On the Computational Studies of Deterministic Global Optimization of Head Dependent Short-Term Hydro Scheduling

Ricardo M. Lima, Marian G. Marcovecchio, Augusto Queiroz Novais, and Ignacio E. Grossmann

***Abstract*—This paper addresses the global optimization of the short term scheduling for hydroelectric power generation. A tailored deterministic global optimization approach, denominated sHBB, is developed and its performance is analyzed. This approach is applied to the optimization of a mixed integer nonlinear programming (MINLP) model for cascades of hydro plants, each one with multiple turbines, and characterized by a detailed representation of the net head of water, and a nonlinear hydropower generation function. A simplified model is also considered where only the linear coefficients of the forebay and tailrace polynomial functions are retained. For comparison purposes, four case studies are addressed with the proposed global optimization strategy and with a commercial solver for global optimization. The results show that the proposed approach is more efficient than the commercial solver in terms of finding a better solution with a smaller optimality gap, using less CPU time. The proposed method can also find alternative and potentially more profitable power production schedules. Significant insights were also obtained regarding the effectiveness of the proposed relaxation strategies.**

***Index Terms*—Global optimization, mixed integer nonlinear programming (MINLP), short term hydro scheduling.**

NOMENCLATURE

A. Indices and sets

i, k, I	Hydro plants.
IC	Pairs of upstream and downstream plants.
qq, G	Grid points for the relaxations of bilinear terms.
j, J	Turbines.
M	Pairs of plants and turbines.
n, N	Grid points for the relaxation of $hdn_{i,t}$.
R	Grid points for supporting hyperplanes.
$T_{t,\tau_{k,i}}$	Time periods to build wrap around constraints.
UI	Turbines with identical features in the same plant.

B. Parameters

$a_{i,l}$	Coefficients for the forebay level polynomials.
$b_{i,l}$	Coefficients for the tailrace level polynomials.
$D_{i,t,n}$	Grid points for the partition scheme for $d_{i,t}$.
H_i	Water head [m].
H_i^L	Minimum water head [m].
H_i^U	Maximum water head [m].
$SC_{i,j}$	Start-up cost of turbine j in plant i [m.u.].
$P_{i,j}^{UP}$	Maximum power of turbine j in plant i [MW].
$Q_{i,j}^L$	Minimum outflow of turbine j in plant i when in operation $[\mathrm{m}^3/\mathrm{s}]$.
$Q_{i,j}^U$	Maximum outflow of turbine j in plant i $[\mathrm{m}^3/\mathrm{s}]$.
$Q_{i,j,t,qq}$	Grid points for the partition scheme for $q_{i,j,t}$.
V_i^{UP}	Target for the maximum storage of the reservoir of plant i at the end of the time horizon $[\mathrm{Hm}^3]$.
V_i^{LO}	Target for the minimum storage in plant i at the end of the time horizon $[\mathrm{Hm}^3]$.
$V0_i$	Initial storage of the reservoir of plant i [%].
VC	Conversion factor from $[\mathrm{m}^3/\mathrm{s}]$ to $[\mathrm{m}^3/\mathrm{h}]$.
WI_i	Forecast natural water inflow of plant i $[\mathrm{m}^3/\mathrm{s}]$.
$\eta_{i,j}$	Average generation efficiency $[\mathrm{MW}/((\mathrm{m}^3/\mathrm{s}).\mathrm{m})]$.
λ_t	Forecast price of energy in period t [m.u./MWh].
$\tau_{i,k}$	Time delay between plant i and plant k [h].
$\varphi_{i,j}$	Penstock head losses as a fraction of the net head.
$\phi_{i,j}$	Constant, where $\phi_{i,j} = \eta_{i,j}(1 - \varphi_{i,j})$.

C. Variables

$c_{i,j,t}$	Start-up cost of unit j in plant i in period t [m.u.].
$d_{i,t}$	Total water discharge of plant i in period t $[\mathrm{m}^3/\mathrm{s}]$.

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R. M. Lima and A. Q. Novais are with the Portuguese National Laboratory for Energy and Geology (LNEG), Lisbon, Portugal (e-mail: ricardo.lima@lneg.pt; augusto.novais@lneg.pt).

M. G. Marcovecchio is with INGAR/CONICET, Instituto de Desarrollo y Diseño and UNL, Universidad Nacional del Litoral, Santa Fe, Argentina (e-mail: mariangm@santafe-conicet.gov.ar).

I. E. Grossmann is with the Chemical Engineering Department, Carnegie Mellon University, Pittsburgh, PA 15213 USA (e-mail: grossmann@cmu.edu).

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$dn_{i,t,n}$	Disaggregated variable for $d_{i,t}$.
$h_{i,t}$	Dummy variable defined as $h_{i,t} = hup_{i,t} - hdn_{i,t}$.
$hdn_{i,t}$	Tailrace level of plant i in period t [m].
$\overline{hdn}_{i,t}$	Overestimator variable for $hdn_{i,t}$.
$hd_{i,j,t,qq}$	Convex-hull variable for water head [m].
$hup_{i,t}$	Forebay level of plant i in period t [m].
$h1_{i,j,t}$	Convex-hull variable for water head [m].
$h2_{i,j,t}$	Convex-hull variable for water head [m].
$p_{i,j,t}$	Power output of unit j in plant i in period t [MW].
$\overline{p}_{i,j,t}$	Dummy variable given by $\overline{p}_{i,j,t} = q_{i,j,t}h_{i,t}$.
$ph_{i,t}$	Total power output of plant i in period t [MW].
$Profit$	Profit [m.u.].
$q_{i,j,t}$	Water discharge of unit j, plant i, period t $[\mathrm{m}^3/\mathrm{s}]$.
$qd_{i,j,t,qq}$	Convex-hull variable for flow $[\mathrm{m}^3/\mathrm{s}]$.
$qh_{i,t}$	Total water discharge in plant i in period t $[\mathrm{m}^3/\mathrm{s}]$.
$s_{i,t}$	Spillage in plant i in period t $[\mathrm{m}^3/\mathrm{s}]$.
$v_{i,t}$	Volume of the reservoir in plant i $[\mathrm{Hm}^3]$.
$x_{i,j,t}$	$= 1$ if turbine j of plant i is online in period t, otherwise 0.
$Z_{i,j,t,qq}$	Boolean variable to establish whether a term in the disjunction is true.
$z_{i,j,t,qq}$	Convex-hull 0–1 variable to assign partition.
$yd_{i,t,n}$	0–1 variable to assign partition.

I. Introduction

HYDRO power generation is the most widely used form of renewable energy, presenting several advantages as compared with other sources: no fuel consumption, long life, and no direct waste or CO_2 emissions [1]. Hydro power plants play an important role in the electricity power sector due to their technical characteristics, reduced operating costs, and their capability of integration with other renewable energy sources. In liberalized electricity power sectors, their technical characteristics provide enough flexibility to participate in the day-ahead electricity markets, as well as to be used by the grid operator to help matching the overall demand and supply. Hydro power plants with pumping capabilities may also store energy, which when integrated with wind power farms increases the relevance of the latter in the power sector. However, a major drawback is that their production is restricted to the availability of water, which may be a problem in dry years. For example in Portugal by the end of June 2012 the total hydro power generation in 2012 had a reduction of 64.9% when compared with 2011 for the same period [2]. Within the framework of a deregulated power sector, the management of a cascade of hydro plants that participates in the day-ahead market involves the scheduling of the hydro power production for a forecast of hourly prices and energy demand, which involves the determination of the startup time of the turbines, duration of operation, the discharged flow, and the respective power produced in order to maximize the operational profit [3]. This problem is denominated as the short-term hydro scheduling (STHS) problem. STHS may be posed as a single problem, for example as for those generating companies that own only hydro plants, or it may be a subproblem integrated within a larger problem where thermal units and/or wind power generators are involved.

In theory, optimization models for the STHS problem are stochastic and nonlinear due to the uncertainty on the: 1) natural inlet flowrates; 2) electricity demand; and 3) electricity prices; and due to the nonlinear relationship between the power produced and the following variables: 1) flow discharged; 2) net water head; and 3) turbine efficiency. In addition, in order to capture the startups and shutdowns, and regions of operation of hydro plants, STHS models involve binary variables, resulting in an overall complex mixed-integer nonlinear stochastic model. The methodologies to address uncertainty in decision support models and the corresponding advantages in the context of electricity markets are discussed in [4]. Since stochastic models can become large, requiring tailored decomposition algorithms and extensive computational times to solve, it is imperative to use as a starting basis a rigorous and computationally efficient deterministic model, which is the major goal of the paper.

This work addresses the deterministic global optimization of head dependent STHS problems using cascades of hydro plants. Mixed-integer linear programming (MILP) models have received a good deal of attention in the literature for the STHS, due to the possibility of: 1) modeling several operating constraints such as ramp constraints or restricted operating regions; 2) using piecewise linear approximation submodels to replace nonlinearities of hydro power functions; and 3) using increasingly efficient MILP solvers such as GUROBI and CPLEX. A detailed MILP model from the point of view of operational constraints is proposed in [5]. These authors considered several of these constraints, and used a simplified linear model to approximate the nonlinear hydro generation function, where the turbines within the plant are considered to have identical characteristics and are aggregated into one single unit. A more accurate piecewise linear sub-model was proposed in [3] to approximate the nonlinear relationship between the power output, the net water head, and the water discharged within the hydro generation function. This sub-model was further improved in [6] by including interpolations between the piecewise functions and by adopting a tighter continuous relaxation. The aforementioned authors have recognized the importance of developing accurate models to describe the hydro power generation function. Their approach has relied on replacing the nonlinearities by piecewise linear approximations represented by sub-MILP models, at the cost of adding extra continuous and binary variables, and equations. The advantage of the MILP approach relies on the fact that it eliminates the numerical difficulties of nonlinear problems such as initialization, convergence and presence of local solutions. The main disadvantage is that the piecewise linear approximations are not exact, and therefore, the results obtained are approximate. This means, that for example in the STHS, the volumes of water in the reservoirs or the power generated profiles calculated from MILP models can show deviations from the real values.

An alternative approach involves building nonlinear models where the nonlinearities are considered explicitly. The main advantage of this approach is that the models are more accurate, i.e. they provide predictions of the variables with a smaller error than the one with linear approximations. The main sources of nonlinearities in STHS are the following: 1) the hydro power generation function; 2) the relation between the forebay level and the volume of water in the reservoir; and 3) the relation between the tailrace level and the total water discharged by the plant. In general, the power generated by a turbine is given by the following equation [7]:

$$p_{i,j,t} = K\eta_{i,j}\zeta_{i,j}q_{i,j,t}H_{i,t} \quad \forall i,t,j \tag{1}$$

where $p_{ij,t}$ denotes the output power of turbine j from plant i in the time period t, K is a constant, $\eta_{i,j}$ is the efficiency of the generator, $\zeta_{i,j}$ is the efficiency of the turbine, $q_{i,j,t}$ the flow discharged by turbine j and $H_{i,t}$ the net water head. Assuming that $\eta_{i,j}$ and $\zeta_{i,j}$ are constants as proposed in [8], (1) involves a bilinear term that is well known to give rise to nonconvexities that may lead NLP solvers to find local solutions [9]. The relations between the forebay and tailrace levels on one hand, and the volume of water in the reservoir and the total water discharged by the plant on the other hand may be represented by linear functions, or more complex polynomial functions [7], [8]. The level of detail used in the definition of these functions is most dictated by the need to increase accuracy, which in some cases may have a significant impact on the economic analysis of the systems.

Detailed nonlinear expressions have been recently included in STHS models with different levels of assumptions [7], [8], [10]–[12]. Nonlinear programming (NLP) models involving continuous variables and nonlinear equations have been also used to model the STHS [13]. However the lack of binary variables restrict the utilization of startup costs or the enforcement of some operational constraints. Mixed-integer nonlinear programming (MINLP) models have been recently proposed for the STHS where detailed nonlinear expressions are considered [7], [8], [10]–[12], [14]. The introduction of these nonlinearities, even using simplified equations as in [13], may lead to nonconvex MINLP models with multiple local optima. Therefore, MINLP solvers that rely on convexity assumptions may not guarantee global optimality of the solutions, or fail to find a feasible solution.

In terms of solution approaches, Lagrangian relaxation (LR) is the most popular approach for solving large scale hydrothermal problems. The superior performance of LR is due to the decomposition of the original model into sub-problems, and the quality of the calculated bounds [15], [16]. A clear and concise review of the advantages and drawbacks of LR is given in [16]. LR has been also applied to the solution of the STHS problems [11], [17]–[19]. However, for a cascade of hydro plants the spatial-temporal interaction between the plants requires additional linking variables when compared with thermal systems. Additional decomposition algorithms involve for example the bi-level decompositions based on two levels of detail [10].

The direct solution of the STHS problem has also relied on dynamic programming methods [20], [21], and on LP-based branch and bound (B&B) solvers for MILP problems [3], [5], [6], [15]. The recent trend on using nonlinearities in the STHS models, and their solution using MINLP solvers has led to the solution of nonconvex MINLP problems with solvers developed for convex MINLP problems [12], [14]. The current technology to solve MINLP problems is not as mature as the technology to solve MILP's. However, several algorithms are available [22]–[24], and implemented into commercial or open MINLP solvers such as DICOPT [25], α-ECP and SBB in GAMS, AAOA in AIMMS, MINOPT [26], and BONMIN [27]. Recently, significant advancements have also been made in the development of theory and algorithms for the deterministic global optimization of NLP and MINLP problems [9], [28]–[30]. This is currently an active area of research in which there are available several solvers, such as BARON [31], LINDOGlobal [32], and Couenne [33], which can address the deterministic global solution of NLP and MINLP problems.

The objective of this work is to address the solution of STHS problems defined by MINLP models using a deterministic global optimization approach. The main contributions of this work are the following: 1) implementation of a spatial B&B algorithm to address the global optimization within a pre-specified tolerance of a detailed STHS MINLP model; 2) use of specific types of constraints suggested by the STHS MINLP model formulation, namely symmetry breaking constraints (SBC), applied to the binary variables associated with the status of the turbines; 3) a specific partition scheme for the relaxation of bilinear terms with semi-continuous variables; 4) comparison of the proposed approach for STHS models with a global optimization solver.

This work is motivated by the current trend to develop more detailed MINLP models for the STHS, and is supported by the following advancements: 1) global optimization for nonconvex MINLP problems; 2) availability of affordable multiple threads computer hardware; and 3) increasing sophistication of MILP solvers as a result of the implementation of new cuts based on polyhedral theory, as well as to the inclusion of heuristics and meta-heuristics within the MILP solvers that help finding integer solutions [34]–[36].

II. Problem Statement

Given is a set of hydro plants in cascade that produce electricity for the day-ahead market. Some plants can store water in a reservoir, while others are run-of-the-river plants. Each plant has a set of turbines with a maximum output flow and power generated, linked to the same reservoir. The problem is to determine the start and duration of operation of each turbine, and the respective power output that maximize the operating profit, subject to the limits of the reservoirs, the mass balances of water, and the operating limits of the turbines. Each reservoir has as inputs a deterministic natural inflow and the discharge from upstream plants, and as outputs the flow discharged by each turbine linked to the reservoir. The profit is calculated as the difference between the revenues of selling electricity minus the start-up costs of the turbines. The value of the water is not considered in the profit, since the volume of each reservoir at the end of the time horizon must be greater than or equal to the initial volume. The system is considered as an electricity price taker, with the price of electricity following a given hourly profile. The time horizon is equal to one day, discretized in periods of one hour. For each pair of plants (i, i') there is a time delay between the total flow discharged from plant i to plant i'. The system does not have to match a specific demand pattern, since all energy produced is delivered, without considering electrical network constraints.

The STHS problem is formulated as a deterministic problem that can be used as a basis for a stochastic programming model in which uncertainties of forecasted prices can be taken into account with an accurate nonlinear model.

III. MINLP MODEL

In this work an MINLP model is proposed for STHS based on the specific hydro parts of the test cases described in [8]. The main mathematical features of this model are the following: 1) the utilization of a nonlinear function for the calculation of the power generated; and 2) the use of polynomial functions to calculate the forebay and tailrace levels. In the current model it is also assumed that each plant may have multiple turbines linked to the same reservoir. Therefore, binary variables are used to account for their startup costs and to enforce ranges of operation. Note that with some exceptions, for example [10] and [11], there are few works in the literature that address multiple turbines per hydro plant.

The objective function of the problem is the maximization of the operational profit of the cascade of hydro plants given by the difference between revenues obtained by selling energy and startup costs:

$$Profit = \sum_i \sum_t \lambda_t ph_{i,t} - \sum_i \sum_j \sum_t c_{i,j,t} \qquad (2)$$

where λ_t, $ph_{i,t}$, and $c_{i,j,t}$ are, respectively, the electricity price in the time period t, the power produced by plant i, and the startup cost of the turbine j in plant i in the same period. The startup cost is given by the following equations:

$$c_{i,j,t} \geq SC_{i,j}(x_{i,j,t} - x_{i,j,t-1}) \quad \forall i, j \in M, t > 1 \qquad (3)$$

$$c_{i,j,t} \geq SC_{i,j}(x_{i,j,t} - x_{i,j,tt}) \quad \forall i, j \in M, t = 1, tt = 24 \qquad (4)$$

where the $SC_{i,j}$ is the startup cost of turbine i in the plant j, and $x_{i,j,t}$ is a binary variable that denotes the on/off status of the turbine. Note that in (4), the initial condition for $t = 0$ is not considered as a fixed initial state. The initial condition is set equal to the condition of the final period, which is a variable.

The forebay level of each plant i in the time period t is calculated using fourth degree polynomials as a function of the volume of water stored

$$hup_{i,t} = a_{i,0} + a_{i,1} v_{i,t} + a_{i,2} v_{i,t}^2 + a_{i,3} v_{i,t}^3 + a_{i,4} v_{i,t}^4 \ \forall i, t. \qquad (5)$$

In the MINLP model adopted in this work the forebay level is not considered constant, unlike most of the work reported in the literature [11], [20]. The tailrace level of each plant i in the time period t is calculated using also fourth degree polynomials as a function of the total discharged water

$$hdn_{i,t} = b_{i,0} + b_{i,1} d_{i,t} + b_{i,2} d_{i,t}^2 + b_{i,3} d_{i,t}^3 + b_{i,4} d_{i,t}^4 \ \forall i, t. \qquad (6)$$

The total output flow of water from plant i in the time period t is equal to the total discharged flow by the turbines of plant i, $qh_{i,t}$, plus the spillage flow, $s_{i,t}$:

$$d_{i,t} = qh_{i,t} + s_{i,t} \quad \forall i, t \qquad (7)$$

where the total discharged flow is equal to the sum of the individual discharged flow of each turbine j in plant i

$$qh_{i,t} = \sum_{j \in M} q_{i,j,t} \quad \forall i, t. \qquad (8)$$

The volume of water stored in each plant i in each time period t results from the mass balance involving the volume of water in the time period $t-1$, the total discharged flow from plant i, the water inflow from the upstream plants and the natural inflow of water:

$$v_{i,t} = v_{i,t-1} + VC\left(WI_i - d_{i,t} + \sum_{k \in IC_{k,i}} d_{k,tt}\right) \ \forall i, t, tt \in T_{t,\tau_{k,i}} \qquad (9)$$

where $v_{i,t}$ denotes the volume of water in the reservoir of plant i in the time period t, VC is a unit conversion factor, WI_i is the deterministic natural inflow of water, $d_{i,t}$ represents the total outflow of water of plant i, $d_{k,tt}$ denotes the total outflow of the upstream plant k in the time period tt that arrives to plant i in the time period t, having elapsed a time $\tau_{k,i}$, which is the time delay for the total output flow from plant k to plant i, and the set $IC_{k,i}$ defines the plants upstream of plant i. In the above equations a wrap-around operator around the time horizon is used to account for the flows of the upstream plants that have a time delay. The power generated by each turbine is defined as a nonlinear function of three variables: 1) the output flow, $q_{i,j,t}$; 2) the forebay level, $hup_{i,t}$; and 3) the tailrace level, $hdn_{i,t}$:

$$p_{i,j,t} = \phi_{i,j} q_{i,j,t}(hup_{i,t} - hdn_{i,t}) \quad \forall i, j \in M, t \qquad (10)$$

with $\phi_{i,j} = \eta_{i,j}(1 - \varphi_{i,j})$, where $\eta_{i,j}$ and $\varphi_{i,j}$ denote the approximate average generation efficiency, and the penstock head losses, respectively. The power generated by each plant is given by the following equation:

$$ph_{i,t} = \sum_{j \in M} p_{i,j,t} \quad \forall i, t. \qquad (11)$$

The following two inequalities enforce a minimum and a maximum value for the water discharged by turbine j in plant i, when $x_{i,j,t} = 1$:

$$q_{i,j,t} \geq Q_{i,j}^L x_{i,j,t} \quad \forall i, j \in M, t \qquad (12)$$

$$q_{i,j,t} \leq Q_{i,j}^U x_{i,j,t} \quad \forall i, j \in M, t \qquad (13)$$

while the maximum power that can be generated by turbine j in plant i is given by the constraint

$$p_{i,j,t} \leq P_{i,j}^{UP} x_{i,j,t} \quad \forall i, j \in M, t. \qquad (14)$$

In this work, the volume of water in each reservoir at the end of the time horizon is set equal to the initial volume:

$$v_{i,t} \geq V0_i \quad \forall i, t = 24. \tag{15}$$

In the above equation, $V0_i$ is a parameter of the model, which means that it can be replaced by a different value that will not change the structure of the model, or the applicability of the proposed solution strategy. Therefore, instead of $V0_i$, a minimum final volume calculated in a previous step from a planning model can be used in (15). These planning models usually comprise hydrothermal systems, and involve larger time horizons than STHS models in order to mitigate end effects in the initial periods. In general, they account for uncertainty in natural water inflows, energy demand and prices, and typically rely on solution approaches based on stochastic dual dynamic programming [37], [38]. Their solutions provide marginal values for the water in the reservoirs during the time horizon, and target values for the initial and final volumes to be used in daily scheduling problems. The utilization of these models is out of the scope of this work, and therefore without loss of generality, $V0_i$ is assumed as the minimum volume at the end of the day.

A. Simplified MINLP Model

The proposed MINLP model can be simplified by neglecting in (5) and (6) the terms of the polynomial function of order greater than one, leading to the linear relationships:

$$hup_{i,t} = a_{i,0} + a_{i,1} v_{i,t} \quad \forall i, t \tag{16}$$

$$hdn_{i,t} = b_{i,0} + b_{i,1} d_{i,t} \quad \forall i, t. \tag{17}$$

This is a straightforward approximation to the original polynomial functions, which is used here with the objective of comparing the computational performance of MINLP models with and without polynomial functions. Note that better approximations of the equations may be obtained by adjusting the coefficients of the linear equation with the original data. However, this is out of the scope of this paper. Therefore, a simplified model (S-MINLP) is proposed by replacing (5) and (6) with (16) and (17) in the MINLP model.

B. MILP Model

In this section an alternative model is proposed by further simplifying the power generation function. Here, the variations on the net water head are neglected, i.e. the difference between the forebay and tailrace levels is assumed to be constant, and denoted by H_i. H_i is defined as the difference between the average forebay level calculated for the lower and upper values of the variable $v_{i,t}$, and the average tailrace level calculated for the lower and upper values of $d_{i,t}$. With these assumptions the power generated is given by the following equation:

$$p_{i,j,t} = \phi_{i,j} q_{i,j,t} H_i \quad \forall i, j \in M, t. \tag{18}$$

In this case, the simplified MILP (S-MILP) model does involve neither bilinear terms nor polynomial functions. This means that from the original MINLP, (5), (6), and (10) are removed and (18) is added.

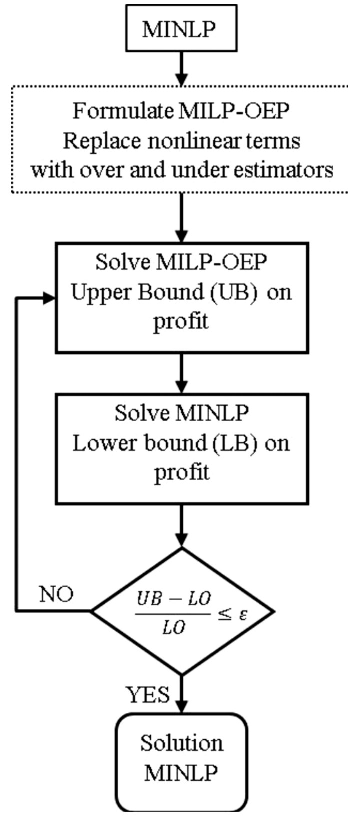

Fig. 1. Overview diagram of the proposed sHBB algorithm.

IV. Solution Approach

A deterministic global optimization strategy denominated spatial Hydro B&B (sHBB) is developed in this section for the STHS. A typical approach to solve deterministic nonlinear STHS problems involves the formulation of an MILP problem that approximates the nonlinear behavior of the system, where the nonlinearities are replaced by piecewise linear approximation sub-models [3], [6]. Likewise, the approach proposed in this work implies also the formulation of an MILP problem from the original MINLP. However, here the formulation relies on the construction of convex linear regions to substitute the nonlinear equations leading to an MILP overestimator problem (MILP-OEP). The solution of the MILP-OEP provides an upper bound on the profit of the system, while simultaneously measuring the quality of the solution of the MINLP model. The solution vector of the MILP-OEP is then used as a starting point to solve the original MINLP model. The MINLP problem provides a lower bound on the profit, and a solution to the original problem.

After the resolution of the MILP-OEP and MINLP problems, the lower and upper bounds on the profit are available that bound the global optimal solution. If the gap between these bounds is within a pre-specified tolerance the search stops, otherwise a spatial B&B framework is applied. A simplified diagram of the proposed sHBB is illustrated in Fig. 1. The B&B algorithm in sHBB is similar to the basic B&B procedure used by MILP solvers to reduce the gap between the linear relaxation and integer solutions. However, sHBB is tailored for hydro systems, and in addition, performs branching on the continuous instead of the binary variables. The constraints of the MILP-OEP provide a tight linear overestimation of the nonconvex region of the original MINLP

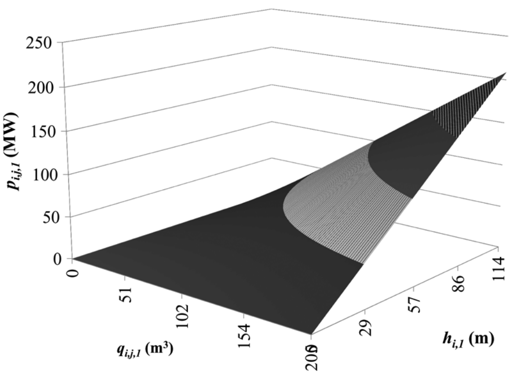

Fig. 2. Power generated as function of $q_{i,j,t}$ and $h_{i,t}$ for the first time period of a specific turbine from one plant used in the case studies.

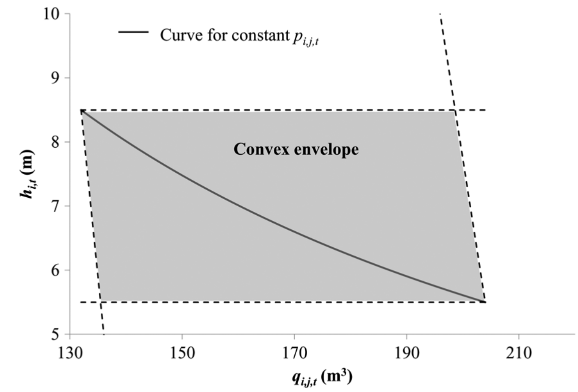

Fig. 3. Projection of the relationship between $q_{i,j,t}$ and $h_{i,t}$ for a constant $p_{i,j,t}$ from Fig. 2, and the convex envelope defined by (20)–(23).

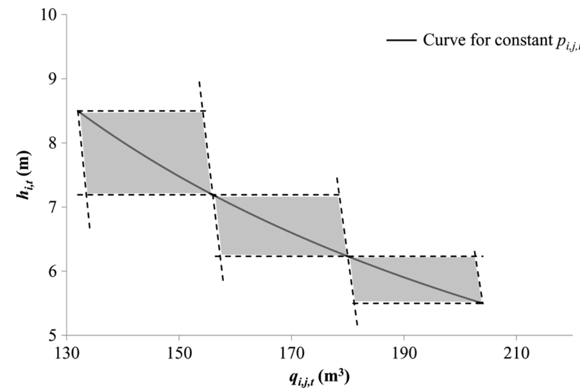

Fig. 4. Relaxation of the nonlinear function using McCormick envelopes defined over partitions of the variable $q_{i,j,t}$.

problem, and hence a valid upper bound on the profit in each node of the sHBB tree. The MILP-OEP is built over the relaxation of the nonlinearities, whereby the bilinear terms and polynomial functions are replaced by polyhedral envelopes that overestimate the feasible region of the original problem.

A. Relaxation of the Bilinear Terms

The bilinear terms in the MINLP and S-MINLP models are present in the power generation function expressed by (10). In this equation, the variables involved are $q_{i,j,t}$, $hup_{i,t}$ and $hdn_{i,t}$, which can be manipulated defining a new variable $h_{i,t} = hup_{i,t} - hdn_{i,t}\ \forall i,\ t$, and thus the bilinear terms to be tackled are defined as $\overline{p}_{i,j,t} = q_{i,j,t}h_{i,t}\ \forall i,\ j \in M,\ t$, which leads to the following linearized equation for the hydro power generation function used in the MILP-OEP:

$$p_{i,j,t} = \phi_{i,j}\overline{p}_{i,j,t} \quad \forall i,j \in M,t. \tag{19}$$

The relaxation of the bilinear terms is built employing the convex envelopes proposed in [39], whereby in the MILP-OEP, $\overline{p}_{i,j,t} = q_{i,j,t}h_{i,t}$ is replaced by the following inequalities, the well-known McCormick inequalities:

$$\overline{p}_{i,j,t} \geq Q^L_{i,j,t}h_{i,t} + q_{i,j,t}H^L_{i,t} - Q^L_{i,j,t}H^L_{i,t} \quad \forall i,j \in M,t \tag{20}$$

$$\overline{p}_{i,j,t} \geq Q^U_{i,j,t}h_{i,t} + q_{i,j,t}H^U_{i,t} - Q^U_{i,j,t}H^U_{i,t} \quad \forall i,j \in M,t \tag{21}$$

$$\overline{p}_{i,j,t} \leq Q^U_{i,j,t}h_{i,t} + q_{i,j,t}H^L_{i,t} - Q^U_{i,j,t}H^L_{i,t} \quad \forall i,j \in M,t \tag{22}$$

$$\overline{p}_{i,j,t} \leq Q^L_{i,j,t}h_{i,t} + q_{i,j,t}H^U_{i,t} - Q^L_{i,j,t}H^U_{i,t} \quad \forall i,j \in M,t. \tag{23}$$

These inequalities provide a convex envelope for each bilinear term associated with turbine j, in plant i and period t. The concept of convex envelope derived from the above inequalities is illustrated in Figs. 2 and 3. In Fig. 2 the nonlinear relationship between the power generated, the turbined flow of water, and the water head is depicted, while in Fig. 3 a projection of the convex envelopes is presented for a fixed value of the power generated. The data used in these figures is based on the input data of one of the case studies.

The error introduced by the relaxation, defined as $|\overline{p}_{i,j,t} - q_{i,j,t}h_{i,t}|$, may be reduced by considering the convex envelopes built over a partition of the domain of the variables $q_{i,j,t}$ as suggested in [29]. This concept is shown in Fig. 4, where three partitions over the domain of $q_{i,j,t}$ are defined.

The formal definition of the convex envelopes presented in Fig. 4 built over a partition scheme with grid points $qq \in G$ is defined by the following disjunction:

$$\bigvee_{qq\in G}\begin{bmatrix} Z_{i,j,t,qq} \\ \overline{p}_{i,j,t} \geq Q_{i,j,t,qq-1}h_{i,t} + q_{i,j,t}H^L_{i,t} - Q_{i,j,t,qq-1}H^L_{i,t} \\ \overline{p}_{i,j,t} \geq Q_{i,j,t,qq}h_{i,t} + q_{i,j,t}H^U_{i,t} - Q_{i,j,t,qq}H^U_{i,t} \\ \overline{p}_{i,j,t} \leq Q_{i,j,t,qq}h_{i,t} + q_{i,j,t}H^L_{i,t} - Q_{i,j,t,qq}H^L_{i,t} \\ \overline{p}_{i,j,t} \leq Q_{i,j,t,qq-1}h_{i,t} + q_{i,j,t}H^U_{i,t} - Q_{i,j,t,qq-1}H^U_{i,t} \end{bmatrix} \quad \forall i,j \in M,t \tag{24}$$

where $Z_{i,j,t,qq}$ is a boolean variable to establish whether the disjunctive term of the partition with index qq is true. This disjunction can be re-written as a sub-MILP model by using a convex-hull reformulation [40]:

$$\overline{p}_{i,j,t} \geq \sum_{qq\in G}\left(Q_{i,j,t,qq-1}hd_{i,j,t,qq-1} + qd_{i,j,t,qq-1}H^L_{i,t} - Q_{i,j,t,qq-1}H^L_{i,t}z_{i,j,t,qq}\right) \quad \forall i,j \in M,t \tag{25}$$

$$\overline{p}_{i,j,t} \geq \sum_{qq\in G}\left(Q_{i,j,t,qq}hd_{i,j,t,qq-1} + qd_{i,j,t,qq-1}H^U_{i,t} - Q_{i,j,t,qq}H^U_{i,t}z_{i,j,t,qq}\right) \quad \forall i,j \in M,t \tag{26}$$

$$\overline{p}_{i,j,t} \leq \sum_{qq\in G} \left(Q_{i,j,t,qq} hd_{i,j,t,qq-1} + qd_{i,j,t,qq-1} H^L_{i,t} - Q_{i,j,t,qq} H^L_{i,t} z_{i,j,t,qq}\right) \quad \forall i,j \in M, t \tag{27}$$

$$\overline{p}_{i,j,t} \leq \sum_{qq\in G} \left(Q_{i,j,t,qq-1} hd_{i,j,t,qq-1} + qd_{i,j,t,qq-1} H^U_{i,t} - Q_{i,j,t,qq-1} H^U_{i,t} z_{i,j,t,qq}\right) \quad \forall i,j \in M, t \tag{28}$$

$$q_{i,j,t} = \sum_{qq\in G} qd_{i,j,t,qq-1} \quad \forall i,j \in M, t \tag{29}$$

$$h1_{i,j,t} = \sum_{qq\in G} hd_{i,j,t,qq-1} \quad \forall i,j \in M, t \tag{30}$$

$$h2_{i,j,t} \leq H^U_{i,t} z_{i,j,t,qq} \quad \forall i,j \in M, qq=1, t \tag{31}$$

$$h_{i,t} = h1_{i,j,t} + h2_{i,j,t} \quad \forall i,j \in M, t \tag{32}$$

$$qd_{i,j,t,qq-1} \geq Q_{i,j,t,qq-1} z_{i,j,t,qq} \quad \forall i,j \in M, t, qq \in G \tag{33}$$

$$qd_{i,j,t,qq-1} \leq Q_{i,j,t,qq} z_{i,j,t,qq} \quad \forall i,j \in M, t, qq \in G \tag{34}$$

$$H^L_{i,t} z_{i,j,t,qq} \leq hd_{i,j,t,qq-1} \leq H^U_{i,t} z_{i,j,t,qq} \quad \forall i,j \in M, t, qq \in G \tag{35}$$

$$\sum_{qq>1} z_{i,j,t,qq} = 1 \quad \forall i,j \in M, t. \tag{36}$$

Increasing the number of partitions reduces the gap between $\overline{p}_{i,j,t}$ and $q_{i,j,t} h_{i,t}$, but also increases the number of equations and continuous and binary variables, and consequently the computational time associated with the solution of the MILP-OEP. In the context of an hydrothermal model, Cerisola *et al.* [41] have recently proposed the same type of linear approximation using piecewise McCormick planes. However, they use a Big-M reformulation, instead of the convex-hull. Preliminary results have shown that in the problems studied in this work the convex-hull provides a tighter relaxation than the Big-M reformulation.

B. Relaxation of the Polynomial Functions

In this work two polynomial relaxations (PR) are considered for building linear envelopes for the polynomial functions: 1) PR1-based on the determination of the inflection points of the polynomials that are nonconvex and nonconcave and calculation of off-set values to be included in the linear over and under estimators functions; and 2) PR2-built by replacing each univariate nonlinear power function of the polynomial function by a new variable, and then for each univariate nonlinear power function an overestimation model is built. The two relaxations are rigorous in the sense that the linearizations do not cut-off any part of the polynomial functions. However, PR1 requires a pre-processing step to identify the characteristics of the polynomial functions, while PR2 does not.

An analysis of the properties of the polynomial functions was made in order to determine the sign of the second derivative through the identification of the inflection points, and hence the polynomial functions that are concave, convex, and nonconvex and nonconcave. Valid over- and under-estimators for the concave and convex polynomial functions are built using piecewise linear approximations between the bounds of the variables and hyperplanes at given points. Due to space limitations, only the construction of these estimators are presented for the concave polynomials associated with the tailrace level. The under-estimators are built over a partition N with grid points n, where the estimator value is represented by $\overline{hdn}_{i,t}$. In order to simplify the equations, $\Theta(D_{i,t,n})$ is defined first as

$$\Theta(D_{i,t,n}) = b_{i,0} + b_{i,1} D_{i,t,n} + b_{i,2} D^2_{i,t,n} + b_{i,3} D^3_{i,t,n} + b_{i,4} D^4_{i,t,n} \quad \forall i,t. \tag{37}$$

The piecewise underestimation is as follows:

$$\overline{hdn}_{i,t} \geq \sum_{n\in N} \left[\Theta(D_{i,t,n}) yd_{i,t,n} + \frac{\Theta(D_{i,t,n+1}) - \Theta(D_{i,t,n})}{D_{i,t,n+1} - D_{i,t,n}} \times (dn_{i,t,n} - D_{i,t,n} yd_{i,t,n})\right] \quad \forall i,t \tag{38}$$

$$d_{i,t} = \sum_{n\in N} dn_{i,t,n} \quad \forall i,t \tag{39}$$

$$D_{i,t,n} yd_{i,t,n} \leq dn_{i,t,n} \leq D_{i,t,n+1} yd_{i,t,n} \quad \forall i,t,n \in N \tag{40}$$

$$\sum_{n\in N} yd_{i,t,n} = 1 \quad \forall i,t. \tag{41}$$

The over estimators are built using the supporting hyperplanes:

$$\overline{hdn}_{i,t} \leq \Theta(D_{i,t,r}) + \nabla\Theta(D_{i,t,r})(d_{i,t} - D_{i,t,r}) \; \forall i,t,r. \tag{42}$$

An equivalent rationale is employed to build under and over estimators for the concave polynomial functions for the forebay levels. As discussed in [16] on the applicability of piecewise linear functions for hydrothermal models, these functions provide an approximation of the real function. However, in this work, they are used to predict bounds.

C. Specific Details and Remarks

The length of the interval of the variables is known to have an impact on the tightness of the relaxation provided by MILP-OEP. Therefore, it is important to eliminate infeasible regions out of the domain of the variables. In this work, a pre-processing step is performed in order to contract the bounds of the variables involved in a nonlinear term of the MINLP by solving two LP problems for each of these variables, whereby a variable is minimized/maximized subject to the constraints of the MILP-OEP. Through this procedure, the lower and upper bounds of the variables are tightened. The formulation of the MILP-OEP is improved by enforcing symmetry breaking constraints over the binary variables $x_{i,j,t}$ for the turbines with identical specifications that operate in parallel and are linked to the same reservoir. These constraints are represented by

$$x_{i,j,t} \geq x_{i,j+1,t} \quad \forall i,j \in M \cap UI_{i,j}, i, j+1 \in M, t \tag{43}$$

where $UI_{i,j}$ represents the subsets of turbines with the same characteristics in the same plant. The sHBB algorithm solves an MILP-OEP and an MINLP model at each node of the tree. The branching process involves splitting the feasible region of the MILP-OEP based on the largest error of the relaxations

TABLE I
COMPARISON BETWEEN sHBB AND BARON FOR THE DETAILED MINLP

	sHBB						BARON		
	PR1			PR2					
Case	P (m.u.)	G (%)	T (s)	P (m.u.)	G (%)	T (s)	P (m.u.)	G (%)	T (s)
Case 1	1,249,705	0.19	404	1,249,705	0.20	52	1,249,243	0.50	106
Case 2	1,372,403	0.15	25	1,372,403	0.22	32	1,371,840	2.67	10,800
Case 3	2,552,090	0.22	71	2,551,783	0.38	65	-426,000	-	10,800
Case 4	2,847,564	2.68	10,800	2,847,392	3.44	10,800	2,845,660	6.04	10,800

P-Profit, G-Gap, T-CPU time, PR1, PR2-Polynomial relaxations. The terminal criteria are set to 0.5% gap and to a maximum CPU time of 10 800 s.

for the bilinear terms and for the polynomial functions. If this error is associated with a bilinear term, the respective semi-continuous variable $q_{i,j,t}$ is split into two regions in the middle point, $q'_{i,j,t}$, of the interval $[Q^L_{i,j,t}, Q^U_{i,j,t}]$, and two new problems are generated, leading to two new nodes. Note that the variable $q_{i,j,t}$ is a semi-continuous variable defined as $q_{i,j,t} \in \{0\} \cup [Q^L_{i,j,t}, Q^U_{i,j,t}]$, which in this work is taken into account when the above convex envelopes are built. This provides a tighter relaxation of the bilinear terms, which is not taken into account for example in the solver BARON.

If $q_{i,j,t}$ is chosen for the branching, the following procedure is applied on each node: 1) the bounds of $q_{i,j,t}$ are updated, and consequently the grid used in the relaxation is updated; 2) the SBC induces an additional bound contraction scheme, which is applied to the turbines in the same plant with identical characteristics of the turbine selected to make the branch. If in one of the generated nodes, the upper bound of $q_{i,j,t}$ is changed, then the upper bounds of the turbines for $j' > j$ are also updated:

$$Q^U_{i,j+1,t} \leq Q^U_{i,j,t} \quad \forall i, j \in M \cap UI_{i,j}, i, j+1 \in M, t. \tag{44}$$

Similarly, for the node where the lower bound is updated, the following bounds are enforced:

$$Q^L_{i,j-1,t} \geq Q^L_{i,j,t} \quad \forall i, j \in M \cap UI_{i,j}, i, j-1 \in M, t. \tag{45}$$

3) If the MILP-OEP is solved to optimality within the maximum CPU time set to solve the MILP-OEP, one additional partition is added to the piecewise partition scheme of all variables $q_{i,j,t'}$ $\forall t'$ in the two new generated nodes; otherwise the number of partitions of the two new generated nodes is set to the number of partitions of the precedent node, and the number of partitions is not increased any further. Other details of the sHBB implementation include: 1) the selection of the next node to solve is made based on the node with the largest upper bound; 2) the MILP-OEPs are solved within a specified CPU time limit. If the problem is solved to optimality, then the upper bound in the node is given by the integer solution; otherwise, the best bound obtained is used as a valid upper bound in the node; and 3) the lower bound of the objective function of the MINLP problem, i.e., a solution of the original problem, is obtained by fixing the binary variables associated with the turbines at the value of the MILP-OEP solution, if they are equal to one, leaving the variables equal to zero free and solving a reduced MINLP problem. This MINLP problem provides a valid lower bound on the objective function at a low computational cost since the number of free binary variables is reduced. This approach proved to be a better option than to fix all the binary variables and solve an NLP problem.

V. COMPUTATIONAL EXPERIMENTS

The computational performance of the proposed models and of the global optimization algorithm are evaluated in this section. We considered four test cases presented in [8] involving cascades of hydro plants with different number of plants, turbines and cascade topology: Case 1–4 plants, 24 turbines; Case 2–5 plants, 22 turbines; Case 3–7 plants, 29 turbines; Case 4–6 plants, 44 turbines. The configuration of the cascades and the topological, reservoir and hydro data are the same as those published in [8]. The size of the four models is presented in Appendix A. The models and the global optimization algorithm are implemented in GAMS [42] and solved on a computer with an Intel Core i7@3.07 GHz CPU, 64 bits, and 8 Gb of RAM. The solvers used are CPLEX 12.4, GAMS/DICOPT 23.8 and BARON 11.1. Table I summarizes the computational performance of sHBB and BARON for Cases 1, 2, 3 and 4 for the detailed MINLP (the indicated CPU time set for sHBB preclude the time spent on bound tightening, which was respectively 21, 15, 23 and 31 min). The termination criteria are set to a 0.5% gap and a maximum CPU time of 10 800 s. For sHBB, the table presents the results with the polynomial relaxations PR1 and PR2, showing that optimality gaps below 0.5% gap are obtained in Cases 1, 2 and 3, in short CPU times, i.e., all below 404 s and 65 s for PR1 and PR2, respectively. However, for the larger Case 4, the performance decreases and solutions with an optimality gap below 0.5% are not reached within 10 800 s. The results obtained with BARON, show that for Case 3 it has difficulty in finding a positive lower bound, and that overall, comparing BARON with the proposed approach, sHBB is found to achieve smaller optimality gaps. BARON is only able to solve Case 1 for the specified optimality gap in 106 s, while for Cases 2, 3, and 4 it cannot reach solutions for a 0.5% optimality gap within 10 800 s, which highlights the better performance of sHBB.

The results obtained with the S-MINLP with sHBB and BARON present the same trend as obtained with the detailed MINLP, but in general with shorter CPU times and smaller optimality gaps for sHBB, see Table II. Comparing the results of the MINLP model with the approximations for the polynomial functions, and without them, S-MINLP, it is clear that the bilinear terms have a major influence on the observed gaps between the lower and upper bounds of the objective function.

TABLE II
COMPARISON BETWEEN sHBB AND BARON FOR THE S-MINLP. THE TERMINAL CRITERIA ARE SET TO 0.5% GAP AND TO A MAXIMUM CPU TIME OF 10 800 s

	sHBB			BARON		
Case	P (m.u.)	G (%)	T (s)	P (m.u.)	G (%)	T (s)
Case 1	1,303,536	0.16	51	1,303,766	0.50	56
Case 2	1,447,638	0.14	16	-312,432	-	10,800
Case 3	2,699,187	0.21	39	2,681,850	2.18	10,800
Case 4	2,907,365	2.32	10800	2,906,790	3.48	10,800

P-Profit, G-Gap, T-CPU time.

TABLE III
MINLP MODELS SOLVED WITH DICOPT, AND S-MILP WITH CPLEX. THE TERMINAL CRITERIA ARE SET TO A MAXIMUM CPU TIME OF 10 800 s, AND A 0.5% GAP FOR CPLEX

	Without SBC		With SBC	
Cases	P (m.u.)	T (s)	P (m.u.)	T (s)
MINLP				
Case 1	1,249,705	9	1,249,705	9
Case 2	1,364,140	10,800	1,369,935	39
Case 3	2,545,173	10,800	2,551,687	8,934
Case 4	2,846,776	10,804	2,848,254	10,654
S-MINLP				
Case 1	1,303,782	5	1,303,782	6
Case 2	1,441,263	10,800	1,436,125	43
Case 3	2,685,520	10,800	2,690,461	291
Case 4	2,906,996	10,803	2,906,496	7,422
S-MILP				
Case 1	1,253,778	0.1	1,253,778	0.1
Case 2	1,363,801	1.1	1,363,801	0.2
Case 3	2,546,971	11.8	2,546,971	1.4
Case 4	2,867,973	31.9	2,867,973	2.4

P-Profit, T-CPU time.

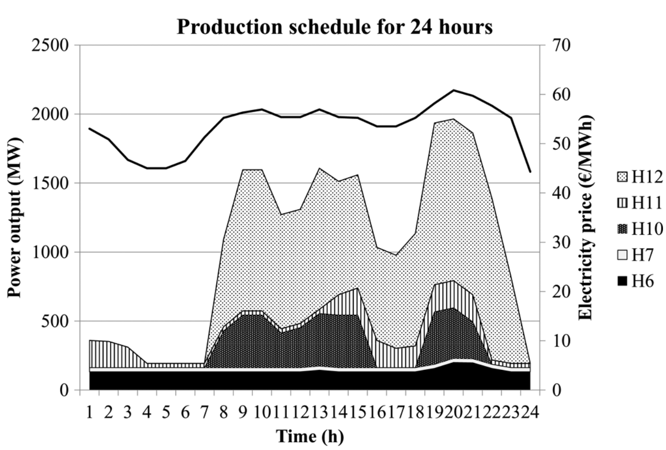

Fig. 5. Case 2 production schedule obtained with sHBB.

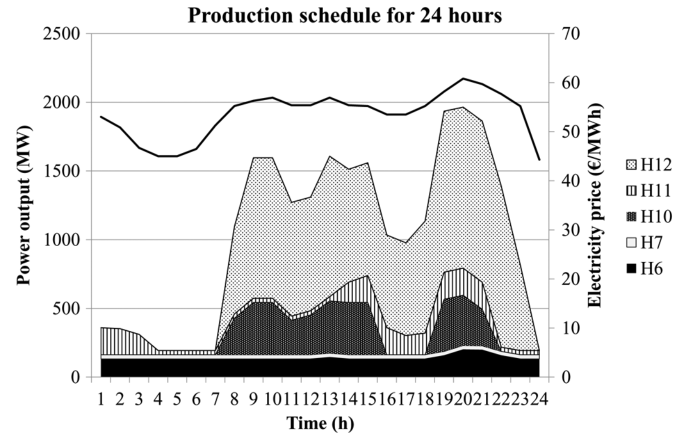

Fig. 6. Case 2 production schedule obtained with BARON.

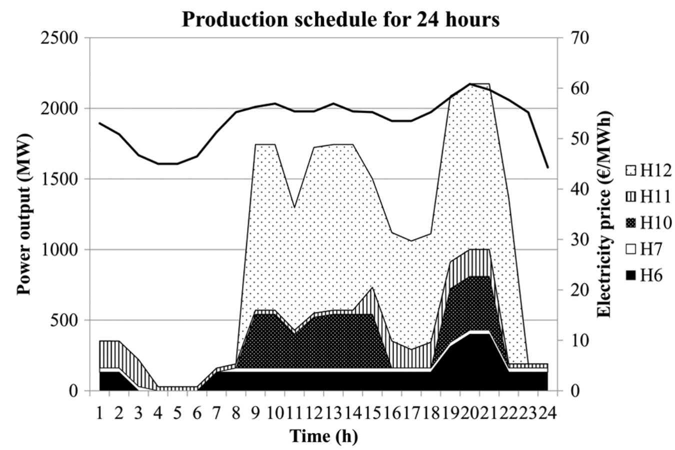

Fig. 7. Case 2 production schedule obtained with S-MILP.

This is supported by the small differences between the performance of sHBB and BARON with the MINLP and S-MINLP models. In order to assess the objective function values obtained with sHBB, a local solver for MINLP problems, DICOPT, is used to solve the detailed MINLP and S-MINLP models. The two models are solved either per se, or including the SBC in order to check the effect of these constraints (see Table III). This table shows the SBCs to have an all-round strong impact on the performance of DICOPT. This is explained by the symmetry breaking imposed by these constraints during the solution of the master problem within DICOPT. Within sHBB these constraints have also an important role on the solution of the MILP-OEP problem and on the solution of the MINLP problem used to calculate the lower bound in each node.

As for the S-MILP model, the CPU times required for solving it with CPLEX are significantly lower than the ones obtained with the global optimization approach, but the SBC still show a positive impact on the CPU times.

A. Scheduling Results

Figs. 5–7 illustrate the power production schedule for Case 2 obtained with sHBB, BARON and the S-MILP model, respectively. These schedules display the same general trend. However a thorough analysis shows differences on the turbines activated and respective water flows. For example, the S-MILP does not activate the turbines from plant H6 in the periods 4, 5 and 6, but does so in the remaining two schedules. As a consequence, different operating conditions in terms of the profile of the volume of water in the reservoir H6 and the spillage of the downstream run of the river plant H7 are obtained with each model (see Figs. 8 and 9).

An interesting result concerns the maximum power output during the peak hours of the day obtained with the different

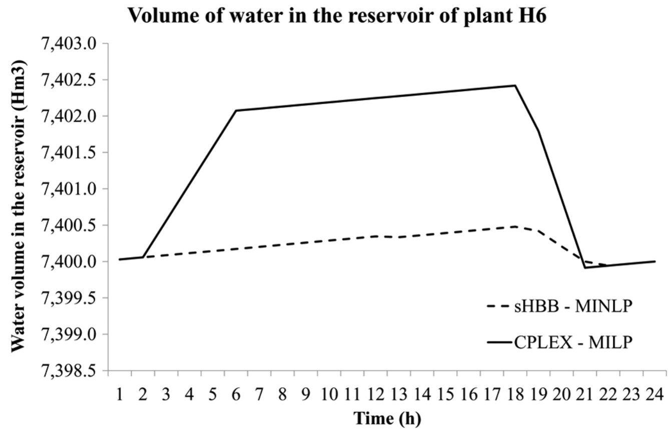

Fig. 8. Case 2 water volumes of reservoir H6 obtained by solving the MINLP problem with sHBB, and the S-MILP problem with CPLEX.

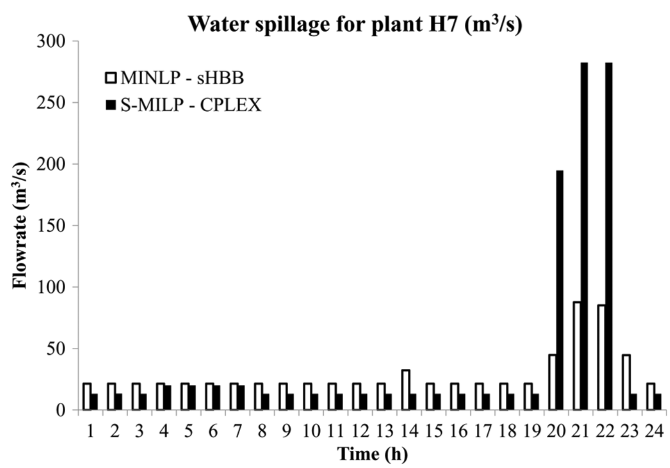

Fig. 9. Case 2 spillage of run of the river plant H7 obtained by solving the MINLP problem with sHBB, and the S-MILP problem with CPLEX.

approaches for the highest electricity prices. The S-MILP model exhibits the highest power output during this period when compared with the sHBB and BARON, but that did not correspond to a higher profit, 1 363 801 m.u. with the S-MILP versus 1 372 403 m.u. with the detailed MINLP using sHBB.

An important remark is that the analysis and comparison of the results obtained should take into consideration that the models have different levels of accuracy and constraints. For example, the detailed MINLP model relates the forebay and tailrace levels with the volume and water discharged, respectively, leading to a variable net water head, while in the S-MILP the net water head is constant. In order to illustrate this feature, the detailed MINLP model was solved considering the schedule of the units fixed to the solution of the S-MILP model. More specifically, the variables $x_{i,j,t}$, $q_{i,j,t}$, and $s_{i,t}$ are fixed, reducing the MINLP to an NLP problem that is completely defined, i.e. with no degrees of freedom. A careful analysis of the resulting model shows that after fixing these decision variables the remaining variables can be calculated in the following sequence: $qh_{i,t}$, $d_{i,t}$, $v_{i,t}$, $hup_{i,t}$, $hdn_{i,t}$, and $p_{i,j,t}$, and $ph_{i,j,t}$. For the sake of expediency, the corresponding NLP problem of each case was solved with the solver CONOPT. For the four cases the solver returned the problem as infeasible at the structure analysis stage and indicated the violation of the maximum power generated by some turbines. In all cases the schedule from the S-MILP model leads to an infeasible schedule using the MINLP model because it forces the turbines to generation values above their limit. This evidence was validated by introducing slack variables in (14) and in the objective function, which confirmed the violation of the maximum limits of the turbines. These results demonstrate that for a specific cascade state, the S-MILP model may incorrectly predict a power generation that is higher than the system maximum generation limit. In this specific case, the S-MILP model considers the water head constant through the time horizon, when in fact due to the water discharges it will change during the 24 hours.

The relative merit of each model in terms of accuracy depends on the nonlinearities involved in the real process, and the assumptions and level of detail of the approximations made. From the point of view of profit, the main drawback of the MINLP lies in the possibility of having the global solution cut-off when it is solved with a local solver, therefore, leading to a significantly sub-optimal profit gain. This provided the motivation for the application of global optimization strategies as proposed in this work.

Analyzing the daily profit of 1 372 403 m.u. found with the proposed approach for Case 2 of MINLP-PR1, it represents a higher profit of 2468 m.u. per day compared to DICOPT with SBC (1 369 935 m.u.), amounting to an additional yearly profit of 900 820 m.u. Similar analysis can be made for the other cases, where the use of sHBB always results in a positive gain. The exception is Case 4, where DICOPT despite being a non-global solver, and thus lacking of global optimality guarantees for non-convex problems, obtained a higher profit. This is most likely a result of the integer cuts that were implemented, forcing the algorithm to fix a different combination of binary variables on each iteration, which are missing in sHBB.

Therefore, for most cases sHBB is able to find better solutions than a non-global solver, with the advantage of guaranteeing that no other solution can be found that improves the objective function above the corresponding optimality gap.

VI. Conclusions

In this paper a tailored global optimization algorithm, sHBB, is proposed for the STHS of a cascade of hydro plants. A non-convex MINLP model for the operation of the cascade is proposed and an overestimator model formulated, which is based on a relaxation framework for the polynomial functions describing the forebay and tailrace levels, as well as for the bilinear terms in the power generation function. The overall performance of the proposed approach is the result of the combination of several algorithmic implementations, such as a specific dynamic partition relaxation of the bilinear terms exploiting the semi-continuous characteristics of one of the variables, and of the symmetry breaking constraints shown to play an important role in the solution of the MILP overestimator problem.

One relevant conclusion, in the case of sHBB, is establishing the bilinear terms as the nonlinear functions that contribute the most for the gap between the lower and the upper bound. This assessment arises from comparing the approximation errors and

TABLE IV
SIZE OF THE MODELS, MINLP/MILP-OEP/S-MILP-OEP

Cases	Equations	Variables	0-1 Variables
Case 1	3,889/22,321/20,017	2,977/14,185/13,081	576/4,200/3,720
Case 2	3,697/13,825/11,209	2,953/ 7,921/ 6,769	528/2,064/1,584
Case 3	4,945/18,673/14,857	3,961/10,729/ 9,001	696/2,808/2,088
Case 4	6,937/24,049/21,865	5,233/13,513/12,769	1,056/3,408/3,168

MILP-OEP—Overestimation model of the original MINLP model with PR1 at the root node. S-MILP-OEP—Overestimation model of the S-MINLP model at the root node.

final gaps obtained with sHBB for the detailed MINLP and simplified model, S-MINLP. This means that in the case studies presented, the polynomials of 4th order can be replaced by the linear approximations without introducing a significant error.

Another important result obtained with sHBB is the identification of alternative power production schedules, which are associated with higher profits, a possibility worth of further investigation.

The proposed algorithm was found to exhibit better computational performance than a current available standard global optimization solver and provide better solutions than a non-global MINLP solver, and thus representing a valid contribution for the application of deterministic global optimization to power systems.

APPENDIX A

Table IV lists the size of the models, MINLP/MILP-OEP/S-MILP-OEP.

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Ricardo M. Lima received the Licentiate degree in 1999, and the Ph.D. degree in 2006, both in chemical engineering from the Faculty of Engineering, University of Porto, Portugal.

He has worked as a post-doc fellow in the Department of Chemical Engineering at the Carnegie Mellon University, Pittsburgh, PA, USA, in 2006–2011, and as an invited researcher in PPG Industries in 2008–2011. He joined the Portuguese National Laboratory of Energy and Geology (LNEG) in Lisbon, Portugal, in 2011. His research interests include mathematical programming, energy systems, and the design, scheduling/planning of industrial processes.

Marian G. Marcovecchio received the B.S. degree in applied mathematics in 2000, and the Ph.D. degree in computational mechanics engineering in 2007 from Universidad Nacional del Litoral (UNL), Santa Fe, Argentina.

She has worked for 2 years as a researcher in the Portuguese National Laboratory of Energy and Geology (LNEG), Lisbon, Portugal. She is currently a researcher at INGAR, Instituto de Desarrollo y Diseño, CONICET and an Associate Professor at UNL. Her research interests include mathematical modeling and global optimization methods; and its applications to the design, operation and planning of industrial processes.

Augusto Queiroz Novais received the Licentiate degree in chemical engineering from IST, Lisbon, Portugal in 1969, the Ph.D. degree in chemical engineering/process systems from the University of Leeds, U.K., in 1978, and the Aggregation degree in chemical engineering from IST in 2007.

He has worked for 2 years as a lecturer in Mozambique, and 5 years as a Research Fellow in the U.K. He is a senior researcher at the Portuguese National Laboratory of Energy and Geology (LNEG), Lisbon, Portugal, where he acts presently as the Head of the Energy Systems Modeling and Optimization Unit. His present interests include mathematical modeling and optimization methods addressing industrial processes and energy systems, and planning, scheduling and optimization of supply chains.

Ignacio E. Grossmann received the B.S. degree in chemical engineering from Universidad Iberoamericana in Mexico in 1977, and the M.Sc. and Ph.D. degrees in chemical engineering from Imperial College in London, U.K.

He is the R. Dean University Professor at Carnegie Mellon University and is currently director of the Center for Advanced Process Decision-making. He is a member of the National Academy of Engineering, and his interests include mixed-integer optimization, design of energy and water systems, and planning, scheduling and supply chain optimization.