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Structural Behavior of Barges in High-Energy Collisions against Bridge Piers

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4 Abstract: The collision of barges against bridge piers is an extreme loading condition that usually governs the design of bridges that span navigable waterways. The magnitude and time variation of impact forces depend on several aspects, such as mass and speed of barges, 5 stiffness of impacted structure, and structural behavior of the barge. The latter has a considerable influence, not only because it defines the 6 maximum possible impact force but also because it defines the energy absorption capacity of the barge. The structural behavior of barges 7 has been studied using scale models and numerical methods. However, the total deformation reached in these studies was limited to the size 8 9 of the barge bow. Hence, there is uncertainty in the behavior for high-energy collisions, where deformations may well exceed this deformation range. This paper studies the structural behavior of barges using detailed nonlinear finite-element (FE) models. Load-10 deformation relationships are established on the basis of the model results for different shapes and sizes of impacted structures. These 11 relationships can be applied in simplified dynamic analyses for design, considering the large deformations expected for high-energy impact 12 scenarios. Simplified analysis methods for symmetric and oblique flotilla impacts are presented and validated against full FE. 13 Q:ADOI: 10.1061/(ASCE)BE.1943-5592.0000789. © 2015 American Society of Civil Engineers.

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16 Introduction

The structural behavior of barges has a significant influence on the 17 analysis of barge collisions against bridge piers or protection 18 structures. The impact-force history during barge-bridge collisions 19 depends on several factors, such as mass and speed of barges, 20 stiffness of the impacted structure, and the configuration of barge 21 flotilla. The structural behavior of the impacting barge defines the 22 23 maximum impact force developed during collision, as well as the 24 energy absorption capacity of the barge. The structural behavior of European Type II and Type IIa barges was studied using scale 25 26 physical models (Meier-Dörnberg 1983), whereas the behavior of Jumbo hopper and oversize tanker barges was studied using high-27 28 resolution finite-element (FE) models (Harik et al. 2008; Consolazio et al. 2008, 2010a). The main goal of these studies 29 30 was to define the load-deformation behavior of the barge bow through simplified force-deformation relationships for the 31 32 development of simplified design procedures. However, the maximum deformation considered by these authors is less than 33 the length of the barge bow, which renders these load-deformation 34 curves applicable to a limited range of energy. Hence, there is 35 36 uncertainty in the structural behavior of barges for high-energy collisions, where the energy absorbed by the barge may lead to 37 deformations exceeding the range considered by these previous 38 studies. This paper focuses on the behavior of barges for a 39

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deformation range larger than that used in previously published studies and proposes analysis methods to account for the forces developed in this deformation range. Simplified methods applicable for high-energy collisions are proposed and validated using detailed FE models.

Scale Physical Models

The guidelines developed by the European Committee for Standardization (CEN 1991) and AASHTO (2012) provide simplified design methods considering a force-deformation relationship obtained by Meier-Dörnberg (1983) on the basis of scale physical models. The force-deformation relationships obtained by Meier-Dörnberg are bilinear and represent mean and upper bound values of the test results (Fig. 1).

The CEN (1991) adopts the mean curve and provides a simplified procedure to estimate a time history of impact forces on the basis of the kinetic energy of the barge. AASHTO (2012), on the other hand, recommends a static impact force, also obtained as a function of the kinetic energy of the barge. The equivalent *static force* concept proposed by AASHTO (2012) has been criticized in the recent literature (e.g., Consolazio et al. 2008; Harik et al. 2008).

The bilinear force-deformation relationships considered by AASHTO and CEN consider a steady increase in load after yielding (Fig. 1). Recent research, however, indicates that the yield load actually remains approximately constant or even decreases with increasing deformations (Consolazio et al. 2008; Harik et al. 2008).

Numerical Models

The structural behavior of Jumbo hopper and oversize tanker68barges was studied using high-resolutionFE models by Consolazio69et al. (2008) and Harik et al. (2008). These authors considered the70influence of different pier shapes and sizes; factors that were found71to have a significant influence on the resulting forces. Consolazio72

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73 et al. (2008) describe that there is a decrease in contact force

⁷⁴ associated with the buckling process of the internal reinforcements

75 and failure of the hull. These authors also found that the contact

⁷⁶ force increases with the size of the impacted pier.

Harik et al. (2008) indicate that barge-bridge collisions are not
 high-speed impact events because the results obtained using



Fig. 1. Load deformation curves for European barges Type II and Type IIa

dynamic and pseudostatic analyses are very similar. Hence, velocity does not significantly influence the crushing resistance of the barge, with the exception of a sharp peak force developed at the beginning of the impact process. Harik et al. (2008) report that this initial peak force is too brief to significantly affect typical bridge structures. Typical force-deformation curves derived by Consolazio et al. (2008) and Harik et al. (2008) for Jumbo hopper barges are shown in Figs. 2 and 3, respectively. On the basis of these results, simplified force-deformation relations are proposed by these authors for developing improved simplified analysis techniques. Consolazio et al. (2008) propose an elastic-perfectly plastic behavior, whereas Harik et al. (2008) derive piecewise linear relationships.

Consolazio et al. (2010a, b) propose several simplified analysis techniques, including a static analysis method (static bracketed impact analysis), a predefined load-history method [applied vessel impact loading (AVIL)], a response spectrum method (impact response spectrum analysis), and a barge–structure interaction approach [coupled vessel impact analysis (CVIA)]. Harik et al. (2008) propose a series of FEM regressions that allow the definition of impact load and collision duration, as well as a spring–mass model for barge flotillas.

These design methods are, however, based on force-deformation relationships only valid for a limited deformation range (less



Fig. 2. Impact force versus deformation for Jumbo hopper obtained using numerical model of barge bow for (a) round piers; and (b) flat piers (data from Consolazio et al. 2008)



Fig 3. Impact force versus deformation for Jumbo hopper obtained using numerical model of barge bow for (a) round piers; and (b) flat piers (data from Harik et al. 2008)

than the length of the barge bow). There are design situations 102 103 where the kinetic energy involved leads to bow deformations that exceed the range considered by these authors. For example, con-104 sidering that average barge tows consist of 15 barges but may go 105 up to 40 barges (e.g., CARIA 2014), and that each barge may have 106 107 a displacement of 1,900 t (e.g., AASHTO 2012), the kinetic energy involved may be on the order of 360 MJ (or more) for a 108 109 velocity of 5 m/s (e.g., Pinto et al. 2008). Even considering that the flotilla may break upon impact, the kinetic energy of a single 110 column of five barges at 5 m/s yields 120 MJ. Considering a 111 kinetic energy of 120 MJ, and the force-deformation relationship 112 recommended by AASHTO (2012), the permanent barge defor-113 114 mation would yield 9.5 m, largely exceeding the deformation 115 range considered by previous studies.

116 Structural Behavior of Barge

The structural behavior of Parana cargo and tanker barges is investigated using nonlinear FE models, on the basis of structural drawings and specifications provided by a regional barge manufacturer. Table 1 shows a comparison of the main features of the Parana, Jumbo hopper, and oversize tanker barges, where it is seen that the Parana cargo is similar to the Jumbo hopper, but provides a greater displacement capacity.

The main purpose of the FE analyses is to extend the forcedeformation relationship up to 14 m of bow deformation to assess previously proposed trends (i.e., elastic-perfectly plastic or constant hardening) for this deformation range.

128 Barge Structure

The Parana cargo and tanker barges are 59.5 m long, with a loaded displacement of 3,100 and 2,900 t, respectively (Table 1). The hulls of these barges are double, and consist of A36 steel plates with L-shaped stiffeners 7.93–19 mm in thickness. These structures include longitudinal and transverse reinforcing sections consisting of L-shaped and U-shaped internal trusses, as well as watertight sections.

Table 1. Typical Barge Characteristics

			Paran	a type
Parameters	Jumbo hopper	Oversize tanker	Cargo	Tanker
Length (m)	59.3	88.2	59.5	59.5
Width (m)	10.7	16.1	16	16
Depth of vessel (m)	3.6	3.6	3.6	3.6
Loaded displacement (t)	1,900	4,300	3,100	2,900

Fig. 4 shows the geometry of tanker and cargo barges, in-137cluding partial views of the reinforcing and watertight sections.138The ASTM A36 stress-strain relationship considered in the FE139models is shown in Fig. 5 (Boyer 2002).140

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Finite-Element Models

The structural behavior of the barges was modeled using SIMULIA 142 (2010). The hull and internal trusses (SR3 and SR4) were modeled 1430 using shell elements with reduced integration, an assumed shear 144 deformation for bending, and hourglass control for membrane 145 behavior. Geometric nonlinearities were also taken into account. 146 Following Harik et al. (2008), a strain rate-independent mate-147 rial definition is considered, as these authors found that strain 148 rate effects are negligible for these types of problems. The pay-149 load was not explicitly modeled, but its inertia was accounted 150 for using an additional mass distributed in the barge body. The 151 welded unions between hull sheets and between the hull and 152 internal trusses were modeled using constraints. Hence, the possi-153 bility of weld rupture is not accounted for in the model. How-154 ever, failure of the steel was considered for an equivalent plastic 155 deformation of 20% by eliminating the elements reaching this 156 threshold. 157

The impacted structure was modeled as a rigid object to eval-158 uate the load-deformation relationship of the barges. However, 159 the dynamic behavior of the impacted structure can be accounted 160 for in subsequent analysis. A general contact algorithm, able to 161 detect contacts among different parts of the model, is considered. 162 An explicit dynamic scheme was preferred over an implicit ana-163 lysis because of the large number of contacts expected. The 164 lashings between barges were defined as tension only, elastic-165





Fig 4. Geometry of Parana cargo and tanker barges

perfectly plastic elements, with failure due to excessive straining.
The lashing models considered herein follow the properties and
usual practices described by Arroyo and Ebeling (2005).

To limit the complexity of the model, hydrodynamic effects were not taken into account. However, hydrodynamic effects can be included in the analysis using a hydrodynamic mass coefficient, as proposed by AASHTO (2012).

Force-deformation relationships for barges were obtained by performing several collision simulations. The analyses considered centered and corner impacts against flat and round piers. Oblique collisions against flat walls with different impact angles were also analyzed.

The FE models were set up following two different approaches for the geometry: (1) the partial-barge model and (2) the fullbarge model.

These two different approaches have been considered by other 181 researchers for the analysis of the structural response of barges 182 183 (Consolazio et al. 2008; Harik et al. 2008). In the first approach, only the front portion of the barge (about 20 m out of a total length 184 185 of 60 m) was modeled. A boundary condition consisting of a constant velocity was imposed in this approach. A uniform mesh 186 was considered because the deformation develops throughout a 187 188 significant part of the model [Fig. 6(a)].

In the second approach, the complete barge was represented, and an initial velocity condition was defined. In this model, the hydrostatic behavior of water was represented using linear springs applied at the bottom of the entire barge along a direction perpendicular to this surface. In this approach, a graded mesh was defined because deformation concentrates in the front portion of the model [Fig. 6(b)].

To isolate the influence of the boundary conditions in the 205 model results, both approaches consider the same mesh size in the 206 barge bow. Fig. 7 shows a comparison of the force-deformation 207 relationships obtained using both approaches. It can be seen that 208 the partial-barge model yields higher forces at the beginning of the 209 deformation process (mainly crushing near the bow), for both 210 cargo and tanker barge types. There are also some differences in 211 forces for greater deformations, where forces obtained using the 212 partial model are similar or less than the values obtained using the 213 full-barge model. The force-deformation relationships reported by 214 Consolazio et al. (2010a) consider a partial-barge model. 215

Results presented in the following sections consider the fullbarge model results, as they are deemed more representative of actual barge behavior. 218

Results

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The following general trends are identified in the different FE model simulations (Figs. 9–12):

- The magnitude of contact forces depends on pier shape and size. In general terms, the mean force increases with pier width and is greater for flat piers versus rounded piers. These trends are also reported by Consolazio et al. (2010a) on the basis of partial-barge models. 226
- An initial peak force is observed, which corresponds to a deformation on the order of 0.05–0.1 m. The deformation corresponding to this initial peak is limited to the zone adjacent to the pier [Fig. 8(a)]. This peak force is higher for flat piers (Figs. 9 and 10).
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Following the initial peak, the contact forces decrease or re-232 main approximately constant up to a deformation of about 5 m. 233 During this stage, the internal trusses in the bow and the hull buckle, whereas the barge body does not undergo considerable 234 deformations [Fig. 8(b)]. 235

236 For deformations greater than 5 m, the contact forces undergo a 237 sharp increase, to a level even greater than the initial peak, 238 mainly owing to the contact of the pier with the barge body [Fig. 8(c)]. This effect is more pronounced for tanker barges, 239 which include a steel structure that covers the payload (Fig. 4). 240

It is observed that contact forces do not significantly change for 241 242 deformations greater than 14 m (Fig. 11).

243 In contrast to the empirical force-deformation curves adopted 244 in codes, such as AASHTO (2012), numerical results show that contact forces do not steadily increase with deformation. Hence, 245 impact forces may be overestimated or underestimated for diffe-246 rent pier geometries or collision energies. 247

248 Some of these trends are consistent with results previously reported by other authors (Consolazio et al. 2008; Harik et al. 249 250 2008). For example, Fig. 12 shows a comparison of the loaddeformation curve for a Parana cargo and Jumbo hopper 251 (Consolazio et al. 2008) barges impacting a flat pier, where the 252 results are quite similar within the deformation range considered 253 254 by previous studies.

255 However, the sharp increase in collision forces for deforma-256 tions larger than 5 m was not identified in previous studies and 257 may have a significant influence on the forces developed for high-258 energy collisions.

259 Typical FE results of centered impacts against flat and rounded 260 piers are shown in Figs. 9 and 10. In these figures, FE results were smoothed for a simpler representation, filtering out sharp force 261 variations at incremental deformations under 0.4 m. 262

Force-deformation relationships for corner impacts were also 263 derived using the FE models. The collision forces obtained for 264 corner impacts are similar to the centered impact results, with the 265 exception of narrow piers, where the mean impact force is lower 266 for corner impacts [Fig. 9(b) versus Fig. 13(a), and Fig. 10(a) 267 versus Fig. 13(b)]. A conservative assumption would consider that 268 an impact of two adjacent barges with a bridge pier occurs at the 269 corner of both barges, as opposed to considering a centered col-270 271 lision of a single barge.

272 Oblique impacts against flat walls at different collision angles 273 were also analyzed using the FE model. In these analyses, the same modeling scheme used for centered impacts was considered. However, for the full barge models, the lateral sides of the barges were prevented from moving along the direction perpendicular to the initial velocity to prevent barge rotation during the collision process (as expected for flotilla collisions).



Fig. 11. Extended load-deformation result for centered impact of cargo barge against 8 m flat pier



Fig. 12. Force versus deformation for Parana cargo barge and Jumbo hopper (data from Consolazio et al. 2008) against flat pier



Fig. 13. Force versus deformation for corner impacts: (a) tanker barge against flat pier; (b) cargo barge against round pier

In the analysis using the partial-barge model, deformations were found to localize near the boundary section for oblique impacts [Fig. 14(a)]. This behavior is considered a fictitious consequence of the boundary condition of the partial model because this behavior was not observed for the full-barge FE models [Fig. 14(b)].

For the case of oblique impacts against flat walls, it was observed that there is a decrease in the collision forces for increasing impact angles with respect to a head-on collision (Fig. 15). The results herein presented consider the deformation measured along a normal direction with respect to the impacted wall.

290 Simplified Force-Deformation Relations

The FE model results for centered, corner, and oblique impacts were approximated by piecewise linear functions to make these results readily available for implementation in simplified analysis methods, as discussed further on in this paper. The forcedeformation relations were defined using 10 points, fitted following the sequential quadratic programming method to minimize the quadratic residual. The following restrictions were imposed:

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- First point is the force-deformation origin (F = 0; $\delta = 0$),
- Force magnitudes must be positive,
- Deformations are given in increasing order, and
- Last value of deformation equals to the maximum deformation reached in the analyses ($\delta = 14 \text{ m}$).

A comparison of the force-deformation relationships considering the full FE model results and the piecewise linear approximation is shown in Fig. 16. The simplified relationships for centered, corner, and oblique impacts are summarized in the Appendix. 307

Simplified Analysis Methods

Full FE models for barge-bridge interaction analyses are not309generally available for design because of the lack of detailed in-
formation on the barge structure and the time cost associated with310111112121312



Fig. 14. Behavior of partial-barge and full-barge models during oblique impacts against flat wall





simplified low-resolution analysis methods are warranted for design. This paper proposes simplified methods that include key
aspects of the barge–bridge collision process.

Two simplified methods, which are able to represent symme-316 trical and oblique impacts, are presented and validated using full 317 318 FE model results. These simplified methods consider the piecewise linear relationships described above to account for the structural 319 320 behavior of the barges. Alternatively, a modification of a simplified procedure previously proposed by Consolazio et al. (2008) is 321 presented to consider a more detailed force-deformation charac-322 terization of the barges applicable to high-energy collisions. These 323 324 methods are applicable to different impact scenarios, as discussed 325 in the following sections.

326 Simplified Coupled Model for Symmetric Impacts

327 Consolazio et al. (2008) proposed an analysis method, referred to 328 as CVIA, in which barges are modeled as single masses connected to the bridge structure using a contact force. This model can be 329 readily implemented in commercially available structural analysis 330 software, such as SAP2000. This approach considers several 331 332 aspects of the collision phenomenon, such as the dynamic response of the impacted structure, barge mass and speed (i.e., kinetic 333 334 energy), and a piecewise linear force-deformation relationship for 335 the barge structure.

A key assumption of this simplified modeling approach is that a barge column can be represented by a single mass. This assumption, however, does not introduce significant differences in results for symmetric impacts because kinetic energy is mainly dissipated by elastoplastic work in the front barge (e.g., Harik et al. 2008). For multicolumn barge flotillas, an equivalent procedure that can account for the influence of barge columns that do not come in contact with the bridge pier was proposed by Luperi and Pinto (2014).

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In the proposed approach, the structural model is augmented with two additional degrees of freedom (DOF), representing interaction with the impacting barges. The first DOF, where the barge mass is assigned, is connected by a piecewise linear (i.e., nonlinear) element to a second DOF (link/support element with multilinear plastic behavior), which in turn is connected by a compression-only spring element to the impact point in the bridge structure (gap element). A schematic representation of this modeling approach is shown in Fig. 17. The stiffness of the compression-only spring is of an order of magnitude greater than the nonlinear element to avoid significantly affecting the results. In addition, a stabilizing mass is assigned to the second DOF.

A nonlinear direct time-integration scheme is considered for the analysis. Because an initial velocity cannot be assigned as an initial condition, two consecutive analysis stages need to be defined. The displacements and velocity of the nodes at the end of the first stage are used by the program as initial conditions for the second stage.

In the first stage, a force is applied to the barge mass, which is accelerated to the desired impact velocity at the end of this stage. The barge mass accelerates freely because an element is defined with a gap equal to the total free displacement needed for the mass to accelerate to the desired velocity.

The results obtained using this simplified analysis procedure 367 are compared with results of a full FE model, where the impacted 368 structure (bridge) is considered as rigid to assess the influence of 369 the barge model only. The final configuration and load history for 370







Fig. 17. Simplified model for symmetric impacts

a column of three cargo barges (2,900 t each) impacting at 5 m/s 371 372 against a 6 m flat pier are shown in Figs. 20 and 21, respectively. It can be seen that there is a very good agreement between the full 373 FE model and simplified model results. In this example, the 374 maximum barge deformation obtained using the FE model ana-375 lyses and simplified model is 12.05 and 12.17 m, respectively, 376 377 whereas the dissipated energy is 96.0 and 95.9 MJ for each 378 modeling approach.

379 Although this proposed modeling approach involves little extra effort on the bridge model for the collision analysis, it is able to 380 accurately represent key aspects of the collision phenomenon, and 381 382 yields force histories that do not substantially differ from full FE 383 analyses, as shown in Fig. 21.

Applied Load History Method Modification for 384 **High-Energy Collisions** 385

A simplified method, referred to as the AVIL, was proposed by 386 Consolazio et al. (2008). In this method —applicable to symmetric 387 impacts-a load history is derived using a set of design parameters 388 389 (e.g., barge velocity and mass) and subsequently applied to the structure. This method is based on principles of conservation of 390 391 energy and linear momentum and assumes an elastic-perfectly plastic behavior for the barge bow. This simplified method has 392 393 been validated using the more elaborate CVIA (Consolazio et al. 394 2008). However, FE analyses for high-energy collisions indicate 395 there is a significant force increase after a certain deformation at the barge bow. Thus, a modification to this method is herein 396 397 proposed to account for the increase in crushing forces for high-398 energy collisions (i.e., large bow deformations).

399 To determine the crushing forces of the barge (controlled both 400 by bow and body), as well as the deformation at which the tran-401 sition of crushing forces occurs, the curves showing the variation of energy with deformation (obtained using the FE models) are 402 403 approximated by bilinear relationships (Fig. 18).

On the basis of these results, the crushing forces at different 404 405 deformation ranges, as well as the transition point, are derived as a function of pier size and shape. The approximations are sum-406 407 marized using the following linear equations:

408	$T_P = A_d + B_d D$
100	$P_1 = A_{F1} + B_{F1}D$
409	$P_2 = A_{F2} + B_{F2}D$
410	$k_{\rm D} = A_{\rm H} + B_{\rm H} D$
	$\kappa_B - \Lambda_K + D_K D$

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412 where T_P = transition point (deformation at which the crushing force increases) in meters; D = pier width in meters; P_1 and $P_2 =$ 413

barge bow and body-crushing loads, respectively; $k_B =$ barge 414 stiffness; and A and B = parameters summarized in Table 2. 415

The load history can thus be estimated using the AVIL 416 method as proposed by Consolazio et al. (2008), where the trial 417 crushing deformation of the barge for inelastic impacts (consi-418 dering a unique crushing force) can be estimated using the 419 following equations: 420

$$a_B = \frac{1}{2} \frac{m_B}{P_1} V_{BY}^2$$
 (2)

$$V_{BY} = \sqrt{V_{Bi}^2 - \frac{P_1^2}{k_S m_B}}$$
(3)

$$k_S = \left(\frac{1}{k_B} + \frac{1}{k_P}\right)^{-1} \tag{4}$$

where a_B = trial barge-crushing deformation; V_{BY} = barge velo-423 city at beginning of yield; V_{Bi} = initial barge velocity; m_B = barge 424 mass; P_1 barge-bow crushing force; t = time until elastic rebound; 425 k_S = effective barge-pier-soil spring stiffness; and k_B and k_P = 426 barge and equivalent pier stiffnesses, respectively. 427

By comparing the trial crushing deformation with the transition 428 point, it can be determined whether the load history can be ob-429 tained using the AVIL procedure, as proposed by Consolazio et al. 430 (2008), considering a single initial crushing force. If the trial 431 crushing deformation is greater than the transition point, then a 432 double-yield load history is determined, where the yield loads 433 represent the crushing forces controlled by the bow and body of 434 the barge. This two-stage load history, herein referred to as 435



Fig. 18 Energy-deformation relationship for Parana cargo barge for centered impacts against flat pier

Table 2. Parameters for Transition Point and Barge Bow and Barge-Crushing Load

		Cargo	barge		Tanker barge						
	Roun	d pier	Flat	pier	Rour	nd pier	Flat pier				
Parameters	A	В	A	В	A	В	Α	В			
<i>t</i> (m)	6.54	- 0.07	6.29	- 0.15	6.81	- 0.03	6.88	- 0.13			
F1 (MN)	3.44	0.08	2.94	0.27	2.7	0.04	2.28	0.08			
F2 (MN)	8.95	0.55	8.15	0.73	15.06	0.85	16.37	0.61			
k_B (MN/m)	264	69	212	81	79	- 3	35	8			

(1)

436 modified applied vessel impact load (MAVIL), is determined as 437 follows:

First, the duration of the elastic loading is determined asproposed by Consolazio et al. (2008):

$$t_Y = \frac{\pi m_B}{2P_1} (V_{Bi} - V_{BY})$$
(5)

441 2. Next, the duration of the plastic phase caused by yielding of442 the barge bow is estimated, as follows:

$$t_1 = m_B \frac{V_{BY} - \sqrt{V_{BY}^2 - 2P_1 T_P / m_B}}{P_1}$$
(6)

444 3. Then, the velocity at which the transition point is reached is 445 calculated:

$$V_1 = V_{BY} - \frac{P_1 t_1}{m_B}$$
(7)

446447 4. The duration of the plastic yielding at the barge body is448 determined as

$$t_2 = \frac{m_B V_1}{P_2} \tag{8}$$

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450 5. Finally, the duration of the elastic unloading is obtained:

$$t_u = \frac{\pi}{2} \frac{m_B}{\sqrt{k_S m_B}} \tag{9}$$

452 On the basis of these parameters, the load history is obtained as 453 shown in Fig. 19.

454 The load history obtained using the MAVIL method is com-455 pared against full FE model results and the simplified coupled method in Fig. 21. Although there are some differences, the 456 MAVIL method can represent the main features of the impact 457 history and the crushing force variation for high-energy collisions. 458 The variation in crushing force would not have been represented 459 by the original AVIL procedure (Consolazio et al. 2008) because it 460 461 was originally developed for a lower deformation range. Hence, it is considered that the MAVIL method is a reasonable approxi-462 463 mation for design in high-energy collisions.

464 Simplified Model for Oblique Impacts

Although not generally a controlling scenario in design, the analysis of oblique impacts may be warranted in different situations,
e.g., collisions with a structure that has slanted sides to deflect
impacting vessels, or an eccentric flotilla collision. One significant



Fig. 19. Modified applied vessel impact load method

assumption for the analysis of symmetric impacts is that the position of barges is given by a single coordinate, which is an unrealistic assumption for oblique impacts. Moreover, other phenomena generally not considered in symmetric-impact models may need to be accounted for, such as failure of lashings, interaction among barges, and geometric interaction with piers. 470

For these situations, a simplified bidimensional analysis 475 method was proposed by Luperi and Pinto (2014). In this app-476 roach, barge and pier are defined using meshes, consisting of 477 a group of points that define their respective contours. The 478 contacts between different elements of the model are detected on 479 the basis of the contour meshes using a contact algorithm (Luperi 480 and Pinto 2014). This algorithm determines which points of a 481 particular barge mesh fall inside the contour defined by an adjacent 482 barge or pier mesh. Hence, a contact zone, its normal direction, a 483 contact overlap, and a relative tangent velocity are derived. On the 484 basis of these parameters, the resulting contact forces within the 485 barge tow are determined. The dynamic barge collision analysis is 486 performed using a numerical integration scheme of the equations 487 of motion of the full barge-structure system. 488

The force-deformation behavior of the barge bows and lashings 489 are represented using nonlinear springs. Barges that do not directly 490 impact the pier are considered to behave as linear elastic. This 491 simplifying assumption is considered a reasonable approximation 492 on the basis of full FE analyses of barge columns reported by Harik 493 et al. (2008), where it is shown that energy dissipation is mainly 494 caused by the plastic deformation of the impacting barge bow. 495

The flotilla model proposed by Luperi and Pinto (2014) can represent oblique impacts, model the behavior of lashings, and account for piecewise linear bow behavior and the dynamic response of the structure.



Fig. 20. Final configuration of a three-barge column impacting a 6 m flat pier at 5 m/s





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500 The oblique force-deformation relationships for high-energy 501 collisions herein presented can be readily incorporated in this bidimensional approach as piecewise linear relations. As an ex-502 503 ample of typical results, the contact-force history for an oblique impact (30°) of a 2×2 barge flotilla against a flat wall, with an 504 505 initial velocity of 5 m/s is considered. The final configuration, load history, and evolution of energy are shown in Figs. 22-24. In this 506 example, the first barge column impacts the flat wall, whereas the 507 second barge column breaks away and later collides with the wall. 508 509 The impact of both barge columns occurs simultaneously at t = 1.9-2.2 s after initial contact. During this time interval, a peak 510 511 impact force caused by the contribution of both columns can be 512 seen in the load history. Some discrepancies exist between the load 513 histories evaluated using each method; however, the impulse delivered to the flat wall is 53.1 and 51.8 MN for the simplified two-514 dimensional (2D) approach and full FE models, respectively 515 (a 2.5% difference). The evolution of energy is shown in Fig. 24, 516 517 where the kinetic and dissipated energies through friction and plastic work are compared. The simplified 2D approach yields 518 519 reasonable results, consistent with general engineering approximations, for oblique impacts of barge flotillas. The total time for 520 the definition of the simplified 2D model and computation is a 521 fraction of the time required for the full FE model setup. There-522 523 fore, it is considered that the simplified 2D model proposed by 524 Luperi and Pinto (2014), including the force-deformation relation-525 ships for oblique impacts presented herein, is a useful tech-526 nique for routine analysis of barge flotilla impacts, particularly for 527 design.



Fig. 22. Final configuration for oblique impact of a 2 × 2 barge flotilla against a flat wall



Fig. 23. Load history for oblique impact of a 2 × 2 barge flotilla against a flat wall

Conclusions

The structural behavior of Parana cargo and tanker barges was 529 studied using high-resolution FE models. Several collision simu-530 lations, considering different shapes and sizes of impacted struc-531 tures, were performed using the numerical models. Nonlinear 532 force-deformation relationships were obtained for an extended 533 range of bow deformation to better define the structural behavior 534 of barges for high-energy collisions. It was found that the impact 535 forces increase considerably for deformations greater than the 536 length of the barge bow. Piecewise linear approximations of force-537 deformation relationships are given for centered, corner, and ob-538 lique impacts. These relationships can readily be incorporated in 539 simplified analysis methods. 540

A simplified modeling approach for symmetric impacts is proposed for its implementation in commercially available structural analysis software. The proposed modeling approach is simple, but able to consider key features of the collision, such as structural response (particularly relevant for flexible protection structures), dynamic amplification, and nonlinear (i.e., piecewise linear) structural response of the barge. Piecewise linear relations are given for high-energy collisions, where the yield load increases for large deformations. This increase in yield load was not identified in previous studies by Consolazio et al. (2010a) and Harik et al. (2008).

The MAVIL method proposed in this paper is applicable to high-energy collisions and allows derivation of the impact-force history, which can later be applied to the structure. This proposed method can represent the crushing-force variation for high-energy collisions, where deformations exceed the range considered by previous studies.

The force-deformation relationships herein presented are also applied to the analysis of oblique impacts of barge flotilla. The proposed force deformations are implemented in a simplified 2D model presented in a previous paper (Luperi and Pinto 2014).

The results obtained using the proposed simplified approaches closely represent the results obtained using more elaborate (and time consuming) FE models.



Fig. 24. Energy evolution for oblique impact of a 2×2 barge flotilla against a flat wall

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Appendix. Simplified Force-Deformation Relations

Piecewise Linear Force-Deformation Relationship for Centered Impact of Parana Cargo Barge

Ι	Dimension ((m)				Load-defo	ormation rel	ationships				
Round pier	2	D	0.01	0.33	0.93	3.92	4.56	4.89	10.92	11.29	12.61	14.00
		F	3.34	3.93	2.44	3.37	6.06	5.37	8.03	11.95	10.67	12.92
	4	D	0.03	0.98	3.75	4.48	4.96	6.15	9.44	11.00	11.28	14.00
		F	4.14	2.77	3.63	6.44	5.87	8.00	10.98	11.46	10.89	11.56
	6	D	0.02	1.15	3.86	4.51	5.29	8.15	9.38	10.43	12.67	14.00
		F	4.16	2.84	4.24	7.12	6.59	12.28	15.82	11.67	12.63	15.34
	8	D	0.01	0.90	3.89	4.42	5.40	8.36	9.51	10.53	13.54	14.00
		F	4.33	3.09	4.78	7.50	7.15	16.24	12.95	13.29	18.18	16.69
	10	D	0.02	1.17	3.94	4.52	5.40	6.93	7.47	8.66	9.22	14.00
		F	4.42	3.25	4.72	8.07	7.96	14.13	12.47	13.05	17.34	18.20
	16	D	0.02	0.47	3.67	5.72	6.27	7.32	8.37	10.51	11.97	14.00
		F	5.67	3.25	5.11	12.24	11.94	17.07	14.77	20.12	13.76	15.12
Flat pier	2	D	0.02	0.29	3.72	4.48	5.41	5.80	6.91	11.01	11.04	13.99
-		F	7.50	2.90	3.47	6.30	5.52	7.46	7.37	10.09	11.44	9.36
	4	D	0.02	0.14	3.76	4.39	5.43	6.59	7.76	8.02	9.96	13.99
		F	12.79	3.10	3.83	6.89	6.85	9.60	9.72	11.82	9.83	14.48
	6	D	0.02	0.13	3.66	4.45	5.06	7.05	8.90	9.68	10.08	13.99
		F	16.55	3.56	4.09	9.08	7.53	14.20	9.56	13.22	11.38	16.55
	8	D	0.02	0.11	3.37	4.33	5.22	5.86	6.99	8.57	13.34	14.00
		F	21.43	4.30	4.06	10.53	10.72	15.34	11.69	12.91	18.35	14.23
	10	D	0.02	0.11	3.19	4.29	4.99	6.34	8.61	10.15	12.25	14.00
		F	26.32	4.75	4.63	12.27	13.26	11.09	13.64	18.57	16.36	17.05
	16	D	0.02	0.11	1.23	4.69	5.31	6.49	7.30	9.57	10.07	14.00
		F	31.94	7.33	3.82	19.21	15.54	28.29	13.70	22.92	15.46	26.90

Note: D = deformation (m); F = force (MN).

Piecewise Linea	r Force-Defor	nation Relati	onship for	Corner Im	pact of Parana	Cargo Barge
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	Dimension ((m)	Load-deformation relationships										
Round pier	2	D	0.21	1.65	3.69	7.48	7.90	10.06	10.14	12.44	12.84	14.00	
		F	3.77	0.95	3.06	4.36	3.02	3.68	2.86	3.70	2.54	5.85	
	4	D	0.13	1.17	1.66	2.76	3.81	6.19	6.67	7.87	10.28	14.00	
		F	4.07	2.24	3.36	3.13	6.95	7.46	10.06	6.45	6.56	7.94	
	6	D	0.10	1.41	1.79	4.86	6.22	6.86	7.64	8.96	10.76	14.00	
		F	4.20	1.90	3.60	9.11	9.07	13.60	6.01	12.58	7.73	13.18	
	8	D	0.09	1.30	6.03	6.34	6.93	7.23	9.13	9.49	11.42	14.00	
		F	4.21	2.61	12.60	9.36	17.08	11.66	16.78	9.35	13.31	16.71	
	10	D	0.08	1.17	4.04	4.21	6.25	6.89	7.39	8.82	11.73	14.00	
		F	4.21	2.68	8.92	10.64	12.65	16.73	10.64	17.09	13.51	20.55	
	16	D	0.03	1.30	3.41	5.58	6.47	6.73	6.95	8.89	10.41	14.00	
		F	4.30	2.92	9.16	11.13	14.48	24.27	13.37	16.57	12.36	22.78	
Flat pier	2	D	0.02	0.78	1.72	2.93	3.21	4.61	6.40	6.94	7.01	14.00	
		F	4.47	1.91	3.73	1.57	4.48	6.71	3.56	8.67	4.19	5.57	
	4	D	0.02	0.51	4.15	4.61	6.04	6.74	7.07	8.92	10.38	14.00	
		F	6.52	2.43	9.05	11.79	9.35	18.77	8.29	14.17	8.63	17.33	
	6	D	0.02	0.53	3.04	5.41	6.19	6.58	7.54	9.17	11.26	14.00	
		F	7.75	1.70	7.71	11.37	12.96	16.45	11.26	17.66	10.17	18.48	
	8	D	0.02	0.56	4.86	5.47	5.93	7.62	8.62	9.17	10.30	14.00	
		F	8.57	1.95	12.73	10.68	13.79	11.73	20.85	15.01	13.30	16.15	
	10	D	0.02	0.58	3.28	3.55	4.07	6.85	7.63	8.54	10.34	14.00	
		F	9.66	1.66	9.75	7.80	10.33	16.39	11.15	18.01	13.37	17.29	
	16	D	0.02	0.35	2.88	3.86	4.17	6.67	7.40	7.89	11.43	14.00	
		F	15.75	2.36	9.50	7.88	11.49	17.98	12.18	14.91	14.76	20.02	

Note: D = deformation (m); F = force (MN).

Piecewise Linear Force-Deformation Relationship for Oblique Impact against Flat Wall of Parana Cargo Barge

Impact angle (degrees) Flat wall											
5	D	0.06	2.40	3.04	6.43	6.92	9.31	9.79	12.01	12.34	14.00
	F	6.76	8.14	16.02	30.40	25.96	32.24	23.59	24.97	31.05	40.71
15	D	0.03	1.30	3.85	5.71	6.31	6.82	8.47	10.30	11.27	13.00
	F	5.42	4.72	15.24	14.80	34.54	21.24	37.44	19.48	21.06	34.06
30	D	0.03	1.20	1.84	2.34	4.03	4.30	5.61	5.85	8.45	11.00
	F	5.29	3.11	8.00	7.35	15.21	12.22	16.21	22.95	19.88	27.16
45	D	0.02	1.35	2.80	4.43	4.91	6.06	6.27	8.46	11.92	13.50
	F	4.17	2.44	10.09	9.56	17.66	15.22	19.72	20.56	18.95	25.08

Note: D = deformation (m); F = force (MN).

Piecewise Linear Force-Deformation Relationship for Centered Impact of Parana Tanker Barge

	Dimension	(m)		Load-deformation relationships								
Round pier	2	D	0.06	2.73	4.02	6.38	6.81	8.76	9.31	11.43	11.63	14.00
		F	2.10	2.07	4.65	3.88	13.70	13.22	15.04	13.14	16.81	19.19
	4	D	0.04	2.37	4.67	6.36	6.80	7.50	8.10	10.96	12.78	14.00
		F	2.07	2.19	5.12	5.55	15.25	17.97	14.16	15.75	21.25	21.54
	6	D	0.01	3.30	3.98	6.37	6.78	9.21	10.02	12.22	13.23	14.00
		F	1.91	2.20	4.55	5.41	16.47	17.61	17.09	29.47	26.35	24.92
	8	D	0.11	1.27	6.00	6.39	6.64	10.00	11.06	11.96	12.85	14.00
		F	2.56	1.75	5.43	7.88	14.90	27.51	21.14	26.70	22.28	27.45
	10	D	0.05	2.84	6.35	6.75	8.03	9.16	9.59	11.42	13.14	14.00
		F	2.08	2.39	7.60	21.37	18.82	24.82	30.73	30.75	20.51	26.83
	16	D	0.18	0.20	1.52	3.19	6.28	6.72	8.33	8.87	13.20	14.00
		F	5.51	2.64	1.78	3.82	9.43	21.64	25.39	31.16	20.70	30.35
Flat pier	2	D	0.10	1.24	4.08	6.20	6.67	7.77	8.42	9.25	9.33	14.00
		F	2.23	1.08	3.12	4.21	13.32	14.49	13.04	15.04	14.16	13.96
	4	D	0.01	2.96	5.65	6.25	6.70	8.11	9.38	9.89	12.26	14.00
		F	1.97	1.96	4.85	6.88	17.57	15.33	20.79	17.34	22.48	23.34
	6	D	0.09	0.22	2.98	6.17	6.71	6.82	9.61	10.65	11.57	14.00
		F	4.93	1.51	2.72	6.38	26.47	15.75	26.10	22.17	24.05	24.22
	8	D	0.15	0.29	3.45	3.99	4.08	6.13	6.69	7.12	10.72	14.00
		F	3.38	2.01	3.44	6.80	3.72	7.96	31.37	18.58	25.16	21.68
	10	D	0.08	0.33	3.12	6.28	6.67	7.26	8.91	9.54	11.04	14.00
		F	8.73	1.94	3.65	11.52	36.74	18.08	25.97	18.99	21.97	24.24
	16	D	0.04	0.65	3.25	6.39	6.81	6.91	8.63	10.23	11.28	14.00
		F	8.51	2.18	2.26	18.74	73.40	37.35	23.61	26.30	16.67	23.36

Note: D = deformation (m); F = force (MN).

Piecewise Linear Force-Deformation Relationship for Corner Impact of Parana Tanker Barge

Di	mension ((m)		Load-deformation relationships								
Round pier	2	D	0.08	0.68	1.24	3.78	4.09	6.52	6.79	8.71	11.35	14.00
		F	1.77	3.28	1.40	2.21	5.04	1.91	5.44	3.51	3.68	3.00
	4	D	0.04	0.59	2.23	4.16	5.45	7.14	7.71	10.56	11.03	14.00
		F	1.80	3.30	1.64	5.22	5.41	9.47	6.89	6.78	7.98	6.83
	6	D	0.01	0.44	3.27	4.08	6.39	6.70	8.74	10.27	12.45	14.00
		F	1.91	3.27	2.02	6.38	6.73	12.88	13.13	7.58	9.83	8.34
	8	D	0.05	0.53	1.01	3.47	3.95	5.98	6.99	10.65	10.79	14.00
		F	1.65	4.75	2.27	3.67	5.98	4.53	17.28	10.47	12.74	9.11
	10	D	0.36	1.18	3.35	3.96	5.13	6.23	6.71	8.20	8.76	14.00
		F	4.62	2.22	3.40	6.25	6.61	10.60	18.61	12.91	18.27	9.82
	16	D	0.17	1.50	2.05	3.48	3.96	5.66	7.25	11.00	13.23	14.00
		F	3.81	2.80	4.80	5.24	7.67	6.68	21.47	15.58	19.55	13.11

	Dimension (m)		Load-deformation relationships									
Flat pier	2	D	0.14	1.66	1.78	2.99	4.08	5.88	6.72	9.08	10.12	14.00	
		F	3.00	1.61	3.35	1.71	4.84	4.47	5.77	6.27	3.86	5.35	
	4	D	0.10	0.87	2.18	4.15	5.72	6.61	9.09	9.28	11.32	14.00	
		F	3.37	2.99	2.40	6.06	5.83	14.52	14.86	9.57	10.65	10.99	
	6	D	0.03	0.26	2.51	3.81	5.37	6.68	9.26	10.27	11.17	14.00	
		F	3.67	3.34	2.47	6.61	6.29	16.76	19.13	12.30	16.15	9.75	
	8	D	0.01	2.81	3.94	4.43	6.06	6.45	8.94	10.40	11.33	14.00	
		F	3.41	3.24	8.35	6.38	10.87	18.58	19.51	12.77	18.98	11.76	
	10	D	0.09	2.25	6.02	6.70	7.62	8.08	9.75	10.36	11.16	14.00	
		F	3.38	3.57	9.79	25.35	15.04	20.46	19.76	14.43	18.07	16.37	
	16	D	0.08	0.39	4.28	6.20	6.74	7.13	8.62	9.67	10.17	14.00	
		F	7.20	2.60	7.29	13.41	37.12	22.74	24.34	19.91	23.93	21.79	

Note: D = deformation (m); F = force (MN).

Piecewise Linear Force-Deformation Relationship for Oblique Impact against Flat Wall of Parana Tanker Barge

Impact (degrees	angle s)					Flat wall					
5	D	0.15	1.18	1.26	3.34	4.85	5.76	8.07	8.43	11.87	14.00
	F	4.40	3.46	4.84	2.87	11.45	12.36	41.36	33.50	20.79	26.71
15	D	0.24	1.01	3.63	3.78	5.77	6.55	8.41	9.18	9.74	14.00
	F	4.64	2.98	4.98	8.58	8.33	21.39	30.41	22.00	30.77	19.52
30	D	0.08	1.36	2.38	3.38	5.51	5.78	7.28	8.68	10.43	14.00
	F	2.63	3.47	3.40	5.99	8.72	19.12	23.12	17.01	31.52	16.72
45	D	0.21	2.63	2.79	4.46	5.31	6.34	6.91	9.97	10.37	14.00
	F	2.77	2.81	8.28	6.38	17.62	16.96	22.21	24.45	21.01	29.97

Note: D = deformation (m); F = force (MN).

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