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Hard candy cooling: Optimization of operating policies considering product quality

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ABSTRACT

To guarantee acceptable hard candy quality during the cooling stage, the distribution of the product's temperature throughout the cooling tunnel must be controlled. Hence, the product's quality depends on the operating conditions of the cooling process and the air conditioning system. In this paper, hard candy quality, operating policies and production planning are integrated in an NLP optimization mathematical model to obtain optimal operating polices under different operating modes, minimizing the annual cost. The resulting model is applied in different case studies in which the production of one, and then six products, is analyzed considering different levels of production, demand and conveyor belt capacities. The study also considers different operating conditions for the air conditioning system under three possible operating modes during the year.

1. Introduction

The cooling stage during the production process of hard candies is one of the most critical unit operations because many quality problems, such as deformation, fragility and aggregation, may appear at this stage. Fig. 1 schematically shows the required unit operations for the hard candy production process. As illustrated in Fig. 1, the cooling tunnel has two air ducts (entrance and exit) and is composed of three conveyor belts (CBs), which are mechanically driven by an engine connected to an adjustable frequency drive (AFD) to vary the residence time of the candies. While the candies are moving along the tunnel, they come into contact with cooling air (CA), which flows parallel to the belts.

Air cooling velocity is regulated by manipulating the operating speed range of the fan. In contrast, the air cooling temperature is set up in a heat exchanger (HE) using auxiliary utilities (cooling/heating), as shown in Fig. 1. The operating conditions of the heat exchanger depend on the air temperature and the optimal value of the cooling temperature.

The heat exchanger feed is formed by mixing (or not) two available streams into a mixer (M1): the cold water stream (CW) from the cooling tower and the hot water stream (HW), which is also used at the tempering stage to temper a stretch of the stainless steel belt. The proportions of the mixture will depend on the model restrictions and the seasonal conditions, which will change the input variables of the cooling tower and, therefore, the temperature of the cold water stream.

As shown in Fig. 1, an alternative air recycling stream at the exit duct is also included in the air conditioning system, which increases the available degrees of freedom to conveniently adjust the operating variables (fluxes and temperatures) and to also reduce the operating costs.

From the point of view of product quality, the size of the hard candy, production level, the dimensions of the cooling tunnel, the temperature and velocity of cooling air, and residence time of the candies inside the tunnel play critical roles in the cooling efficiency. For example, a high air velocity may lead to a non-uniform temperature profile, which increases the product's fragility and causes the production of misshapen candies and their consequent rejection, resulting in significant financial losses. In contrast, higher candy temperatures at the end of the tunnel and incorrect residence times lead, respectively, to deformation and the candy aggregation. Therefore, the operational mode of the cooling tunnel is crucial for the final candy's quality.

Higher product quality is definitely obtained when the tunnel is operated in such a manner that the difference in temperature between the center and the surface of the candy is minimized (the more uniform transient temperature behavior), ensuring an appropriate temperature for the wrapping stage (28–40 °C).

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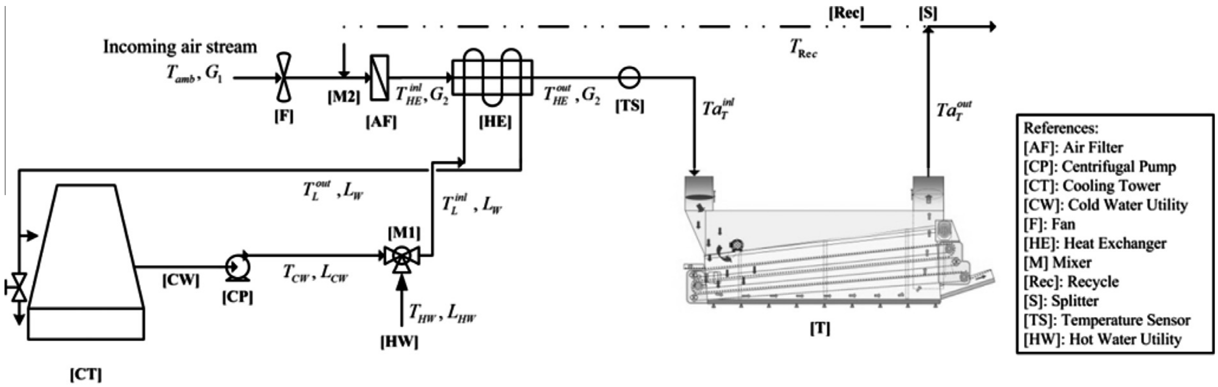

Fig. 1. Flow sheet of the air conditioning system coupled with the cooling tunnel.

Temperatures higher than 40 °C lead to stickiness or deformation problems (non-conforming products). In contrast, temperatures lower than 40 °C lead to higher residence times, requiring an "infinite" length cooling tunnel. Finally, the thermo-physical properties of the products and their composition also influence the product's quality (Reinheimer et al., 2010).

The aim of this work is to present an optimization model to solve a dynamic optimization problem for the revamping of a hard candy production line and taking into account a measure of the product's quality. The dynamic optimization model is based on a previous work in which the operating conditions of the cooling tunnel were optimized to minimize hard candy temperature differences, which are, in turn, associated with quality problems (Reinheimer et al., 2012). The model proposed in that article is now properly extended to include a quality model, which explicitly penalizes product rejection in the economic objective function. The model is also extended to determine the operating schedule of the cooling tunnel to manufacture six different types of hard candy products (of different flavors) as a function of the product demand throughout a time horizon of 1 year. Thus, the trade-offs associated with product quality, temperature difference profiles, production level, and production and reprocessing costs are simultaneously optimized. The annual production has been divided into three distinct seasons: defined as winter ($k = w$), summer ($k = s$) and mid-season ($k = m$). Therefore, three different scenarios or operating modes have been investigated in relation to the seasonal operating cost. The model can be used for operation planning and for the optimization of the operating conditions when a revision of the existing line (production increment) must be satisfied. Candy size and composition, ambient air properties, heat transfer area, and operating costs are the main input data for the optimization model. In this case, the heat transfer area of the heat exchanger, the air duct section in the cooling tunnel, and the capacity of the cooling tower are fixed because the equipment already exists.

The resulting model is implemented in GAMS (General Algebraic Modeling System) and solved using CONOPT, a local optimization algorithm based on the reduced gradient method.

2. Assumptions and mathematical model

2.1. Assumptions

The main assumptions of the model can be summarized as follows:

- Candies are considered homogeneous and isotropic spheres.
- Model 1-D. Temporal variations of the temperature in the radial direction are considered.
- Choi and Okos' models (1986) were used for the estimation of thermo-physical properties, such as density, thermal capacity, and thermal conductivity.
- Variations in thermo-physical properties with temperature are neglected for the temperature range considered in this work.
- There is no moisture loss. Due to the low water content of hard candy (2.5%), water loss during the cooling of candies is insignificant. Accordingly, moisture intake from the air is also negligible because air humidity is monitored, and profiles can be considered constant between the values at the tunnel entrance and exit.
- Changes in air humidity are small enough to produce a negligible effect on the thermo-physical properties of the air. The properties of air flow are calculated as the average temperature of the cooling air flow stream.
- The convective heat transfer coefficient is computed as the area averaged value of the local heat transfer coefficient (Becker and Fricke, 2004).

2.2. Mathematical model

The complete mathematical model includes the heat transfer model (energy balance of the unsteady heat transfer process), including the corresponding boundary conditions, the air conditioning model, and the equations necessary to compute the enthalpies and physical properties of the air, which are described in detail in Reinheimer et al. (2012).

In this section, the main constraints that were coupled into the previous model and that are related to the proposed cost model for flavored hard candy products are presented. The actual model is based on the following two indexes: k and i which are used, respectively, to refer to the season ($k = w,s,m$) and type of hard candy product ($i = 1–6$).

Seasonal capacity, production, and operating cost are calculated to achieve annual production capacity constraints. Different seasonal production plans can be obtained according to the operating modes selected for the proposed objective function (minimizing the annual operating costs).

As mentioned earlier, the maximum temperature difference between the center and the surface of pieces of hard candies is associated with fragility problems during the wrapping stage. As can be observed from previous results (Reinheimer et al., 2012), the risk of product fragility is high at the beginning of the cooling process because the temperature difference reaches its maximum value and then decreases slowly.

Hence, a quality variation constraint, defined as the product rejection rate, φ, is included in the optimization model. The type of function was adopted considering historical data for the product rejection rate due to fragility problems, which are closely related to the maximum internal temperature. By analyzing historical data, it was concluded that a linear relationship between the product rejection rate and the maximum internal temperature in the range of internal temperature differences arising under the different operating conditions of the cooling tunnel (cooling air temperature and velocity) can be proposed with a satisfactory estimated error.

The product rejection rate function can be defined as:

$$\varphi_i = a_{1,i} \cdot \Delta T_{\max i} - a_{2,i} \tag{1}$$

where $\Delta T_{\max i}$ depends on the operating conditions of the cooling tunnel, and $a_{1,i}$ and $a_{2,i}$ are obtained from annual historical data for each line.

One of the main problems in food process optimization is considering product quality as an economic objective function. For the model presented here, this function can be performed by penalizing product rejection. Hence, a reprocessing cost is applied to the respective production, which is higher when the maximum temperature difference is higher. Therefore, a trade-off is expected between economic and quality aspects because higher residence times, lower air cooling velocities, and higher air cooling temperatures could be expected to reduce internal temperature differences, as demonstrated by the optimization model previously analyzed (Reinheimer et al., 2012). However, in this previous work, cost functions and the quantification of product rejection as a function of product quality measures were not considered. Thus, only a performance analysis was performed. Another missing point in the previous model was the performance of the cooling tower, which is dependent on the operating mode.

The model equations are stated below.

The total operating time is:

$$OT = OT_w + OT_s + OT_m \tag{2}$$

and the seasonal operating time is computed as:

$$OT_k = OD_k \cdot DT_k \tag{3}$$

where OD_k and DT_k are the operating days and the daily operating time per season, respectively. Therefore, the daily operating time is computed as:

$$DT_k = \sum_{i=1}^{6} DT_{k,i} \tag{4}$$

where $DT_{k,i}$ is the daily operating time per product.

The conveyor belt velocity is:

$$v_{CBk,i} = \frac{CBL}{\theta_{k,i}} \tag{5}$$

where CBL is the conveyor belt length and $\theta_{k,i}$ is the residence time during each season and for each product.

The seasonal production of each product is calculated as:

$$P_{k,i} = \frac{CBL \cdot CBC_{k,i}}{\theta_{k,i}} \tag{6}$$

where CBC is the conveyor belt capacity, which can be different for each product (i) in each operating mode (k). After inspection, which is also based on the rejection function, the production rate is divided into acceptance and rejection (for reprocessing) rates, which are defined as:

Reprocessed product (rejection rate):

$$REP_{k,i} = P_{k,i} \cdot \frac{\varphi_i}{100} \tag{7}$$

Acceptable product:

$$AP_{k,i} = P_{k,i} - REP_{k,i} \tag{8}$$

The production capacity is defined as:

$$CAP_{k,i} = P_{k,i} \cdot OT_{k,i} \tag{9}$$

Then, the seasonal capacity and annual production capacity per product are respectively expressed as:

$$CAP_k = \sum_{i=1}^{6} CAP_{k,i} \tag{10}$$

$$CAP_i = CAP_{w,i} + CAP_{s,i} + CAP_{m,i} \tag{11}$$

The equation for the stock control at the end of the season per product and season is formulated as follows:

$$ST_{fk,i} = ST_{0k,i} + CAP_{k,i} - DEM_{k,i} \tag{12}$$

where $ST_{0k,i}$ and $DEM_{k,i}$ are the initial stock and demand per product and season. Then, the annual demand per product is calculated as:

$$DEM_i = DEM_{w,i} + DEM_{s,i} + DEM_{m,i} \tag{13}$$

The seasonal demand and stock equations are respectively formulated as:

$$DEM_k = \sum_{i=1}^{6} DEM_{k,i} \tag{14}$$

$$ST_{fk} = \sum_{i=1}^{6} ST_{fk,i} \tag{15}$$

$$ST_{0k} = \sum_{i=1}^{6} ST_{0k,i} \tag{16}$$

As expected, the final stock of one season corresponds to the initial stock of the following season. In this example, winter is the initial season, and the mid-season period is taken as one season to illustrate the schedule and considering that:

$$ST_{fw,i} = ST_{0s,i} \tag{17}$$

$$ST_{fs,i} = ST_{0m,i} \tag{18}$$

$$ST_{fm,i} = ST_{0w,i} \tag{19}$$

A restriction of storage capacity, SC, also exists in every season, that is:

$$ST_{fk} \leqslant SC_{k\max} \tag{20}$$

This value is an estimate because sales forecasting is not contemplated in this case.

The costs associated with raw material and labor costs are estimated as:

$$C_{RMk,i} = CAP_{k,i} \cdot C_{uRMi} \tag{21}$$

where $C_{uRM\ i}$ is the unitary raw materials and labor cost per kilogram of product. Then, the total raw material cost per operating mode is computed as:

$$C_{RMk} = \sum_{i=1}^{6} C_{RMk,i} \tag{22}$$

The product rejection rate, which expresses the amount of product that has to be reprocessed, is charged with an additional cost, defined as:

$$C_{REPk,i} = REP_{k,i} \cdot C_{uREPi} \cdot OT_{k,i} \tag{23}$$

where $C_{uREP\ i}$ is the unitary reprocessing cost per kilogram of product. This unitary cost, obtained from historical data, is an average cost that considers the wasting of raw materials (including flavoring and coloring agents), and energy and labor costs. Then, the total reprocessing cost per operating mode is defined as:

$$C_{REPk} = \sum_{i=1}^{6} C_{REPk,i} \tag{24}$$

The following equations are defined to represent the consumption of utilities and their respective operating costs.

Total power consumption:

$$E_{k,i} = E_{FA1,k,i} + E_{FA2,k,i} + E_{CB,k,i} + E_{FI,k,i} \tag{25}$$

where E_{FA}, E_{CB}, and E_{FI} refer to the power consumption of the fan, conveyor belt, and rotary valves of the bag filters, respectively. The power consumption of the rotary valves is assumed to be constant for the operating modes. In contrast, consumption functions are defined to compute the energy consumption of the fan and conveyor belt, which depend on the operating conditions. In fact, the following constraints were derived by fitting information available from catalogues covering a wide range of air flow rates and conveyor belt velocities.

$$E_{FA1,k,i} = 135.3 \cdot G^3_{2,k,i} \tag{26}$$

$$E_{FA2,k,i} = 135.3 \cdot \mathrm{Rec}^3_{k,i} \tag{27}$$

$$E_{CB,k,i} = 0.6 + 84.8 \cdot v_{CB,k,i} \tag{28}$$

The independent term in Eq. (28) corresponds to the driving power of the conveyor belt, which depends on the belt length.

Then, the equation for the power cost is given by:

$$C_{Ek} = E_k \cdot C_{uE} \cdot OT_k \tag{29}$$

where C_{uE} is the unitary power cost.

The hot and cold water utility costs are defined as:

$$C_{HWk} = L_{HWk}.C_{uHW} \cdot OT_k \tag{30}$$

$$C_{CWk} = L_{CWk}.C_{uCW} \cdot OT_k \tag{31}$$

where L_{HW} and L_{CW} refer to the hot and cold water utility mass flow rates, and C_{uHW} and C_{uHW} are their respective unitary costs.

Finally, the operating costs for each operating (seasonal) mode and the annual cost are respectively given by:

$$C_{OPk} = C_{RMk} + C_{REPk} + C_{Ek} + C_{HWk} + C_{CWk} \tag{32}$$

$$CT = C_{OPw} + C_{OPs} + C_{OPm} \tag{33}$$

The total production rate is subject to energy and utility costs but not to the reprocessing costs, which are only affected by the product rejection rate. Therefore, the operating cost of a unit of product can be defined as:

$$c_{k,i} = C_{uRMi} + \frac{C_{Ek} + C_{HWk} + C_{CWk}}{CAP_k} \tag{34}$$

Then, to compute the unitary cost of the reprocessed product, REP_k, it is necessary to add the unitary reprocessing cost, C_{uREP}, to the product cost (Eq. (34)).

Because the equipment units already exist, Eq. (33) is the total cost approach, and operating costs are only considered, without taking into account fixed capital costs and the respective capital recovery factor.

The operating cost per unit of product can vary between seasons because different utility costs are incurred depending on the properties of the ambient air (temperature, relative humidity and mass flow), which enters the air conditioning system, and operating conditions (cooling air and conveyor belt velocities).

2.3. Operating constraints

(1) Air convection is directed by a horizontal fan with a safe operating speed range of 1.2–3 m/s, which is imposed by lower and upper bounds, and therefore:

$$1.2 \leqslant v_{ak,i} \leqslant 3$$

(2) In the air conditioning system, it is possible to recycle up to 60% of the air stream at the tunnel's exit to reduce utility costs (if this is possible) because power costs are associated with the recycled flow stream. Hence:

$$0 \leqslant Y_{k,i} \leqslant 0.6$$

(3) The conveyor belt capacity is restricted by the stamp-forming machine used in the previous process stage of the forming process, and an average value of 0.5 is frequently achieved. Instead, maximum and minimum loads of 0.9 and 0.3, respectively, are considered in the model:

$$0.3 \leqslant CBC_{k,i} \leqslant 0.9$$

(4) The air cooling temperature is set up in a heat exchanger using auxiliary utilities (cooling/heating). The operating conditions of the heat exchanger depend on the external ambient temperature, and this temperature also depends on the season and on the optimal value of the cooling temperature. From the point of view of operating costs, it is not convenient for the cooling air temperature to be lower than the lowest ambient air temperature, and, to assure a glassy structure, it also cannot be higher than the glass's transition temperature. Hence:

$$10 \leqslant Ta_{Tk,i} \leqslant 34$$

Other operating constraints are used in the case studies, fixing bounds to satisfy a specific minimum demand or to produce a specific product and considering a maximum admissible processing time per day.

2.4. Objective function

One of the main goals of this paper is to determine the optimal operating conditions of the cooling unit's operation during the manufacture of hard candies, which satisfy an economic objective function under product quality requirements.

The objective function, *OF*, consists of minimizing the annual operating costs:

$$Min\ OF = Min\ CT \tag{35}$$

Therefore, production policies can be obtained based on economic aspects (seasonal production costs, utility consumption and costs).

In summary, the optimization problem states:

- Min $CT\,(v_{a\ k,i}, Ta_{T\ k,i}, \theta_{k,i}, G_{1\ k,i}, G_{2\ k,i}, Y_{k,i}, L_{HW\ k,i}, L_{CW\ k,i}, CAP_{k,i}, \varphi_i)$, subject to heat transfer, air conditioning, and cost model equations.

The analysis of the trade-offs that exists among quality aspects, operating costs, and operating conditions will be explored in the next section.

The optimization model involves 4138 constraints (equalities and inequalities). The total number of variables is 1596. It should be noted that global optimization solutions cannot be guaranteed due to the presence of bilinear terms and logarithms that lead to non-convex constraints.

Table 1
Seasonal values.

	Scenarios		
	Winter ($k = w$)	Summer ($k = s$)	Mid-season ($k = m$)
Seasonal operating days, OD_k (days)	90	90	120
Seasonal operating time, OT_k (h)	2160	2160	2880
Daily operating time, DT_k (h)	24	24	24
Average ambient air temperature, $T_{amb,\ k}$ (°C)	10	27	17
Average relative humidity (%)	65	75	70
Wet bulb temperature, T_{WB} (°C)	7.5	22.8	13.7
Cold water utility temperature, $T_{CW,k}$ (°C)	17	27.5	21

Table 2
Model parameters.

Parameter	Value
Hard candy composition (w/w%):	0.276
Carbohydrates	97.13
Water	2.5
Ash	0.18
Hard candy thermal conductivity, $k_{c,i}$ (W/m °C)	0.276
Hard candy specific heat, $Cp_{c,i}$ (J/kg °C)	1700
Hard candy thermal diffusivity, $\alpha_{c,i}$ (m^2/s)	1.106 E-7
Hard candy radio, R_i (m)	0.008
Hard candy inlet temperature, $T_{inl,i}$ (°C)	80
Hard candy glass transition temperature, Tg_i (°C)	34
Final hard candy temperature difference, $\Delta T_{f\ max,i}$	3
Air duct cross section, S (m^2)	0.2025
Specific heat capacity of water, Cp_W (J/kg °C)	4180
Specific heat capacity of air, Cp_a (J/kg °C)	1005
Overall heat transfer coefficient, U (W/m^2 °C)	200
Heat transfer area, A (m^2)	2.9
Temperature of hot water utility, T_{HW} (°C)	50
Conveyor belt length, CBL (m)	30
Reprocessing unitary cost, C_{uREP} ($/kg)	3
Hot water utility unitary cost, C_{uHW} ($/1000 kg)	1.02
Cold water utility unitary cost, C_{uCW} ($/1000 kg)	0.185
Power unitary cost, C_{uE} ($/kW h)	0.06
Rotary valves energy consumption, E_{Fl} (kW)	0.4
Maximum storage capacity, $SC_{k\ max}$ (kg/sn)	100,000

An Intel Core i7 2230 M 2.20 GHz processor with 8 GB RAM was used to perform the simulations and optimizations.

3. Results and discussion

3.1. Case study I: model validation

For validation purposes, the model implemented in GAMS was used as a simulator in a predictive manner. For this simulation, it was necessary to fix the model's degrees of freedom. Operating variables not considered in the previous models (Reinheimer et al., 2010; 2012), such as the operating variables arising from the cooling tower, the air conditioning system, the blowers and other equipment units, are fixed for validation purpose. The simulation results obtained using the actual model were compared with the experimental data for the heat transfer model used in Reinheimer et al. (2010). It is worth noting that both implemented models showed the same performance and results, as expected. Here, different case studies are presented to discuss the results obtained for the optimization problem described in Section 3.

Representative weather conditions and operating times, considered in each seasonal operating mode, are presented in Table 1. The main parameters associated with the hard candies, the air conditioning model, and the cost model are enumerated in Table 2.

The global hard candy composition is similar for the six products. Hence, the physico-thermal properties can be considered equal. Differences in additives exist among the six products, which have an impact on the reprocessing costs and also on the rejection rate of each product.

Table 3 presents the parameters and constraints that correspond to each hard candy product.

3.2. Case study II: analysis of optimal solutions for the production of one product

To obtain optimal temperature profiles, the production of product number 1 will be studied. To analyze the optimal temporal temperature profiles, it is important to keep in mind that the final temperature is critical for the wrapping stage (it must be equal to or lower than the glass transition temperature), whereas the temperature difference between the surface and the center at the end of the tunnel is crucial to avoid deformation problems. The maximum temperature difference is the optimization variable associated with fragility problems, which is related to the production and quality levels by the product rejection function and is also related to the optimization variables of the cooling tunnel.

To facilitate the analysis of the trade-off between costs and the distribution of the production time among the operating modes, a fixed conveyor belt capacity (CBC_k, equal to 0.5 kg/m) and a maximum admissible time of 4 h per day for the fabrication of this product are considered in this case study. Therefore:

$$DT_{k,1} \leqslant 4$$

A detailed description of the main variables obtained when solving the optimization problem is provided in Table 4.

When analyzing the distribution of the annual planned production among the seasonal scenarios, different conclusions can be achieved. As a result of the optimization, the operating conditions of the cooling tunnel are the same for the three scenarios, and therefore, the same residence times and maximum temperature differences are achieved. However, the production plan is concentrated on the summer season because of the lower operating cost per unit of product during this season and also because the expected demand can be satisfied by production in just one scenario or operating mode. This production plan is possible under the

Table 3
Parameters corresponding to each hard candy product.

	Product					
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
a_{1i}	0.289	0.251	0.291	0.321	0.267	0.302
a_{2i}	1.9203	1.813	1.9528	1.9087	1.9098	1.9677
Unitary raw material cost, C_{URMi} ($/kg)	7.0	7.5	7.8	7.7	7.3	7.5
Unitary reprocessing cost, C_{UREPi} ($/kg)	3.0	3.1	3.5	3.7	3.3	3.6
Annual demand, DEM_i (kg/year)	15,000	35,000	40,000	40,000	65,000	57,000
Initial stock, $ST_{0w,i}$ (kg)	0	1500	750	2000	500	2500

Table 4
Optimal values of the main variables to satisfy a demand of 15,000 kg/year of product number 1 for a fixed CBC = 0.5 kg/m.

	Min CT		
	Winter ($k = w$)	Summer ($k = s$)	Mid-season ($k = m$)
va_k (m/s)	N/A	1.2[a]	N/A
$Ta_{T\ k}$ (°C)	N/A	33.66	N/A
$Ta_{T,k}^{inl}$ (°C)	N/A	30.41	N/A
$Ta_{T,k}^{inl}$ (°C)	N/A	36.91	N/A
Y_k	N/A	0.338	N/A
θ_k (s)	N/A	1064.62	N/A
$\Delta T_{\max\ k}$ (°C)	N/A	14.81	N/A
DT_k (h)	N/A	3.286	N/A
P_k (kg/h)	N/A	50.71	N/A
CAP_k (kg/sn)	N/A	15,000[a]	N/A
CT_k ($/sn)	N/A	1.0619E5	N/A
φ_k (%)	N/A	2.36	N/A
E_k (kW)	N/A	6.92	N/A
q (W)	N/A	16.7	N/A
L_{CW} (kg/h)	N/A	3.08	N/A
L_{HW} (kg/h)	N/A	1.18	N/A
c_k ($/kg)	N/A	7.00822	N/A
CAP (kg/year)	15,000[a]		
CT ($/year)	1.0619E5		

N/A – not applicable.
[a] Lower bound.

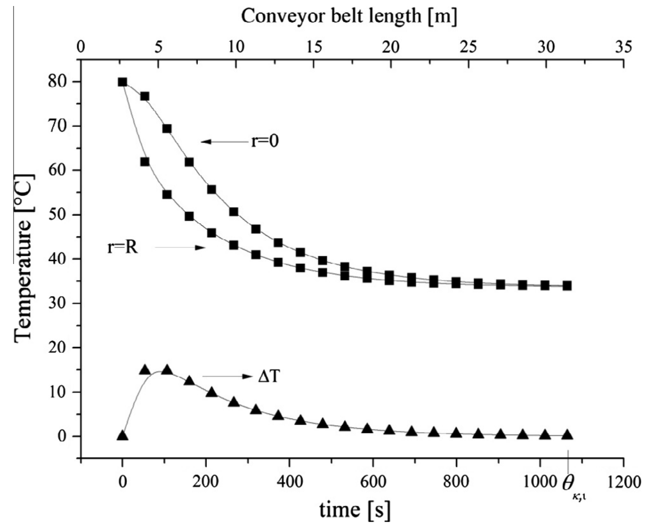

Fig. 2. Temperature profiles and differences between the center and the surface of the candy vs. residence times.

Table 5
Optimal values of the main variables to satisfy a demand of 50,000 kg/year for a fixed CBC = 0.5 kg/m.

	Min CT		
	Winter ($k = w$)	Summer ($k = s$)	Mid-season ($k = m$)
DT_k (h)	1.62	4[b]	4[b]
CAP_k (kg/sn)	7397	18,258	24,345
CT_k ($/sn)	52,376.88	1.2925E5	1.7236E5
CAP (kg/year)	50,000[a]		
CT ($/year)	3.5399E5		

[a] Lower bound.
[b] Upper bound.

Table 6
Optimal values of the main variables to satisfy a demand of 50,000 kg/year for variable CBC_k.

	Min CT		
	Winter ($k = w$)	Summer ($k = s$)	Mid-season ($k = m$)
$Ta_{T,k}^{inl}$ (°C)	N/A	27.81	27.81
$Ta_{T,k}^{inl}$ (°C)	N/A	39.51	39.51
Y_k	N/A	0.062	0.446
CBC (kg/m)	N/A	0.9[b]	0.9[b]
DT_k (h)	N/A	4[b]	1.74
P_k (kg/h)	N/A	91.29	91.29
CAP_k (kg/sn)	N/A	32,865	17,135
CT_k ($/sn)	N/A	2.3253E5	1.2124E5
φ_k (%)	N/A	2.36	2.36
E_k (kW)	N/A	6.79	6.85
q (W)	N/A	11.13	501.12
L_{CW} (kg/h)	N/A	1.17	12.26
L_{HW} (kg/h)	N/A	0.44	21.72
c_k ($/kg)	N/A	7.00447	7.0053
CAP (kg/year)	50,000[a]		
CT ($/year)	3.5377E5		

N/A not applicable.
[a] Lower bound.
[b] Upper bound.

hypothesis that there are no restrictions with respect to storage room capacity, storage conditions, or product shelf life. For this optimization problem, the difference in the operating costs with respect to the other operating modes comes from the costs of the air conditioning system and power consumption because the reprocessing costs, which are associated with the temperature difference and operating conditions, are the same in all seasons. Also, the production only satisfies the demand (15,000 kg/year) due to the applied objective function (minimization of annual operating costs).

Regarding the operating conditions of the cooling tunnel, v_{as} reached its lower bound. Observing the set of values for the operating conditions during the cooling stage, a clear trade-off between quality and operating costs exists. From previous results, it was concluded that lower values for the air cooling velocity lead to lower temperature differences (Reinheimer et al., 2012), and this effect is reflected in lower rates of product rejection. Conversely, regarding the production level and costs, lower air velocities result in higher residence times and, hence, lower production and higher power costs.

In general, the optimal value for the cooling air velocity takes its lower bounds from the fact that for higher air cooling velocities, the maximum temperature difference increases. Hence, the product rejection rate also rises. In contrast, when the air cooling velocity has higher values, the cooling air temperature also tends to have higher values to diminish internal temperature differences by reducing the heat transfer driving force. For higher air velocity values, the seasonal total costs are higher for a fixed conveyor belt capacity because of the increment in the reprocessing and power costs.

When the cooling tunnel operates under other non-optimal conditions, such as a $va_{k,i} = 3$ m/s, $Ta_{T\ k,i} = 25$ °C, a significant increment of the seasonal cost (23%) is evidenced. Under these operating conditions for the cooling stage, the product rejection rate is 5.19%, almost twice that corresponding to the optimal conditions (Table 4).

The optimal temperature and temperature difference temporal profiles for the optimization problem are presented in Fig. 2. As can be clearly observed from Fig. 2, the risk of product fragility is minimized because the temperature profiles are the most uniform when considering the internal heat transfer resistance (low product thermal conductivity), and therefore, the temperature differences are lower, which is preferred from the point of view of quality aspects. However, this phenomenon is achieved with longer residence times, as mentioned above.

3.2.1. Increment of product demand

A possible increment of the annual demand is analyzed for this case study to investigate the distribution of the annual production. For this case, a higher minimum demand of 50,000 kg/year is proposed for the same conveyor belt capacity (CBC = 0.5 kg/m). Table 5 reports the new optimal values. In this scenario, the annual production is distributed such that the maximum production capacity is used during the summer and mid-season because of the lower operating costs (power and utility costs) per kilogram of product, and, as expected, the remainder of the production takes place under the winter scenario.

If the effect of the conveyor belt capacity is analyzed, the maximum and minimum loads per meter of belt are introduced in the model as upper and lower bounds, respectively. Table 6 reports the optimal values for a product demand of 50,000 kg/year.

As expected, the value of the conveyor belt capacity raises its upper bounds in all of the operating modes to minimize the total operating costs. Therefore, the production rate per hour is higher than the previous results reported in Table 5. Then, as a consequence, the operating costs per kilogram of product are lower due to the increment of product per meter of belt and decreased for the required operating time for fabrication at a specific rate. As in the previous results, the annual production is accumulated in summer by operating at total capacity using the maximum operating time per day, and the remainder of the demand is met under the mid-season scenario.

It is possible to conclude that the model captures the existing trade-off between costs and operating time for a given capacity. The desired production rate is first accomplished under the operating mode with the lowest operating costs and then in the following operating mode with the next lowest operating costs, depending on the product demand.

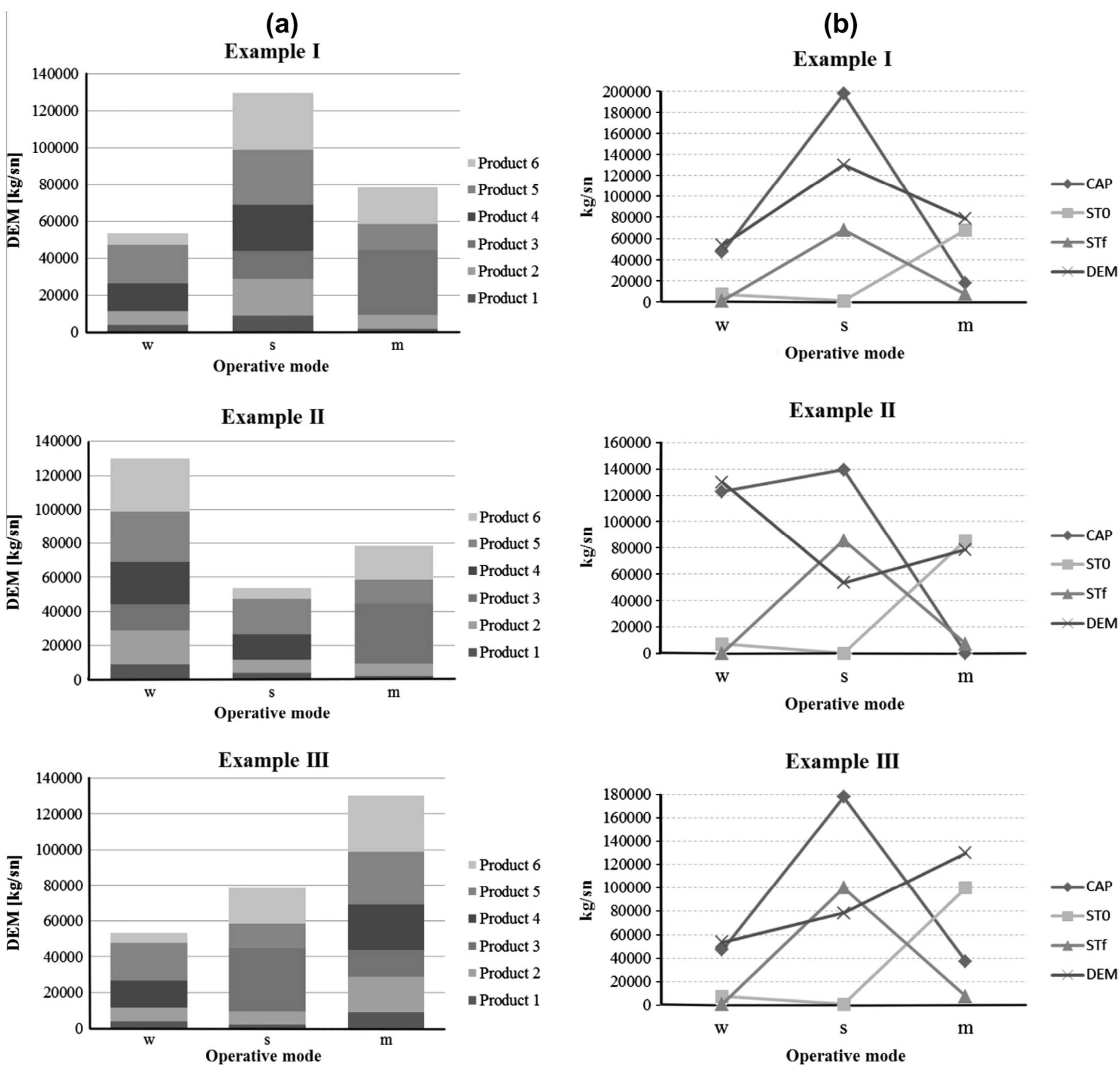

Fig. 3. (a) Distribution demand for each product. (b) Cumulative demand, production, and stock profiles.

3.3. Case Study III: production planning for the six products

In this section, the use of the model for a preliminary scheduling of the total annual production, for determining the level of production for each product during each period and for minimizing the operating costs is presented. For this case study, the following hypotheses for the cumulative production approach are:

- Demand and production rates are average values during each operating mode.
- Physical resources are fixed during the annual horizon.
- The total annual demand is fixed; however, the cumulative demand fluctuations among the seasons are considered.
- There is no cost involved in holding seasonal inventories.
- The inventory restrictions are provided at the end of each season.

The cooling equipment unit, as mentioned earlier, is used to produce six different flavored hard candy products along the annual time horizon.

For this optimization problem, the storage capacity constraint per season reported in Table 4 is used, which is provided as a mean value and fixed at the end of each season. Therefore, the production planning will report the rate of production per season and the total production to satisfy the annual demand reported in Table 3 and considering the initial stock.

In fact, the same optimal operating conditions, as in previous sections, are obtained for all the operating modes: $va_{k,i} = 1.2$ m/s, $Ta_{T\ k,i} = 33.66$ °C, $\theta_{k,i} = 1064.72$ s. Because the global composition of the hard candies is considered approximately equal, the operating conditions of the cooling tunnel during the cooling stage are the same for the fabrication of the six products.

Different cumulative demands for each product, shown in Fig. 3a, are proposed in three examples to illustrate the case study. Fig. 3b corresponds to the production and stock profiles required to satisfy different demand profiles. As can be observed from Fig. 3b, the production and stock profiles are the same independent of the distribution of the annual demand and according to the hypothesis set and data provided in Fig. 3a. Thus, because the operating costs are the lowest during the summer, the model is solved by producing the maximum possible capacity during this season, depending on the demand profile, and producing only the amount of product required to satisfy the demand during the winter operating mode because of its higher operating costs. Therefore, the initial stock profile always has the same tendency. In the winter, its value corresponds to the reported and initial data (Table 3), and, in the summer, the initial stock is null because of the higher operating costs during the winter season. Then, during the mid-season, the value of the stock is high because the production is higher than the demand during the summer because of the exploitation of resources and the lower operating costs, as mentioned before.

In addition, for all of the examples presented in Fig. 3, the model solves production at the maximum conveyor belt capacity for all of the operating modes and products and, therefore, at the maximum production rate and, consequently, for a shorter operating time per day. These results evidence the trade-off between the operating costs and the operating time when scheduling the production plan.

Also, when analyzing the operating time schedule for the examples, the model solves the production rate for the maximum operating time per day (24 h), leaving idle days in some operating modes and depending on the level of production. Table 7 presents these results for the examples. Furthermore, the daily operating time per product is distributed in proportion to the level of production of the operating mode.

Table 7
Optimal values of the main variables for the different examples.

	Example I	Example II	Example III
$CBC_{k,i}$ (kg/m)	0.9[aa]	0.9[a]	0.9[a]
OD_w (days)	22	56	22
OD_s (days)	90[a]	64	81
OD_m (days)	8	0	17
DT_w (h)	24[a]	24[a]	24[a]
DT_s (h)	24[a]	24[a]	24[a]
DT_m (h)	24[a]	0	24[a]

[a] Upper bound.

3.3.1. Influence of the stock capacity on the optimal solutions

The annual operating costs associated with the optimal production plan for each example previously presented are: $1,989,780 (Example I), $1,989,926 (Example II), and $1,989,789 (Example III). If the production plans are optimized without considering the possibility of stock capacity, the annual operating costs increase by 2.38%, 3.43%, and 2.91%, respectively. Thus, it is concluded that the stock capacity has a weaker influence on the total operating cost than the operation of the cooling tunnel under conditions that differ from the optimal ones, as discussed in the previous case study.

4. Conclusions

This paper presents a nonlinear programming model for the production planning of hard candies as a function of the product demand throughout the time horizon and considering quality aspects related to temperature distribution during the cooling stage. By applying a mathematical programming technique and robust optimization algorithms, the optimal production plans for six different types of hard candies and operating conditions were determined.

The main advantages of the developed model are that it allows for the simultaneous optimization of the trade-offs among quality aspects, operating costs, operating conditions, production level, and stock. A phenomenological model for the heat transfer in the hard candies using the central finite difference method as a discretization method, including the variations of the air conditioning system under different operating modes or seasons and cost aspects related to utilities, raw materials, process conditions, and product quality, was developed.

The optimization model has been precisely applied in different case studies to analyze:

- The annual production distribution for one product under the three operating modes analyzing the possible increment of production.
- The production of six products considering different cumulative demands.

The optimal solution depends on the specific costs used. For the cost data considered in this paper, the results demonstrate that depending on the cumulative demand, the model solved production at a higher level than the corresponding demand during the summer season because, in this operating mode, the operating costs are the lowest. Therefore, final stocks always exist in this season. In contrast, the winter season presents higher operating costs, and hence, the model is solved by producing just enough to satisfy the demand, with the final stock always being null.

Also, independent of the weather conditions on the operating modes, the model solved that the operating conditions in the cooling tunnel that satisfied the internal temperature constraints were: $va_{k,i} = 1.2$ m/s (its lower bound, which corresponds to the minimum safe operating velocity of the fan), $Ta_{T\ k,i}$ = 33.66 °C and $\theta_{k,i}$ = 1064.72 s. If the cooling tunnel is operated under conditions that differ from the optimal ones, the seasonal cost of producing one product increases by 23% compared with the cost corresponding to the optimal operating conditions. These results reflect the importance of having an optimization model that takes into account quality and cost variables simultaneously. In contrast, the possibility of considering stock capacity has less impact on the annual operating cost.

Further research should focus on the implementation of the model considering the costs of holding seasonal inventories and their possible immediate distribution.

Independent of the results presented in this work, the optimization model, under the mentioned assumptions, takes into account the implicit composition and thermo-physical properties of hard candies and the quality aspects and operating costs involved in the task of obtaining preliminary production plans, which is a novel approach in the food engineering field.

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