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On the nature of the conjunction fallacy

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Abstract In a seminal work, Tversky and Kahneman showed that in some contexts people tend to believe that a conjunction of events (e.g., Linda is a bank teller *and* is active in the feminist movement) is *more likely* to occur than one of the conjuncts (e.g., Linda is a bank teller). This belief violates the *conjunction rule* in probability theory. Tversky and Kahneman called this phenomenon the "conjunction fallacy". Since the discovery of the phenomenon in 1983, researchers in psychology and philosophy have engaged in important controversies around the conjunction fallacy. The goal of this paper is to explore the most important of these controversies, namely, the controversy about the *nature* of the conjunction fallacy. Is the conjunction fallacy mainly due to a *misunderstanding* of the problem by participants (misunderstanding hypothesis) or is it mainly due to a genuine *reasoning bias* (reasoning bias hypothesis)? A substantial portion of research on the topic has been directed to test the misunderstanding hypothesis. I review this literature and argue that a stronger case can be made *against* the misunderstanding hypothesis. Thus, I indirectly provide support for the reasoning bias hypothesis.

Keywords Cogntive psychology · Human reasoning · Conjunction fallacy

1 Introduction

In the last couple of decades cognitive psychologists have shown several disquieting phenomena about the way people reason. Particularly, they have shown that in certain contexts people tend to reason in ways that violate standard rules in logic and

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probability theory (see Hastie and Dawes 2001; Samuels et al. 2002; Gilovich et al. 2002 for reviews on the topic). However, the *interpretation* of these empirical results and the *criteria* used to assess people's performance are not straightforward matters. As a result, these issues have been source of important debates not only among psychologists but also among philosophers (see, for example, Kahneman and Tversky 1996; Gigerenzer 1996; Samuels and Stich 2004; Cohen 1981, 1982; Adler 1984, 1991).

In this paper I will focus on a particular error people seem to be prone to commit. The tendency to commit this particular error has been called "the conjunction fallacy". The story of this phenomenon begins in 1983. In that year, Tversky and Kahneman showed that under certain circumstances, people tend to believe that the probability of a conjunction of events (A&B) is higher than the probability of one of its conjuncts (let's say, A). Here is the most famous example that tends to elicit the conjunction fallacy:

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Rank the following statements according to their probability: using 1 for the most probable and 2 for the least probable[1]:

(a) Linda is a bank teller.
(b) Linda is a bank teller and is active in the feminist movement.

Most people (between 80% and 90%) rank the conjunction (b) as more probable than the base event (a), despite the fact that this violates the conjunction rule in probability theory: the probability of a conjunction cannot exceed the probability of any of the conjuncts.[2] The justification goes as follows. If event (b) (in symbols, B&F) occurs, then, necessarily, event (a) (in symbols, B) will occur as well, that is, if Linda is a feminist bank teller, necessarily, she will be a bank teller. But event (a) may occur without (b) occurring, that is, Linda may be a non-feminist bank teller. Thus, (b) can never be more likely to occur than (a).

Although the existence of this phenomenon became widely accepted, many researchers engaged in substantive controversies around the conjunction fallacy.

The goal of this paper is to analyze the most important of these controversies, namely, the debate about the *nature* of the conjunction fallacy. The dispute focuses on whether the conjunction fallacy is the product of people's *misunderstanding* of the problem or is rather a case of genuine *reasoning bias*. Let me be clear about this point. The central question at stake is: Why do people tend to commit the conjunction fallacy? Maybe it is because in the context of Linda's description, they tend to interpret the option "bank teller" as "bank teller who is not a feminist", in which case there would be no conjunction fallacy. Besides, the typical ranking (B&F, B) would be reasonable since Linda does sound like a feminist. Or maybe people do not interpret the word "probability" (as it occurs in the question of the Linda problem) in the same way

[1] The original version of the problem contained eight options. For simplicity purposes, I will only mention the relevant ones.

[2] I am here implicitly assuming that probability theory is the right tool to approach and solve this problem. I will take this assumption for granted (see Moro 2007, Chap. 5 for a defense of this assumption). For now, I will only mention that the conjunction rule also holds in the relevant non-standard models of probability (e.g. Cohen 1977; Shafer 1976).

researchers do. Clearly, researchers are using such a word in the sense of *mathematical probability*. But most people ignore this theory, so they may understand the question in a completely different way. If this is so, again, their answers may turn out to be perfectly reasonable. Or even if they understand the options and question as researchers assume, they may not take the conjunction task as asking for the "correct" answer but rather as asking for the answer that is most *informative* given the evidence provided in the problem (i.e., Linda's description). But given this description, the option of Linda being a feminist bank teller is more informative than the option of Linda being just a bank teller. So, again, the typical answer would be both non-fallacious and reasonable. All these possibilities are different versions of the misunderstanding hypothesis: the conjunction fallacy is mainly due to a misunderstanding of the problem or the task.

However, there is another set of possibilities. Perhaps people do interpret the options, the question, and the task as researchers assume. If this is the case and they still commit the conjunction fallacy, they do not adequately reason about the problem. Maybe they fail to notice the *inclusion relation* between the options. Or maybe they do notice this relation but fail to see its *relevance* for the question at stake. Or maybe people just use *reasoning strategies* that work fine in other situations but they do not work well in the context of conjunction problems. These possibilities are different versions of the reasoning bias hypothesis: the conjunction fallacy is mainly due to a failure in reasoning.

Much of the research efforts in the area were (and are) directed to test the misunderstanding hypothesis. The empirical evidence provided so far is mixed. Some studies seem to provide evidence in favor of such a hypothesis while others seem to provide grounds for its rejection. I will offer a survey of this evidence and the debate around it. I will then argue that a stronger case can be made *against* the misunderstanding hypothesis. Thus, I will indirectly offer some support for the reasoning bias hypothesis.

2 Different versions of the misunderstanding hypothesis

Gerd Gigerenzer and others have argued that the conjunction fallacy might be due to pragmatic factors that lead subjects to interpret certain problems (as the Linda problem) differently from the way the experimenters intend. Four factors have been proposed as responsible for such misunderstanding: (1) misunderstanding of the conjunction (i.e., due to the word "and")[3]; (2) misunderstanding of the conjunct option (e.g. "bank-teller" in the Linda problem); (3) misunderstanding about the word "probability"; and (4) misunderstanding about the goal or goals of the task.

I will deal with the first factor in Sect. 3. I will focus on the second factor in Sect. 4. Finally, I will discuss the last two factors in Sect. 5.[4]

But it is important to determine, first, which predictions follow from each side of the debate when testing the misunderstanding hypothesis. The existence of four potentially

[3] I want to thank an anonymous reviewer who called my attention on this possible source of misunderstanding.

[4] Of course, it is necessary to control for the four factors at the same time. At the end of Sect. 5, I will go over conditions that are designed to do that. But for clarity purposes, I will analyze one factor at a time.

confusing factors opens different possibilities for the advocates of such a hypothesis. They might adhere to any particular combination of the four factors mentioned above. Regardless of the particular combination chosen, the structure of their predictions is as follows. When the alleged factor or factors are clarified, most of the subjects will reason in accordance with the conjunction rule in probability theory. The advocates of the *reasoning bias hypothesis*, in turn, will predict the opposite result. Even when the factor or factors at stake are clarified, most of the people will keep committing the conjunction fallacy.[5]

3 Misunderstanding of the conjunction

The first factor to analyze as possible source of misunderstanding is the word "and". It has been argued (see, for example, Bar-Hillel and Neter 1993) that such a word is ambiguous and can elicit misinterpretations. More specifically, it has been pointed out that the word "and" can be interpreted as a disjunction, that is, as suggesting a union rather than an intersection (as in the phrase 'John invited colleagues *and* friends for his party'). If participants interpret the word "and" in such a way, there would be no conjunction fallacy. The reason is that the probability of a disjunction is always equal or higher than the probability of each disjunct.

However, the relevant studies in the literature found negative evidence against this version of the misunderstanding hypothesis. Let me briefly review the literature on the topic.

Bar-Hillel and Neter (1993) report high percentages of conjunction fallacy in contexts where the word 'and' does not appear. For example, most subjects consider that it is more likely for certain person to have died of lung cancer than to have died of any type of cancer (including lung cancer). Additionally, Sides et al. (2002) argue that the disjunctive reading of 'and' is not conversationally justified when joining whole sentences rather than categories. For example, it does not make sense to propose a disjunctive reading for the following sentence: 'A tax cut will be passed by Congress *and* it will be supported by most Democrats'. However, Sides and colleagues show that most participants still commit the conjunction fallacy with this and similar examples. Furthermore, Bonini et al. (2004) uses the following strategy to discourages a disjunctive reading of 'and': to include an explicit reminder of the conjunctive meaning of 'and'. They use a betting format (I will discuss this format in detail in Sect. 5). After they present subjects an option of the form (X and Y), it reads 'both events must happen for you to win the money on this bet' (p. 204). Again, most participants still continue committing the conjunction fallacy in this context. Finally, Tentori et al. (2004) run a control task to check whether participants (after receiving

[5] An important qualification is in order. Tversky and Kahneman and their followers do *not* claim that under every single condition subjects will view a conjunction as more probable than one of the conjuncts. What they claim is that under *certain conditions* (see Wedell and Moro 2007 for a discussion of such conditions) most people will commit the conjunction fallacy. But they have also pointed out several conditions under which most people correctly follow the conjunction rule when solving this type of problems. For example, if the subjects only receive the information about Linda's age, most of them will follow the conjunction rule (I want to thank Michael Dickson for a very valuable discussion on this point.).

information of the form (X and Y) believe that X is true, on the one hand, and Y is true, on the other. They report that around 91% of participants hold these beliefs. And among these participants, 70% of them commit the conjunction fallacy.

Thus, this possible source of misunderstanding has been empirically explored but the results are clearly negative.

4 Misunderstanding of the conjunct option

The possible misunderstanding about the conjunct factor (e.g. "bank teller" in the Linda problem) was critically analyzed by Adler (1984) who suggests that subjects would tend to assume that Linda's description is highly relevant for solving of the problem. This assumption, in turn, might lead people to interpret 'Linda is a bank teller' as 'Linda is a bank teller who is not a feminist'. The same implicature also follows from assuming a strong contrast between the options. Of course, for someone who interprets the conjunct option that way, there is no conjunction violation. Several experimental studies have tried to block this implicature by using different techniques. In the next subsections, I will provide a summary of the techniques used and the results found.

4.1 Technique 1: rephrasing the conjunct option

The first utilization of this technique was by Tversky and Kahneman (1983) themselves. They requested to rate (by using a 9-point scale ranging from 1—extremely unlikely to 9—extremely likely) the following two statements:

(a) Linda is bank teller and is active in the feminist movement.
(b) Linda is a bank teller *whether or not she is active in the feminist movement* (p. 299. The emphasis is mine).

They found that 57% of the subjects committed the conjunction fallacy by rating option (a) higher than (b). Thus, although there was a big improvement of around 30% points with respect to the original version,[6] most people still continued committing the fallacy in one of the most transparent versions of the problem. This result seems to provide negative evidence for the misunderstanding hypothesis—at least in this particular version.

The best study that uses the same technique but offers positive evidence is Dulany and Hilton (1991). They argue that Tversky and Kahneman's phrase is ambiguous. People might interpret their clarified phrase as "Linda is a bank teller even if she is not a feminist". This view does not seem very convincing to me[7] but let us accept it

[6] The original version contains each conjunct, the conjunction, and five filler items.

[7] I recognize that in *some contexts* the "whether or not" phrase could be read as Dulany and Hilton suggest. For example, Hilton (1995) shows that the phrase "we are going to the zoo tomorrow whether or not it rains" can be interpreted as "we are going to the zoo tomorrow even if it rains". But a crucial factor for the interpretation of an option in the context of conjunction problems is the *contrast* with other options. Thus, the misunderstanding of the option "bank teller" as "non feminist bank teller" is derived from the contrast with the other presented option "bank teller and feminist". Under the discussed condition, the contrast

for the sake of argument. So, they propose a change. In experiment 3 of their study, they replace the conjunct with the following phrase:

> Linda is a bank teller *and regardless of the actual likelihoods, there are simply two logical possibilities: she must either be or not be active in the feminist movement.* (p. 98, my emphasis)

Under this framework, only 38% of the subjects committed the conjunction fallacy. Although this percentage of fallacies is still high,[8] most people correctly followed the conjunction rule. So, this result seems to support the misunderstanding hypothesis.

However, this last condition contains some extra factors besides clarification of the conjunct that might be responsible for the facilitation:

(1) The very mention of "possibilities" might have helped because now subjects are explicitly told that the last option involves *two* possibilities, one of which is related to the previous option. This presentation in terms of possibilities, in turn, might have elicited a set-based representation of the problem. If this is the case, Tversky and Kahneman would be very happy with the result. The reason is that they do not claim that people will commit the conjunction under every single condition. They argue that experimenters can include certain elements—as frequency phrasing for example—that can make people understand the set structure of the problem. When this happens, they argue, people will tend to follow the conjunction rule. Therefore, if the responsible factor for the facilitation was the wording in terms of possibilities rather than the clarification of the conjunct, the result would support the reasoning bias hypothesis—at least, as defended by Tversky and Kahneman.
(2) The very mention of words like "likelihood" or "logical" makes the second option very *sophisticated* and, thus, very attractive to pick, even if subjects do not have a clear understanding of the meaning of such words.
(3) Something similar might have occurred with the tautology mentioned in the rephrased option. The mere presence of a tautology in such an option might have also make it very attractive to pick, even when participants miss the inclusion relation which is key for the solution of the problem. If that is the case, they might have chosen the correct option for the wrong reason.

Thus, it is not obvious to me that the misunderstanding hypothesis has been strongly confirmed by this study. It is an open question whether the facilitation is due (a) to the clarification of the conjunct; or (b) to any of the extra factors mentioned above (or to some combination of them, or perhaps to all of them); or (c) partially due to (a) and partially to (b). Until we find out what was the main cause of facilitation, this

Footnote 7 continued
occurs between "Linda is a bank teller and a feminist" and "Linda is a bank teller whether or not she is a feminist". It is clear that the contrast lies in the property of being of feminist. This property is positively asserted in the first option and presented as one of two possibilities in the second option. Thus, I do not see how the postulated misunderstanding can be justified in this particular context.

[8] Under the same condition, they found more dramatic improvements in problems other than the Linda problem. More specifically, the percentages of conjunction fallacies of the other three problems they tested were 29%, 22% and 11%.

study does not provide a completely compelling case in favor of the misunderstanding hypothesis.

Similarly, it is still unclear whether the results by Tversky and Kahneman are due to the fact that people do not understand the relevance of the inclusion relation or that their rephrasing of the conjunct was not clear enough.

Therefore, so far we are left with no clear winner.

What about other attempts to rephrase the conjunct option?

Messer and Griggs (1993) try another possible rephrasing of the conjunct. They replace "bank teller" for "bank teller *regardless of whether or not she is also active in the feminist movement*" (my emphasis). They found an improvement of 21% points. The rate of conjunction fallacies drop from 77% to 56%. Still most of the subjects committed the conjunction fallacy. This result seems to provide some evidence *against* the misunderstanding hypothesis. However, the phrase they chose seems somewhat confusing. Thus, the relevance of the result can be called into question. So, we should consider a better wording of the conjunct.

This is provided by Macdonald and Gilhooly (1990) who use the following rephrasing: "Linda is a bank teller who *may or may not be active in the feminist movement*" (my emphasis). I find this alternative very attractive. On the one hand, the phrase sounds less artificial than the "whether or not" phrase. On the other hand, although it clearly suggests the inclusion relation as in previous cases, it does not seem to contain extra factors that make it difficult to interpret the results. What did MacDonald and Gilhooly find? The result was that only around 20% of the people committed the fallacy. This result seems to favor the misunderstanding hypothesis. But a key feature of their study is that they asked the question about Linda in the future (10 years from now). MacDonald and Gilhooly did that because of the following reason. They argue that subjects might entertain the idea that the author of the problem is suggesting that Linda is indeed a bank teller. The question about 10 years from now somehow eliminates this influence. Notice, though, that from a normative point of view, even if the suspicion were correct, the conjunction rule should still hold. Anyway, it seems that the improvement found depends crucially on the fact that the question is about the future. What happens when the question is asked about Linda in the present time? The study by Agnoli and Krantz (1989) provides an answer to this question. They used the "may or may not" phrase but the question was asked about Linda in the present time. They found an improvement of around 10% but still 69% of the group who received the clarified conjunct committed the fallacy. So, it still remains the fact that most of the people commit the fallacy when the key option is clarified but the question is asked about the present time. So, this rephrasing also seems to provide negative evidence for the misunderstanding hypothesis.

However, it can be claimed the advocates of the reasoning bias hypothesis should explain why people do not reason correctly about the present time while reasoning correctly about the future. I completely agree with this request. But although the request is perfectly reasonable, it misses the point. Why? The reason is that the condition *in the future* does not actually clarify the conjunct—or anything in the problem for that matter. The "may or may not" phrase, on the other hand, does clarify the conjunct. So, it is this last modification the relevant one for our debate. When the conjunct is clarified in this way, still most of subjects commit the conjunct fallacy. Therefore, under this

condition, we have a stronger case for rejecting the misunderstanding hypothesis rather than for supporting it.

However, I do not find this particular technique very appealing. First, it is difficult to evaluate, *on objective grounds*, the quality of a phrase to replace the conjunct. Second, when the conjunct is clarified, the inclusion relation becomes explicitly exposed. Thus, this technique eliminates one of the reasoning steps to solve this type of problem. This, in turn, creates a problem: any improvement in performance might be due to the explicit exposition of the inclusion relation or to the clarification of the conjunct as such or to a combination of both.

On the other hand, it is true that this technique seems to eliminate the possibility of misunderstanding of the conjunct.

What conclusion should we draw from this analysis of the evidence?

Although the evidence shows mixed results, I think the reasoning bias hypothesis seems slightly better justified than its rival. The only clear result for the misunderstanding hypothesis is the one found by Dulany and Hilton (1991). They showed that with a specific rephrasing of the conjunct option, the majority of subjects follow the conjunction rule. I argued that this result admits more than one interpretation. The facilitation that Dulany and Hilton found might be caused for factors other than the clarification of the conjunct. If the result were due to these extra factors, it would support Tversky and Kahneman's reasoning bias hypothesis.

Furthermore, all the other rephrases of the conjunct elicit the performance that the advocates of the reasoning bias hypothesis predict: even when the conjunct is clarified, most of the subjects still keep committing the conjunction fallacy.

Still, it seems perfectly reasonable to think that, so far, there is no clear winner of the debate. It is time, then, to explore other techniques.

4.2 Technique 2: changing the set of options

When one faces only the options (B) and (B&F) in the Linda problem, it seems tempting to interpret the conjunct as "bank teller who is not a feminist". But if the set of options is changed, this interpretation might be overruled. For example, Hertwig and Chase (1998), who are aware of the problem, use a different set of options. They avoid fillers and add the option "Linda is active in the feminist movement" (F) as an additional option. This is subtle way to avoid the misinterpretation of the conjunct. Why? Because it is now evident that one faces a set of options with a clear structure: the set has two individual options and the combination of both. Besides, given the "feminist" option, the information about Linda becomes relevant for solving the problem. Furthermore, this technique does not explicitly state the inclusion relationship. I think this is an improvement in comparison with the previous technique. However, it is not completely clear that this technique will succeed in blocking the misunderstanding of the conjunct. After all, people may keep using a similar contrast with the conjunction (B&F) and interpret the feminist option as "feminist who is not a bank teller".

Having seen the theoretical pros and cons of this technique, let us see what Hertwig and Chase (1998) found. Actually, they report very surprising results. They focus on the *response mode* factor. When they ask people to *rank* the probability of the

alternatives (in the Linda problem, for example), most people commit the conjunction fallacy. This is the usual result. But when they ask people to *estimate* the probability of each alternative, most people correctly follow the conjunction rule.[9]

Why are these results relevant for the evaluation of the misunderstanding hypothesis? The basic point is that the same conversational-pragmatic analysis used to derive the misunderstanding of the conjunct hypothesis can be applied to the estimation instruction. When comparing (B) and (B&F), subjects would expect that Linda's description is relevant for solving the problem. Or they might just assume a strong contrast between these options. Then, they might interpret "bank teller" as "bank teller who is not feminist". In other words, if people tend to misinterpret the conjunct in the ranking instruction, one will expect that they continue to do so under estimation instruction. But if they interpret "bank teller" as "non-feminist bank teller", they should give higher probability estimates to the conjunction (B&F). After all, Linda does sound like the prototype of a feminist! However, people tend to assign lower estimates to the conjunction. Thus, subjects do not seem to misinterpret the conjunct after all—at least, under estimation instruction. But if that is the case, why should we think that they misinterpret the conjunct under ranking instruction? Of course, it might not be impossible to come up with an explanation for either (a) why people give good probability estimates despite of misinterpreting the conjunct; or (b) why the estimation instruction inspire the right interpretation of the conjunct. But as far as I am aware, no advocate of the misunderstanding of the conjunct hypothesis has provided such an explanation.

What about other applications of this technique?

Morier and Borgida (1984) provide another instance of application of this technique. They use a different set of options to avoid the misunderstanding. They also use the estimation response mode. In addition to both conjuncts and the conjunction, they included the disjunction "Linda is a bank teller *or* is active in the feminist movement".

Again, the purpose of presenting the fourth extra option is to avoid any misunderstanding. This set of options clearly contains two individual possibilities, the conjunction of both and the disjunction of both. They found that only 48% committed the fallacy. This result seems to provide support for the misunderstanding hypothesis.

However, three objections immediately arise. First, notice that almost half of the subjects still keep committing the conjunction fallacy under this clarified set of options. So, even if the result falls within the predicted values, it does not seem like an extremely strong justification for the misunderstanding hypothesis. Second, Morier and Borgida use the estimation response mode, which, as we saw above, tend to boost people's performance. Thus, it is not clear whether the facilitation can be due to the clarification or to the response mode or to a combination of both. Third, the result is not stable. When four fillers were added to the list, 69% of the subjects committed the fallacy. I do not think that people start misunderstanding options under this condition. But the inclusion relation is now a little bit more difficult to spot, so the problem becomes a little bit harder.

[9] A similar result was found by Hertwig and Gigerenzer (1999). Using the same set of options, these researchers found that when people *estimate frequencies*, they usually follow the conjunction rule, but they usually violate that rule when *ranking probabilities*. I will analyze this study in detail in Sect. 5.

Which conclusion should we draw from the review of this technique? As with the previous technique, the evidence does not point in only one direction. Under a given condition, most people who received a clarified set of options follow the conjunction rule. But under a slightly different condition, most people tend to commit the conjunction fallacy again. Neither group has been able to provide any satisfactory explanation for these effects.

Still, as I mentioned above, it is not completely clear—at least to me—that if there is any misunderstanding of the conjunct, this technique will be able to reduce it or eliminate it. The main point is that there is no clear winner of the debate. So, in order to solve our dilemma, we should move on and explore the application of new techniques.

4.3 Technique 3: avoiding the word "and"

Politzer and Noveck (1991) compare people's performance under *explicit* conjunctions (e.g. Lendl will play the finals *and* win) with performance under *implicit* conjunctions (e.g. Lendl will win the finals). Based on a conversational analysis, they argue that under the explicit-conjunction condition people will tend to interpret the conjunct option, i.e., "Lendl will play the finals" as "Lendl will play the finals and lose". They also argue that this will not happen under implicit conjunctions.[10] For the sake of argument, I will accept their analysis. They found that among people who dealt with implicit conjunctions only 28% committed the conjunction fallacy.

This result seems to provide a good support for the misunderstanding hypothesis. However, I argue that this is not the case. The main evidence for this point is that the average percentage of conjunction error for explicit conjunction was only 40%. This result was higher (although not significantly higher) than for implicit conjunctions. But the crucial point is that the percentage of error under explicit conjunction was still below the 50% mark. That means that most people reasoned according to the conjunction rule even under the explicit-conjunction condition. This result suggests that their analysis about how people interpret the conjunct option under explicit conjunction is wrong. Had people generally interpreted the conjunct as predicted, most of them would have shown conjunction effects. Why? The reason is simple. The information in the cover stories is such that the presented conjunction (e.g. B&F) always seems more probable that the omitted conjunction involving the complement (e.g. B&∼F). Thus, if subjects generally interpret the conjunct as this last omitted conjunction, they will tend to choose the other conjunction, the one that was explicitly presented. The empirical result was, however, the opposite. Thus, Politzer and Noveck's finding does not properly support the misunderstanding proposal.

[10] Politzer and Noveck (1991, p. 91) argue that the implicature (X & not Y) is triggered *only* when there is contrast between two items such that one item (e.g., X) is identical to a second one except for the addition of a qualifier (e.g., X *and Y*). It is the explicit absence of such qualifier in the first item what generates the implicature 'and not Y'. This hypothesis has the consequence that the implicit conjunction format will not generate the implicature simply because the necessary conditions for the implicature do not hold: the implicit conjunction presupposes without asserting the element in common and, more importantly, contains no qualifier.

There is, however, a point worth considering before we move on to the next technique. It should be conceded that there was a 12% point difference in conjunction errors between the implicit and the explicit conjunction conditions (28–40). This difference might indeed be due to the misunderstanding of the conjunct. It might also be due to other factor but the advocates of the reasoning bias hypothesis have not proposed any alternative explanation. So, researchers on both sides should be aware that the use of the word "and" might be responsible for some misunderstanding of the conjunct. Even if the effect is not large, it should be controlled for when running this type of experiment.

4.4 Technique 4: focusing on the selected event of a bet[11]

Sides et al. (2002) explore the debate around the misunderstanding hypothesis by introducing several *changes in the instructions* given to participants (I will focus on their Experiment 4 as it is the most relevant for our purposes). Two of these changes are, in turn, especially relevant to reduce or eliminate people's misunderstanding of the conjunct. First, they use a betting framework. They give participants the typical pair of options (X) and (X&Y) and ask them to choose the event for which *they would like to bet on* (I will discuss the use of the betting framework in detail in Sect. 5). The idea is that the betting framework may help people focus on the truth conditions of the bet. If this goal is accomplished, participants should interpret X as X rather than as X&∼Y. The second key modification is as follows. Participants are told that a judge (unfamiliar with the experiment) will pay the bets by looking only at the selected event. In order to reinforce the instruction, participants are explicitly requested to scratch out the unselected event. Thus, each participant knows that the judge's interpretation of his or her selected event cannot be influenced by the other event. Again, the idea is to make participants interpret X as X rather than as X& ∼Y.

Sides and colleagues used two conjunction fallacy problems plus other eight problems with irrelevant logical forms as fillers. Another feature of the study was that they used conjunctions of complete sentences (e.g., a tax cut will be passed by Congress between January 1, and March 31, 2000 (X) and it will be supported by most Democrats (Y)). What was the result? The authors reported that around 64% of participants at least once (out of two opportunities) decided to bet on the conjunction, committing, thus, the conjunction fallacy. This result seems to provide strong evidence against the misunderstanding hypothesis.

The advocates of the misunderstanding of the conjunct hypothesis have not responded to this challenge and it is difficult to imagine how they can do it.

Still, one may wonders if there is more compelling evidence against this version of the misunderstanding hypothesis. In the next section, I will argue for an affirmative answer. Thus, we need to explore the last technique used to avoid misunderstanding of the conjunct.

[11] I am grateful to an anonymous reviewer for calling my attention on the use of this technique and for extremely valuable comments on the topic.

4.5 Technique 5: explicitly including the problematic interpretation

I think the best way to avoid the misunderstanding of the conjunct is to present the possible misinterpreted conjunction as one of the options. Tentori et al. (2004) and Bonini et al. (2004) systematically use this technique. For example, Tentori and her collaborators present subjects with the following problem:

> The Scandinavian peninsula is the European area with the greatest percentage of people with blond hair and blue eyes. This is the case even though (as in Italy) every combination of hair and eye color occurs. Suppose we choose at random an individual from the Scandinavian population. Which event do you think is the most probable? (Check your choice)
> (a) The individual has blond hair.
> (b) The individual has blond hair and blue eyes.
> (c) The individual has blond hair and does not have blue eyes. (p. 470)

In this context, it does not make sense to interpret the conjunct "blond hair" as "blond hair without blue eyes" because one already has this option in (c)! From a conversational point of view, it would be uncooperative to repeat one of the options in an altered form. Thus, the possibility of misunderstanding of the conjunct seems very implausible.

Notice, also, that this framework does not hide the inclusion relationship (there is no filler) but it does not explicitly expose it by using some rephrasing technique either. The inclusion relationship is suggested by the structure of the set of options: option (a) clearly covers possibilities (b) and (c). In addition to the Scandinavian problem, Tentori and colleagues use the Volleyball problem[12] with identical structure and other three problems as fillers. Let us focus on the two conjunction fallacy problems. Tentori and colleagues report that on average 74% of people pick either options (b) or (c). So, under a condition that seems to eliminate the possibility of misunderstanding, most people still commit the fallacy.

Given the importance of this result, Dr. Douglas Wedell and I ran an experimental study with the Scandinavian problem, the Volleyball problem, and other six ones with the same structure (see Wedell and Moro 2007). We were able to replicate the results found by Tentori and colleagues. Besides, we controlled for the possible misunderstanding of the conjunct. We used the following technique. The last problem of our study was the Linda problem. Some of the participants face a set of options equivalent to our problems. For example, the options were:

[12] The Volleyball problem (Tentori et al. 2004, p. 471) reads as follows: professional volleyball players have greatly changed in the course of the last decade. In particular, they have grown younger, yet taller. Women players in the first Italian division are on average taller than 1.80 m, ranging from 1.75 m for some setters to more than 1.90 m for many spikers. Suppose we choose at random a female volleyball player from the Italian first division. Which do you think is the most probable? (Check your choice.)

- The woman is less than 21 years old.
- The woman is less than 21 years old and is taller than 1.77 m.
- The woman is less than 21 years old and is not taller than 1.77 m.

- Linda is a bank teller.
- Linda is a bank teller and active in the feminist movement.
- Linda is a bank teller and is not active in the feminist movement.

But the crucial point was in the following question. By using a similar technique than the one used by Dulany and Hilton (1991), we asked:

In the problem above, which of the two statements below best captures what you think was meant by the statement: "Linda is a bank teller". (Check the one that you thought the author meant in writing the statement. Note that there is no "correct" answer as this is simply how you interpreted the statement.)

– Linda is bank teller and is not active in the feminist movement.
– Linda is a bank teller whether or not she is active in the feminist movement.[13]

The result we found is that only around 10% of the subjects picked the first option showing evidence of misinterpretation of the conjunct. But even if 10% of the conjunction errors can be accounted for the misinterpretation of the conjunct, still more than half of the subjects commit the fallacy without apparently misunderstanding the conjunct.

We also run a very similar condition, except that the Linda problem did not contain the third option, that is, contained only (B) and (B&F). We found that the two-option format tends to elicit a high percentage of misunderstandings (see Dulany and Hilton 1991 for a similar finding). However, we found some negative evidence against the misunderstanding of the conjunct hypothesis. When we move from the two-option format to the three-option format, the tendencies are as follow. Although the percentage of misunderstanding is significantly reduced, the percentage of conjunction fallacies is still increased a little bit. Thus, a correct understanding of the conjunct does not seem to bring a reduction of conjunction errors as the misunderstanding hypothesis predicts.

As far as I know, the advocates of the misunderstanding have not responded to Tentori et al.'s challenge.

[13] By applying a similar technique, Dulany and Hilton (1991) found very low percentages of the intended interpretation of the conjunct. However, the reliability of such results can be called into question since Dulany and Hilton's technique contained an important methodological problem: it asks participants to choose among several possible interpretations of the conjunct but only two of them were well justified by conversational analysis. The inclusion of irrelevant alternatives may, in turn, have lead to confusion. For example, a possible interpretation read "bank-teller who has a high probability of being feminist". The participants who committed the fallacy, by trying to give an answer consistent with the previous response, may have checked this option, although they never interpreted the conjunct option that way. In order to avoid this type of problem, we present participants only with the options that were justified by conversational analysis.

However, there is no guarantee that *our* version of Dulany and Hilton's technique delivers accurate results. The main reason is that participants may not be conscious of their own interpretations. Furthermore, the way they respond to the Linda problem may still interfere in the control task—even if our version does not contain confusing irrelevant alternatives. Nevertheless, I think this technique may still provide a good initial approximation of how people interpret one of the key options.

Morier and Borgida (1984) also found the same result. They added "Linda is a bank teller who is not a feminist" in the list of options. There was a slight difference in the setup of the experiment, though: they used several fillers, which can make the inclusion relationship more difficult to spot. They found that 77% of subjects committed the conjunction fallacy under this condition.

Thus, we have a condition that seems to eliminate the possibility of misunderstanding of the conjunct and still elicit a high percentage of conjunction fallacies. This is the strongest result against the misunderstanding of the conjunct hypothesis.

What could be a reasonable response from the advocates of the misunderstanding hypothesis? At this point, they could argue that even if there is no misunderstanding of the conjunct, there might be other sources of misunderstanding. For example, there are two concrete possibilities that have been mentioned in other contexts: (1) maybe people misunderstand the word "probable"; and (2) people might interpret the goal (or one of the goals) of the task as requiring answers as informative as possible (I will explore both possibilities in the next section.).

5 Misunderstanding of the word "probability"

Even if the conjunction fallacy is not mainly due to a misunderstanding of the options, subjects might still tend to interpret the Linda problem, for example, differently from the way experimenters do. One particular proposal is that there is a miscommunication about the word "probability". Gigerenzer and others (e.g. Gigerenzer 1996; Hertwig and Gigerenzer 1999; Fiedler 1988) argue in favor of this hypothesis. They claim that when people are asked to rate or rank the *probability* of the different alternatives, they might not understand the term 'probability' in its mathematical sense but as equivalent to the term 'plausibility' or 'believability' or 'with the appearance of truth', which are, after all, legitimate meanings according to well-respected dictionaries. If subjects interpret 'probability' in this way, the typical response would be very reasonable rather than fallacious. As Klaus Fiedler puts it:

> With respect to the second conjecture, that the [conjunction] fallacy may (partly) represent a verbal misunderstanding of the probability concept, it should be noted that the meaning of this term in everyday language is very vague. The prevailing statistical interpretation of probability (as relative frequency) does not appear to apply to colloquial language because everyday experience is seldom based on semantic-frequency counts. Rather, the usual interpretation of "probability" may come close to such subjective criteria as "believability", "degree of confidence", "imaginability" or "plausibility". [...] If "probability" is just a technical term used by statisticians or experimenters while real-life problems may be conceived more naturally in different terms, then the external validity of the conjunction fallacy is put into doubt. (Fiedler 1988, pp. 123–124)

The message is clear. If experimenters and subjects assign different meanings to the word "probability", the study would show a clear case of miscommunication between subjects and experimenters. Furthermore, the claim that the participants are still committing a fallacy would become very hard to justify.

5.1 The response: dropping the word probability does not solve the problem

The previous objection seems very reasonable. However, Tversky and Kahneman (1983) themselves did consider this possibility. In order to avoid any misunderstanding with the word 'probability', they completely dropped it, asking people to rate alternatives according to their "willingness-to-bet" on them. For example, instead of requesting a probability ranking, Tversky and Kahneman (1983) give subjects the following instructions for the Linda Problem:

> If you could win $10 by betting on an event, which of the following would you choose to bet on? (Check one).
> (a) Linda is a bank teller.
> (b) Linda is bank teller and is active in the feminist movement. (p. 300)

The implicit rationale is that, under betting instruction, subjects will be willing to make as much money as possible. And in order to do so, they should bet on the most *probable* event (that is, "probable" in the intended mathematical meaning of the word). Furthermore, under betting instructions, people should focus on the truth conditions of the bet rather than in possible conversation implicatures.[14] What did Tversky and Kahneman find? The result was better. There was a drop in the conjunction fallacy of around 30% points, but still most people (56%) committed the fallacy. Thus, the misunderstanding of the word "probability" might explain part of the phenomenon, but it is not the whole story. The phenomenon is robust and it does not just go away by omitting the word "probability".

This result has been replicated under different conditions and with a variety of examples (e.g., Bar-Hillel and Neter 1993; Messer and Griggs 1993; Sides 2000; Sides et al. 2002; Bonini et al. 2004; Wolford et al. 1990). In all of these cases, researchers have found high percentages (always more than 50%) of conjunction violations. Here are two of the clearest examples under betting instruction. The first one is from Wolford et al. (1990):

> The third race at Churchill Downs is a stakes race featuring nine 3-year-olds. The race is over a mile and an eighth and the track is in good condition. We are going to focus on two of the horses entered in the race. The first one, HardRock, has run seven previous races and only finished on the money once, and that was over a muddy track. According to the handicapper for the Daily Racing Form, HardRock is picked to finish seventh. Avenger is the favorite for the third race. He has finished in the money seven times in nine races, winning five of them. He is well rested and appears to be in excellent form for today's race. Listed bellow are three betting tickets for the third race; each ticket cost $2 and pays $6 if successful. Please rank the order the tickets with (1) being the one that you would most like to have and (3) being the one that you would least like to have.

[14] However, in the betting framework of the Linda example, it is reasonable to suspect that people might still misinterpret the conjunct as "bank teller who is not a feminist". But this objection is not applicable in general. The next two examples will show cases where the misinterpretation of the conjunct is indeed implausible.

– Ticket 1: HardRock to finish second.
– Ticket 2: Avenger to win.
– Ticket 3: Avenger to win and HardRock to finish second. (p. 49)

The result Wolford and his coworkers found is that most of the subjects prefer ticket 3 to ticket 1, committing, thus, the conjunction fallacy. Furthermore, the fact that the options are tickets probably discourages (although it does not completely eliminate) the possibility of misinterpretation of the conjunct.

The second example is from Bonini et al. (2004). They make people bet on future events (1 year from now) by using the following setup. They give each subject 7 euros per problem. Subjects have to distribute these 7 euros among three options. They are told that can assign the money in whatever way they find most convenient. For example, they might assign 1/3 of the money to each option; or they might assign 2 euros to the first option, 5 euros to the second option and nothing to the third option; or they might assign all the money to one single option. The only restriction they impose is that the addition of assignments has to equal 7 euros. The prize is as follows. After 1 year, one of their problems is going to be selected at random. The organizer will examine the three options, one at a time. If the event mentioned in option 1 has occurred, they will give whatever the subject assigned to that option. Then, they will follow the same procedure with option 2. If the event mentioned there has occurred, they will give the participant whatever he or she assigned to that option. Finally, they repeat the same procedure with option 3.

The researchers included several problems of irrelevant logical form as fillers. Here is the Internet example with the relevant logical form:

In Italy...
(a) more than 90% of private schools will be connected to the Internet.
(b) more than 90% of private schools will be connected to the Internet and *less* than 70% of public schools will be connected to the Internet (both events must happen for you to win the money placed on this bet).
(c) more than 90% of private schools will be connected to the Internet and *at least* 70% of public schools will be connected to the Internet (both events must happen for you to win the money placed on this bet). (p. 209)

Notice, first, that the structure of the options (A, A&B and A&$\sim$B) discourages the misinterpretation of the conjunct because the possible misinterpreted phrase is presented as one of the options.[15] In this type of problems, the best way to maximize profits is to place all the money in the first option. Actually, this is the only strategy that does not commit the conjunction fallacy. Any amount of money placed in options (b) or (c) is unnecessarily at risk of being lost. This is the case unless one is completely sure that the added event will occur or completely sure that it will not occur. But given the nature of this type of event, it is reasonable to assume that no subject would believe any of these options with certainty. In this particular example, 78% of the subjects placed some money on the second option (average: 2.39 euros), and 73% place some

[15] Actually, this technique was also used by Tentori et al. (2004) in the Scandinavian example of Sect. 4.

money on the third option (average: 1.87 euros). Thus, most people committed the conjunction fallacy with respect to each of the alternatives!

These last two examples show high percentages of conjunction fallacy in frameworks where the word "probability" does not appear. This seems to provide negative evidence against the misunderstanding hypothesis.

However, a general objection can be raised. The advocates of the reasoning bias hypothesis make assumptions about subjects' intentions and interpretations of the task that might not be accurate. As for intentions, one might call into question that people are really trying to maximize monetary gain. And even if this is so, money might not be the only goal people have in mind. As for the interpretation of the task, not all frameworks are completely transparent. For example, Bonini's setup, although neat, might have been very hard to capture, at least regarding its logical consequences. How do subjects actually interpret the betting task? What are their goals?[16] In the next section, I will present a concrete rival proposal by the advocates of the misunderstanding hypothesis.

5.2 Reply: people might also misinterpret the goal or goals of the task

How can the advocates of the misunderstanding hypothesis respond to this challenge? The betting framework has the advantage of creating a stimulus for picking the most probable option without mentioning the word "probability". The advocates of the misunderstanding hypothesis need to look for another possible source of misunderstanding. This brings us to the forth factor that might be responsible for misinterpretations and errors: the goal or goals of the task. The Heuristics and Biases program has assumed that under probability frameworks, people want to get correct answers. Under betting instructions, it is assumed that people want to make as much money as possible. These assumptions are reasonable (especially, the last one) but they may be wrong. And even if they are right, subjects might have additional goals in mind when solving conjunction problems.

This objection is defended by Hertwig and Gigerenzer (1999) among others (e.g. Adler 1991; Baird, pers. commun.). The basic point is that subjects may interpret the goal (or one of the goals) of a conjunction task as the request to be as *informative*

[16] Let me propose alternative goals that may be interfering with the goal of maximizing monetary profit. Let me concede, first, that all (or almost all) participants probably prefer to make more than less money. Or even if they do not care about money, they may still want to select the most probable event. For example, they may want to show that they figure what is the most rewarded option. But at the same time, each of them probably wants to spend as little time as possible participating in the experiment. Now, these two goals are not necessarily incompatible with each other but they go in different directions. Notice that the less time a participant spends, the less likely it is that she will discover the key logical relation. Moreover, the monetary incentive Bonini and colleagues offered to the participants, although good, was not *extremely* attractive (each participant could win at most 7 euros). Furthermore, participants perhaps wanted to make sure of not leaving empty-handed, that is, make sure of winning something—at least a couple of euros. Of course, the best strategy would still be to put all the money in the general option. But if one had not discover the logical structure of the options—pressed by the time-saving goal—it seems reasonable to put some money on every option as long as each seems plausible. Of course, in order to be taken seriously, this proposal should be tested empirically (I want to thank an anonymous reviewer for suggestions on this topic.).

as possible.[17] Usually, the conjunction is more informative than the conjunct. In fact, cover stories seem to be created to produce that very effect. So, if subjects are trying to be informative, it makes perfect sense to choose a conjunction over one of the conjuncts. For example, given Linda's profile, it is clearly more informative to say that she is a feminist bank teller than just a bank teller.[18]

[17] An advocate of the reasoning bias hypothesis may argue that this problem only occurs in the probability task. Even if probability questions may elicit intentions of being informative, this does not happen in betting formats. Under these formats, it is reasonable to assume that people just want to make as much money as possible. A possible answer to this objection—from an advocate of the misunderstanding hypothesis—may go as follows. People may entertain *several* goals in mind when solving a problem. In addition to try to make as much money as possible, participants probably want to finish as quickly as possible, and—Gigerenzer and others will argue—they want to provide answers as informative as possible as well. Of course, the problem is that the money goal and the informativeness goal go in opposite directions in the conjunction fallacy context. Notice, tough, that it is not necessary to assume that subjects take the informativeness goal as more important than the money goal when committing the fallacy. Participants probably do not even imagine that their goals can go in opposite directions. They may even assume that both goals always go together. Furthermore, consider the following two points. On the one hand, the fact that the conjunction is more informative than the conjunct can be noticed right away after reading the problem. On the other hand, for figuring out that the conjunction is less likely to occur, it is necessary to discover the consequences of the logical relationship between the options, and this takes time and effort. Thus, they may choose the informativeness goal over the money goal just because it is easier to satisfy. Nevertheless, all these considerations are only based on speculations and, in order to be taken seriously, they should be tested empirically (I want to thank an anonymous reviewer for very valuable comments on the topic.).

[18] Some reader may wonder about the difference between the notions of informativeness and representativeness. The question is relevant because representativeness is—according to Tversky and Kahneman—the heuristic used by participants when committing the conjunction fallacy. If informativeness is just another name for representativeness, Tversky and Kahneman would then support what we can call the informativeness hypothesis—participants are trying to be informative when solving conjunction fallacy problems. However, as I point out above, this hypothesis is used against Tversky and Kahneman's proposal. So, let's establish the distinction. I will argue that key aspect of the distinction is the *intention* of the user. But before I justify this point, let us briefly analyze each notion in turn. On the one hand, representativeness is the heuristic used to *evaluate probability* of events by assessing the *degree of correspondence or similarity* between a sample and a population, an instance and a category or an outcome and a model (Tversky and Kahneman 1983, p. 295). In the case of the Linda problem, for example, participants would try to evaluate the probability of each option—Tverky and Kahneman would argue—by assessing the degree of similarity between Linda's profile and (a) the stereotype of a feminist, (b) the stereotype of a bank teller and (c) the stereotype of a feminist bank teller. Informativeness, on the other hand, is linked to Grice's conversational analysis (Grice 1975). The idea is that in normal conversation—where the purpose is to convey information—the hearer will assume that the speaker is cooperative. In order to be so, the speaker should satisfy several conversational maxims as quality (tell the truth), quantity (tell all of it) and relevance (tell what the hearer wants to know). In the case of the Linda problem, for example, participants will assume that Linda's profile is relevant for solving the problem. They may, in turn, assume that they are also required to give a relevant answer, that is, an answer that makes use of such information given in the problem. Of course, these assumptions are wrong. But by assuming relevance, the argument goes, the participant will be forced to reinterpret the task. One possibility is to reinterpret the option (B) as (B & not F). Another possibility is to reinterpret the very goal of the task. Participants—and this is the key point—may not even try to evaluate probabilities. Rather, they may be interpreting the task as the requirement of ordering options according to their informational value given Linda's profile. And the informational value is related to the specificity of the options (e.g., If I think (but I'm not quite sure) that John was born in Paris and I'm asked "Where is John from?", the answer "France" is more informative yet less probable than "Europe"). To sum up, when using representativeness, one is trying to answer a *probability* question based in some degree of similarity or correspondence. When trying to give an informative answer, one may still use similarity as criterion but, crucially, one is *not* trying to answer a probability question, but just trying to provide an answer that makes good use of the information given in the problem. This alternative interpretation of the goal of the task has an important consequence: a participant pursuing informativeness is *not* committing a reasoning error. If a

This position seems reasonable but it faces at least three problems. First, neither Gigerenzer nor any other researcher has provided any empirical evidence that this is how people actually tend to interpret the goal or goals of the task at stake. Thus, although the objection is somewhat plausible, it lacks evidential support. It is true that when people only receive information about Linda's age, they tend to follow the conjunction rule. This might count as evidence in its favor, but no systematic study has investigated this factor.

Second, even granting that the conjunction is usually more informative than the conjunct, there are cases in which this is far from clear. Take, for example, the Internet case mentioned above. Given the percentage of people that put money on each option, it seems that most subjects did not have strong beliefs about the issue of Internet access in public schools. So, even if they did want to be informative, there was no much information available to base their bets. So, why do not subjects bet on the most general hypothesis that assures the highest amount of money? The advocates of this version of the misunderstanding hypothesis should tell a story to make sense of this type of example.

Finally, there is a third and more serious objection to this version of the misunderstanding hypothesis. There are contexts in which people tend to follow the conjunction rule (for example, estimation contexts or frequency contexts). How do the advocates of this type of misunderstanding hypothesis explain these improvements? Is it the case that people do not want to be informative anymore? This possibility does not sound very plausible. So, again, the advocates of this hypothesis should provide some story to explain the difference in performance.

5.3 Evidence in favor of the misunderstanding hypothesis

Let's grant for the sake of argument that the betting format is not very reliable to judge people's judgments. Gigerenzer and other evolutionary psychologists have proposed another format to test the "probability" misunderstanding hypothesis: the frequency format. The idea is that after Linda description, subjects are asked:

There are 100 women that fit Linda's description. Please, give your best estimate of the following values:

(a) How many of these women are bank tellers?
(b) How many of these women are active in the feminist movement?
(c) How many of these women are bank tellers and active in the feminist movement?

In this framework, most of the people (between 75% and 89%) correctly follow the conjunction rule. Fiedler (1988) systematized this last finding. The phenomenon in question has been called the 'frequency effect'.

Gigerenzer (1994) summarizes this result by claiming that the fallacy *disappears* under frequency formats. Furthermore, he argues that this frequency format overcomes

Footnote 18 continued
participant is not assessing the probability of the relevant events, it is impossible for this person to violate the rules of the probability calculus. I want to thank an anonymous reviewer who helped me to clarify this point.

the ambiguity of traditional probability formats: if asked for frequency judgments, people infer the intended mathematical meaning.[19] When this happens—he argues—most of the people give correct answers. Thus, Gigerenzer concludes, this evidence clearly supports the misunderstanding hypothesis.

5.4 Evidence against the misunderstanding hypothesis

Unfortunately for Gigerenzer, there is one context in which the conjunction fallacy does not disappear under frequency format. Tentori et al. (2004) offer evidence for this point. They report a big percentage of conjunction fallacies when subjects solve the following version of the Scandinavian problem[20]:

> The Scandinavian peninsula is the European area with the greatest percentage of people with blond hair and blue eyes. This is the case even though (as in Italy) every combination of hair and eye color occurs. Suppose we choose at random 100 individuals from the Scandinavian population. Which group do you think is the most numerous? (Check your choice)
> (a) Individuals who have blond hair.
> (b) Individuals who have blond hair and blue eyes.
> (c) Individuals who have blond hair and do not have blue eyes. (Tentori et al. 2004, pp. 473–474)

This is a clear case of frequency format. Again, in addition to the Scandinavian problem, Tentori and colleagues use the Volleyball problem with identical structure[21] and other three problems as fillers. Let us focus on the two conjunction fallacy problems. The result is that on average 66% of the subjects commit the conjunction fallacy by either picking the second or the third option.[22] No member of the evolutionary

[19] The justification of such a prediction goes as follows. Frequency judgments are obtained by asking participants to estimate and to compare the number of objects (usually, people) of different kinds. Such instructions are as transparent as you can get. And by doing these estimations and comparisons, people establish the relative frequency of different events. And this relative frequency, in turn, is the very meaning of "probability" that researchers have in mind when assessing people's performance.

[20] I mentioned this problem in the previous section. This is the frequency version of the problem.

[21] The frequency version of the Volleyball (Tentori et al. 2004, p. 474) problem read as follows: Professional volleyball players have greatly changed in the course of the last decade. In particular, they have grown younger, yet taller. Women players in the first Italian division are on average taller than 1.80 m, ranging from 1.75 m for some setters to more than 1.90 m for many spikers. Suppose we choose at random 100 female volleyball players from the Italian first division. Which group do you think is the most numerous? (Check your choice.)

- Women who are less than 21 years old.
- Women who are less than 21 years old and are taller than 1.77 m.
- Women who are less than 21 years old and are not taller than 1.77 m.

[22] From a developmental psychology perspective, it may seem quite surprising that Tentori et al. (2004) found that adults chose the group of Scandinavians with blond hair and blue eyes to be more numerous than the group of Scandinavian with blond hair, as this choice clearly violates the subset rule. In effect, Inhelder and Piaget (1964) show that 8 year old children can recognize and make use of subset relations, so that they no longer pick the subset as more numerous than a more inclusive set. How is it possible, then, that the adults in Tentori et al.'s experiment are not able apply the subset rule, even when the task format is similar

approach seems to have responded to this challenge and I cannot see how they might do it.

There is one additional point worth mentioning. As mentioned in Sect. 3, given the importance of this result, Dr. Wedell and I ran an experimental study with several different examples and we were able to replicate the same effect. Most people (72%) committed the conjunction fallacy under this frequency format. We also controlled for the misinterpretation of the conjunct. Our estimation is that only around 10% of the subjects might have misunderstood the conjunct (see Wedell and Moro 2007 for details). Again, even if 10% of conjunction errors can be accounted for this misinterpretation, still more than half of the subjects still commit the fallacy, apparently perfectly understanding the problem.

5.5 Explanation of opposite empirical results under frequency format

It might seem odd that different experiments on frequency formats have produced such opposite results. On the one hand, Gigerenzer finds that frequency formats produce a big improvement in people's performance. On the other hand, Tentori finds the opposite result. How to explain this disagreement? Let me offer a proposal.

The idea is that the key difference between the two studies is the *response mode* of the problem, that is, the *type of answer* the subjects are required to provide. In Hertwig and Gigerenzer (1999) subjects are required to give a *frequency estimate*. In Tentori et al. (2004) subjects are required to choose *the group with the highest frequency*.

There is empirical evidence showing that the response mode greatly affects performance. As mentioned in Sect. 3, Hertwig and Chase (1998) explored the difference between two response modes: estimates and rankings. They showed that people are very good at providing estimates but very bad at providing rankings. For example, they showed that if the Linda problem requests a probability estimate rather than a ranking, most people follow the conjunction rule. Hertwig and Gigerenzer (1999) also recognize the importance of the response mode. They estimate that half of the frequency effect is due to the frequency format itself, but the other half is due to the response mode (i.e. to provide an estimate).

The response format used in Tentori et al. (2004) is neither a ranking nor an estimate. Nevertheless, I think it is closer to the ranking format than to the estimate format. Subjects are requested to choose the *most numerous group*. Thus, they are asked to provide the top frequency option. This seems to require the same type of comparative operations that are required for coming up with a ranking.

Actually, in our experimental study, we found some empirical evidence for this idea. We created estimation versions for each of our problems. It turned out that most people

Footnote 22 continued
to that used in developmental studies? This is a question at the heart of much of the research on conjunction errors and is not easily answered. It is clear that adults are able to see and use subset relations, but that under some specific contexts such as those described in the literature on conjunction errors, people's responses do not reflect the importance or relevance of the implied subset relations. The purpose of many studies in the literature is to systematically explore conditions under which responses do or do not reflect those subset relations (see Wedell and Moro 2007 for a review on the topic).

provided frequency estimates that are in accordance with the conjunction principle. Thus, the response mode seems to be the factor that explains the difference between Gigerenzer and Tentori et al.'s results (see Wedell and Moro 2007).

Regardless of the explanation of this phenomenon, it seems that the study provided by Tentori and her collaborators shows a clear case of the conjunction fallacy under frequency format. Thus, the conjunction fallacy does *not* seem to disappear under frequency format.

The upshot is then that the empirical evidence seems to go against the misunderstanding hypothesis. Of course, this does not prove that the conjunction fallacy is a genuine reasoning bias. It is still possible that subjects misinterpret the instructions in a different way. However, we can argue that the empirical evidence *provided so far* seems to go against the misunderstanding hypothesis. The intuition that the probability of a conjunction is higher than one of the conjuncts seems to be very robust and it does not go away by just rephrasing the problem.

6 Conclusion

I have explored the debate over the nature of the conjunction fallacy. More specifically, I showed that a substantial portion of research has been directed to test the misunderstanding hypothesis. As we saw, some studies seem to provide support for such a hypothesis while others provide grounds for its rejection. The question is whether the empirical support of these sides is balanced or one of them is in a better position against its rival. I tried to suggest that the side that rejects the misunderstanding hypothesis seems better justified. The main argument comes down to this. It is true that some of the examples in the literature are ambiguous, and sometimes (although not always), when they are clarified, people perform better. But there is at least one case that is so clearly presented that it is hard to argue that people are misinterpreting it. This is the Scandinavian problem under frequency-choice format. In this case, the word 'probability' does not appear, so it cannot be blamed for the high rate of conjunction fallacies. Moreover, the presence of the third option (i.e., blond without blue eyes) makes implausible the misinterpretation of the conjunct.

The only possible objection is that people might be interpreting the goal of the task as being as informative as possible. Unfortunately, there is no systematic study that empirically explores this possibility. Thus, any conclusion about the issue will be speculative. Still, Wedell and Moro (2007) found evidence against this possibility. In this study, we asked subjects to provide frequency estimates for each group. As mentioned above, under this condition, most people followed the conjunction rule. It would be implausible to say that subjects do not want to be informative anymore. This possibility seems hard to believe. Hence, this version of the misunderstanding hypothesis cannot be used against the Scandinavian example either.

In the end, we are left with at least one condition that does not seem open to misinterpretation. However, most people commit the conjunction fallacy in this case. For this reason, it seems to me that the misunderstanding hypothesis does not provide a good picture on the nature of the conjunction fallacy. Of course, more research

is needed to reach a well supported verdict on the matter. Thus, I admit that the misunderstanding hypothesis is still an open possibility.

References

Adler, J. (1984). Abstraction is uncooperative. *Journal for the Theory of Social Behaviour, 14*, 165–181. doi:10.1111/j.1468-5914.1984.tb00493.x.

Adler, J. (1991). An optimist's pessimism: Conversation and conjunction. *Posnam Studies in the Philosophy of the Sciences and Humanities, 21*, 251–282.

Agnoli, F., & Krantz, D. (1989). Suppressing natural heuristic by formal instruction: The case of the conjunction fallacy. *Cognitive Psychology, 21*, 515–550. doi:10.1016/0010-0285(89)90017-0.

Bar-Hillel, M., & Neter, E. (1993). How alike is it versus how likely is it: A disjunction fallacy in probability judgments. *Journal of Personality and Social Psychology, 65*(6), 1119–1131. doi:10.1037/0022-3514.65.6.1119.

Bonini, N., Tentori, K., & Osherson, D. (2004). A different conjunction fallacy. *Mind & Language, 19*(2), 199–210. doi:10.1111/j.1468-0017.2004.00254.x.

Cohen, L. J. (1977). *The probable and the provable*. Oxford: Clarendon Press.

Cohen, L. J. (1981). Can human irrationality be experimentally demonstrated? *The Behavioral and Brain Sciences, 4*, 317–370.

Cohen, L. J. (1982). Are people programmed to commit fallacies? Further thoughts about the interpretation of experimental data and probability judgment. *Journal for the Theory of Social Behaviour, 12*, 251–274. doi:10.1111/j.1468-5914.1982.tb00450.x.

Dulany, D., & Hilton, D. (1991). Conversational implicature, conscious representation and the conjunction fallacy. *Social Cognition, 9*, 85–110.

Fiedler, K. (1988). The dependence of the conjunction fallacy on subtle linguistic factors. *Psychological Research, 50*, 123–129. doi:10.1007/BF00309212.

Gigerenzer, G. (1994). Why the distinction between single-event probabilities and frequencies is important for psychology (and vice versa). In G. Wright & P. Ayton (Eds.), *Subjective probability* (pp. 129–161). New York: Wiley.

Gigerenzer, G. (1996). On narrow norms and vague heuristics: A reply to Kahneman and Tversky (1996). *Psychological Review, 103*(3), 592–596. doi:10.1037/0033-295X.103.3.592.

Gilovich, T., Griffin, D., & Kahneman, D. (2002). *Heuristics and biases: The psychology of intuitive judgment*. Cambridge: Cambridge University Press.

Grice, H. P. (1975). Logic and conversation. In P. Cole & J. L. Morgan (Eds.), *Sintax and semantics 3: Speech acts* (pp. 41–58). New York: Academic Press.

Hastie, R., & Dawes, R. (2001). *Rational choice in an uncertain world: The psychology of judgment and decision making*. Thousand Oaks: Sage Publications.

Hertwig, R., & Chase, V. (1998). Many reasons of just one: How response mode affects reasoning in the conjunction problem. *Thinking & Reasoning, 4*(4), 319–352. doi:10.1080/135467898394102.

Hertwig, R., & Gigerenzer, G. (1999). The conjunction fallacy revisited: How intelligent inferences look like reasoning errors. *Journal of Behavioral Decision Making, 12*, 275–305. doi:10.1002/(SICI)1099-0771(199912)12:4<275::AID-BDM323>3.0.CO;2-M.

Hilton, D. (1995). The social context of reasoning: Conversational inference and rational judgment. *Psychological Bulletin, 118*(2), 248–271. doi:10.1037/0033-2909.118.2.248.

Inhelder, B., & Piaget, J. (1964). *The early growth of logic in the child: Classification and seriation*. New York: Norton.

Kahneman, D., & Tversky, A. (1996). On the reality of cognitive illusions. *Psychological Review, 103*(3), 592–591. doi:10.1037/0033-295X.103.3.582.

Macdonald, R., & Gilhooly, K. (1990). More about Linda or conjunctions in contexts. *The European Journal of Cognitive Psychology, 2*, 57–70. doi:10.1080/09541449008406197.

Messer, W., & Griggs, R. (1993). Another look at Linda. *Bulletin of the Psychonomic Society, 31*, 193–196.

Morier, D., & Borgida, E. (1984). The conjunction fallacy: A task specific phenomenon? *Personality and Social Psychology Bulletin, 10*, 243–252. doi:10.1177/0146167284102010.

Moro, R. (2007). *Linda is back: Controversies around the conjunction fallacy in the psychology of reasoning*. Dissertation, University of South Carolina.

Politzer, G., & Noveck, I. (1991). Are conjunction rule violations the result of conversational rule violations? *Journal of Psycholinguistic Research, 20*, 83–103. doi:10.1007/BF01067877.

Samuels, R., & Stich, S. (2004). Rationality and psychology. In A. Mele & P. Rawling (Eds.), *The Oxford handbook of rationality* (pp. 279–300). Oxford: Oxford University Press.

Samuels, R., Stich, S., & Faucher, L. (2002). Reason and rationality. In I. Niiniluoto, M. Sintonen, & J. Wolenski (Eds.), *Handbook of epistemology* (pp. 131–179). Dordrecht: Kluwer.

Shafer, G. (1976). *A mathematical theory of evidence*. Princeton, NJ: Princeton University Press.

Sides, A. (2000). *The conjunction fallacy under probability and betting instructions*. MA Thesis, Rice University.

Sides, A., Osherson, D., Bonini, N., & Viale, R. (2002). On the reality of the conjunction fallacy. *Memory & Cognition, 30*(2), 191–198.

Tentori, K., Bonini, N., & Osherson, D. (2004). The conjunction fallacy: A misunderstanding about conjunction? *Cognitive Science, 28*, 467–477.

Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. *Psychological Review, 90*(4), 293–315. doi:10.1037/0033-295X.90.4.293.

Wedell, D. H., & Moro, R. (2007). Testing boundary conditions for the conjunction fallacy: Effects of response mode, conceptual focus and problem type. *Cognition* (in press). doi:10.1016/j.cognition.2007.08.03.

Wolford, G., Taylor, H. A., & Beck, J. R. (1990). The conjunction fallacy? *Memory & Cognition, 18*, 47–53.