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Effect of charge–spin separation on the conductance through interacting low-dimensional rings

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ABSTRACT

We calculate the conductance through Aharonov–Bohm chain and ladder rings pierced by a magnetic flux which couples with the charge degrees of freedom. The system is weakly coupled to two leads and contains strongly interacting electrons modeled by the prototypical $t-J$ and Hubbard models. For a wide range of parameters we observe characteristic dips in the conductance as a function of magnetic flux which are a signature of spin and charge separation. We also show how the dips evolve when the parameters of the models depart from the ideal case of total spin–charge separation. The ladder ring can be mapped onto an effective model for large anisotropy which can be easily analyzed. These results open the possibility of observing this peculiar many-body phenomenon in anisotropic ladder systems and in real nanoscopic devices.

1. Introduction

New advances in fabrication techniques in nanoscience and technology allow the preparation of samples that constitute an interesting playground to test established and new ideas in physics, in particular, the effect of strong electronic correlations. Important examples can be given in experiments with quantum dots (QDs), relevant to our problem, such as the realization of the Kondo effect in systems with one QD [1–4] and the study of the metal–insulator transition in a chain of 15 QDs [5]. The synthesis of new materials of quasi-1D electronic character has led in the last decade to a variety of experiments which seek evidence of spin–charge separation (SCS), predicted theoretically for one-dimensional (1D) strongly interacting systems in the framework of the Luttinger liquid (LL) theory [6–8], such as the observation of non-universal power-law $I-V$ characteristics [9], the search for characteristic dispersive features by angle-resolved photoemission spectroscopy (ARPES) [10], the violation of the Wiedemann–Franz law [11]and the analysis of spin and charge conductivities [10,12]. Candidate materials to present SCS [13] are the organic Bechgaard and Fabre salts, molybdenum bronzes and chalcogenides [9], cuprate chain and ladder compounds [14], laterally confined two-dimensional electron gases, cleaved-edge overgrowth systems [15] and also carbon nanotube systems [16,17].

Several theoretical methods for detecting and visualizing SCS were proposed and demonstrated many years ago. For example, direct calculations of the real-time evolution of electronic wave packets in finite Hubbard rings revealed different velocities in the dispersion of spin and charge densities as an immediate consequence of SCS [18]. Also, Kollath and coworkers have repeated this calculation for larger systems using the density matrix renormalization group (DMRG) technique [19]and observed distinct features of SCS in a model for one-dimensional cold Fermi gases in a harmonic trap, proposing quantitative estimates for an experimental observation of SCS in an array of atomic wires.

Another approach was adopted in Ref. [20], where the authors analyzed the transmission through infinite Aharonov–Bohm (AB) rings. The motion of the electrons in the ring was described by a LL propagator with different charge and spin velocities, v_c and v_s, included explicitly. With this assumption the flux-dependence of the transmission has, in addition to the periodicity in multiples of the flux quantum $\Phi_0 = hc/e$, new structures which appear at fractional values of the flux which are determined by the ratio v_s/v_c. Recent results which go beyond the single pole approximation used there, claim that the number of dips is not determined approximately by v_c/v_s but by v_J/v_s, where v_J is the current velocity [21]. The results, however, agree for small integer values of p and q, for $v_s/v_c = p/q$. Numerical calculations of the transmittance through finite AB rings described by the $t-J$ model show clear dips at the fluxes that correspond to the ratio v_s/v_c [22,23]. This discrepancy arises due to the finiteness of the system. We have recently discussed the extension of these results to ladders of two legs as a first step to higher dimensions [24].

In essence, these structures arise because transmission requires the separated spin and charge degrees of freedom of an

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injected electron to recombine at the drain lead after traveling through the ring in the presence of the AB flux.

In this paper we will briefly review previous results on the transmittance through finite one-dimensional strongly interacting rings and ladders represented by the $t-J$ model. Additionally we will show new results in which the electrons in the ring are described by the Hubbard model, showing the robustness of the characteristic dips in the conductance as the local interaction U is diminished from the completely charge–spin separated case for infinite U. We will also present results for an effective model that describes the ladder system for large interchain coupling.

2. The model

The system is sketched in Fig. 1 and consists of a finite ring threaded by a magnetic flux, weakly connected to two leads. The electrons inside the ring are strongly correlated, while the leads are non-interacting.

Our model Hamiltonian reads $H = H_{\text{leads}} + H_{\text{link}} + H_{\text{ring}}$, where H_{leads} describes free electrons in the left and right leads,

$$H_{\text{link}} = -t' \sum_{\sigma} (a^{\dagger}_{-1,\sigma} c_{0,\sigma} + a^{\dagger}_{1,\sigma} c_{L/2,\sigma} + \text{H.c.}) \tag{1}$$

describes the exchange of quasiparticles between the leads ($a_{i,\sigma}$) and particular sites of the ring ($c_{i,\sigma}$)and H_{ring} depends on the particular model considered in the ring.

Following Ref. [20], the transmission from the left to the right lead can be calculated to second order in t' where the ring is integrated out. The resulting effective Hamiltonian is equivalent to a one-particle model for a non-interacting chain with two central sites modified by the interacting ring, with effective on-site energy $\varepsilon(\omega) = t'2G^{R}_{0,0}(\omega)$ and effective hopping between them $\tilde{t}(\omega) = t'2G^{R}_{0,L/2}(\omega)$. $G^{R}_{i,j}(\omega)$ denotes the Green function of the isolated ring.

At zero temperature, the transmittance and conductance of the system may then be computed using the effective impurity problem. The transmittance $T(\omega)$ is given by [20]

$$T(\omega, V_g, \phi) = \frac{4t^2 \sin^2 k |\tilde{t}(\omega)|^2}{|[\omega - \varepsilon(\omega) + te^{ik}]^2 - |\tilde{t}^2(\omega)||^2}, \tag{2}$$

where $\omega = -2t\cos k$ is the tight-binding dispersion relation for the free electrons in the leads which are incident upon the impurities. These equations are exact for a non-interacting system. However, for a more precise calculation of the transmittance, one should resort to non-perturbative approaches [25]. We nevertheless expect the dip structure to remain since they stem from more general symmetry considerations [23].

From Eq. (2), $T(\omega, V_g, \phi)$ may be calculated from the Green functions of the isolated ring. We consider holes incident on a ring of L sites and $N_e + 1$ electrons in the ground state, obtaining the Green functions from the ground state of the ring calculated using numerical diagonalization [26] and substituting these in Eq. (2). We fix the energy $\omega = 0$ to represent half-filled leads and explore the dependence of the transmittance on the threading flux, obtained by integration over the excitations in a small energy window, which accounts for possible voltage fluctuations and temperature effects [22].

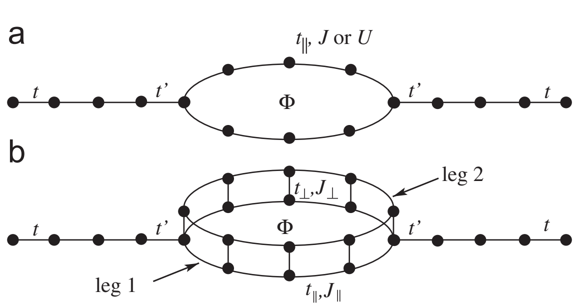

Fig. 1. Schematic representation of the interacting systems on a chain (a) and ladder (b) rings connected by links t' to free-electron leads.

3. Conductance through a single ring

In this section we will calculate the transmittance through a ring (Fig. 1a) for which the Hamiltonian reads

$$H_{\text{ring}} = -eV_g \sum_{l,\sigma} c^{\dagger}_{l,\sigma} c_{l,\sigma} - t_{\|} \sum_{l,\sigma} (c^{\dagger}_{l,\sigma} c_{l+1,\sigma} e^{-i\phi/L} + \text{H.c.}) + H_{\text{int}}, \tag{3}$$

where the flux is given in units of the flux quantum $\phi = 2\pi\Phi/\Phi_0$ and the system is subjected to an applied gate voltage V_g.

For the $t-J$ model $H_{\text{int}} = J\sum_l \mathbf{S}_l \cdot \mathbf{S}_{l+1}$, $\mathbf{S}_l = \sum_{\alpha\beta} c^{\dagger}_{l\alpha} \sigma_{\alpha\beta} c_{l\beta}$ is the spin at site l and no double occupancy is allowed. For the Hubbard model $H_{\text{int}} = \sum_{l,\sigma\neq\sigma'} U n_{l,\sigma} n_{l,\sigma'}$, with $n_{l,\sigma} = c^{\dagger}_{l,\sigma} c_{l,\sigma}$. Both models are related in the very large interaction limit by $J = 4t_{\|}^2/U$.

In the case of infinite on-site repulsion U (or equivalently $J = 0$), the wave function can be factorized into a spin and a charge part [22,27,28]. Therefore, charge–spin separation becomes evident. For each spin state, the system can be mapped into a spinless model with an effective flux which depends on the spin. Considering a non-degenerate ground state containing $N = N_e + 1$ particles and analyzing the part of the Green function that enters the transmittance when a particle is destroyed, it is shown that the dips occur when two intermediate states cross at a given flux and interfere destructively. These particular fluxes depend on the spin quantum numbers and are located at [23]

$$\phi_d = \pi(2n+1)/N_e \tag{4}$$

with n integer. If the integration energy window includes these levels, a dip in the conductance arises. If the ring is connected to the leads at a distance $M \neq L/2$, the dips at ϕ_d are less intense [23].

In Fig. 2 we show the results for the transmittance through an eight-site ring described by the $t-J$ model for small J with different fillings where we can clearly see the dips located at the positions given by Eq. (4).

As a matter of comparison, in Figs. 3 and 4 we show results for the transmittance using the Hubbard model in the ring. For large

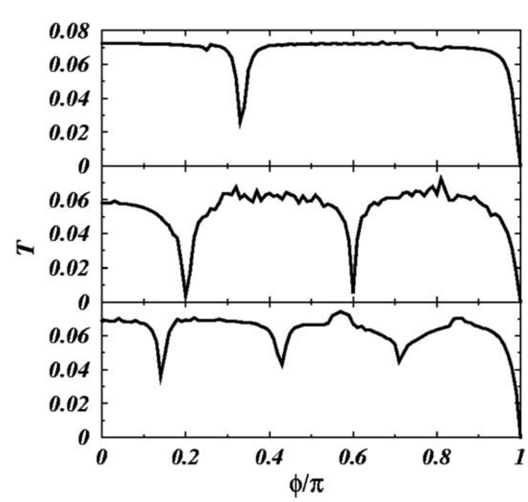

Fig. 2. Transmittance as a function of flux for a $t-J$ model with $J = 0.001t_{\|}$, $t_{\|} = t$, $t' = 0.3t$ and $L = 8$ sites. The filling of the ring is, from top to bottom, $N_e + 1 = 4$, $N_e + 1 = 6$ and $N_e + 1 = 8$. The transmission occurs through intermediate states with $N = 3$, 5 and 7 particles, respectively, which lead to minima at flux values $\phi_d = \pi(2n+1)/N_e$ (see text).

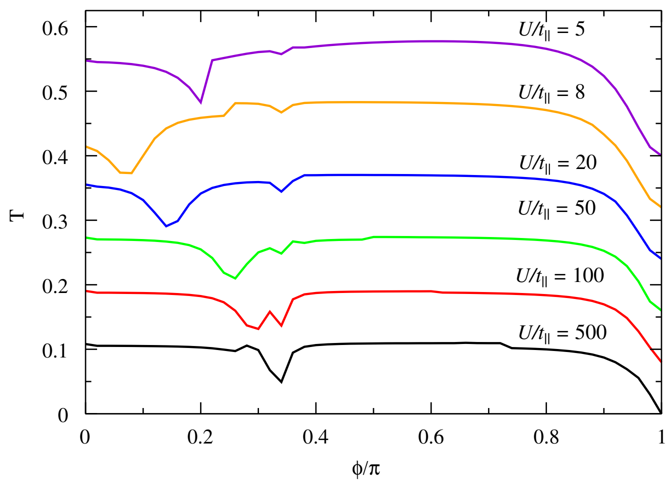

Fig. 3. Transmittance vs flux for the Hubbard model in the 1D ring for $L = 6$ sites, $N_e + 1 = 4$ particles in the ground state and several values of U (curves are shifted vertically for better visualization).

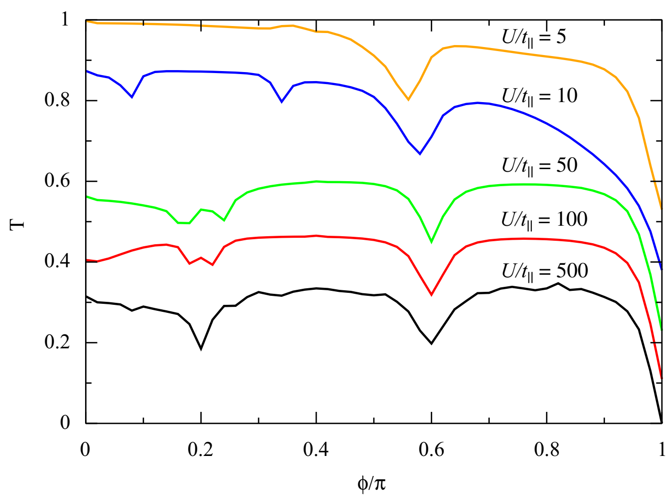

Fig. 4. Transmittance vs flux for the Hubbard model in the 1D ring for $L = 6$ sites, $N_e + 1 = 6$ particles in the ground state and several values of U (curves are shifted vertically for better visualization).

interactions, $U \gg t_\parallel$, dips are found at the positions given by Eq. (4). However, when U is reduced increasing the mixture between different spin sectors, thus weakening the spin–charge separation, we observe the appearence of new dips and a shift of some of them. This is due to the fact that the destructively interfering level crossings which lead to the reduction in the conductance occur at different values of the flux, as explained in Ref. [23].

4. Conductance through a ladder ring

In this section we will present results for a system consisting of a ring formed by an anisotropic ladder containing interacting electrons modeled by the following Hamiltonian (Fig. 1b):

$$H_{\rm ring} = -eV_g \sum_{i,l,\sigma} c^\dagger_{i_l,\sigma} c_{i_l,\sigma} - t_\parallel (c^\dagger_{i_l,\sigma} c_{i_l+1,\sigma} e^{-i\phi/L} + {\rm H.c.})$$
$$-t_\perp \sum_{i,\sigma} (c^\dagger_{i_1,\sigma} c_{i_2,\sigma} + {\rm H.c.}) + J_\parallel \sum_{i,l} \mathbf{S}_{i_l} \cdot \mathbf{S}_{i_l+1} + J_\perp \sum_i \mathbf{S}_{i_1} \cdot \mathbf{S}_{i_2}. \qquad (5)$$

The AB ring has L rungs, the fermionic operators $c^\dagger_{i_l,\sigma}$ create electrons at sites $i = 1, L$ of leg $l = 1, 2$ with spin σ and

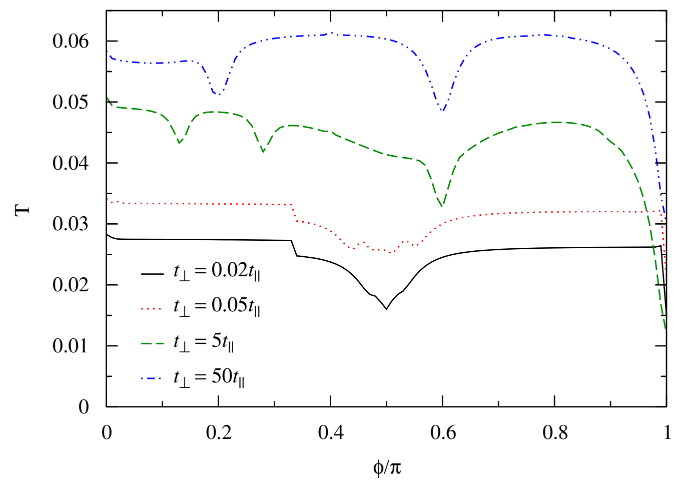

Fig. 5. Flux-dependent transmittance through an anisotropic ladder showing the transition from weakly to strongly coupled chains for $L = 6$ rungs and $N = 6$ particles in the ground state for $J_\perp = J_\parallel = 0$.

$\mathbf{S}_{i_l} = \sum_{\alpha\beta} c^\dagger_{i_l\alpha} \sigma_{\alpha\beta} c_{i_l\beta}$ is the spin at site i and leg l. No double occupancy is allowed.

For this ladder case the transmittance will depend on the anisotropy parameters. We find signs of charge–spin separation, i.e., dips in the transmittance for fractional values of the magnetic flux, for two extreme cases: weakly ($t_\perp \ll t_\parallel$) and strongly ($t_\perp \gg t_\parallel$) coupled chains (Fig. 5).

For the ladder with $t_\perp = 0$ and a total even number of electrons N in the ground state, the lowest-lying state has $N/2$ electrons in each leg. As we are calculating the transmittance through one leg only and the intermediate state has one particle less, from the condition for ϕ_d with $N_e = N/2 - 1$, one expects to see dips at $\phi_d = \pi((2n+1)/N/2 - 1)$. In Fig. 5, for the weakly coupled case with $N = 6$ particles, each leg has, predominantly, $N_e + 1 = 3$ electrons leading to a dip at $\phi = \pi/2$. When $t_\perp$ increases the dips disappear until the opposite strongly coupled chain regime is reached. In this limit and for $J = 0$, the bands corresponding to the bonding and antibonding states of each rung are very far apart leading to an effective one-dimensional system in one of the bands, where dips might reappear.

Now the total number of electrons in the lower band corresponds to the total filling N (for a less than half-filled band) and the transmittance will involve $N_e = N - 1$ electrons. Hence, if the physics is similar to the infinite-U Hubbard chain, the dips will be found at fluxes $\phi_d = \pi((2n+1)/N - 1)$. In Fig. 5 we find that for large values of $t_\perp/t_\parallel$, the dips correspond indeed to these fluxes. For smaller values of $t_\perp$, we find a shift in the location of the minima and sometimes the appearence of new dips.

4.1. *Effective model for a strongly coupled ladder*

When the ratio between inter and intrachain couplings $t_\perp/t_\parallel \gg 1$, our model (5) can be mapped to an effective model in the subspace of the bonding states of each rung, using degenerate perturbation theory up to second order in $t_\parallel$ [29]. The model is valid for energies lower than $t_\perp$ and a less than half-filled system:

$$H_{\rm eff} = -t_\parallel \sum_{i\sigma} (\hat{c}^\dagger_{i\sigma} \hat{c}_{i+1,\sigma} + {\rm H.c.}) + J \sum_i (\mathbf{S}_i \cdot \mathbf{S}_{i+1} - 1/4)$$
$$+ t'' \sum_{i\sigma} (\hat{c}^\dagger_{i+2,\sigma} \hat{c}_{i\sigma} (\mathbf{S}_i \cdot \mathbf{S}_{i+1} - 1/4) + {\rm H.c.}), \qquad (6)$$

where $J = J_\parallel/2 + 2t_\parallel^2/(t_\perp - 3J_\perp/4)$, $t'' = t_\parallel^2/(t_\perp - 3J_\perp/4)$ and $\hat{c}_{i\sigma} = 1/\sqrt{2}(c_{i_1,\sigma} + c_{i_2,\sigma})$, the bonding operator. The "second"

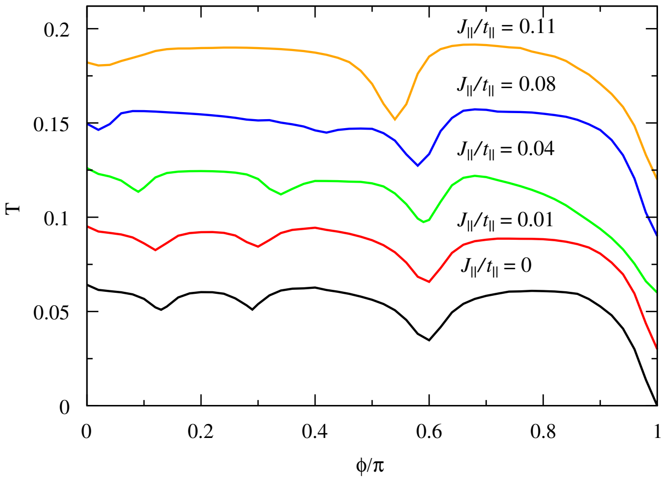

Fig. 6. Transmittance through a strongly coupled ladder described by H_{eff} for finite interactions $J_{\|}/J_{\perp} = t_{\|}^2/t_{\perp}^2$ for $t_{\perp} = 10t_{\|}$, $L = 6$ rungs and $N = N_e + 1 = 6$ electrons (curves are shifted vertically for better visualization).

dimension of the original ladder is reflected by the second nearest-neighbor term which tends to destroy the dips.

In Fig. 6 we show the conductance in this limit calculated using this effective model where the dips are visible and correspond to the fluxes given by Eq. (4) for small interactions J. Taking into account the fact that the J's are obtained perturbatively from the large-U Hubbard model, we keep their relation as $J_{\perp}/J_{\|} = t_{\perp}^2/t_{\|}^2$. Here we observe that the dips are still present for finite J's, however, as for the $J = 0$ case where the dips were affected by the interchain hopping parameter, in this case we also find shifts and reductions in the their depth caused by the interactions.

5. Conclusions

To conclude, in this paper we have shown the existence of dips in the conductance through finite strongly correlated low-dimensional systems which arise as a consequence of non-trivial destructive interference effects at fractional values of the flux quantum Φ_0. This feature is a strong indication of the existence of SCS in these systems. We have presented results for the transmittance through Hubbard and $t-J$ one-dimensional and anisotropic ladder rings. In all cases we find that the dip structure is robust against finite interactions (small J's or large U). However, we find new dips and shifts of their positions with respect to the ideal scenario of complete charge–spin separation. We also find that the dip structure, originally predicted for 1D systems, is still present in the presence of a second transmission channel modeled by a ladder system in the anisotropic limit. For a wide range of parameters, in particular for weak and strong hoppings across the rungs $t_{\perp}$, the dips remain, but they disappear for intermediate values of this parameter. These findings open the possibility of measuring this peculiar phenomenon in real nanoscopic systems or artificial structures, such as rings of quantum dots on the sub-μm scale, where the magnetic fields needed for this kind of experiments become accessible.

Acknowledgments

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