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A Mixed Integer Linear Programming model for simultaneous design and scheduling of flowshop plants

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ABSTRACT

Models representing batch plants, especially flowshop facilities where all the products require the same processing sequence, have received much attention in the last decades. In particular, plant design and production scheduling have been addressed as disconnected problems due to the tremendous combinatory complexity associated to their simultaneous optimization. This paper develops a model for both design and scheduling of flowshop batch plants considering mixed product campaign and parallel unit duplication. Thus, a realistic formulation is attained, where industrial and commercial aspects are jointly taken into account. The proposed approach is formulated as a Mixed Integer Linear Programming model that determines the number of units per stages, unit and batch sizes and batch sequencing in each unit in order to fulfill the demand requirements at minimum investment cost. A set of novel constraints is proposed where the number of batches of each product in the campaign is an optimization variable. The approach performance is evaluated through several numerical examples.

1. Introduction

Batch processes are characterized by their flexibility and ability to produce low-volume products, sharing the same equipment. The main classification of batch processes is based on the production path involved for products manufacture: flowshop or multiproduct batch plants are employed when all the products require all the stages following the same sequence of operations, while, in jobshop or multipurpose batch plants, products can follow different processing sequences, not necessarily employing all the stages.

In this paper, the study is focused on flowshop or multiproduct batch plants. The general design problem of this type of facilities consists of determining: (a) the plant configuration, i.e. the number of parallel units required for each stage and, sometimes, the assignment of intermediate storage between stages; (b) the unit and storage vessel sizes; and (c) the number and size of batches for all the products, in order to optimize an economic performance measure while satisfying constraints on the production requirements in the available time horizon. This problem has been generally formulated as a mixed-integer non linear programming (MINLP) model [1].

On the other hand, taking into account that all products, usually with similar recipes, are processed using the same stages, production must be scheduled in order to improve the plant performance and avoid large inventory levels. According to Papageorgiou and Pantelides [2], the campaign mode operation is particularly appropriate for plants working under stable demand patterns over long planning horizons. The plant can be operated with mixed product campaigns (MPC), where in

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Nomenclature

Sets

I	set of products
J	set of stages of batch plant
K_j	set of available identical parallel units in each batch stage j
L	set of production slots
SV_j	set of discrete sizes for stage j

Indices

i	product
j	stage
k	unit
L	slot
n	number of batches of a product
p	discrete size for batch unit
P_j	number of available discrete sizes for a unit of stage j

Parameters

CCF	capital charge factor
CT^{UP}	upper bound for the variable CTC (cycle time)
H	time horizon
NBC_i^{UP}	maximum number of batches of product i in the composition of a campaign
Q_i	demand of product i in the time horizon H
SF_{ij}	size factor of product i in stage j
t_{ij}	processing times for each product i in stage j
VF_{jp}	discrete size for batch units in stage j

Binary variables

v_{jp}	binary variable that denotes if the units of stage j have size p
w_{ijpn}	binary variable that represents the bilinear term $v_{jp}\, X_{in}$
ww_{ijpn}	binary variable that represents the cross product $w_{ijpn}CT$
X_{in}	binary variable that denotes if the campaign has n batches of product i
Y_{ijkl}	binary variable that assigns product i to slot l of unit k in stage j
Z_{jk}	binary variable that specify if unit k of stage j is employed

Continuous variables

B_i	continuous variable that denotes the batch size of product i
CTC	continuous variable that denotes the cycle time of the campaign
CT_{jk}	continuous variable that denotes the cycle time of unit k at stage j
e_{jkp}	continuous variable that represents the bilinear term $Z_{jk}\, v_{jp}$
NBC_i	continuous variable that represents the number of batches of product i included in the campaign
NB_i	continuous variable that represents the total number of batches of product i in the time horizon
NC	continuous variable that represents the number of times that the campaign is repeated
TF_{jkl}	continuous variable that denotes the finishing time of slot l in unit k of stage j
TI_{jkl}	continuous variable that denotes the starting time of slot l in unit k of stage j
V_j	continuous variable that denotes the size of a batch unit in stage j

each campaign various batches of different products are manufactured and the same batches arrangement is cyclically repeated over the time horizon. In this case, several decisions must be made at the scheduling level: the number of batches of each product involved in the production campaign and their sequencing in order to optimize a suitable performance measure. This problem represents an important challenge given the combinatorial nature of scheduling decisions. Most formulations for scheduling belong to the set of NP-complete problems [3] and, despite significant advances in optimization approaches, there is still a number of major challenges and questions that remain unsolved [4]. When the plant design is not a priori provided, the problem becomes worse, because both the number of batches of each product and the available equipment are unknown. In this last case, in order to simplify the model, most of the formulations assume single product campaigns (SPC), where all batches of a given product are manufactured before switching to another product. However, this proposal is not appropriate for the production or commercial points of view.

The multistage nature of a batch plant allows four different storage options: (i) unlimited intermediate storage (UIS); (i) finite intermediate storage (FIS); (iii) no intermediate storage (NIS); (iv) zero wait (ZW). In both the NIS and ZW modes, there is no storage between stages, while for UIS and FIS modes intermediate storage is provided. Intermediate materials can wait in storage with unlimited capacity in UIS and limited capacity in FIS, or in the current processing unit in NIS. In the ZW mode, the batch must be immediately transferred to a unit of the downstream stage after being processed.

In this work, a detailed MILP mathematical formulation for the simultaneous design and scheduling of flowshop plants is addressed. Unlike the previous proposed approaches, the number and size of batches of each product in the campaign, the number and size of process units for each stage and the production sequence in each unit are model variables. Considering that the number of units in each stage and the number of batches in the campaign are unknown, the scheduling formulation represents a great challenge from the modeling point of view. A slot-based continuous-time representation for modeling the assignment of batches to units and their sequencing is employed. Taking into account the combinatorial nature of the scheduling problem, appropriate constraints must be included in the model to assess the cycle time of the production campaign, fulfilling demands over the time horizon. In order to attain a general formulation, multistage facilities are assumed, including out of phase unit duplication at every stage. The unit sizes are restricted to discrete values and an upper bound for the number of batches of each product in the campaign is provided, in order to keep the linearity of the problem.

Therefore, the proposed MILP model represents a novel approach where the simultaneous optimization of plant design and scheduling considering MPCs can be solved to global optimality with reasonable computational effort. The approach performance is assessed through the several examples where different values for the key problem parameters are analyzed.

This paper is organized as follows. In the next section a literature review is presented. In Section 3 the problem description and its main assumptions are stated; in Section 4 the mathematical formulation is described in detail, while in Section 5 several examples are solved in order to illustrate the proposed approach. Finally, conclusions and future works are discussed in Section 6.

2. Literature review

Many papers have been published dealing with design and scheduling of multiproduct batch plants, but usually as decoupled problems. Moreover, in order to simplify the formulation, when both problems have been simultaneously addressed, the simplest scheduling policy has been considered, namely SPC [1,5,6]. Although this assumption greatly simplifies the design model, it is inappropriate from the commercial point of view, where a more steady supply of products is required.

Among the few contributions that simultaneously have considered design and scheduling with MPC, Birewar and Grossmann [7] proved that MPC can reduce idle times, increasing equipment utilization. They proposed Mixed Integer Linear Programming (MILP) models for the minimization of the cycle time with MPC, involving sequence-dependent transition times. In order to simplify the formulation, only one unit per processing stage was admitted. In a later work [8], they incorporated new design alternatives in order to attain savings in the capital cost of batch plants. They considered simultaneous synthesis, design and scheduling in a MINLP formulation. However, their model for the case of MPC with Zero Wait (ZW) transfer policy did not assume parallel units. Later, in order to achieve MILP models, Voudouris and Grossmann [9] reformulated these problems assuming a set of available discrete sizes for the units to be installed in the plant, but maintaining previous assumptions. Finally these authors extended that approach in order to incorporate parallel units. However this last approach was limited with an approximation for the cycle time [10]. Corsano et al. [11] proposed an approach for the simultaneous synthesis and design of multiproduct batch plant considering MPC, but they used a heuristic resolution strategy where a limited set of preselected campaigns were included.

The scheduling problem of flowshop plants has also gained much attention in the literature. There are several excellent review papers [4,12–17] where different modeling approaches and resolution strategies have been analyzed. Most often scheduling formulations are expressed as mathematical programming models, especially through MILPs. Several papers have been published dealing with short-term scheduling of multiproduct batch plants with parallel units but considering Unlimited Intermediate Storage (UIS), which simplifies the formulation but is an unreal policy [18–23].

Many works has also resorted to heuristics approaches taking into account the NP nature of scheduling problem. For example, Soewandi and Elmaghraby [24] compared several heuristics procedures to minimize the makespan. On the other hand, Nowicki and Smutnicki [25] as well as Wardono and Fathi [26] used Tabu Search for this problem, while Hop and Nagurur [27] proposed a resolution strategy based on genetic algorithms.

Recently, focusing on mathematical programming based-approaches, Liu and Karimi [28,29] developed and evaluated a series of slot-based and sequence-based MILP formulations for scheduling of multistage, multiproduct batch plants with parallel units considering UIS and ZW policies. Given the plant structure and the number of batches of each product to be processed in the available time horizon, the proposed model assigns batches to units in each stage in order to minimize the makespan. Each stage is treated as a black box in which batches enter and exit without distinguishing the units involved. Hence, these kinds of formulations cannot be incorporated to the design problem of multiproduct batch plants where the units sizing must be also determined and makespan is not an appropriate performance measure. Taking into account that long time horizons are used, formulations using MPC are more suitable and, therefore, the calculation of the cycle time must be modeled in order to attain a more appropriate representation.

Summarizing the literature review in this area, articles have often focused on one problem, design or scheduling. Design works have strongly simplified the formulation resorting to SPC. On the other hand, scheduling articles have assumed a given plant and a predetermined number of batches. Though scheduling is a difficult problem, if the number of batches and the structure of the plant are unknown, the model complexity is severely increased. Previous approaches have not posed solutions considering MPC, typically appropriate for stable scenarios and a good approximation when a new plant has to be designed. Also, different performance measures have been used (makespan, earliness, tardiness, etc.) but not the minimization of the cycle time, appropriate measure when long term time horizons and MPC are contemplated.

3. Problem description

Fig. 1 shows a flowshop batch plant with N_J stages. The plant processes a set of N_I products that follow the same production sequence.

The plant can be operated using a SPC or MPC approach. Fig. 2 shows an example of SPC for a plant composed by three stages and one unit per stage. It can be noted that the total number of batches of product i_1, NB_{i_1}, are consecutively manufactured before changing to product i_2. In the MPC case, a campaign can include several batches of all the products. This campaign is repeated NC times over the time horizon. Fig. 3 shows an example of a multiproduct batch plant that produces three products and involves three stages, with one unit per stage. The campaign composition is two batches of i_3, and one of i_1 and i_2, and the batches sequencing is i_3–i_2–i_1–i_3 on each unit.

The integrated problem for design, assignment and sequencing of batches for a flowshop plant can be stated as follows:

Given:

(a) a batch plant composed by a set J of processing stages,
(b) a set K_j of identical parallel units that can be allocated in each batch stage j,
(c) a set I of products that must be manufactured,
(d) the processing time of product i at stage j, t_{ij}, and the size factor of product i for stage j, SF_{ij}, i.e. the size needed at stage j to produce 1 kg of final product i,

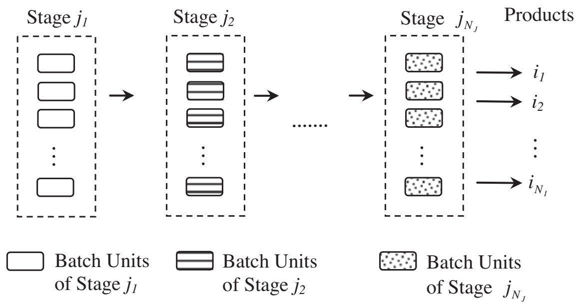

Fig. 1. Flowshop plant.

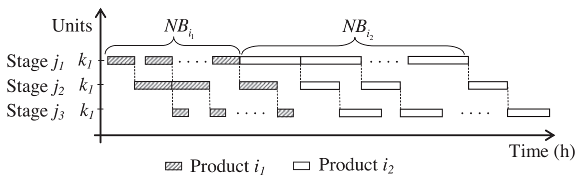

Fig. 2. Gantt chart for single product campaign.

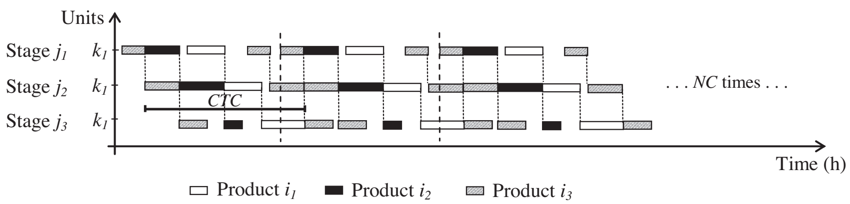

Fig. 3. Gantt chart for mixed production campaign.

(e) a set $SV_j = \{VF_{j1}, VF_{j2}, \ldots, VF_{jP_j}\}$ of P_j available discrete sizes for stage j,
(f) a maximum number NBC_i^{UP} of batches of product i in the composition of a campaign,
(g) a time horizon H over which product demands must be satisfied, and
(h) Q_i, the demand of each product i over the time horizon H.

Determine:

(a) the batch plant design (number of parallel units operating out of phase in each stage and their corresponding sizes, number and size of batches for each product),
(b) the configuration of the production campaign,
(c) the assignment of batches to processing units,
(d) the sequencing of batches in each processing unit,
(e) the number of times that the campaign is cyclically repeated over the time horizon.

The performance measure is minimizing the batch plant investment cost fulfilling the product demands over the time horizon.

In the proposed model, the ZW transfer policy between stages is adopted. Also, batch transfer times between units are very small and consequently negligible or included in the processing times.

The plant operates in MPC mode, i.e. the production campaign is composed by a set of batches of the different involved products. The number of batches of each product i in the campaign is a decision variable. Only an upper bound for the number of batches of product i in the campaign, NBC_i^{UP}, is established.

In order to model the assignment of batches to units and its sequencing, an asynchronous slot-based representation is introduced. The slots correspond to time intervals of variable length where the batches will be assigned. If a batch of product i must be processed on unit k of stage j, then it must be assigned to exactly one slot l in that unit. Moreover, in each slot of a specific unit at most one batch of product can be processed.

The asynchronous representation requires postulating a priori an appropriate number of slots for each unit that integrates the plant. This is not a trivial decision and represents an important trade-off between optimality and computational performance. Taking into account that the number of batches of each product i in the campaign is a decision variable, with upper bound equal to NBC_i^{UP}, the same number of production slots for each unit is defined. Let L be the set of postulated slots for each unit of each stage, then the cardinality of L is $|L| = \sum_i NBC_i^{UP}$. This assumption can be improved in order to reduce the resolution time. It is also assumed that the assignment of batches to slots follows the same order in all the stages.

Taking into account the numerous ways in which batches can be assigned to units and slots, the sequencing of batches represents a problem of combinatorial nature. In this paper, in order to reduce alternative solutions and hence decrease computational effort, assumptions about batches preordering and utilization of units and slots in each stage have been included in the model formulation.

4. Model formulation

As already mentioned, the proposed model simultaneously solves the design and scheduling of multiproduct batch plants considering MPC, which have been usually treated in decoupled form. This section describes the basic constraints and major characteristics of the mathematical formulation.

In the following subsections, constraints for units and batches assignment, plant design, and production scheduling are described. The objective function that minimizes the investment cost for the plant is also presented.

4.1. Allocation constraints

In order to assign products to slots and units, the following binary variables are defined:

$$Z_{jk} = \begin{cases} 1 & \text{if unit } k \text{ of stage } j \text{ is employed} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ijkl} = \begin{cases} 1 & \text{if product } i \text{ is assigned to slot } l \text{ and} \\ & \text{processed in unit } k \text{ of stage } j, \\ 0 & \text{otherwise} \end{cases}$$

Since parallel units in each stage j are identical and the number of postulated slots for each unit is the same, a batch of product i can be processed at any slot of any unit k ($k \in K_j$) with the same processing time and size factor. In order to reduce the search space, it is assumed that units and slots of each stage are utilized in ascending order. Then, the following constraint establishes that unit $k+1$ is used only if unit k has been already allocated:

$$Z_{jk} \geqslant Z_{jk+1}, \quad \forall j,\ k, k+1 \in K_j \tag{1}$$

Similarly, the following inequality assures that slot $l+1$ is occupied only if slot l has been already allocated:

$$\sum_i \sum_k Y_{ijkl} \geqslant \sum_i \sum_k Y_{ijkl+1}, \quad \forall j, 1 \leqslant l \leqslant |L| - 1 \tag{2}$$

Taking into account that if $Z_{jk} = 0 \Rightarrow Y_{ijkl} = 0$, i.e. if unit k of stage j is not utilized then this unit is not employed to process any batch, the assignment variables Z_{jk} and Y_{ijkl} can be linked in such a way that Z_{jk} is activated when Y_{ijkl} is equal to one. This condition is guaranteed by the constraint:

$$Y_{ijkl} \leqslant Z_{jk}, \quad \forall i, j,\ l,\ k \in K_j \tag{3}$$

Also, when unit k of stage j is employed, then at least a batch of product in a slot must be processed in that unit. Thus,

$$\sum_l \sum_i Y_{ijkl} \geqslant Z_{jk}, \quad \forall j, k \in K_j \tag{4}$$

In order to reduce the search space, assumptions about slots utilization are introducing in this formulation. Slot l can be only assigned for processing one product at the most in a unit of stage j. Hence, the following constraint must be considered:

$$\sum_i \sum_k Y_{ijkl} \leqslant 1, \quad \forall j, l \tag{5}$$

In particular, the constraint

$$\sum_i \sum_l 2^l Y_{ijkl} \geqslant \sum_i \sum_l 2^l Y_{ijk+1l}, \quad \forall l, j, k, k+1 \in K_j \tag{6}$$

is used in order to reduce the search space, and it establishes that the succession formed by the weighted sum of the slots occupied in each unit of a stage forms a decreasing succession [30].

As a result of Eq. (5), the total number of slots used in each stage j is $\sum_i NBC_i$, where NBC_i is the number of batches of product i included in the campaign. Due to the overestimation of the postulated slots in each unit, some slots may be empty.

Finally, a preordering constraint on batches of different products is established. Each product batch must be assigned to the same slot l in different stages

$$\sum_i \sum_k iY_{ijkl} = \sum_i \sum_k iY_{ij'kl} \quad \forall l, j, j', (j < j') \tag{7}$$

This constraint assures that the assignment of batches to slots follows the same order in all the stages, i.e. a product batch is processed in exactly the same slot in all stages, but not necessarily in the same unit.

In order to define the number of batches of product i, NBC_i, that compose the production campaign, the following binary variable is defined:

$$X_{in} = \begin{cases} 1 & \text{if } \ n \text{ batches of product } i \text{ are processed in a campaign} \\ 0 & \text{otherwise} \end{cases}$$

The following constraint is posed to ensure that exactly one of the options is selected:

$$\sum_{n=1}^{NBC_i^{UP}} X_{in} = 1, \quad \forall i \tag{8}$$

Therefore,

$$\sum_{n=1}^{NBC_i^{UP}} nX_{in} = NBC_i, \quad \forall i \tag{9}$$

Given the process topology, NBC_i batches of product i must be processed in each stage j. This constraint is represented through the following equation:

$$\sum_l \sum_k Y_{ijkl} = NBC_i, \quad \forall i, j \tag{10}$$

4.2. Timing constraints

In order to obtain the initial and final times of the slots utilized in the plant stages, the following nonnegative continuous variables are introduced: TI_{jkl} and TF_{jkl}. They denote the starting and finishing times, respectively, of slot l in unit k of stage j. The relation between these variables and Y_{ijkl} is represented by the equation:

$$TF_{jkl} = TI_{jkl} + \sum_i t_{ij} Y_{ijkl}, \quad \forall j, l, k \in K_j \tag{11}$$

When no product is assigned to slot l of unit k in stage j, $Y_{ijkl} = 0\ \forall i$, and hence TI_{jkl} and TF_{jkl} are equal.

In order to avoid slots overlapping in each unit, the following constraint is added:

$$TF_{jkl} \leqslant TI_{jkl'}, \quad \forall j, k \in K_j, l, l', (l < l') \tag{12}$$

Besides, if no product is assigned to slot l+1 of unit k at stage j ($Y_{ijk,l+1} = 0\ \forall i$), then the starting time of this slot is enforced to be equal to the finishing time of slot l. Therefore, taking into account that Eq. (12) is satisfied for successive slots in a unit, this new condition is represented by:

$$TF_{jkl} - TI_{jkl+1} \geqslant -M_1 \sum_i Y_{ijkl+1}, \quad \forall j, k \in K_j, 1 \leqslant l \leqslant |L| - 1 \tag{13}$$

where M_1 is a sufficiently large number that makes the constraint redundant when a product is assigned to slot $l + 1$.

According to the adopted ZW transfer policy, when a product i utilizes units k in stage j and k' in stage j+1, the following equation must hold:

$$TF_{jkl} = TI_{j+1k'l}, \quad \forall l, j, j+1 \in J, k \in K_j, k' \in K_{j+1} \tag{14}$$

Taking into account that this constraint must be only satisfied when a batch is assigned to those units, this condition can be expressed through Big-M constraints, as:

$$TF_{jkl} - TI_{j+1k'l} \geqslant -M_2 \left(2 - \sum_i Y_{ijkl} - \sum_i Y_{ij+1k'l} \right) \quad \forall l, j, j+1 \in J, k \in K_j, k' \in K_{j+1} \tag{15a}$$

$$-TF_{jkl} + TI_{j+1k'l} \geqslant -M_2 (2 - \sum_i Y_{ijkl} - \sum_i Y_{ij+1k'l}) \quad \forall l, j, j+1 \in J, k \in K_j, k' \in K_{j+1} \tag{15b}$$

where M_2 is a sufficiently large number that relaxes this constraint when product i does not utilize unit k in stage j or k' in stage $j + 1$.

Considering that the plant operates in MPC mode during the time horizon, the same production sequence will be executed repeatedly. Hence, it is necessary to obtain an expression for the campaign cycle time. Thus, the number of times that the campaign can be repeated in the time horizon is determined.

First of all, having defined the initial and final times of slots for each unit in different stages, the cycle time of a unit k corresponding to stage j is calculated by the following expression:

$$CT_{jk} = TF_{jkl_n} - TI_{jk\tilde{l}_{k_j}}, \quad \forall j, k \in K_j, \tilde{l}_{k_j} = \min \left\{ 1 \leqslant l \leqslant |L| : \sum_i Y_{ijkl} = 1 \right\} \tag{16}$$

where $\tilde{l}_{k_j}$ represents the first slot assigned to unit k of stage j and l_n the last slot in each unit k of stage j.

This equation is rewritten using a Big-M formulation as:

$$CT_{jk} \geqslant TF_{jkl_n} - TI_{jkl} + M_3 \left(\left(\sum_i Y_{ijkl} - 1 \right) - \sum_i \sum_{1 \leqslant l' < l} Y_{ijkl} \right), \quad \forall j, k \in K_j, l \in L \tag{17}$$

$$-CT_{jk} \geqslant -TF_{jkl_n} + TI_{jkl} + M_3 \left(\left(\sum_i Y_{ijkl} - 1 \right) - \sum_i \sum_{1 \leqslant l' < l} Y_{ijkl} \right), \quad \forall j, k \in K_j, l \in L \tag{18}$$

where M_3 is a sufficiently large number.

Therefore, the cycle time of the campaign, CTC, is given by:

$$CTC \geqslant CT_{jk}, \quad \forall j, k \in K_j \tag{19}$$

4.3. Design constraints

In this section appropriate constraints are presented for selecting the number of units in each stage, their sizes, the batch sizes of each product in the campaign, and the number of times that the MPC is cyclically repeated over the time horizon.

Let V_j be the batch unit size of stage j, B_i the batch size of product i and SF_{ij} the size factor of product i in stage j, i.e. the required size in the stage j to produce a mass unit of product i, then:

$$V_j \geqslant SF_{ij} B_i, \quad \forall i, j \tag{20}$$

which means that the unit size of stage j has to be large enough to accommodate all products.

The total number of batches NB_i for each product i in the time horizon depends on the demand of product i, Q_i, and the batch size of product i, B_i, and it is defined by:

$$NB_i = \frac{Q_i}{B_i}, \quad \forall i \tag{21}$$

The number of times that the MPC will be repeated in the time horizon H, NC, is a model variable, and the relation with NB_i is given by:

$$NBC_i NC = NB_i, \quad \forall i \tag{22}$$

In this way, substituting Eq. (21) and Eq. (22) into Eq. (20), the following nonlinear inequality is obtained:

$$V_j \geqslant \frac{SF_{ij}Q_i}{NBC_i NC}, \quad \forall i,j \tag{23}$$

As has been mentioned in the problem description section, a set $SV_j = \{VF_{j1}, VF_{j2}, \ldots, VF_{jP_j}\}$ of available discrete sizes for units of stage j is proposed. Therefore, the following binary variable selects a discrete value for the variable V_j:

$$v_{jp} = \begin{cases} 1 & \text{if units of stage } j \text{ have size p} \\ 0 & \text{otherwise} \end{cases}$$

Then, the unit size of stage j is given by:

$$V_j = \sum_p v_{jp} VF_{jp}, \quad \forall j \tag{24}$$

where

$$\sum_p v_{jp} = 1, \quad \forall j \tag{25}$$

Substituting Eq. (9) and Eq. (24) into Eq. (23), the following constraint is obtained:

$$NC \geqslant \sum_p \sum_{n=1}^{NBC_i^{UP}} \frac{SF_{ij}Q_i}{VF_{jp}n} v_{jp} X_{in}, \quad \forall i,j \tag{26}$$

This constraint is nonlinear because of the bilinear term $v_{jp}X_{in}$. In order to eliminate this non linearity, a new binary variable w_{ijpn} is defined, and it takes value 1 if both v_{jp} and X_{in} are 1, and 0 otherwise. Using propositional logic, this condition is enforced by the expression:

$$w_{ijpn} \geqslant v_{jp} + X_{in} - 1, \quad \forall i,j,p, 1 \leqslant n \leqslant NBC_i^{UP} \tag{27}$$

Therefore, constraint (26) is reduced to the linear inequality:

$$NC \geqslant \sum_p \sum_{n=1}^{NBC_i^{UP}} \frac{SF_{ij}Q_i}{VF_{jp}n} w_{ijpn}, \quad \forall i,j \tag{28}$$

As previously mentioned, the campaign is cyclically repeated over the time horizon. Then, the cycle time of the campaign CTC multiplied by the number of times that the campaign is repeated, NC, must to be less than or equal to the time horizon H. Therefore, using Eq. (28), the following constraint is posed:

$$CTC \sum_p \sum_{n=1}^{NBC_i^{UP}} \frac{SF_{ij}Q_i}{VF_{jp}n} w_{ijpn} \leqslant H, \quad \forall i,j \tag{29}$$

As cycle time not depends of p and n, Eq. (29) is rewritten in the following way:

$$\sum_p \sum_{n=1}^{NBC_i^{UP}} \frac{SF_{ij}Q_i}{VF_{jp}n} w_{ijpn} CTC \leqslant H, \quad \forall i,j \tag{30}$$

In order to avoid non linearity in Eq. (30), a new nonnegative continuous variable ww_{ijpn} is introduced to represent the cross product $w_{ijpn}CTC$ [9]. Substituting in Eq. (30), the obtained expression is:

$$\sum_p \sum_{n=1}^{NBC_i^{UP}} \frac{SF_{ij}Q_i}{VF_{jp}n} ww_{ijpn} \leqslant H, \quad \forall i,j \tag{31}$$

with the following additional constraints:

$$\sum_{p}\sum_{n=1}^{NBC_i^{UP}} ww_{ijpn} = CTC, \quad \forall i, j \tag{32}$$

$$ww_{ijpn} \leqslant CTC^{UP} w_{ijpn}, \quad \forall i, j, p, 1 \leqslant n \leqslant NBC_i^{UP} \tag{33}$$

where CTC^{UP} is an upper bound for CTC.

4.4. Objective function

The objective function minimizes the annual investment cost of batch plant, CI, given by:

$$CI = CCF\sum_{j}\sum_{k} Z_{jk}\alpha_j V_j^{\beta_j} \tag{34}$$

where α_j and β_j are appropriate cost coefficients for unit j and CCF is a capital charge factor on the time horizon, which includes an amortization term.

Considering Eq. (24) and taking into account Eq. (25), Eq. (34) can be re-written as:

$$CI = CCF\sum_{j}\sum_{k}\sum_{p} \alpha_j VF_{jp}^{\beta_j} Z_{jk}\, v_{jp} \tag{35}$$

A new variable e_{jkp} is defined to eliminate the bilinear term $Z_{jk}\, v_{jp}$ in Eq. (35). This variable has to be linked to the assignment variables v_{jp} and Z_{jk} such that e_{jkp} take value 1 if both are one, and 0 otherwise. Using propositional logic, this condition is enforced by the expression:

$$e_{jkp} \geqslant v_{jp} + Z_{jk} - 1, \quad \forall j, k \in K_j, p \tag{36}$$

Nevertheless, this new variable can be settled as continuous if the following upper and lower bounds are incorporated:

$$0 \leqslant e_{jkp} \leqslant 1, \quad \forall j, k \in K_j, p \tag{37}$$

Thus, a lineal objective function is obtained:

$$CI = CCF\sum_{j}\sum_{k}\sum_{p} \alpha_j VF_{jp}^{\beta_j} e_{jkp} \tag{38}$$

This objective function subject to the constraints (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (15a), (15b), (17), (18), (19), (24), (25), (27), (28), (31), (32), (33), (36), (37) defines a novel MILP model to obtain simultaneously the optimal campaign sequencing and the plant design.

5. Numerical results

In this Section two examples are developed in order to show the attained solutions and the performance of the proposed approach. All the examples were implemented and solved in GAMS [31] in an Intel Core 2 Duo, 2.64 GHz. The CPLEX solver was employed for solving the MILP problems, with a 0% optimality gap. The number of continuous and binary variables and constraints strongly depends on the number of available units in each stage, the maximum number of batches allowed for each product in the campaign, and the number of discrete options considered for the batch unit sizes. When the number of binary variables is increased, the computational complexity of the problem and consequently the computational resolution time is also increased. Example 2 shows how the model performance and the results vary according to the values of these parameters.

5.1. Example 1

This example considers a batch plant with three stages (j_1,j_2,j_3). The first and second stages can be duplicated up to three units while the third stage can include up to two in parallel units. Four products (i_1, i_2, i_3, i_4) have to be processed in the plant during a time horizon H = 7000 h. The processing time and size factor of each product in each stage, and the product

Table 1
Problem parameters for Example 1.

Product	Processing time: t_{ij} (h)			Size factor: SF_{ij}			NBC_i^{UP}	Demand: Q_i
	j_1	j_2	j_3	j_1	j_2	j_3		
i_1	14	25	7	0.7	0.6	0.5	2	400,000
i_2	16	18	5	0.6	0.7	0.45	2	480,000
i_3	12	15	4	0.7	0.65	0.55	3	850,000
i_4	10	20	5	0.65	0.7	0.5	3	1,200,000

Table 2
Discrete sizes and cost coefficients for Example 1.

	VF_{j1}	VF_{j2}	VF_{j3}	VF_{j4}	VF_{j5}	α_j	β_j
j_1	650	1300	2600	5200	7800	6000	0.6
j_2	700	1400	2800	5600	8400	6000	0.6
j_3	1000	2000	3000	4000	6000	7000	0.7

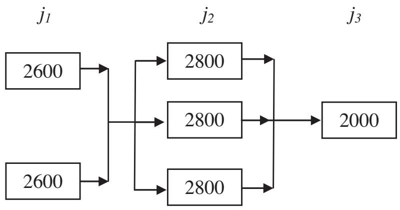

Fig. 4. Optimal plant design for Example 1.

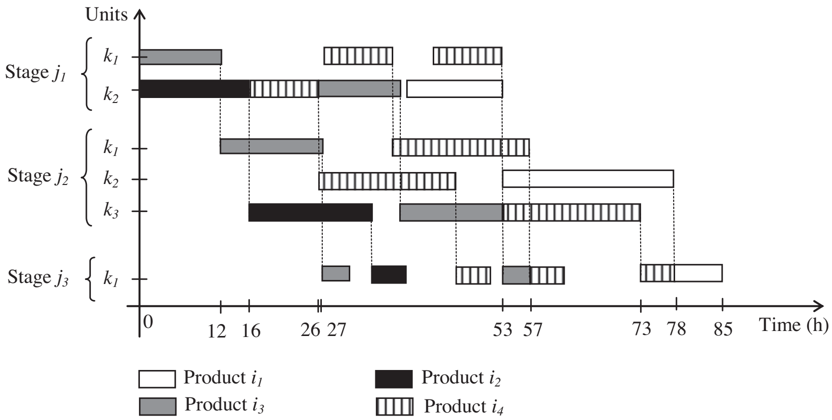

Fig. 5. Optimal Grantt chart for Example 1.

demands are shown in Table 1. The maximum numbers of batches for each product (NBC_i^{UP}) have been proposed taking into account the product demands and they are included in Table 1. The sets of available unit sizes considered in this example are displayed in Table 2 as well as the unit cost coefficients involved in the investment cost.

The optimal solution of the proposed approach for this example consists in a plant with the design shown in Fig. 4. The number of batches in the campaign is 1, 1, 2 and 3 for product i_1, i_2, i_3 and i_4 respectively, i.e., the upper bound for the number of batches in the campaign is reached only for product i_4. The optimal production sequence in each batch unit for the different stages is illustrated in Fig. 5. The cycle time of the optimal campaign is equal to 58 h and the campaign is repeated 120 times.

The total investment cost is $1,220,348 and the model computational performance is displayed in Table 3.

As previously mentioned, simultaneous optimization of design and scheduling problems allows assessing all the trade-offs among the involved decisions. The optimal solution chooses the best plant configuration and production scheduling, by jointly evaluating the different optimization variables. In this case, stage j_2 is time limiting. Therefore, the use of three out-of-phase parallel units reduces the cycle time for this stage. On the other hand, demands should be fulfilled while keeping unit sizes as small as possible. Thus, the number of batches of product i_4 in the campaign is equal to its upper bound,

Table 3
Computational resume for Example 1.

Constraints	1840
Binary variables	503
Continuous variables	909
CPU time (s)	847.3

Table 4
Model parameters for Example 2.

Product	Processing time: t_{ij} (h)			Size factor: SF_{ij} (L/kg)			Demand: Q_i (kg)
	j_1	j_2	j_3	j_1	j_2	j_3	
i_1	14	5	3	0.7	0.6	0.5	750,000
i_2	16	6	2	0.6	0.5	0.4	550,000

Table 5
Maximum number of batches and unit duplication allowed for each instance of Example 2.

	NBC_i^{UP}		$\lvert K_j \rvert$		
	i_1	i_2	j_1	j_2	j_3
Instance 1	4	3	3	3	3
Instance 2	6	4	3	3	3
Instance 3	8	8	3	3	3
Instance 4	8	8	1	1	1

since its demand is the largest one. Therefore, trade-offs like unit duplication vs. cycle time, unit duplication vs. investment cost, batch sizes vs. number of batches of each product in the campaign, etc. are simultaneously evaluated and the model is capable of solving them together.

5.2. Example 2

This example presents four instances for producing two products (i_1, i_2) with different maximum number of batches allowed for each product in the campaign in order to evaluate the performance of the proposed model and the different solutions that can be obtained when MPC is considered. The considered batch plant has three stages (j_1,j_2,j_3) for processing both products in a time horizon H = 7000 h. For instances 1–3, up to three units can be duplicated in each stage while for instance 4 only one unit per stage is allowed. This last instance is performed in order to remark the impact caused by unit duplication in the plant design and consequently in the objective function. Table 4 shows the model parameters and Table 5 the characteristics for each instance. The set of available unit sizes for all the instances is $SV_j = \{500, 650, 750, 875, 1000, 1500, 2000\}$, expressed in litres. It is assumed that the same set is valid for all the stages.

Table 6
Optimal plant design for each instances of Example 2.

	Units out-of-phase			Unit sizes		
	j_1	j_2	j_3	j_1	j_2	j_3
Instance 1	3	1	1	750	650	650
Instance 2	2	1	1	1000	875	650
Instance 3	2	1	1	1000	875	650
Instance 4	1	1	1	2000	2000	1500

Table 7
Optimal scheduling for each instance of Example 2.

	NBC_i		CTC	NC
	i_1	i_2		
Instance 1	3	2	30	233
Instance 2	5	3	60	115
Instance 3	5	3	60	115
Instance 4	7	4	162	41

Table 8
Computational results for each instance of Example 2.

	Constraints	Binary variables	Continuous variables	Objective function	CPU time (s)
Instance 1	1244	310	660	499,300	45.6
Instance 2	1865	430	897	468,700	83.5
Instance 3	3350	670	1371	468,700	931.4
Instance 4	1494	472	933	627,300	1.1

The optimal plant design and campaign composition for each instance are shown in Table 6 and Table 7 respectively, while Table 8 displays the computational performance.

The best economical solution is reached at instances 2 and 3, where the optimal plant design consist in two units out of phase for the first stage, one unit for the second and third stages, and the unit sizes are 1000 L, 875 L, and 650 L, respectively.

Although the optimal number of batches of each product in the campaign is the same in both optimal instances (5 and 3 batches for product i_1 and i_2, respectively), the campaign scheduling is different, and therefore alternative solutions are achieved. Computational performance for instance 3 is worse than for instance 2 since the number of variables and constraints is increased due to a greater maximum number of product batches is considered.

First, analyzing the results of Table 6 and 7, the trade-offs between the different problem elements are clear. In instance 1, the maximum number of batches for the campaign composition is not enough for fulfil the product demands in a plant like that obtained in instances 2 and 3. Therefore, three duplicated units have to be used for stage j_1, and thus, the investment is increased 6.5%. Depending on the parameters allowed for the campaign composition, the cost of the plant changes. Hence, the inclusion of scheduling constraints in this problem is justified due to a strong trade-off among design and scheduling decisions.

The last instance only permits one unit per stage with the objective of showing the impact caused by unit duplication in the plant design and the performance of the production process. The optimal solution for this instance employs larger unit sizes than those of the previous instances since unit duplication is not allowed and the product demands have to be reached in the time horizon. The attained campaign uses seven batches of i_1 and four batches of i_2. In this case, there is no idle time in stage j_1 which is limiting time. Idle times in stages j_2 and j_3 involve the use of larger unit sizes. Therefore, the investment cost for this instance is increased 33.8% compared with instances 2 and 3.

Finally, a single product campaign approach for the same problem (see Table 4 for problem data) was solved in order to compare the solutions. In this case, the optimal solution adopts the same plant configuration of instances 2 and 3. Although both optimal objective functions coincide, through the use of MPC more steady supply of products and reduced stock levels are achieved, as was previously mentioned. If the objective function only considers investment cost like in this formulation, no improvement can be shown. Therefore, new considerations about supply, inventory costs, etc. must be included in future models.

6. Conclusions

This article presents a new approach for simultaneous design and scheduling of flowshop plants. Until now, previous works had generally solved these problems resorting to decoupled models, without taking into account the trade-offs between them. Usually, when design problem is formulated, they assumed SPC and, thus, the formulation is strongly simplified. However, this kind of solutions is not suitable from the commercial point of view since large product stocks are required.

The approach proposed in this work addresses the joint design and scheduling of flowshop plants with MPC production mode. For batch plant design, a maximum number of out of phase duplicated units is considered, and batch unit sizes are selected according to a set of available discrete sizes, as usually found in commercial and industrial practices. For scheduling, a maximum number of batches for each product in the campaign composition is established. In this way, the nonlinearities in the mathematical model are avoided. Therefore, the proposed formulation is addressed as a MILP one, which determines the global optimal solution for the simultaneous flowshop plant design and scheduling.

Taking into account the combinatorial complexity of scheduling models, appropriate preordering constraints have been developed and incorporated in the proposed formulation. This aims at reducing alternative solutions, search space, and thus computational effort.

In short, the proposed formulation simultaneously determines plant configuration, including the number of units at each stage and their sizes, and the number and size of the batches of every product. These batches are produced through MPCs that are cyclically repeated over the time horizon. The campaign configuration includes the allocation of batches to the selected units and their sequencing.

This formulation has been applied to several problems in order to highlight the trade-offs among the different involved decisions and the mathematical characteristics of the model. Also, the performance has been assessed.

Thus, a new modeling strategy has been presented to solve an optimization problem not previously approached with the assumptions here adopted. From the mathematical point of view, future works will be focused on computational performance including major problems.

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