

# Integrated Modeling Framework for Supply Chain Design Considering Multiproduct Production Facilities

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**ABSTRACT:** Significant benefits can be obtained if the interactions among different decision levels are appropriately addressed and simultaneously solved. In this work, a MILP formulation for the supply chain design is presented which simultaneously takes into account considerations of multiproduct batch production facilities. Usually SC design models used to assume a constant performance and design of the involved plants. Our proposal allows assessment of the trade-offs between decisions of different management levels: from the strategic perspective (nodes selection, supplier selection, material flows among nodes, etc.) until the operative one (production scheduling using campaigns). From several examples, this approach shows that decisions about supply chain and plants are tightly related among them and a general performance cannot be assumed for the production facilities.

## 1. INTRODUCTION

A supply chain (SC) is a network of firms and distribution channels organized to acquire raw materials, convert them to finished products, and distribute these products to customers. Several decisions must be addressed in order to achieve an efficient SC coordination. They can be classified into three categories according to their importance and the length of the considered planning horizon. First, decisions regarding the location, capacity, and technology of plants and warehouses are generally seen as strategic with a planning horizon of several years. Second, supplier selection, product assignment, as well as distribution channel and transportation mode selection belong to the tactical level and can be revised every few months. Finally, raw material, semifinished, and finished product flows in the network are operational decisions that are easily modified in the short term.<sup>1</sup>

Several authors have referred to the integration of SC decisions as an important and still open issue.<sup>2–6</sup> Significant benefits can be obtained by addressing the network as a whole, considering its various components and the interactions among decision levels simultaneously.

In general, most previous published papers have considered the SC optimization problems, taking into account different perspectives separately, and then, the trade-offs between the various decisions involved are squandered. However, in the last years, there have been some attempts to combine decisions in SC models, particularly strategic and tactical ones. Sundarmoorthy and Karimi<sup>7</sup> presented an approach for new product introduction and planning in pharmaceutical supply chains. Guillén et al.<sup>8</sup> integrated planning and scheduling decisions of chemical SC, taking into account financial management issues. Amaro and Barbosa-Póvoa<sup>9</sup> presented a modeling approach for the sequential planning and scheduling of SC. Lainez et al.<sup>10</sup> proposed a mixed integer linear program (MILP) for SC design and planning integration. They showed that significant improvements can be achieved when decisions are integrated, but it increases the computational complexity. You and Grossmann<sup>11</sup> formulated a mixed-integer nonlinear program (MINLP) model for simultaneously considering inventory optimization and SC network design under demand

uncertainty. Guillén and Grossmann<sup>12</sup> addressed the optimal design and planning of sustainable chemical processes through a bicriterion stochastic MINLP. They proposed a decomposition methodology separating the problem into two subproblems and iterating between them. Lainez et al.<sup>13</sup> present a flexible multiperiod formulation for the design and planning problems, by translating a recipe representation to the SC environment. They consider all the feasible links and material flows among the potential SC members. Naraharsetti and Karimi<sup>14</sup> developed a MILP model for SC redesign in multipurpose plants. The proposed approach serves as a tool for deciding where to introduce a new production process. You et al.<sup>15</sup> proposed a multiperiod MILP model for the simultaneous capacity, production, and distribution planning for a multisite system. They considered potential capacity modifications in a production facility in order to produce different product families. Pinto-Varela et al.<sup>16</sup> addresses the SC planning and design problems introducing environmental aspects. They combine MILP and symmetric fuzzy linear programming to solve the model. A representation, using the Resource-Task-Network (RTN), is posed to generate an integrated formulation of the SC design and planning.

Despite the abundant and growing body of literature about SC design and planning, SC planning and scheduling, and SC redesign and planning, including many reviews,<sup>2,17,18</sup> to the best of our knowledge, only a few papers deal with the integrated design of SC and involved plants. Corsano et al.<sup>19</sup> presented a detailed nonlinear programming (NLP) model for the design of a multisite plant complex, considering integration between plants simultaneously with the optimal operation and production planning of each plant involved in the multiplant complex. Corsano et al.<sup>20</sup> presented a MINLP optimization model for a sustainable design and operation analysis of sugar/ethanol SC. A detailed model for the ethanol plant design was

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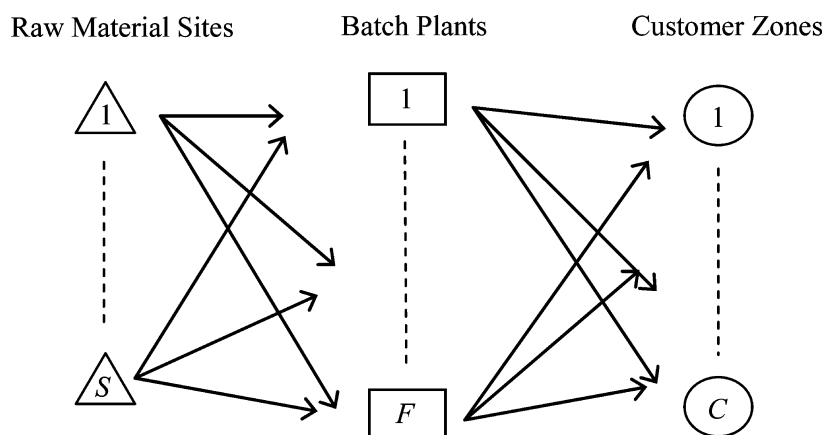


Figure 1. SC representation considered for the proposed approach.

embedded in the SC model and, therefore, plant and SC designs were simultaneously obtained. In a more general work, Corsano and Montagna<sup>21</sup> presented a MILP model for the simultaneous optimization of SC and involved plants design. In that work, decisions regarding SC network, such as nodes selection and materials distribution, are together considered with multiproduct batch plants design decisions in order to attain a more integrated perspective of the SC design problem. The advantage of that approach is that simultaneous optimization allows assessing the trade-offs between different decision variables, evaluations that cannot be carried out when sequential methodologies are considered.

However, these previous approaches have not considered operational aspects. They have only been focused on long-term decisions. Therefore, it is worth assessing the influence of operations about the global supply chain design. There are several operational elements that could be included. In this work, scheduling is chosen as a critical decision taking into account that an appropriate scheduling policy influences the global SC performance and allows efficient use of facilities, adequate transport and distribution, suitable inventory levels, and procurement policies, etc.

When the relationship among decisions of different levels must be a priori assessed, a tight short-term scheduling cannot be considered. That approach is focused on satisfying specific requirements. Conversely, a general scheduling should be posed in order to consider production flows in the plant from a broad perspective. On the other hand, when the global and long-term design problem is posed, several suppositions have to be assumed. Usually in deterministic models, the more usual or frequent scenario is posed to solve the problem. Many previous works in multiproduct batch plant design (Barbosa-Póvoa<sup>22</sup>) used a single product campaign, where the requirements for a product had to be fulfilled before following with the next product. This is the simplest scheduling policy. In this way, design models are simplified, but from the operational and commercial point of view, the production policy adopted is not realistic, since, for example, huge inventories should be kept to support this approach. If the impact of the attained solution on the operation management should be analyzed, inventory and logistics decisions could not be appropriately evaluated. Thus, all the operational aspects could not be assessed.

In this work, an operation based on mixed product campaigns (MPCs) is introduced. In this case, a campaign includes several batches of different products that are going to

be manufactured in the plant and the same sequence is cyclically repeated over the time horizon. Obviously, this approach is completely suitable when plants work under stable demand patterns over long planning horizons.<sup>23</sup> Nevertheless, in general, stability cannot be assured for a long-term context and those cases are few. Nowadays, market conditions are continuously changing. However, when the relationship between operational and strategic decisions must be considered and the links between them had to be assessed, this kind of scenario provides an appropriate context. In order to analyze the performance of the global SC as well as the behavior of the involved elements, more frequent or general cases can be posed. Thus, for example, production flows in facilities can be estimated, which is a very important result, since they affect several decisions: inventory, transport, procurement, etc. Generally, previous deterministic approaches also assumed a particular scenario. Now, this article works with the same concept, but adding operational elements for the more frequent cases. Therefore, this formulation provides a better comprehension of the supply chain behavior and the operation of each plant, assuming production facilities have different operation policies to achieve an optimal global performance.

Recently, Fumero et al.<sup>24</sup> presented a MILP model for the simultaneous design and production scheduling of multiproduct batch plants. This formulation determines the optimal plant configuration, unit sizes, number of batches of each product in the production campaign, and its sequencing in order to fulfill specific product demands over the time horizon. Taking into account the model presented by these authors, constraints are reformulated in order to incorporate them into a global SC design model. In the previous model, the plant is given and the product demands are known. Now, the plants must be allocated and the product requirements must be determined for each plant. Therefore, the required reformulations are not trivial, resulting, first, in a MINLP model. Then, several constraints are treated in order to keep the linearity of the model to guarantee the global solution and an efficient computational performance.

Thus, this work presents a novel approach in order to assess the relationship among decisions involved in the SC design. The integration and simultaneous evaluation of the trade-offs between strategic and operational considerations allow considering the impact of the different decisions. The capabilities of the presented formulation are highlighted

through different examples, which are solved in reasonable computational times.

This paper is organized as follows. First, the description of the problem and the main assumptions are presented in section 2. In section 3, the general MILP model is formulated. In section 4, several examples for the proposed approach are presented, in order to show the effect and the influence of plant performance models on the overall SC design model. Finally, conclusions of the work are drawn in the last section of this paper.

## 2. PROBLEM DEFINITION

The SC under study considers three-echelons: raw material sites, multiproduct batch production plants, and customer zones (Figure 1). The raw material sites are known, and they provide the required inputs for each plant. At each raw material site  $s$  ( $s = 1, \dots, S$ ), one or more types of raw materials  $r$  ( $r = 1, \dots, R$ ) are available to be delivered to plants. The number, location, and design of multiproduct batch plants have to be determined. Each possible plant location  $f$  ( $f = 1, \dots, F$ ) has  $J_f$  stages  $j$  ( $j = 1, \dots, J_f$ ) to produce product  $i$  ( $i = 1, \dots, N_p$ ). The plant  $f$  operates over the time horizon  $H_f$ . Each customer zone  $c$  ( $c = 1, \dots, C$ ) has a known product  $i$  demand  $DM_{ic}$  to be fulfilled.

For each stage of the installed batch plant  $f$ , up to  $K_{jf}$  out-of-phase identical units can be duplicated. Unit sizes are restricted to take discrete values. Following the usual procurement policy in industry, a set  $SV_{jf} = \{VF_{jf1}, VF_{jf2}, \dots, VF_{jfP_{jf}}\}$  is provided, where  $VF_{jfP_{jf}}$  represents the discrete size  $p$  for batch equipment of stage  $j$  of plant  $f$  and  $P_{jf}$  is the given number of available standard sizes for stage  $j$  of plant  $f$ . Since the parallel units in each stage  $j$  are assumed to be identical, a batch of product  $i$  can be processed on any unit with the same processing time  $t_{ijf}$  and size factor  $SF_{ijf}$ . The size factor  $SF_{ijf}$  represents the required capacity of a unit in stage  $j$  at the plant  $f$  to produce a unit of mass of final product  $i$ .

Taking into account that the production of product  $i$  in each installed plant,  $Q_{if}$  is a model variable, the number of batches of product  $i$  in the campaign,  $NBC_{if}$  must be determined as well as the number of campaign repetitions along time horizon  $H_f$  denoted by  $NN_f$ . In order to attain a linear formulation, a maximum number of  $NBC_{if}^{UP}$  batches of product  $i$  in the MPC composition is suggested, and, for  $NN_f$ , an appropriate discretization is proposed, considering the minimum and maximum number of times that the campaign can be cyclically repeated over the time horizon, expressed by  $NN_f^{LO}$  and  $NN_f^{UP}$ , respectively.

Then, the problem consists of simultaneously determining the following:

(a) SC design: (i) plants installation; (ii) raw material supply from each raw material site; (iii) product amount produced in each installed batch plant; and (iv) material flows among SC nodes.

(b) Installed batch plants design: (i) the configuration of each plant (the number of in-parallel units operating out-of-phase in each stage); (ii) unit sizes; and (iii) the number and size of the batches for each product in each plant.

(c) Installed batch plants production scheduling: (i) the composition of the MPC (number of batches for each product in a campaign) for each installed plant; (ii) the assignment of batches to units in each stage; (iii) production sequence on each unit; (iv) initial and final processing times for the batches that compose the MPC in each processing unit; and (v) the number of repetitions of the MPC along the time horizon.

The performance measure is minimizing the total annual cost, given by the cost associated with plants installation, equipment investment cost, production cost, and transportation cost between the SC nodes.

## 3. MODEL FORMULATION

In order to simplify the model formulation, this article assumes a SC configuration with three echelons. However, this formulation can be easily extended to include a fourth echelon corresponding to warehouses.

**3.1. SC Design Constraints.** Taking into account the decisions involved in the network design, the following binary variables for plants and products assignment are defined:

$$ex_f = \begin{cases} 1 & \text{if plant } f \text{ is installed} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{if} = \begin{cases} 1 & \text{if product } i \text{ is produced in plant } f \\ 0 & \text{otherwise} \end{cases}$$

Between these variables, the following logical constraint is held: if plant  $f$  is not installed ( $ex_f = 0$ ), no product is produced, i.e.  $z_{if} = 0$ . Then,

$$z_{if} \leq ex_f \quad \forall i, f \quad (1)$$

The production of each product in each plant is bounded according to operative, commercial, or marketing conditions. Then:

$$z_{if}Q_{if}^{LOW} \leq Q_{if} \leq z_{if}Q_{if}^{UP} \quad \forall i, f \quad (2)$$

where  $Q_{if}$  represents the amount of product  $i$  produced in plant  $f$ , and  $Q_{if}^{LOW}$  and  $Q_{if}^{UP}$  are known bounds. In the same way, raw material site  $s$  has a limited capacity of raw material  $r$  to be transported to all the installed production plants:

$$\sum_i \sum_f QR_{srif} \leq QR_{rs}^{UP} \quad \forall s, r \quad (3)$$

where  $QR_{srif}$  is the amount of  $r$  transported from  $s$  to  $f$  for producing  $i$ , and  $QR_{sr}^{UP}$  is the available amount of  $r$  at site  $s$ . Moreover, if plant  $f$  is not installed or product  $i$  is not produced at plant  $f$ ,  $QR_{srif}$  has to be zero:

$$QR_{srif} \leq z_{if}QR_{sr}^{UP} \quad \forall s, r, i, f \quad (4)$$

Let  $fc_{rif}$  be a conversion factor that indicates the relation between the raw material  $r$  required to produce one unit of final product  $i$ . Then,

$$\sum_s QR_{srif} = fc_{rif}Q_{if} \quad \forall r, i, f \quad (5)$$

expresses the amount of  $r$  needed to produce  $i$  in plant  $f$ .

For the mass balances between production plants and customer zones, the continuous variable  $QC_{ifc}$  represents the amount of product  $i$  delivered from plant  $f$  to customer zone  $c$ . Then, assuming that the total amount of product  $i$  manufactured at plant  $f$  is delivered to customer zones, the following constraint is posed:

$$\sum_c QC_{ifc} = Q_{if} \quad \forall i, f \quad (6)$$

If product  $i$  is not produced in plant  $f$ , then the amount delivered to each customer zone  $c$  has to be zero. Otherwise,

the total amount delivered is at most the demand of that product in that customer zone:

$$QC_{ifc} \leq z_{if}DM_{ic} \quad \forall i, f, c \quad (7)$$

Equation 7 is redundant, taking into account expressions 2 and 6. However, it has been included in order to improve the computational performance.

Finally, the demand of each product in each customer zone has to be fulfilled:

$$\sum_f QC_{ifc} = DM_{ic} \quad \forall i, c \quad (8)$$

**3.2. Plants Design Constraints.** In this work, the constraints for plants design are largely inspired from Fumero et al.<sup>24</sup> Following, the necessary reformulations of that model are presented in order to embed the plants design and scheduling formulations into the SC design model.

Let  $x_{inf}$  and  $v_{jpf}$  be the binary variables used for selecting the number of batches of each product in the production campaign and the units size for each stage at each installed plant. Then, for each installed plant  $f$ , the number of batches of product  $i$  in the campaign,  $NBC_{if}$  and the unit size for each stage  $j$ ,  $V_{jf}$  are determined by the following equations:

$$NBC_{if} = \sum_{n=1}^{NBC_{if}^{UP}} nx_{inf} \quad \forall i, f \quad (9)$$

$$V_{jf} = \sum_{p=1}^{P_{jf}} v_{jpf}VF_{jfp} \quad \forall j, f \quad (10)$$

The following constraint ensures that exactly one option is selected for  $NBC_{if}$  if product  $i$  is produced in the installed plant  $f$ . Otherwise, i.e.  $z_{if} = 0$ , the number of batches of product  $i$  in the campaign is zero from eq 9.

$$\sum_{n=1}^{NBC_{if}^{UP}} x_{inf} = z_{if} \quad \forall i, f \quad (11)$$

Analogously, if the plant  $f$  is installed, the following constraint is stated to ensure that only one option is selected for the unit size of stage  $j$  of this plant:

$$\sum_{p=1}^{P_{jf}} v_{jpf} = ex_f \quad \forall j, f \quad (12)$$

Taking into account that for each plant  $f$ , the unit size of stage  $j$ ,  $V_{jf}$  must be sufficient to process a batch of each product, the following constraint must be satisfied:

$$V_{jf} \geq SF_{ijf}B_{if} \quad \forall i, j, f \quad (13)$$

Then, considering that the batch size of product  $i$  of plant  $f$ ,  $B_{if}$  depends on the total production of that product, the number of batches of product  $i$  in the campaign and the number of cycles of the campaign, namely,  $B_{if} = Q_{if}/NB_{if}$  where  $NB_{if} = NBC_{if} NN_{if}$  and using eqs 9 and 10, eq 13 is rewritten as follows:

$$NN_f \geq \sum_{p=1}^{P_{jf}} \sum_{n=1}^{NBC_{if}^{UP}} \frac{SF_{ijf}Q_{if}}{VF_{jfp}n} v_{jpf}x_{inf} \quad \forall i, j, f \quad (14)$$

In contrast to formulation presented by Fumero et al.,<sup>24</sup> production requirements of each installed plant are unknown. Therefore, in order to avoid the non linear factor  $Q_{if} v_{jpf} x_{inf}$  the following continuous variable and constraints are defined:

$$w_{ijpnf} = \begin{cases} Q_{if} & \text{if } v_{jpf} \text{ and } x_{inf} \text{ are simultaneously 1} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{p=1}^{P_{jf}} w_{ijpnf} \leq Q_{if}^{UP} x_{inf} \quad \forall i, j, f, n \quad (15)$$

$$\sum_{n=1}^{NBC_{if}^{UP}} w_{ijpnf} \leq Q_{if}^{UP} v_{jpf} \quad \forall i, j, f, 1 \leq p \leq P_{jf} \quad (16)$$

$$\sum_{n=1}^{NBC_{if}^{UP}} \sum_{p=1}^{P_{jf}} w_{ijpnf} = Q_{if} \quad \forall i, j, f \quad (17)$$

Therefore, eq 14 is expressed as follows:

$$NN_f \geq \sum_{p=1}^{P_{jf}} \sum_{n=1}^{NBC_{if}^{UP}} \frac{SF_{ijf}}{VF_{jfp}n} w_{ijpnf} \quad \forall i, j, f \quad (18)$$

Assuming that the head and tail of the schedule are negligible in each installed plant  $f$ , the product between the campaign cycle time  $CTC_f$  and the number of times that the campaign is repeated must be less than or equal to the time horizon:

$$CTC_f NN_f \leq H_f \quad \forall f \quad (19)$$

Due to the fact that  $CTC_f$  and  $NN_f$  are optimization variables, this expression is reformulated to avoid nonlinearities. Variable  $NN_f$  is discretized with an appropriate detail level. For each plant  $f$ , the interval  $[NN_f^{LP}, NN_f^{UP}]$  is uniformly discretized through  $N_f$  points proposed by the designer, called  $T_{mf}$ ,  $m = 1, \dots, N_f$ . Then, the binary variable  $NNC_{mf}$  is introduced to select the number of times that the production campaign is repeated over the time horizon of plant  $f$ :

$$NNC_{mf} = \begin{cases} 1 & \text{if campaign is repeated } T_{mf} \text{ times in plant } f \\ 0 & \text{otherwise} \end{cases}$$

In order to guarantee a unique selection of campaign repetition for each installed plant and to force to zero  $NNC_{mf}$  variables if plant  $f$  is not installed, the following constraint must be held:

$$\sum_{m=1}^{N_f} NNC_{mf} = ex_f \quad \forall f \quad (20)$$

Then, the number of times that the campaign is cyclically repeated over the time horizon in plant  $f$  is given by the following:

$$NN_f = \sum_{m=1}^{N_f} T_{mf} NNC_{mf} \quad \forall f \quad (21)$$

Therefore, replacing eq 21 into eq 19, the following constraints hold:

$$CTC_f \sum_{m=1}^{N_f} T_{mf} NNC_{mf} \leq H_f \quad \forall f \quad (22)$$

As  $CTC_f$  does not depend on subscript  $m$ , eq 22 can be rewritten in the following way:

$$\sum_{m=1}^{N_f} T_{mf} NNC_{mf} CTC_f \leq H_f \quad \forall f \quad (23)$$

In order to attain a linear expression, a non-negative continuous variable is defined and new constraints are added:

$$ww_{mf} = \begin{cases} CTC_f & \text{if binary variable } NNC_{mf} \text{ takes value 1} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{m=1}^{N_f} ww_{mf} = CTC_f \quad \forall f \quad (24)$$

$$ww_{mf} \leq CTC_f^{UP} NNC_{mf} \quad \forall m, f \quad (25)$$

Therefore, constraint 23 can be linearized as follows:

$$\sum_{m=1}^{N_f} T_{mf} ww_{mf} \leq H_f \quad \forall f \quad (26)$$

**3.3. Batch Plants Scheduling Constraints.** Decisions regarding the campaign scheduling of each installed plant have been modeled using an asynchronous slot-based representation. Taking into account that, for each plant, the number of batches of each product in the MPC composition is a model variable and the number of parallel units in each stage is unknown, the required slots number is not a trivial decision. For each installed plant, an appropriate number of production slots for each unit has been postulated in order to reduce the computing time, where reasonable assumptions for assignment of batches to units and slots have been considered. A detailed description of these assumptions can be found in Fumero et al.<sup>24</sup>

Let  $L_{kif}$  be the number of slots postulated for unit  $k$  of stage  $j$  in plant  $f$ . Then, if the sum  $\sum_i NBC_{if}^{UP}$  is defined by  $L_f$ , according to Fumero et al.,<sup>24</sup>  $L_{kif} = L_f - k + 1$ , for  $1 \leq k \leq K_{if}$ . Next, the binary variables of the previous article are extended to all plants embedded in the SC and the main relationships among them are presented in order to facilitate the readability of the model.

$$U_{jkf} = \begin{cases} 1 & \text{if unit } k \text{ of stage } j \text{ of plant } f \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ijkf} = \begin{cases} 1 & \text{if product } i \text{ is assigned to slot } l \\ & \text{and processed in unit } k \text{ of stage } j \text{ in plant } f \\ 0 & \text{otherwise} \end{cases}$$

$$X_{jkf} = \begin{cases} 1 & \text{if slot } l \text{ of unit } k \text{ of stage } j \text{ in plant } f \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{ilf} = \begin{cases} 1 & \text{if product } i \text{ is processed in slot } l \text{ of plant } f \\ 0 & \text{otherwise} \end{cases}$$

Although assignment variable  $Y_{ijkf}$  is sufficient for modeling the scheduling decisions of the problem, variables  $X_{jkf}$  and  $Z_{ilf}$  are introduced in order to improve the model computational performance.

The relations among previous binary variables are stated:

$$Y_{ijkf} \leq Z_{ilf}, \quad \forall i, j, 1 \leq l \leq L_{kif}, 1 \leq k \leq K_{if}, f \quad (27)$$

$$Y_{ijkf} \leq X_{jkf}, \quad \forall i, j, 1 \leq l \leq L_{kif}, 1 \leq k \leq K_{if}, f \quad (28)$$

$$Y_{ijkf} \geq X_{jkf} + Z_{ilf} - 1, \quad \forall i, j, f, 1 \leq l \leq L_{kif}, 1 \leq k \leq K_{if} \quad (29)$$

$$Y_{ijkf} \leq U_{jkf}, \quad \forall i, j, f, 1 \leq l \leq L_{kif}, 1 \leq k \leq K_{if} \quad (30)$$

$$X_{jkf} \leq U_{jkf}, \quad \forall j, f, 1 \leq l \leq L_{kif}, 1 \leq k \leq K_{if} \quad (31)$$

$$\sum_{k=1}^{K_{if}} Y_{ijkf} = Z_{ilf}, \quad \forall i, j, f, 1 \leq l \leq L_f, 1 \leq k \leq K_{if} \quad (32)$$

$$\sum_i Y_{ijkf} = X_{jkf}, \quad \forall j, f, 1 \leq l \leq L_{kif}, 1 \leq k \leq K_{if} \quad (33)$$

$$\sum_i \sum_{l=1}^{L_{kif}} Y_{ijkf} \geq U_{jkf}, \quad \forall j, f, 1 \leq k \leq K_{if} \quad (34)$$

Equations 27–29 allow defining variable  $Y_{ijkf}$  as continuous on interval  $[0, 1]$ . Thus, although the model introduces extra variables ( $X_{jkf}$  and  $Z_{ilf}$ ), the total number of binary variables in the formulation is reduced.

Also, in this formulation are introduced new logical relations among previous binary variables and the binary variables  $ex_j$  and  $z_{if}$  used for SC design. They are as follows:

$$U_{jkf} \leq ex_j \quad \forall j, f, 1 \leq k \leq K_{if} \quad (35)$$

$$X_{jkf} \leq ex_j \quad \forall j, f, 1 \leq k \leq K_{if}, 1 \leq l \leq L_{kif} \quad (36)$$

$$Y_{ijkf} \leq z_{if} \quad \forall i, j, f, 1 \leq k \leq K_{if}, 1 \leq l \leq L_{kif} \quad (37)$$

$$Z_{ilf} \leq z_{if} \quad \forall i, f, 1 \leq l \leq L_f \quad (38)$$

Constraint 35 assures that if plant  $f$  is not installed, then no units are used in all the stages of that plant. Moreover, no slot is used if plant  $f$  is not installed (eq 36). On the other hand, if product  $i$  is not produced in plant  $f$ , variables  $Y_{ijkf}$  and  $Z_{ilf}$  must be zero. It is worth noting that if plant  $f$  is not installed, variables  $Y_{ijkf}$  and  $Z_{ilf}$  are also zero for each product  $i$  due to eq 1.

With the aim of reducing the search space, assumptions about units and slots utilization of each installed plant are considered in this formulation. Without loss of generality, the following constraints are imposed:

$$U_{jkf} \geq U_{j,k+1,f}, \quad \forall j, f, 1 \leq k \leq K_{if} - 1 \quad (39)$$

$$\sum_i Z_{ilf} \geq \sum_i Z_{i,l+1,f}, \quad \forall f, 1 \leq l \leq L_f - 1 \quad (40)$$

$$Y_{i'jk'lf} \leq 1 - Y_{ijkf}, \quad \forall i, i', j, f, 1 \leq l \leq L_{kif}, 1 \leq l \leq L_{k'if}, 1 \leq k \leq K_{if}, 1 \leq k' \leq K_{if}, (k \neq k') \quad (41)$$

$$\sum_i \sum_{\substack{k \\ 1 \leq k \leq K_{jf} \\ k/l \leq L_{k,jf}}} Y_{ijklf} \leq ex_f, \quad \forall j, f, 1 \leq l \leq L_f \tag{42}$$

$$\sum_i Z_{ijf} \leq ex_f, \quad \forall f, 1 \leq l \leq L_f - 1 \tag{43}$$

Constraint 39 establishes that units of each stage are utilized in ascending order, while constraint 40 assures that, on each stage, a slot is occupied only if the previous slot has been used for processing a batch on some unit of this stage. Constraint 41 guarantees that if slot  $l$  of unit  $k$  at stage  $j$  of plant  $f$  is utilized to process one product, then this slot cannot be occupied by the remainder units of this stage. Moreover, eqs 42 and 43 enforce that the slot  $l$  can only be assigned for processing at most one product in each stage of each plant, if it is installed.

Analogous to eq 41, the following constraint must be satisfied:

$$\begin{aligned} X_{jk'lf} &\leq 1 - X_{jklf}, \\ \forall j, f, 1 \leq l \leq L_{k,jf}, 1 \leq l \leq L_{k'jf}, 1 \leq k \leq K_{jf}, \\ 1 \leq k' \leq K_{jf}, (k \neq k') \end{aligned} \tag{44}$$

In order to eliminate alternative solutions, without affecting the model optimality, the following constraint is used:

$$\sum_{l \leq L_{k,jf}} 2^l X_{jklf} \geq \sum_{l \leq L_{k+1,jf}} 2^l X_{jk+1lf}, \quad \forall j, f, 1 \leq k < K_{jf} \tag{45}$$

This inequality establishes that the succession formed by the weighted sum of the slots occupied in each unit of a stage forms a decreasing succession.

As was showed in Fumero et al.,<sup>25</sup> the resolution can be improved if a preordering constraint in the scheduling is imposed. This simplification can significantly decrease the computational time when several decisions are optimized simultaneously, as happens in this approach. The following constraint assures that, for each installed plant, the assignment of batches to slots follows the same order in all the stages; that is, for each plant, a product batch is processed in exactly the same slot in all stages:

$$\begin{aligned} \sum_i \sum_{\substack{k \\ 1 \leq k \leq K_{jf} \\ k/l \leq L_{k,jf}}} i Y_{ijklf} &= \sum_i \sum_{\substack{k \\ 1 \leq k \leq K_{j'f} \\ k/l \leq L_{k,j'f}}} i Y_{ij'klf} \\ \forall j, j', (j < j'), f, 1 \leq l \leq L_f \end{aligned} \tag{46}$$

Although suboptimal solutions can be obtained, the computational effort is drastically reduced. This assumption provides a good solution which coincides with the global optimum of the exact scheduling model in most of the solved cases.<sup>24</sup>

Variable  $Z_{ijf}$  allows expression of the number of batches of product  $i$  included in the campaign of installed plant  $f$  as follows:

$$\sum_i Z_{ijf} = NBC_{if}, \quad \forall i, f \tag{47}$$

The timing constraints presented by Fumero et al.<sup>24</sup> are reformulated for all plants involved in the SC:

$$\begin{aligned} TF_{jklf} &= TI_{jklf} + \sum_i t_{ijf} Y_{ijklf} \\ \forall j, f, 1 \leq k \leq K_{jf}, 1 \leq l \leq L_{k,jf} \end{aligned} \tag{48}$$

$$TF_{jklf} \leq TI_{jk+1lf} \quad \forall j, f, 1 \leq k \leq K_{jf}, 1 \leq l < L_{k,jf} \tag{49}$$

$$\begin{aligned} TF_{jklf} - TI_{jk+1lf} &\geq -M_1 X_{jk+1lf} \\ \forall j, f, 1 \leq k \leq K_{jf}, 1 \leq l < L_{k,jf} \end{aligned} \tag{50}$$

$$\begin{aligned} TF_{jklf} - TI_{j+1k'lf} &\geq M_2 (X_{jklf} + X_{j+1k'lf} - 2) \\ \forall f, 1 \leq j < J_f, 1 \leq k \leq K_{jf}, 1 \leq k' \leq K_{j+1f}, \\ 1 \leq l \leq \min\{L_{k,jf}, L_{k'+1f}\} \end{aligned} \tag{51}$$

$$\begin{aligned} -TF_{jklf} + TI_{j+1k'lf} &\geq M_2 (X_{jklf} + X_{j+1k'lf} - 2) \\ \forall f, 1 \leq j < J_f, 1 \leq k \leq K_{jf}, 1 \leq k' \leq K_{j+1f}, \\ 1 \leq l \leq \min\{L_{k,jf}, L_{k'+1f}\} \end{aligned} \tag{52}$$

where  $M_1$  and  $M_2$  are sufficiently large numbers.

Constraint 48 defines the final processing time of each proposed slot in unit  $k$  at stage  $j$  of plant  $f$  as a function of the initial time and the processing time of the assigned product, if this slot is used. Inequality 49 avoids slots overlapping on a given unit. Moreover, if no product is assigned to slot  $l + 1$  of unit  $k$  at stage  $j$  of plant  $f$  ( $X_{jk+1lf} = 0$ ), then the starting time of this slot and the finishing time of slot  $l$  must be equal. Through Big-M type constraint 50 and taking into account constraint 49, the previous condition is represented. The batch transfer policy adopted in this work is the Zero-Wait, which assumes that a batch, after finishing its processing at a stage, must be transferred immediately to the next stage. Big-M constraints 51 and 52 allow expression of this transfer policy.

In order to calculate the cycle time of the campaign of plant  $f$ ,  $CTC_f$ , the last slot of each unit  $k$  of stage  $j$  in plant  $f$ ,  $L_{k,jf}$ , and the first slot effectively assigned to unit  $k$  of stage  $j$  in plant  $f$ ,  $\tilde{l}_{k,jf}$  ( $\tilde{l}_{k,jf} = \min\{1 \leq l \leq L_f / X_{jklf} = 1\}$ ), are taken into account:

$$CTC_f = \max_j \{ \max_{1 \leq k \leq K_{jf}} \{ TF_{jkL_{k,jf}} - TI_{jk\tilde{l}_{k,jf}} \} \} \tag{53}$$

This equation can be represented using a Big-M formulation, as follows:

$$\begin{aligned} CTC_f - TF_{jkL_{k,jf}} + TI_{jklf} &\geq M_3 ((X_{jklf} - 1) - \sum_{1 \leq l' < l} X_{jk'l'f}) \\ \forall j, f, 1 \leq k \leq K_{jf}, 1 \leq l \leq L_{k,jf} \end{aligned} \tag{54}$$

where  $M_3$  is a sufficiently large number that makes the constraint redundant for all the previous and subsequent slots, if any, to the first nonempty one in unit  $k$  of stage  $j$  in plant  $f$ .

**3.4. Objective Function.** The objective function is the total cost minimization, which includes the following: plants installation cost, equipment investment cost for each installed plant, raw material and production costs, and transportation cost between SC nodes.

This is a simple and initial objective function. Given that results about operational aspects are obtained, more comprehensive functions can be proposed to assess different trade-offs. For example, knowing the product flows, more explicit expressions could be included to evaluate the transport cost, the impact of inventory level, etc.

For installation cost, the following expression is considered:

$$CINST = \sum_f CP_f ex_f \quad (55)$$

where  $CP_f$  is the annualized fixed cost for plant  $f$  installation.

The annualized investment cost of each plant  $f$  is expressed as follows:

$$IC_f = CCF \sum_j \sum_k U_{jkf} \alpha_{jf} V_{jf}^{\beta_{jf}} \quad \forall f \quad (56)$$

where  $\alpha_{jf}$  and  $\beta_{jf}$  are appropriate cost coefficients for units of stage  $j$  of plant  $f$  and  $CCF$  is a capital charge factor on the time horizon, which includes an amortization term. Considering eq 10, eq 56 can be rewritten as follows:

$$IC_f = CCF \sum_j \sum_k \sum_p \alpha_{jf} VF_{jfp}^{\beta_{jf}} U_{jkf} v_{jpf} \quad \forall f \quad (57)$$

A new variable  $e_{jkpf}$  is defined to eliminate the bilinear term  $U_{jkf} v_{jpf}$  in eq 57. This variable has to be linked to the decision variables  $v_{jpf}$  and  $U_{jkf}$  such that  $e_{jkpf}$  takes value 1 if both are 1, and 0 otherwise. Then, the following constraint enforces this logic relation:

$$e_{jkpf} \geq v_{jpf} + U_{jkf} - 1, \quad \forall j, f, 1 \leq k \leq K_{jf}, 1 \leq p \leq P_{jf} \quad (58)$$

Thus, a linear function is obtained:

$$IC_f = CCF \sum_j \sum_k \sum_p \alpha_{jf} VF_{jfp}^{\beta_{jf}} e_{jkpf} \quad \forall f \quad (59)$$

Therefore, the total investment cost of installed plants is the following:

$$CINV = \sum_f IC_f \quad (60)$$

Raw material and production costs are given by the following:

$$CPROD = \sum_f \sum_s \sum_r \sum_i CRAW_{sr} QR_{srif} + \sum_f \sum_i CPR_{if} Q_{if} \quad (61)$$

where  $CPR_{if}$  and  $CRAW_{sr}$  are cost coefficients, per mass unit, of product  $i$  produced in plant  $f$ , and raw material  $r$  produced in site  $s$ , respectively.

The transportation cost of raw materials from sites to batch plants, and transportation cost of final products from production plants to customer zones are as follows:

$$CTRANS = \sum_f \sum_s \sum_r \sum_i CTRAW_{srif} QR_{srif} + \sum_f \sum_c \sum_i CTIFC_{ifc} QC_{ifc} \quad (62)$$

where the parameters  $CTRAW_{srif}$  and  $CTIFC_{ifc}$  represent transportation costs, per mass unit, of raw material  $r$  from site  $s$  to plant  $f$ , and transportation costs of final product  $i$  from plant  $f$  to customer zone  $c$ .

Finally, in order to reduce the number of alternative solutions and consequently the search space, a penalty term that involves the campaign cycle time of each installed plant is considered:

$$CPEN = \sum_f \lambda_f CTC_f \quad (63)$$

The coefficient  $\lambda_f$  is appropriately selected by taking into account the involved model parameters. In this way, the approach becomes the more economical solution with minimum campaign cycle time, avoiding alternative feasible campaigns for each chosen plant structure. Also, the incorporation of this penalization in the objective function improves the computational performance.

Therefore, the objective function  $TCOST$  to be minimized is the following:

$$TCOST = CINST + CINV + CPROD + CTRANS + CPEN \quad (64)$$

In short, the model MILP for the simultaneous SC design and installed plants design considering production scheduling is given by the minimization of eq 64 subject to constraints 1–12, 15–18, 20–21, 24–52, 54–55, and 58–63.

#### 4. EXAMPLES

In this section, the capabilities of the proposed approach are highlighted through examples. All the examples were implemented and solved using GAMS<sup>26</sup> on an Intel i7, 2.8 GHz processor, and GUROBI 2.0.1 was used for solving the MILP problems with a 0% optimality gap. The model constraints and variables number strongly depend on the number of SC nodes, the different design and product options considered for each plant (number of products, number of stages in each plant, maximum number of units duplications admitted for each stage, number of discrete sizes for units in the different plant stages, maximum number of product batches allowed in the campaign, number of postulated slots in each unit), and the number of discrete options proposed for campaign repetitions in each plant.

**4.1. Example 1.** In the first example, the SC topology considered has three raw material sites with three different types of consumables, three customer zones, and a maximum of three locations where production plants may be installed. Each plant can produce three products (A, B, and C) through three batch stages, which admit up to three units duplicated out-of-phase, for stages 1 and 2, and up to two duplicated units for stage 3.

Next, two cases are presented in order to illustrate the close interaction among problem elements and decision levels. They show how small changes in some problem parameters can lead to significant modifications in different decisions such as the procurement of raw materials, the productions of each installed plant, the plants design, the production campaigns, etc. Therefore, all these aspects have to be included in the whole and simultaneous representation of the problem.

**4.1.1. Case 1.** Table 1 shows some plant design parameters, while Table 2 displays the discrete sizes for batch units and unit cost coefficients. The  $CCF$  coefficient for equipment invest-

**Table 1. Example 1—Case 1: Problem Parameters for Each Plant  $f$** 

$i$	processing time: $t_{ij}$ (h)			size factor: $SF_{ij}$ (L/kg)			conversion factor: $fc_{rjf}$		
	$j$			$j$			$r$		
	1	2	3	1	2	3	1	2	3
A	14	25	7	0.7	0.6	0.5	1.5	1.25	1
B	16	18	5	0.6	0.7	0.45	1.2	1	0
C	12	15	4	0.7	0.65	0.55	0	1.5	1.2

**Table 2. Example 1—Case 1: Available Unit Sizes and Cost Coefficients for Each Stage of Plant  $f$** 

$j$	unit discrete sizes: $VF_{jfp}$ (L)					cost coefficient: $\alpha_{jf}$	cost exponent: $\beta_{jf}$
	1	2	3	4	5		
1	650	1300	2600	5200	7800	6000	0.6
2	700	1400	2800	5600	8400	6000	0.6
3	1000	2000	3000	4000	6000	7000	0.7

ment cost is adopted equal to 0.225. Table 3 shows the production cost coefficients for each plant and the product demands that must be fulfilled in the time horizon ( $HT_f = 7000$  h).

**Table 3. Example 1—Case 1: Production Costs and Product Demands**

$i$	production cost in each plant: $CPR_{jf}$ (\$/kg)			demands: $DM_{ic}$ (kg)		
	$f$			$c$		
	1	2	3	1	2	3
A	0.5	0.6	0.8	200000	200000	400000
B	0.6	0.2	0.9	150000	130000	200000
C	0.3	0.4	0.9	310000	220000	320000

The maximum number of batches for each product in the campaign is equal to three in all plants ( $NBC_{jf}^{UP} = 3, \forall i, f$ ).

The raw materials availability at each site and their costs are shown in Table 4, and the fixed plant installation costs and the

**Table 4. Example 1—Case 1: Raw Material Acquisition Cost and Availability**

$s$	raw material acquisition cost: $CRAW_s$ (\$/kg)			raw material availability: $QR_s^{UP}$ (kg)		
	$r$			$r$		
	1	2	3	1	2	3
1	0.2	0.1	0.3	950000	960000	820000
2	0.1	0.2	0.2	1230000	1580000	1280000
3	0.3	0.3	0.1	860000	550000	650000

transportation costs between different SC nodes are shown in Table 5. For transportation costs, distances (km), fuel prices (\$/L), and fuel economy (km/L) are taken into account.

Variable  $NN_f$  is uniformly discretized, taking into account 31 elements, where  $NN_f^{LOW} = 100$  and  $NN_f^{UP} = 250$ . In other words, the step size is equal to 5, and the recurrence relation  $T_{mf} = T_{m-1f} + 5$  for  $m = 2, \dots, 31$  with  $T_{1f} = 100$ , allows defining the discrete multiple choice for the variable that determines the

**Table 5. Example 1—Case 1: Plant Installation Cost, Transportation Cost for Each Raw Material  $R$  from Sites to Plants, and Transportation Cost for Each Product  $i$  from Plants to Customer Zones**

$f$	plant installation annualized fixed cost: $CP_f$ (\$)	distribution costs: raw material sites—plants $CTRAW_{sf}$ (\$/kg)			distribution costs: plants—customer zones $CTIFC_{jfc}$ (\$/kg)		
		$s$			$c$		
		1	2	3	1	2	3
1	9000	0.2	0.1	0.2	0.5	0.9	0.5
2	9000	0.1	0.2	0.3	0.6	0.4	0.9
3	10000	0.2	0.1	0.3	0.6	0.8	0.9

number of repetitions of the campaign over the time horizon in plant  $f$ .

The model under these assumptions comprises 9412 constraints, 2273 continuous variables, and 477 binary variables. It was solved in 625.23 CPU seconds, and the total annual cost is equal to \$4958906.07. An itemized list of costs is shown in Table 6.

**Table 6. Economical Results for Example 1 (\$/year)**

costs	optimal solution	
	example 1—case 1	example 1—case 2
investment	1236224.25	1236224.25
plants installation	18000.00	18000.00
production	888818.18	792818.18
raw material procurement	1075263.64	1071063.64
transportation from raw material sites to plants	718690.91	747145.45
transportation from plants to customer zones	1021909.09	929909.09
total	4958906.07	4795160.61

In the optimal solution, two plants are installed (plants 1 and 2), and they are supplied from the three raw material sites as is shown in Figure 2. Sites 1 and 2 provide the three types of raw materials, while site 3 only supplies raw materials 2 and 3. Both plants produce products A, B, and C. More specifically, productions of plant 1 are  $Q_{A1} = 600000$  kg,  $Q_{B1} = 240000$  kg, and  $Q_{C1} = 631818$  kg, while for plant 2 they are  $Q_{A2} = 200000$  kg,  $Q_{B2} = 240000$  kg, and  $Q_{C2} = 218182$  kg. The design of each plant is depicted in Figures 3 and 4.

Plant 1 has two out-of-phase parallel units in stage 1, three out-of-phase units in stage 2, and one unit in stage 3. The campaign composition comprises three batches of A and C, and one batch of B. The batch sequencing in each unit of this plant is shown in Figure 5. The campaign cycle time is reached at unit 1 of the first stage, and it is equal to 56 h. The campaign is cyclically repeated  $T_{\delta 1} = 125$  times over the time horizon. Plant 2 has only one unit per stage, and the campaign composition is equal to one batch of each product. The batch sequencing on each unit is shown in Figure 6, where it can be noted that the campaign cycle time is reached at stage 2, which is time limiting, and it is equal to 58 h. The campaign is repeated  $T_{\delta 2} = 120$  times over the time horizon.

Table 7 summarizes the different consumption of raw materials at each site, where the used up raw materials are



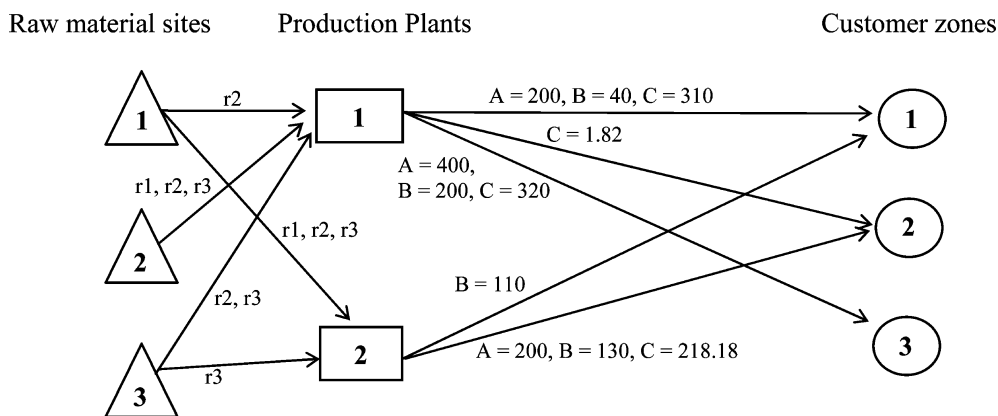


Figure 2. Example 1—Case 1: SC design (amounts  $\times 10^3$  kg).

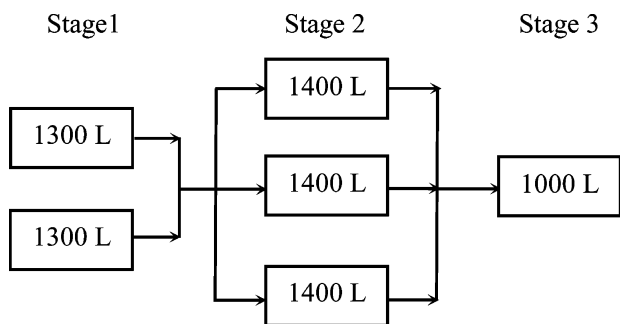


Figure 3. Example 1—Case 1: Plant 1 optimal design.

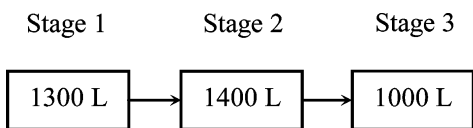


Figure 4. Example 1—Case 1: Plant 2 optimal design.

highlighted in gray. Taking into account the distribution costs shown in Table 5, sites 1 and 2 supply the three raw materials to plants 2 and 1, respectively. In particular, the total raw materials 2 and 3 available at site 2 are used for production in plant 1. However, as these are not sufficient to fulfill the requirements of production of plant 1, other sources of raw

materials must be used. First, the availability of raw material 2 is used up from site 1, since it does not provide the total raw material 2 to plant 2. Then, plant 1 uses part of the availability of raw material 2 from site 3. Also, due to trade-offs between the costs of procurement and transport of raw materials, plant 2 partially consumes raw material 3 from sites 1 and 3.

It is worth mentioning that this model is flexible in the sense of no production or distribution policies are imposed. Therefore, all the variables are simultaneously evaluated and different trade-offs among SC configuration, plants design, and production campaigns design can be assessed.

In this example, the attained plant structures and the campaign for each of them are different, which shows that these elements cannot be a priori assumed. That is, there is a strong relationship among all variables and parameters of the problem, which is assessed by the proposed approach that simultaneously considers all these aspects. From the operational point of view, both allocated plants are very different. For example, production rates are not similar and a standard performance for facilities cannot be assumed. Moreover, from these results several elements can be evaluated: inventory levels, transport policies, etc.

4.1.2. Case 2. In this case, only the final product transportation costs from plants to customer zones are modified from case 1 presented above, as is shown in Table 8. In particular, the transportation cost of product B from plant

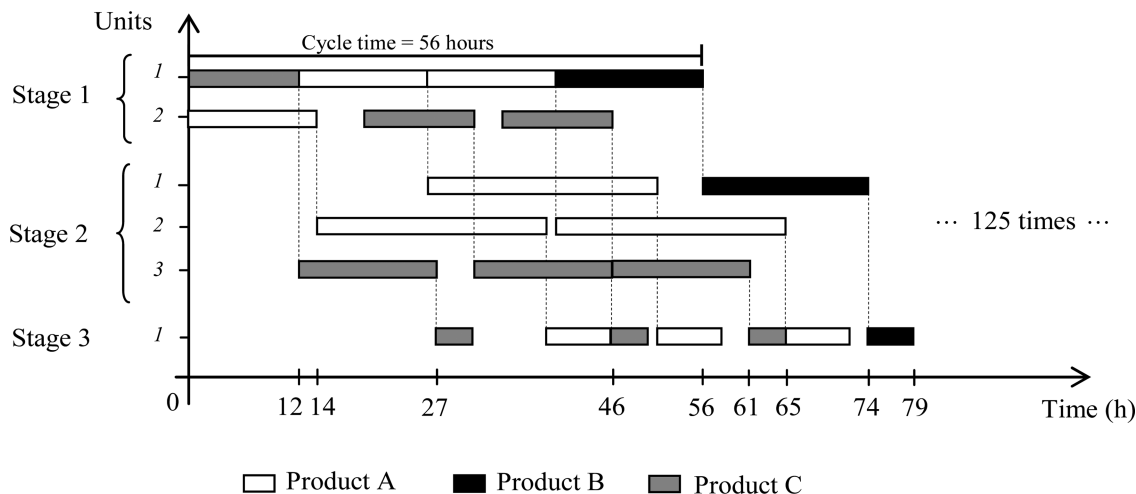


Figure 5. Example 1—Case 1: Gantt chart for the optimal production sequencing of plant 1.

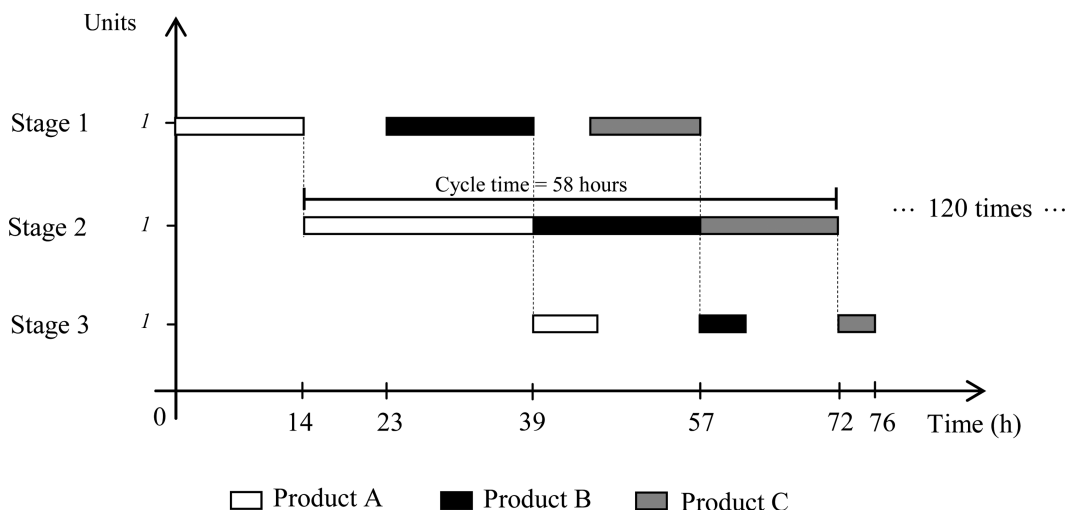


Figure 6. Example 1—Case 1: Gantt chart for the optimal production sequencing of plant 2.

Table 7. Example 1—Case 1: Amount of Raw Material Transported from Each Site to All Plants

consumables ( <i>r</i> )	raw material sites ( <i>s</i> )		
	1	2	3
1	588000	1188000	0
2	960000	1580000	215003
3	261820	1280000	278182

Table 8. Example 1—Case 2: Transportation Cost for Each Final Product from Plants to Customer Zones

<i>f</i>	transportation costs: plants—customer zones $CTIFC_{ifc}$ (\$/kg)								
	product <i>i</i> = A			product <i>i</i> = B			product <i>i</i> = C		
	<i>c</i>			<i>c</i>			<i>c</i>		
	1	2	3	1	2	3	1	2	3
1	0.5	0.9	0.5	0.5	0.9	0.5	0.5	0.9	0.5
2	0.6	0.4	0.9	0.4	0.2	0.3	0.6	0.4	0.9
3	0.6	0.8	0.9	0.6	0.8	0.9	0.6	0.8	0.9

2 to customer zones 1–3 has been decreased about 33%, 50%, and 66%, respectively. The rest of the model parameters and discrete options for the number of repetitions of the campaign

of each installed plant are not changed, and therefore, the model size is the same as in case 1.

In this instance the model was solved in 169 CPU seconds. The optimal objective value is equal to \$4795160.61, and in the second column of Table 6, a detailed list of costs is presented.

Figure 7 shows the optimal SC design, while in Figures 8 and 9 the installed batch plant designs are illustrated. From these

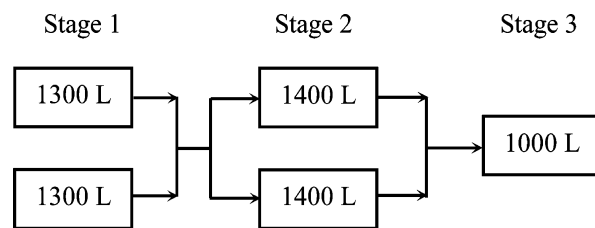


Figure 8. Example 1—Case 2: Plant 1 optimal design.

figures can be noted that the raw material and final product distributions and the productions of each installed plant are different from the previous case, as well as the facilities designs. However, the overall investment cost is the same since the unit sizes and the number of equipments used in both cases is equal.

Since the production cost of product B in plant 2 is significantly lower than in plant 1, as well as the transportation cost from plant 2 to different customers, the total required

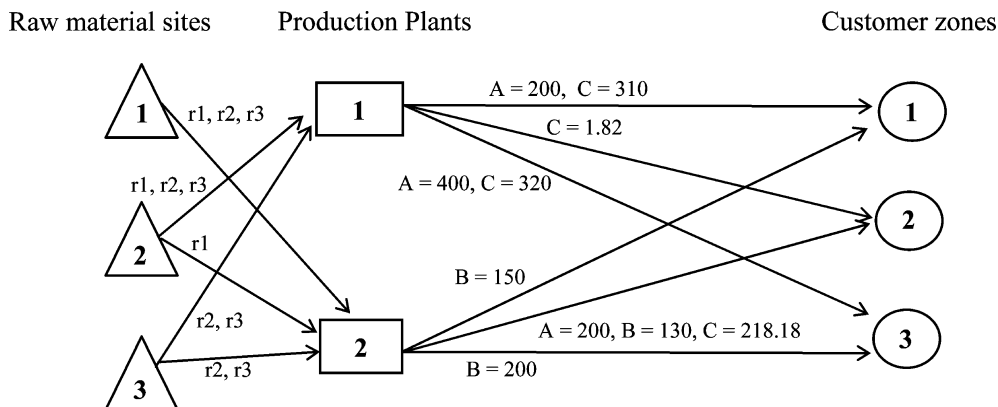


Figure 7. Example 1—Case 2: SC design (amounts  $\times 10^3$  kg).

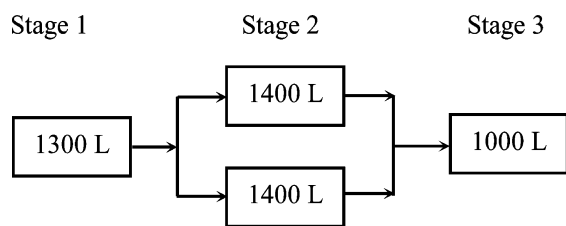


Figure 9. Example 1—Case 2: Plant 2 optimal design.

amount of B is produced only in that plant, while the other productions are the same as in case 1. In this way, production cost and transportation cost from plants to clients is reduced by 11% and 9%, respectively, from the values obtained in the previous instance.

The campaign composition, the batches sequencing, and the number of repetitions of the campaigns are shown in Figures 10 and 11 for each installed plant.

Finally, comparing cases 1 and 2, it is clear that the only change made on the transportation cost for product B from plant 2 to all customers leads to modifications in the solution of both instances, including the procurement of raw materials, the productions of each installed plant, the plants design, the production campaigns, etc. Though the difference between the total costs of both optimal solutions is small, from the operational point of view changes are significant. Flows are very different and, therefore, more detailed formulations and expressions could be included to assess the impact of the introduced modification with respect to inventory levels, transport policies, etc.

**4.2. Example 2.** In order to show the efficiency of the proposed approach and the impact of the different decisions on the computational complexity of the model, two larger cases are analyzed.

**4.2.1. Case 1.** In this case, the SC topology considered has four raw material sites with three different types of consumables, four customer zones, and a maximum of four locations where production plants may be installed. Each plant can produce three products (A, B, and C) through three batch stages, which admit up to three units duplicated out-of-phase for each stage.

Parameters for plant design are shown in Tables 9 and 10. The CCF coefficient for equipment investment cost is adopted equal to 0.225. Table 11 displays the production cost

coefficients for each plant and the product demands that must be fulfilled in the time horizon ( $HT_f = 7000$  h).

The maximum number of batches for each product in the campaign is equal to three in all plants ( $NBC_{ij}^{UP} = 3, \forall i, f$ ).

The raw materials availability at each site and their costs are shown in Table 12. The fixed plant installation costs and the transportation costs between different SC nodes are shown in Table 13.

The variable  $NN_f$  is uniformly discretized, considering a step size equal to 10, over the interval [100, 250]. Then the recurrence relation  $T_{mf} = T_{m-1f} + 10$  for  $2 \leq m \leq 16$  with  $T_{1f} = 100$  allows defining the discrete options for  $NN_f$ .

The model comprises 12560 linear constraints, 2956 continuous variables, and 576 binary variables, and it was solved in 752.36 CPU seconds. Although, in this instance a more complex supply chain structure is addressed regarding the previous example, the resolution time is slightly increased. The optimal solution has a value of \$3542173.09, and an itemized list of costs is shown in Table 14.

Figure 12 shows the SC design. Plants 1 and 2 are installed, and products A, B, and C are produced in both plants. More specifically, productions of plant 1 are  $Q_{A1} = 425429$  kg,  $Q_{B1} = 370000$  kg, and  $Q_{C1} = 741176$  kg, while for plant 2 they are  $Q_{A2} = 324571$  kg,  $Q_{B2} = 380000$  kg, and  $Q_{C2} = 178824$  kg. The plants design is depicted in Figures 13 and 14. Plant 1 is supplied from three raw material sites (1, 3, and 4) while plant 2 is only supplied from site 3. Table 15 shows the consumption of raw materials at each site, where the totally consumed raw materials are highlighted in gray.

The campaign for plant 1 comprises one batch of A and B and two batches of C. The batch sequencing in each unit of this plant is shown in Figure 15. The campaign cycle time is reached at unit of the third stage, and it is equal to 33 h. The campaign is cyclically repeated 210 times over the time horizon. For plant 2, the campaign composition includes one batch of each product and the batch sequencing on each unit is shown in Figure 16. From this figure, it can be noted that the campaign cycle time is reached at stage 2, which is time limiting, and it is equal to 36 h. The campaign is repeated 190 times over the time horizon.

**4.2.2. Case 2.** In this case, the SC structure considered is the same as case 1 for raw material sites and customer zones, but a maximum of three production plants may be installed. In contrast to the previous instance, each plant can produce four

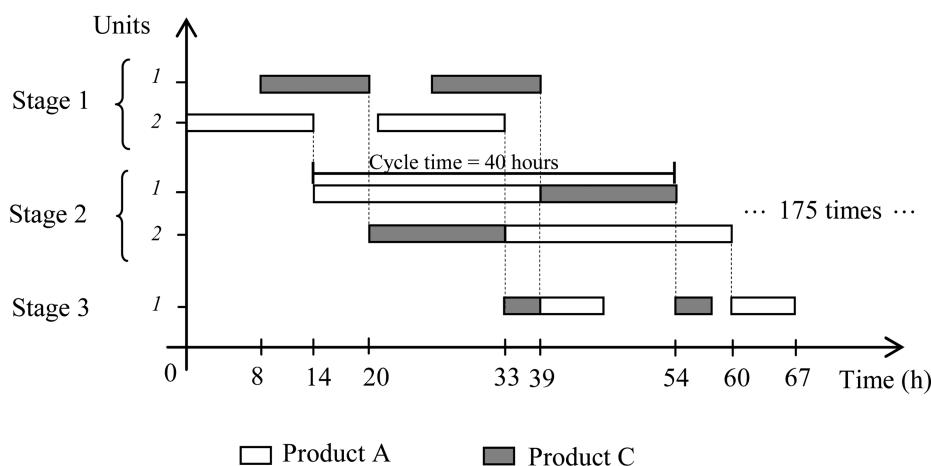


Figure 10. Example 1—Case 2: Gantt chart for optimal production campaign of plant 1.

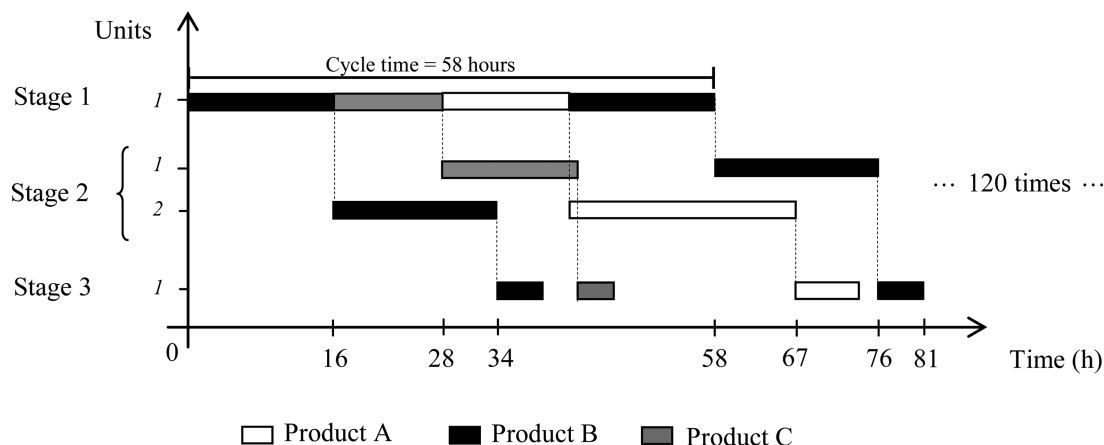


Figure 11. Example 1—Case 2: Gantt chart for optimal production campaign of plant 2.

Table 9. Example 2—Case 1: Problem Parameters for Each Plant *f*

<i>i</i>	processing time: $t_{ij}$ (h)			size factor: $SF_{ij}$ (L/kg)			conversion factor: $fc_{rjf}$		
	<i>j</i>			<i>j</i>			<i>r</i>		
	1	2	3	1	2	3	1	2	3
A	4	9	6	0.7	0.6	0.5	0.53	0.48	0.35
B	3	12	4	0.6	0.7	0.45	0.42	0.35	0
C	4	15	9	0.7	0.65	0.85	0	0.53	0.42

Table 10. Example 2—Case 1: Available Unit Sizes and Cost Coefficients for Each Stage of Plant *f*

<i>j</i>	unit discrete sizes: $VF_{jfp}$ (L)					cost coefficient: $\alpha_{jf}$	cost exponent: $\beta_{jf}$
	1	2	3	4	5		
1	600	1200	1800	2400	4800	6000	0.6
2	700	1400	2100	2800	5600	6000	0.6
3	500	1000	1500	3000	4500	7000	0.7

products (A, B, C, and D) through three batch stages, which admit up to three units duplicated out-of-phase for each stage.

The processing times, size, and conversion factors for products A, B, and C are the same as those for case 1. For product D, the processing times considered are 3, 10, and 5 h, and the size factors are 0.7, 0.6, and 0.5 L/kg for stages 1–3, respectively, while the conversion factors for the new product are 0.46, 0.42, and 0.35 for raw materials 1, 2, and 3, respectively. The maximum number of batches of each product in the campaign is 3 for all products. The production costs of products A, B, and C in plants 1, 2, and 3 are the same as in previous cases, while for product D they are 0.25, 0.36, and 0.68, respectively. Table 16 shows the product demands that must be fulfilled in the time horizon ( $HT_f = 7000$  h). The rest

Table 12. Example 2—Case 1: Raw Material Acquisition Cost and Availability

<i>s</i>	raw material acquisition cost: $CRAW_{sr}$ (\$/kg)			raw material availability: $QR_{sr}^{UP}$ (kg)		
	<i>r</i>			<i>r</i>		
	1	2	3	1	2	3
1	0.2	0.1	0.2	350000	500000	330000
2	0.3	0.3	0.2	215000	450000	410000
3	0.1	0.2	0.1	330000	540000	315000
4	0.2	0.2	0.2	215000	440000	410000

of the model parameters are not changed regarding to instance 1.

The optimal objective value is equal to \$4792107.87, and in the second column of Table 14, a detailed list of costs is presented.

Figure 17 shows the optimal SC design. Plants 1 and 2 are installed: plant 1 produces products A, B, and C, while plant 2 produces products A, B, and D. More specifically, the productions of plant 1 are  $Q_{A1} = 570000$  kg,  $Q_{B1} = 330000$  kg, and  $Q_{C1} = 1320000$  kg, while those for plant 2 are  $Q_{A2} = 180000$  kg,  $Q_{B2} = 420000$  kg, and  $Q_{D2} = 500000$  kg. The raw material and final product distributions and the productions of each installed plant are different from the previous example, as well as the facilities designs. Figures 18 and 19 illustrate the installed batch plant designs.

For plant 1, the campaign and the number of times that it is repeated over the time horizon are the same as in case 1 presented above; however, as unit sizes in all stages are larger, the batch sizes of products A and C are increased to fulfill the production demands in the time horizon. Plant 2 has one unit in stages 1 and 3 and two out-of-phase parallel units in stage 2. The campaign composition comprises ones batch of A, two batches of B, and three batches of D. The batch sequencing in

Table 11. Example 2—Case 1: Production Costs and Product Demands

<i>i</i>	production cost in each plant: $CPR_{jf}$ (\$/kg)				demands: $DM_{ic}$ (kg)			
	<i>f</i>				<i>c</i>			
	1	2	3	4	1	2	3	4
A	0.35	0.28	0.56	0.78	300000	100000	250000	100000
B	0.42	0.14	0.63	0.53	300000	100000	200000	150000
C	0.21	0.35	0.63	0.55	440000	120000	100000	260000

**Table 13. Example 2—Case 1: Plant Installation Cost, Transportation Cost for Each Raw Material *r* from Sites to Plants, and Transportation Cost for Each Product *i* from Plant to Customer Zones**

<i>f</i>	plant installation annualized fixed cost: $CP_f$ (\$)	distribution costs: raw material sites—plants $CTRAW_{sf}$ (\$/kg)				distribution costs: plants—customer zones $CTIFC_{ifc}$ (\$/kg)			
		<i>s</i>				<i>c</i>			
		1	2	3	4	1	2	3	4
1	9000	0.1	0.1	0.2	0.3	0.4	0.6	0.5	0.4
2	9000	0.4	0.2	0.1	0.5	0.6	0.4	0.9	0.3
3	10000	0.2	0.1	0.3	0.2	0.9	0.8	0.8	0.7
4	10000	0.2	0.2	0.3	0.3	0.8	0.7	0.9	0.6

**Table 14. Economical Results for Example 2 (\$/year)**

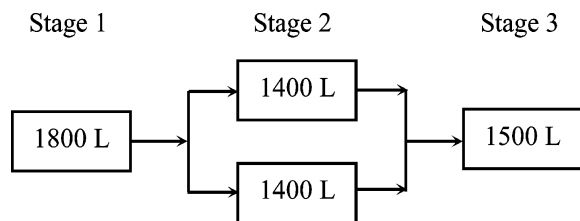
costs	optimal solution	
	example 2—case 1	example 2—case 2
investment	1100627.63	1288922.87
plants installation	18000.00	18000.00
production	666615.29	904500.00
raw material procurement	371755.00	575342.50
transportation from raw material sites to plants	279496.18	467342.50
transportation from plants to customer zones	1105678.99	1538000.00
total	3542173.09	4792107.87

each unit of this plant is shown in Figure 20. The campaign cycle time is reached at the third stage, and it is equal to 32 h. The campaign is cyclically repeated 210 times over the time horizon.

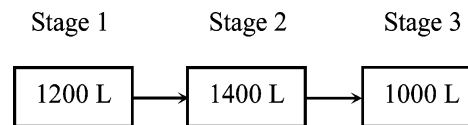
Taking into account that the total amount to be produced of product C is larger than in case 1 and a new product is elaborated, the required raw materials levels are increased. In the optimal solution, raw materials from the four sites are used.

Table 17 summarizes the different consumption of raw materials at each site, where the used up raw materials are highlighted in gray.

Finally, for this case the model size involves 15356 constraints, 3181 continuous variables, and 579 binary variables, and it was solved in 3213.62 CPU seconds. Although, the principal difference in the model size, with regard to case 1, is the number of constraints (approximately increased by 22%), the increase in the required computational time is mainly due to the scheduling decisions. In this instance, the increase in the



**Figure 13. Example 2—Case 1: Plant 1 optimal design.**



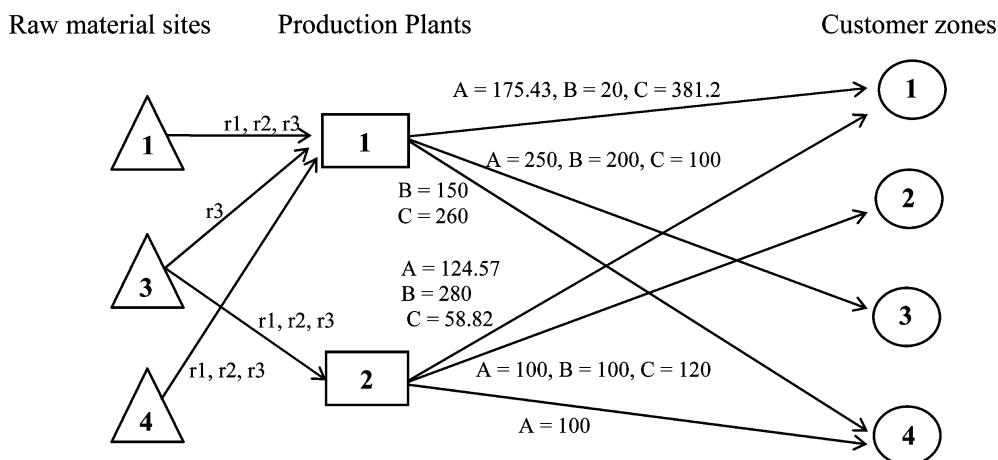
**Figure 14. Example 2—Case 1: Plant 2 optimal design.**

**Table 15. Example 2—Case 1: Amount of Raw Material Transported from Each Site to All Plants**

consumables ( <i>r</i> )	raw material sites ( <i>s</i> )			
	1	2	3	4
1	350000	0	330000	28750
2	500000	0	368882	204740
3	330000	0	315000	3900

number of products to be elaborated strongly impacts on the number of slots postulated for each unit of installed plants.

**4.3. Example 3.** In this example, the objective is to highlight the impact of the simultaneous assessment of SC design and plants design and scheduling, showing that



**Figure 12. Example 2—Case 1: SC design (amounts  $\times 10^3$  kg).**

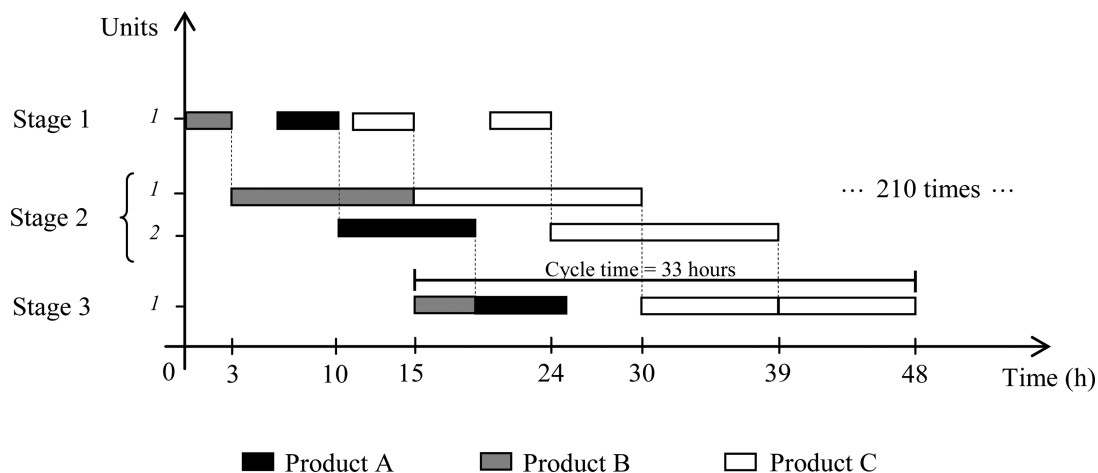


Figure 15. Example 2—Case 1: Gantt chart for optimal production campaign of plant 1.

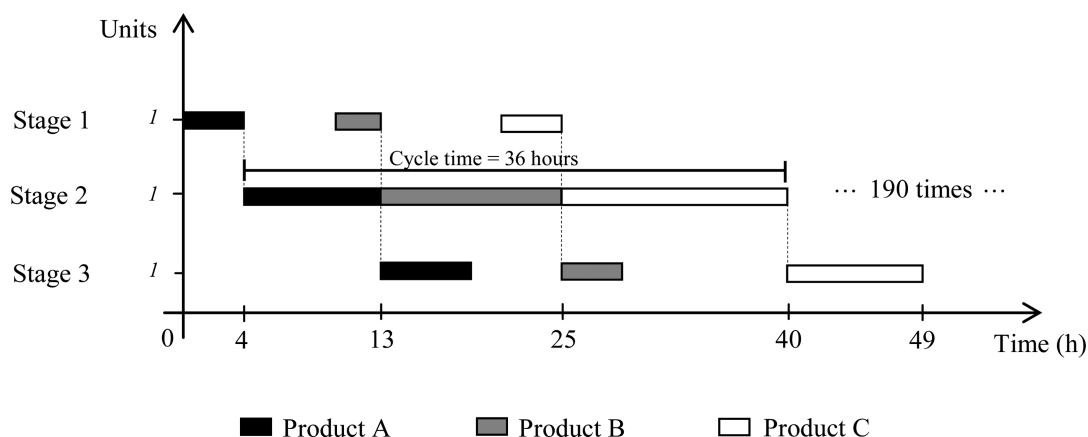


Figure 16. Example 2—Case 1: Gantt chart for optimal production campaign of plant 2.

Table 16. Example 2—Case 2: Product Demands

i	demands: $DM_{ic}$ (kg)			
	c			
	1	2	3	4
A	300000	100000	250000	100000
B	300000	100000	200000	150000
C	440000	120000	500000	260000
D	200000	100000	100000	100000

hierarchical methodologies give different solutions, at both SC and plant level and, also, from the strategic and operational perspectives.

The considered SC topology and the structural options for involved plants are the same as in example 1 of this manuscript. Data on processing times, size and conversion factors, products demands, raw material availabilities at each site, available discrete sizes for units, fixed plant installation costs, and cost exponents involved in the equipment cost are taken from example 1. The cost coefficients relating to production in each plant, raw material acquisition in each site, and transportation among different SC nodes are shown in Tables 18 and 19. Also, batch unit cost coefficients have been increased by 10% with respect to example 1. Finally, the maximum number of batches for each product in the campaign is equal to three in all plants ( $NBC_{if}^{UP} = 3, \forall i, f$ ).

Variable  $NN_f$  is uniformly discretized, considering a step size equal to 5, over the interval [100, 400]. Then the recurrence relation  $T_{mf} = T_{m-1f} + 5$  for  $2 \leq m \leq 61$  with initial condition  $T_{1f} = 100$  allows defining the discrete options for  $NN_f$ .

The proposed approach allows evaluating the different trade-offs generated between all parameters in order to meet demands at minimum cost required. In this case, the model comprises 10660 constraints, 2614 continuous variables, and 591 binary variables, and it was solved in 487.3 CPU seconds. The total annual cost is equal to \$2998985.37, and in the first column of Table 20, a detailed list of costs is presented.

The optimal SC design is shown in Figure 21. Only one plant is selected (plant 2), and it is supplied from the three raw material sites. Table 21 summarizes the different consumption of raw materials at each site, where the used up raw materials are highlighted in gray.

The design of the installed plant is shown in Figure 22. The optimal campaign for that plant comprises two batches of products A and C and one batch of product B, and it is repeated 125 times over the time horizon. The campaign cycle time is equal to 51 h, and the batch sequencing is shown in Figure 23.

The solution obtained through the simultaneous approach is compared with the attained results using a sequential approach, based on the hierarchical optimization of the decisions relative to SC design and, then, design and production scheduling of installed plants. The sequential approach involves two steps: in

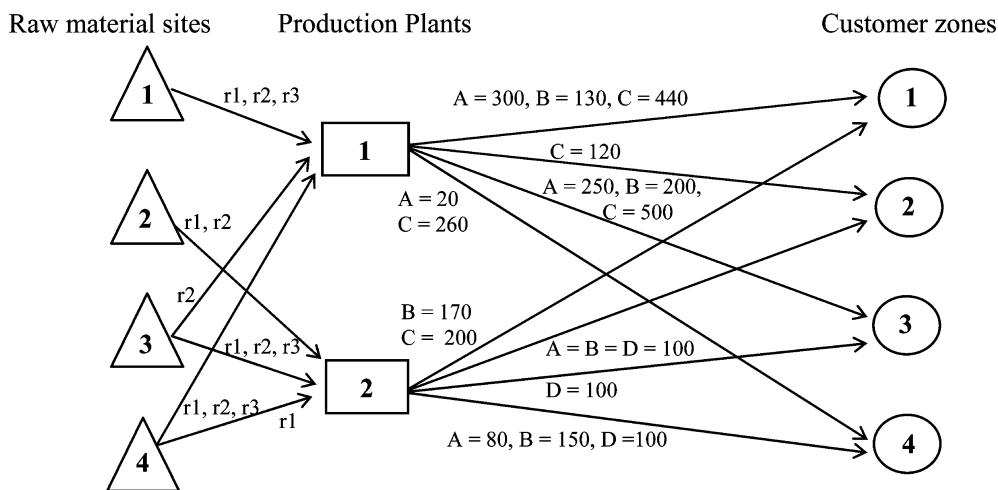


Figure 17. Example 2—Case 2: SC design (amounts  $\times 10^3$  kg).

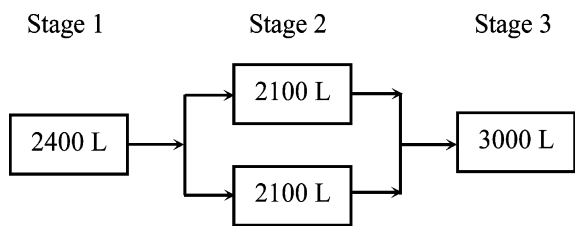


Figure 18. Example 2—Case 2: Plant 1 optimal design.

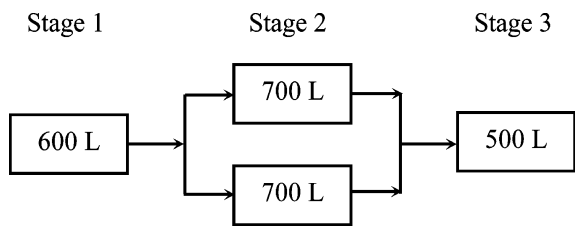


Figure 19. Example 2—Case 2: Plant 2 optimal design.

Table 17. Example 2—Case 2: Amount of Raw Material Transported from Each Site to All Plants

consumables ( <i>r</i> )	raw material sites ( <i>s</i> )			
	1	2	3	4
1	350000	41250	330000	215000
2	500000	13625	540000	440000
3	330000	0	315000	346900

Table 18. Example 3—Production Costs and Transportation Costs of Products from Plants to Customers

<i>f</i>	production cost in each plant: $CPR_{jf}$ (\$/kg)			distribution costs: plants— customer zones $CTIFC_{jfc}$ (\$/kg)		
	<i>i</i>			<i>c</i>		
	A	B	C	1	2	3
1	0.3	0.4	0.15	0.2	0.18	0.3
2	0.1	0.3	0.2	0.1	0.08	0.1
3	0.45	0.4	0.45	0.12	0.16	0.18

the first, the SC design is solved, where the model formulation involves eqs 1–8 and the objective function minimizes plants installation, production, raw material procurement, and transportation costs, in order to fulfill the product demands in the

time horizon. The optimal solution of this step allows obtaining the network configuration (plants number and localization), the flows among SC nodes, and production of each installed plant,  $Q_{jf}$ . In the second step, taking into account production

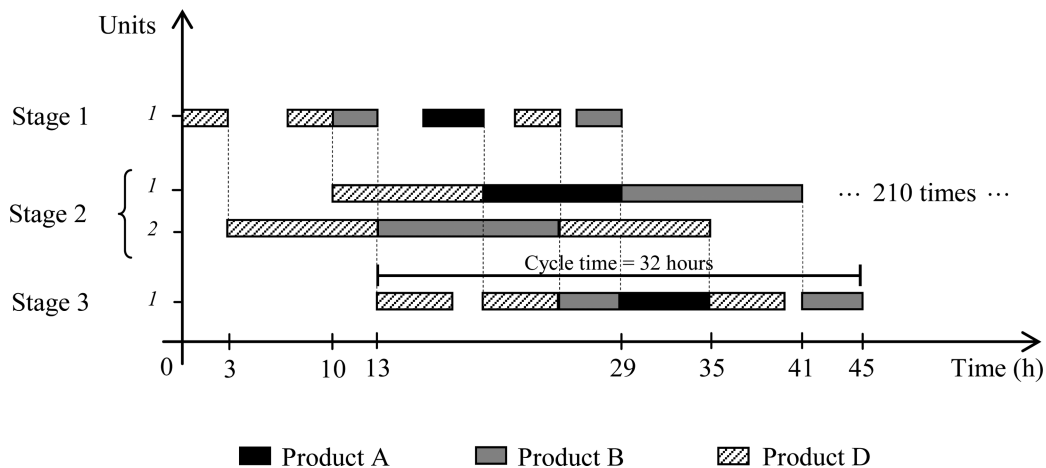


Figure 20. Example 2—Case 2: Gantt chart for optimal production campaign of plant 2.

Table 19. Example 3—Raw Material Costs and Transportation Costs of Raw Materials from Sites to Plants

s	raw material acquisition cost: $CRAW_r$ (\$/kg)			distribution costs: raw material sites—plants $CTRAW_{sf}$ (\$/kg)		
	r			f		
	1	2	3	1	2	3
1	0.1	0.05	0.15	0.05	0.05	0.1
2	0.05	0.1	0.1	0.05	0.15	0.05
3	0.15	0.15	0.05	0.1	0.15	0.15

Table 20. Example 3 - Economical Results (\$/year)

costs	optimal solution	
	simultaneous approach	sequential approach
investment	1149285.37	1894033.79
plants installation	9000.00	28000.00
production	394000.00	382000.00
raw material procurement	565050.00	531550.00
transportation	881650.00	784450.00
total	2998985.37	3620033.79

requirements in each plant, i.e. fixing variable  $Q_{ip}$ , the problem is focused on determining the design and the optimal production campaign for each plant selected in the first stage, considering the minimization of the investment cost.<sup>24</sup>

The optimal SC design is shown in Figure 24. All plants are installed, and they are supplied from the three raw material sites. Plant 1 produces product C, plant 2 produces products A and B, while plant 3 produces product B. More specifically, production of plant 1 is  $Q_{C1} = 850000$  kg, productions of plant 2 are  $Q_{A2} = 800000$  kg and  $Q_{B2} = 175000$  kg, while that for plant 3 is  $Q_{B3} = 305000$  kg. The model posed in the first step includes 199 constraints, 139 continuous variables, and 12 binary variables, and it was solved in 0.09 s.

Then, fixing the product amount that must be manufactured in each installed batch plant, the decisions of design and production campaign scheduling for each installed plant are determined in a second step. The design of each plant is solved through separate models using the first formulation presented by Fumero et al.,<sup>24</sup> and its solutions are depicted in Figures 25–27. Plant 1 has two out-of-phase parallel units in stage 2 and one unit in stages 1 and 3. The campaign composition comprises two batches of C. The batch sequencing in each unit of this plant is shown in Figure 28. The campaign cycle time is reached at the first stage, and it is equal to 24 h. The campaign

Table 21. Example 3—Amount of Raw Material Transported from Each Site to All Plants for the Simultaneous Approach

consumables (r)	raw material sites (s)		
	1	2	3
1	950000	826000	0
2	960000	1580000	215000
3	820000	350000	650000

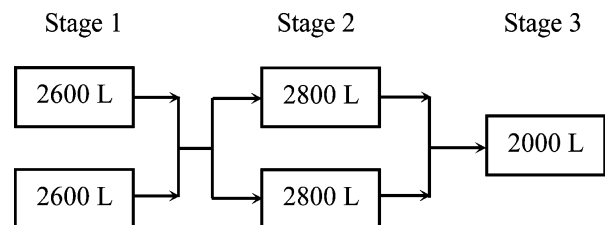


Figure 22. Example 3—Plant 2 optimal design.

is cyclically repeated 290 times over the time horizon. Plant 2 has two out-of-phase units in stage 1, three out-of-phase units in stage 2, and one unit in stage 3. The campaign composition includes two batches of product A and one batch of product B. The batch sequencing on each unit is shown in Figure 29, where it can be noted that the campaign cycle time is reached at unit 1 of stage 1, and it is equal to 28 h. The campaign is repeated 220 times over the time horizon. Plant 3 has one unit in each stage, and it is dedicated to the production of a single product, B, with a cycle time of 18 h. Finally, the models for all plants are independent and the numbers of constraints, continuous variables, and binary variables for each installed plant design and scheduling model are 584, 259, and 102 for plants 1 and 3, and 1599, 524, and 186 for plant 2. For each plant, the model was solved in 0.13, 1.76, and 0.14 CPU seconds, respectively.

From Figures 21–29, it is worth highlighting that the incorporation of plants design and campaign scheduling to the SC design model affects not only the decisions at the plant level but also the SC design. That is, the simultaneous approach allows evaluating the trade-offs among the problem variables and parameters. Obviously, the integrated model is more complex and its solution requires a greater computational time.

A detailed list of costs involved in both steps of the sequential approach is specified in the second column of Table 20. As is noted in this table, the logistic cost in the sequential

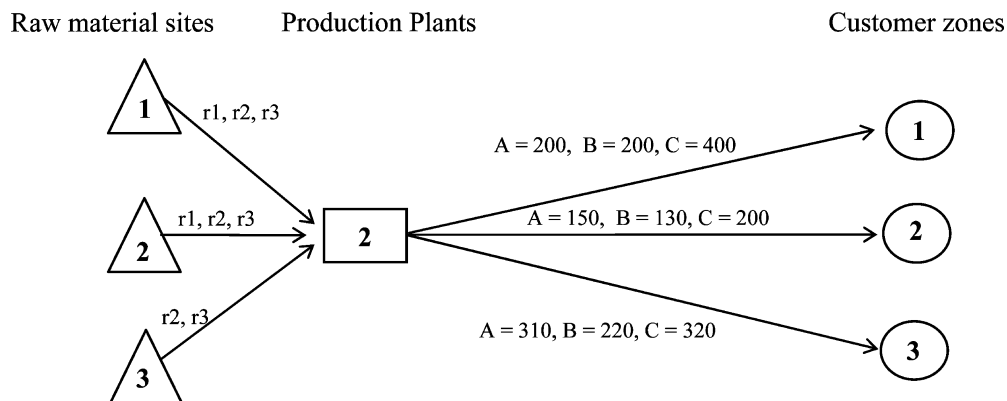


Figure 21. Example 3—SC Design for the Simultaneous Approach (Amounts  $\times 10^3$  kg).



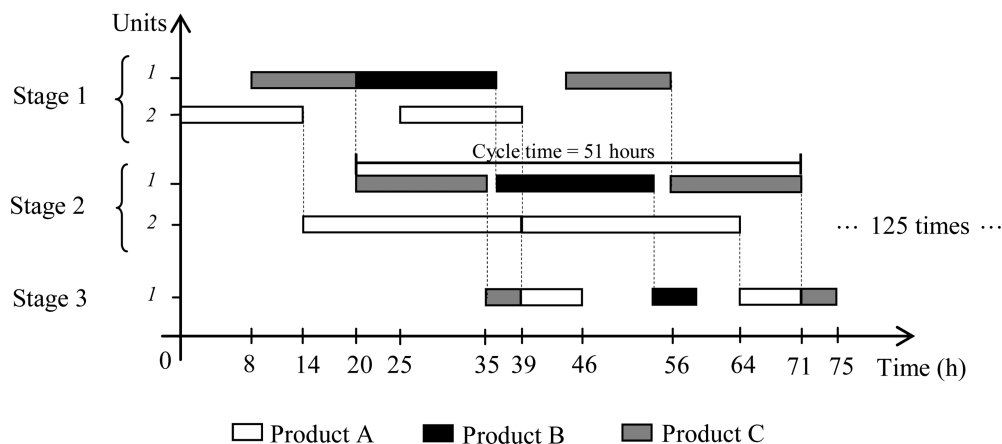


Figure 23. Example 3—Gantt chart for optimal production campaign of plant 2 for the simultaneous approach.

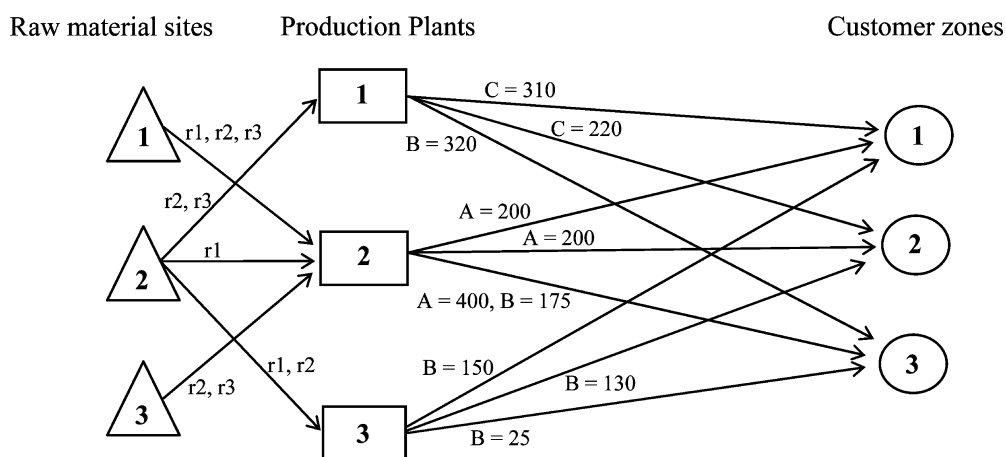


Figure 24. Example 3—SC design for the sequential approach (amounts  $\times 10^3$  kg).

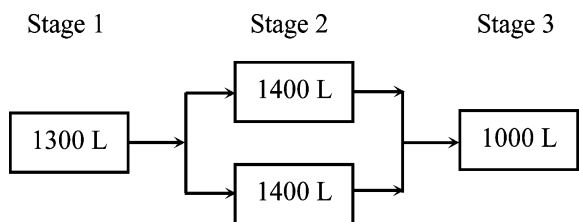


Figure 25. Example 3—Plant 1 optimal design for the sequential approach.

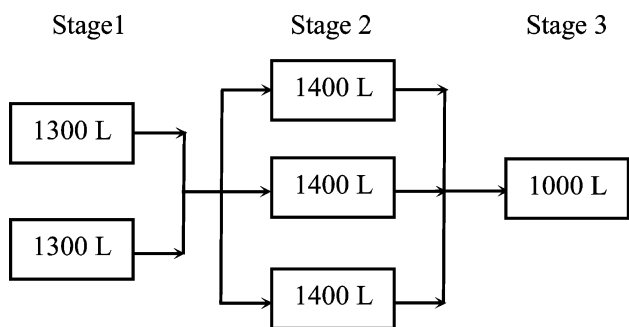


Figure 26. Example 3—Plant 2 optimal design for the sequential approach.

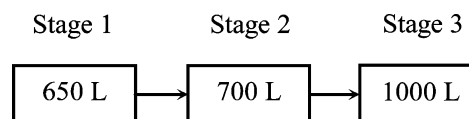


Figure 27. Example 3—Plant 3 optimal design for the sequential approach.

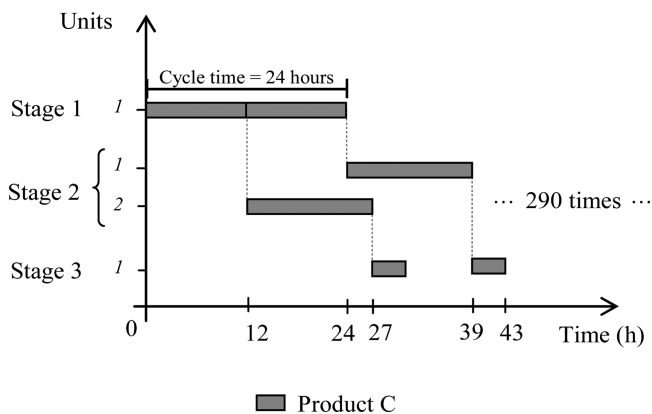


Figure 28. Example 3—Gantt chart for optimal production campaign of plant 1 for the sequential approach.

approach is slightly less than that in the simultaneous approach, because this cost is optimized in the first step of the hierarchical

approach. However, the investment cost in the sequential approach is 65% higher than the same cost of the simultaneous approach addressed in this paper, since the plants' design must

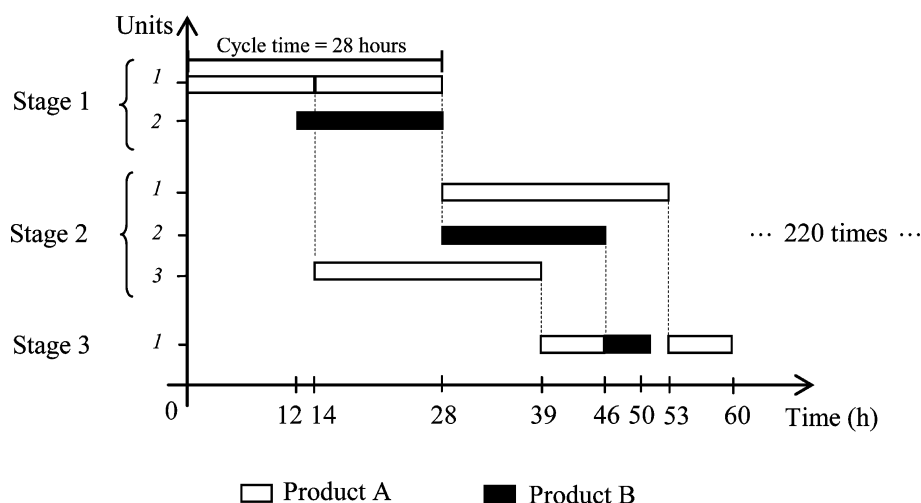


Figure 29. Example 3—Gantt chart for optimal production campaign of plant 2 for the sequential approach.

be performed on a fixed network configuration with fixed production levels. Lastly, the total cost of the sequential approach is 20% higher than the total cost of the simultaneous approach. It is important to note that the difference between the objective functions of the simultaneous and sequential approaches strongly depends on the magnitudes of the involved costs. In particular, it can be very significant when the investment cost is higher than others costs, as happens in this example. Besides, this example shows that achieved solutions are very different with both approaches, and the simultaneous consideration of the involved elements allows assessment of the trade-offs among SC and plant aspects and, on the other hand, among decision levels.

## 5. CONCLUSIONS

The integration of decision levels in SC optimization has been referred to by several authors in the literature as a challenging and still open issue. An integrated approach allows simultaneously assessing the different trade-offs between several decision variables, which is not achieved when hierarchical methodologies are applied.

In this work a novel formulation for the simultaneous SC design and plants design including production scheduling was proposed. Assuming a stable scenario, which is a good approximation in an initial and strategic assessment, the links among the several considered decisions can be evaluated. In this way, a more realistic approach is attained. In order to appropriately include operational characteristics into the model, the production planning based on mixed production campaigns of cyclical repetition was considered. From the operational perspective, this approach allows drawing conclusions about other aspects such as inventory levels, transporting plans, etc., taking into account optimal flows are attained. The problem was formulated as a MILP model, where some assumptions were posed in order to maintain the linearity of the problem and ensure the optimality of the solution.

A highlighted feature of the proposed approach is the capability of the model for evaluating simultaneously different decisions that are usually treated in a separate manner. The SC design is approached in a more integrated perspective, where the network configuration, the material flows between nodes, the embedded plants design, and the production through campaigns in each installed plant are jointly determined. The

presented examples proved that there are close links among the different decisions, and all the parameters and variables should be simultaneously treated to appropriately assess all the trade-offs involved.

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### Notes

The authors declare no competing financial interest.

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## NOMENCLATURE

### Indices

- $c$  customers zone
- $f$  production plant
- $i$  product
- $j$  stage
- $k$  unit
- $l$  slot
- $n$  number of batches of a product
- $p$  discrete size for batch unit
- $P_{jf}$  number of available discrete sizes for a unit of stage  $j$  of plant  $f$
- $r$  raw material
- $s$  raw materials site

### Set

- $SV_{jf}$  available discrete sizes for units of stage  $j$  in plant  $f$

### Parameters

- $CCF$  capital charge factor
- $CP_f$  fixed cost for plant  $f$  installation
- $CP_{R_{if}}$  production cost, per mass unit, of product  $i$  produced at plant  $f$
- $CRAW_{sr}$  procurement cost, per mass unit, of raw material  $r$  in site  $s$

$CTC_f^{UP}$  upper bound for variable  $CTC_f$   
 $CTIFC_{ifc}$  transportation cost, per mass unit, of final product  $i$  from plant  $f$  to customer  $c$   
 $CTRAW_{srf}$  transportation cost, per mass unit, of raw material  $r$  from site  $s$  to plant  $f$   
 $DM_{ic}$  demand of product  $i$  from customer zone  $c$   
 $fc_{rif}$  conversion factor that indicates the relation between raw material  $r$  required to produce one unit of final product  $i$  at plant  $f$   
 $H_f$  time horizon for plant  $f$   
 $K_{jf}$  maximum number of identical parallel units that can be allowed at batch stage  $j$  of plant  $f$   
 $L_{kif}$  number of slots postulated for unit  $k$  of stage  $j$  in plant  $f$   
 $NBC_{if}^{UP}$  maximum number of batches of product  $i$  in the campaign of plant  $f$   
 $NBC_f^{LOW}$  left end of discretization interval of variable  $NN_f$   
 $NBC_f^{UP}$  right end of discretization interval of variable  $NN_f$   
 $Q_{if}^{LOW}$  low bound for the production of product  $i$  in plant  $f$  in the case that binary variable  $z_{if} = 1$   
 $Q_{if}^{UP}$  upper bound for the production of product  $i$  in plant  $f$  in the case that binary variable  $z_{if} = 1$   
 $QR_{sr}^{UP}$  availability of raw material  $r$  in site  $s$   
 $SF_{ijf}$  size factor of product  $i$  in stage  $j$  of plant  $f$   
 $t_{ijf}$  processing time for product  $i$  in stage  $j$  of plant  $f$   
 $T_{mf}$   $m$ -th point obtained from the discretization of variable  $NN_f$  over interval  $[NN_f^{LOW}, NN_f^{UP}]$   
 $VF_{ijp}$  discrete size  $p$  for batch units in stage  $j$  at plant  $f$   
 $\lambda_f$  weighting factor for variable  $CTC_f$  in the objective function  
 $\alpha_{if}$  cost coefficient for batch units of stage  $j$  at plant  $f$   
 $\beta_{if}$  cost exponent for batch units of stage  $j$  at plant  $f$

### Binary Variables

$ex_f$  indicates if plant  $f$  is installed  
 $NNC_{mf}$  specifies if the campaign of plant  $f$  is repeated  $T_{mf}$  times over the time horizon  $H_f$   
 $U_{jkf}$  specifies if unit  $k$  of stage  $j$  at plant  $f$  is employed  
 $v_{jpf}$  denotes if the units of stage  $j$  at plant  $f$  have size  $p$   
 $x_{inf}$  denotes if  $n$  batches of product  $i$  are processed in the campaign of plant  $f$   
 $X_{ljkf}$  indicates if slot  $l$  of unit  $k$  in stage  $j$  at plant  $f$  is employed  
 $z_{if}$  indicates if product  $i$  is produced at plant  $f$   
 $Z_{ijf}$  specifies if product  $i$  is assigned to slot  $l$  at plant  $f$

### Continuous Variables

$B_{if}$  batch size of product  $i$  at plant  $f$   
 $CINST$  annualized installation cost  
 $CINV$  annualized investment cost  
 $CPEN$  penalty term used in the objective function that involves the campaign cycle time of installed plants  
 $CPROD$  raw materials procurement and production cost  
 $CTC_f$  cycle time of the campaign of plant  $f$   
 $CTTRANS$  transportation cost of raw materials from sites to batch plants and of final products from production plants to customer zones  
 $CTRAW_{srf}$  transportation cost, per mass unit, of raw material  $r$  from site  $s$  to plant  $f$   
 $e_{jkpf}$  represents the bilinear term  $U_{jkf}v_{jpf}$   
 $IC_f$  investment cost of plant  $f$   
 $NB_{if}$  total number of batches of product  $i$  processed at plant  $f$  in the time horizon  $H_f$   
 $NBC_{if}$  number of batches of product  $i$  included in the campaign of plant  $f$   
 $NN_f$  number of times that the campaign of plant  $f$  is cyclically repeated over the time horizon  $H_f$

$Q_{if}$  amount of product  $i$  produced in plant  $f$   
 $QC_{ifc}$  amount of product  $i$  sent from plant  $f$  to customer zone  $c$   
 $QR_{srijf}$  amount of raw material  $r$  sent from site  $s$  to plant  $f$  to produce product  $i$   
 $TCOST$  total cost  
 $TF_{ljkf}$  final processing time of slot  $l$  in unit  $k$  of stage  $j$  at plant  $f$   
 $TI_{ljkf}$  initial processing time of slot  $l$  in unit  $k$  of stage  $j$  at plant  $f$   
 $V_{jf}$  size of a batch unit in stage  $j$  of plant  $f$   
 $w_{ijpnf}$  variable that denotes  $Q_{if}$  if the binary variables  $v_{jpf}$  and  $x_{inf}$  simultaneously take the value 1  
 $ww_{mf}$  represents the cross product of variables  $NNC_{mf}$  and  $CTC_f$   
 $Y_{ijklf}$  continuous variable on interval  $[0, 1]$  that indicates if product  $i$  is assigned to slot  $l$  of unit  $k$  in stage  $j$  of plant  $f$

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