t_3	t_4
p_1	3	3.5	4	3.1
p_2	3.7	4.8	5.8	4.9
p_3	4.7	5.8	7.2	5.5
p_4	7 1	8 8	11	8.4
p_5	6.5	7.7	9	7.4

Table 21
Minimum amount to order from each supplier j signing contract c.

$Qmin_{cj}$	j_1	j_2	j_3
c_1	0	0	0
c_2	85	90	87
c_3	110	125	120
c_4	170	180	170

Table 22
Average cost of material family f in period t.

$COSTavg_{ft}$	t_1	t_2	t_3	t_4
f_1	0.667	0.829	1.08	0.833
f_2	1.275	1.585	2.064	1.592
f_3	2.391	2.971	3.869	2.985

Table 18
Material provision capacity in each period t from each supplier j.

$Qmax_{jkt}$	j_1				j_2				j_3			
	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
k_1	500	200	100	150	450	200	100	200	440	150	50	200
k_2	550	300	100	200	490	280	120	200	500	300	150	180
k_3	600	600	600	600	550	550	550	550	555	555	555	555
k_4	400	200	150	200	420	200	140	210	380	180	90	200
k_5	460	300	200	250	470	310	190	240	466	290	185	250
k_6	550	400	300	350	560	410	290	345	555	400	280	350
k_7	600	600	600	600	600	600	600	600	570	570	570	570
k_8	350	200	180	250	360	200	155	260	340	180	150	250
k_9	500	350	250	300	520	345	200	250	510	355	180	250
k_{10}	600	600	600	600	550	550	550	550	520	520	520	520

Table 23
Discount or interest rate offered by supplier j signing contract c.

δ_{jc}	c_1	c_2	c_3	c_4
j_1	0	0.1	0.2	−0.23
j_2	0	0.15	0.22	−0.35
j_3	0	0.13	0.23	−0.35

Table 24
Unit shipment costs.

DC_1	10
DC_2	20
DC_3	30

Table 25
Maximum amount for the delivery orders.

EOQ_1	150
EOQ_2	400
EOQ_3	5000

References

Al-Ameri, T. A., Shah, N., & Papageorgiou, L. G. (2008). Optimization of vendor-managed inventory systems in a rolling horizon framework. *Computers and Industrial Engineering*, *54*, 1019.

Bansal, M., Karimi, I. A., & Srinivasan, R. (2007). Optimal contract selection for the global supply and distribution of raw materials. *Industrial and Engineering Chemistry Research*, *46*, 6522.

Björk, C., & Carlsson, C. (2007). The effect of flexible lead times on a paper producer. *International Journal of Production Economics*, *107*, 139.

Bowersox, D. J., Closs, D. J., & Cooper, M. B. (2007). *Supply chain logistics management*. Singapore: McGraw-Hill.

Chen, K. K., & Chang, C.-T. (2007). A seasonal demand inventory model with variable lead time and resource constraints. *Applied Mathematical Modelling*, *31*, 2433.

Gao, Z., & Tang, L. (2003). A multi-objective model for purchasing of bulk raw materials of a large-scale integrated steel plant. *International Journal of Production Economics*, *83*, 325.

Gjerdrum, J., Shah, N., & Papageorgiou, L. G. (2002). Fair transfer price and inventory holding policies in two-enterprise supply chains. *European Journal of Operational Research*, *143*, 582.

Grossmann, I. E. (2005). Enterprise-wide optimization: A new frontier in process systems engineering. *AIChE Journal*, *51*, 1846.

Grossmann, I. E., Meeraus, A., & Vecchietti, A. R. (2005). Computational experience with LogMIP solving linear and nonlinear disjunctive programming problems. In *Proceedings of the 9th INFORMS computers society conference* Annapolis, MA, USA.

Guillén, G., Mele, F. D., Bagajewicz, M. J., Espuña, A., & Puigjaner, L. (2005). Multiobjective supply chain design under uncertainty. *Chemical Engineering Science*, *60*, 1535.

Haimes, Y. Y. (1973). Integrated system identification and optimization. *Control and Dynamic Systems: Advances in Theory and Applications*, *10*, 435.

Harjunkoski, I., Westerlund, T., & Pörn, R. (1999). Numerical and environmental considerations on a complex industrial mixed integer non-linear programming (MINLP) problem. *Computers and Chemical Engineering*, *23*, 1545.

Jia, Z., & Ierapetritou, M. (2007). Generate Pareto optimal solutions of scheduling problems using normal boundary intersection technique. *Computers and Chemical Engineering*, *31*, 268.

Mohebbi, E., & Hao, D. (2006). When supplier's availability affects the replenishment lead time—An extension of the supply-interruption problem. *European Journal of Operational Research*, *175*, 992.

Naraharisetti, P. K., Adhitya, A., Karimi, I., & Srinivasan, R. (2008). Decision support for resilient enterprises—from suppliers to shareholders. In *Proceedings of the foundations of computer-aided process operations, (FOCAPO 2008)* Boston, MA, USA.

Park, M., Park, S., Mele, F., & Grossmann, I. E. (2006). Modeling of purchase and sales contracts in supply chain optimization. *Industrial and Engineering Chemistry Research*, *45*, 5013–5026.

Pörn, R., Harjunkoski, I., & Westerlund, T. (1999). Convexification of different classes of non-convex MINLP problems. *Computers and Chemical Engineering*, *23*, 439.

Rodriguez, M. A., & Vecchietti, A. (2008). Enterprise optimization for solving an assignment and trim-loss non-convex problem. *Computers and Chemical Engineering*, *32*, 2812.

Rodriguez, M. A., & Vecchietti, A. (2009). Logical and generalized disjunctive programming for supplier and contract selection under provision uncertainty. *Industrial and Engineering Chemistry Research*, *48*, 5506.

Sarker, B. R., & Diponegoro, A. (2009). Optimal production plans and shipment schedules in a supply-chain system with multiple suppliers and multiple buyers. *European Journal of Operational Research*, *194*, 753.

Sharpe, W. F. (1964). Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance*, *19*, 425.

Sousa, R., Shah, N., & Papageorgiou, L. (2008). Supply chain design and multilevel planning—An industrial case. *Computers and Chemical Engineering*, *32*, 2643.

van Ryzin, G. J. (2001). *Analyzing inventory cost and service in supply chains*. Columbia Business School, available at http://www.columbia.edu/~gjv1/invnote4.PDF.

Varma, V. A., Reklaitis, G. V., Blau, G. E., & Pekny, J. F. (2007). Enterprise-wide modeling & optimization—An overview of emerging research challenges and opportunities. *Computers and Chemical Engineering*, *31*, 692.

You, F., & Grossmann, I. E. (2008). Design of responsive supply chains under demand uncertainty. *Computers and Chemical Engineering*, *32*, 3090.

Zadeh, L. (1963). Optimality and nonscalar-valued performance criteria. *IEEE Transactions on Automatic Control*, *8*, 59.