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Abstract In this work, a novel algorithm for trajectory tracking in mobile robots is presented. For the purpose of tracking trajectory, a methodology based on the interpolation of trigonometric functions of the wheeled mobile robot kinematics is proposed. In addition, the convergence of the interpolation-based control systems is analysed. Furthermore, the optimal controller parameters are selected through Monte Carlo Experiments (MCE) in order to minimize a cost index. The MCE is able to find, the best set of gains that minimizes the tracking error. Experimental results over a mobile robot Pionner 3AT are conclusive and satisfactory. In addition, a comparative study of control performance is carried out against another controllers.

Keywords Tracking control · Interpolation · Non-linear multivariable control · Mobile robot

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1 Introduction

Tracking control of wheeled mobile robots (WMR) is one of the most attractive research areas for several decades [4, 8, 20, 42] . As a part of research interests, the trajectory tracking problem is particularly relevant in practical applications. The use of WMR is extensive in fields where transportation, inspection and operation tasks are required (industry, assembly, mining, safety). Another promising application is the design of robotic systems for the assistance of disabled, handicapped or elderly people. Trajectory tracking is important also in cooperative tasks. In formation problems, two or more robots must fulfill together a particular task, such as moving a load or inspect an area [32, 36].

Many researchers have been working on this field for a long period, they have proposed control techniques for target tracking problems, which consist of state feedback control, fuzzy logic control [9, 16, 33, 39], potential field [10], neural network [11], etc. Many WMR models and control schemes have been presented for trajectory tracking purposes. The aim of such schemes is either to utilize a kinematic trajectory tracking controller [4, 30, 38] or to construct an integrated kinematic and dynamic controller [22, 25, 26] for the robot to track a desired trajectory. In order to utilize kinematic trajectory tracking controllers, the kinematics of the nonholonomic WMR are used to generate linear and angular velocity references applied to the robot.

Finite-time tracking control method [27, 40, 41, 44] is a fast control technique, which achieves the desired trajectory in finite time. The authors in Zhang et al. [44] proposes a class of control laws based on cascaded control design. By using cascaded control design, they obtain two subsystems. One subsystem is stabilized by an improved global finite-time control law to relax the strict constraints on the desired velocities. The other subsystem is stabilized by a finite-time sliding mode control law. However, the low efficiency is still a problem of these methods, and the strong constraints on the desired velocities should be strictly satisfied.

Fuzzy logic control can be considered as an effective tool for nonlinear controller design [5, 18, 19, 28]. Traditionally, the fuzzy logic controller has been applied to the development of a complete navigation problem of a mobile robot [1, 13]. In Resende et al. [28] propose a controller that combines the heuristic knowledge of the problem, the sector non linearity approach and the inverse kinematic of the mobile platform. In Li et al. [14] propose a fuzzy target tracking control unit (FTTCU), which comprises a behavior network for each action of the tracking control and a gate network for combining all the information of the infrared sensors. The disadvantages of these methods relies on the amount of information that must be retrieved from the system in order to construct the knowledge base for the control laws.

Furthermore, there are many results that are based on the look-ahead methods [6, 20], where, instead of the center of mass in the wheeled mobile robots, the intersection point of a straight line passing through the middle of the vehicle and an axis of the two wheels is chosen in the configuration of the posture to make use of the feedback linearization technique. However, this approach has the following problem: as the distance between the center of mass and the intersection point becomes larger, the tracking performance will deteriorate. On the other hand, when it becomes smaller, the control input tends to become much larger as it involves the inverse of almost the singular matrix. Thus, it is also desirable to develop the analytic uncertain kinematic model that adopts the center of mass as the configuration of the posture.

All the previous works have reported good results, but the implementation of their controllers can become a not so simple task, due to the prior knowledge that one should have in order to design these algorithms. One different direction in the controller design have been delimited by the use of interpolation methods applied over the kinematic model of the vehicle, combined with numerical methods such as Euler and Runge-Kutta to solve the equations between sample time [29, 30].

This paper provides a positive answer to the above challenging problem. In this work a tracking controller design way based on interpolation methods for trajectory tracking on a wheeled mobile robot is presented. First, the kinematic robot model is approximated through an interpolation method using a series of Taylor with zero order for the trigonometric functions. Then, a controller based on a proportional tracking error approach is proposed. Therefore, the control signals are obtained by solving a system of linear equations. One of the advantages of the proposed approach is that the methodology is based upon easily understandable concepts, and that there is no need of complex calculations to attain the control signal. The trajectory tracking controller structure arises naturally derived through a handcrafted procedure that is inferred by analyzing the continuous mathematical model of the system.

The main contribution of this paper is a methodology for trajectory tracking in mobile robots. In addition, in this work the Monte Carlo (MC) based sampling experiment is implemented for tuning the proposed controllers. The controller parameters can be computed to minimize a cost index, here being determined in simulations using the Monte Carlo Experiment (MCE). In addition, an empirical analysis is included and the theoretical results are validated. The proposed technique is implemented in real time in a Pionner 3-AT robot with good results. The results obtained showed an improvement compared to others controllers of the literature [12, 21, 28]. Another contribution of this paper is the analysis of convergence of the interpolation-based control system.

The paper is organized as follows: Section 2 describes the interpolation based controller design. Section 3 present the convergence analysis of the Proposed Control Algorithm. In Section 4, simulation for tunning parameters through Monte Carlo Experiment are presented. Section 5 presents the experimental results using a mobile robot Pionner 3-AT. Finally, Conclusions are detailed in Section 6.

2 Interpolation Based Controller

Now a method based on interpolation and algebraic conditions for design trajectory tracking controllers is presented. The design method is introduced with a simple interpolation and afterward the order of this is increased as a sort of design parameter. This design does not require any a-priori information of dynamics parameters and model linearization.

To this end, consider the coordinate system in Fig. 1 and the kinematic robot model from Eq. 1, thus

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos(\psi) & 0 \\ \sin(\psi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} \tag{1}$$

where x, y represents the cartesian position of the mobile robot; ψ is the robot orientation; u represents the linear velocity of the mobile robot and r is the angular velocity of the mobile robot. This model has been used in several recent papers [15, 28, 31, 37, 43].

The goal is to find the values of u and r so that the mobile robot may follow a pre-established trajectory (x_{ref}, y_{ref}) with a minimum error. Then, if the reference trajectory $x_{ref}, y_{ref}, \psi_{ref}$ is known, $\dot{x}$, $\dot{y}$, $\dot{\psi}$ in Eq. 1 can be substituted assuming an approaching proportional to the tracking error as in Eq. 2. Where, k_x, k_y and k_ψ are positive constants that allow us to adjust the performance of the proposed control system, and fulfill: $k_x > 0$ $k_y > 0$ and $k_\psi > 0$ to the tracking errors tends to zero, see convergence analysis in Section 3.

$$\begin{bmatrix} \cos(\psi) & 0 \\ \sin(\psi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} \dot{x}_{ref} + k_x(x_{ref} - x) \\ \dot{y}_{ref} + k_y(y_{ref} - y) \\ \dot{\psi}_{ref} + k_\psi(\psi_{ref} - \psi) \end{bmatrix};$$

$$e(t) = \begin{bmatrix} e_x \\ e_y \\ e_\psi \end{bmatrix} = \begin{bmatrix} x_{ref} - x \\ y_{ref} - y \\ \psi_{ref} - \psi \end{bmatrix} \tag{2}$$

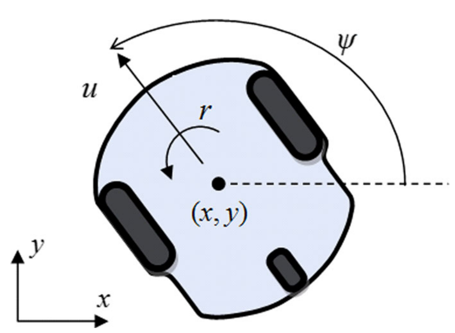


Fig. 1 Geometric description of the mobile robot

As the matrix columns in Eq. 1 are orthonormal, the kinematic state vector can be decomposed as a linear combination in the column b as is space and in the null space associated. From it can be deduced the control action u and r that must be applied such that the mobile robot reaches the desired trajectory.

Considering the above idea, we propose replaced the orientation ψ_{ref} by ψ_{ez},where ψ_{ez} represents the necessary orientation to make the mobile robot tend to the reference trajectory (see Section 3). Then, the trajectory tracking problem is set as solving a system of linear equations,

$$\underbrace{\begin{bmatrix} \cos(\psi) & 0 \\ \sin(\psi) & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} u \\ r \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{x}_{ref} + k_x(x_{ref} - x) \\ \dot{y}_{ref} + k_y(y_{ref} - y) \\ \dot{\psi}_{ez} + k_\psi(\psi_{ez} - \psi) \end{bmatrix}}_{b} \tag{3}$$

Equation 3 is a system with three equations and two unknown variables. The optimum solution of it from mean squares is obtained from normal equation (Eq. 3) (see Strang [34]).

$$B \begin{bmatrix} u \\ r \end{bmatrix} = b \Rightarrow B^T B \begin{bmatrix} u \\ r \end{bmatrix} = B^T b;\ B^T B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^T b = \begin{bmatrix} \Delta_x \cos\psi + \Delta_y \sin\psi \\ \Delta_\psi \end{bmatrix};$$

$$where \begin{cases} \Delta_x = \dot{x}_{ref} + k_x(x_{ref} - x) \\ \Delta_y = \dot{y}_{ref} + k_y(y_{ref} - y) \\ \Delta_\psi = \dot{\psi}_{ez} + k_\psi(\psi_{ez} - \psi) \end{cases} \tag{4}$$

In order to guarantee that the system shown in Eq. 3 has exact solution, constants B_1 and B_2 in Eq. 5 must exist. In other words, **b** EC**B** (Column Space of **B**), then:

$$B_1 \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \\ 0 \end{bmatrix} + B_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_\psi \end{bmatrix} \tag{5}$$

$B_1, B_2 \in \Re$

Thus, from Eq. 5,

$$B_1 \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta_x \\ \Delta_y \\ 0 \end{bmatrix} \Rightarrow \frac{\sin(\psi)}{\cos(\psi)} = \frac{\Delta_y}{\Delta_x} \tag{6}$$

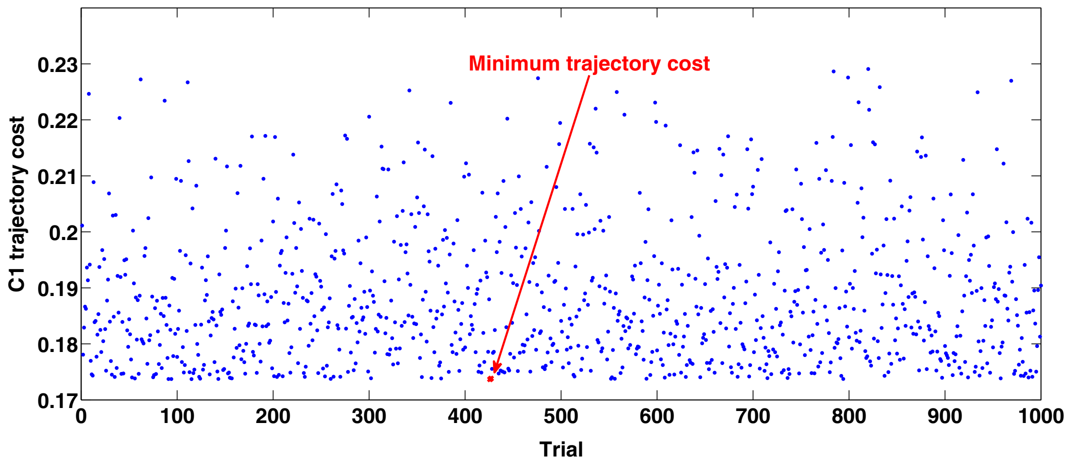


Fig. 2 Trajectory cost of C1 for the 1000 trials

The orientation ψ in Eq. 6 ensures that the system (3) has exact solution, this orientation will be called ψ_{ez} and is computed according to Eq. 7.

$$\psi_{ez} = \text{atan2}\left(\Delta_y, \Delta_x\right) \tag{7}$$

Next, the control actions, that make that the mobile robot reaches and follows the reference trajectory, can be computed solving the system (3) by least squared

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} \Delta_x \cos(\psi) + \Delta_y \sin(\psi) \\ \dot{\psi}_{ez} + k_\psi(\psi_{ez} - \psi) \end{bmatrix} \tag{8}$$

Now, an improvement based on a numerical approach is introduced. Using a series of Taylor with zero order for the trigonometric functions in Eq. 8 about ψ_{ez}, it is valid exactly, for instance, for

$$\cos(\psi) = \cos(\psi_{ez}) - \sin\left(\psi_{ez} + \zeta e_\psi\right) e_\psi \tag{9}$$

with $e_\psi = \psi - \psi_{ez}$ and $0 < \zeta < 1$. So, the control vector in Eq. 8 is constructed with $cos(\psi) \approx cos(\psi_{ez})$ and $sin(\psi) \approx sin(\psi_{ez})$ and referred to as an interpolation of order zero.

Finally, the control action which fulfills the tracking goal of this work is:

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} \Delta_x \cos(\psi_{ez}) + \Delta_y \sin(\psi_{ez}) \\ \dot{\psi}_{ez} + k_\psi(\psi_{ez} - \psi) \end{bmatrix} \tag{10}$$

The control system with controller laws (10) is asymptotic stable as proved later. This controller of order zero is in future referred to as controller 1 (C1).

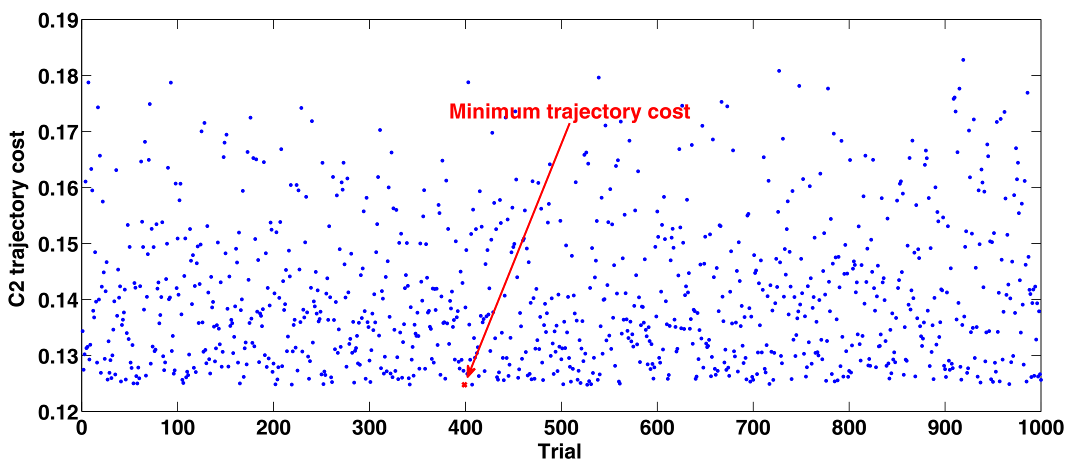


Fig. 3 Trajectory cost of C2 for the 1000 trials

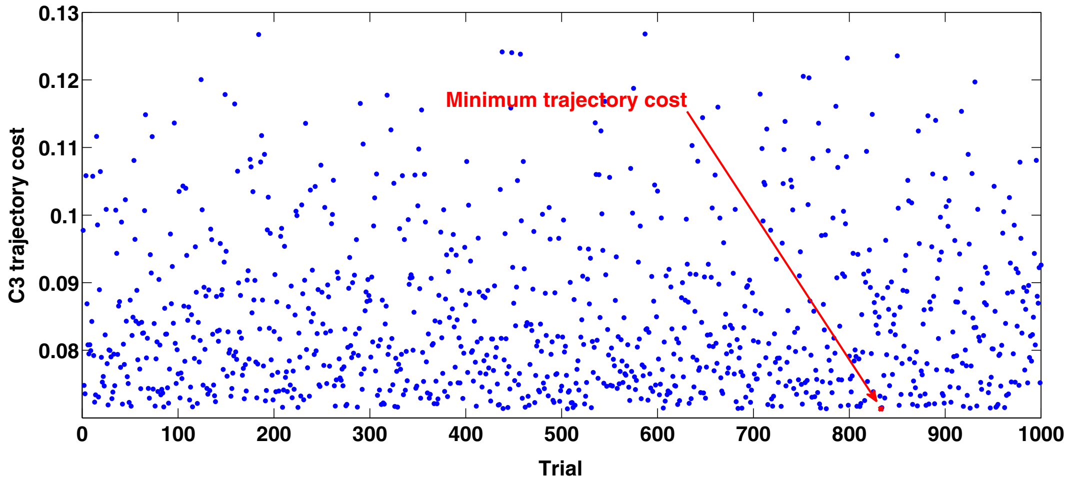


Fig. 4 Trajectory cost of C3 for the 1000 trials

Similarly, if the Taylor expansion for the $cos(\psi)$ and $sin(\psi)$ is truncated at the second term, thus, for instance

$$\cos(\psi) = \cos(\psi_{ez}) - \sin(\psi_{ez})\, e_\psi - \cos\left(\psi_{ez} + \zeta e_\psi\right) \frac{e_\psi^2}{2} \tag{11}$$

then the control action vector (8) can be redefined as

$$\begin{aligned} u &= \Delta_x \left[\cos(\psi_{ez}) - \sin(\psi_{ez})\, e_\psi\right] \\ &\quad + \Delta_y \left[\sin(\psi_{ez}) - \cos(\psi_{ez})\, e_\psi\right] \\ r &= \dot{\psi}_{ez} + k_\psi (\psi_{ez} - \psi) \end{aligned} \tag{12}$$

where ψ_{ez} is calculated numerically using explicit equations as

$$\frac{\sin(\psi_{ez}) + \cos(\psi_{ez})(\psi - \psi_{ez})}{\cos(\psi_{ez}) - \sin(\psi_{ez})(\psi - \psi_{ez})} = \frac{\Delta_y}{\Delta_x} \tag{13}$$

This controller of first order with Eq. 12 is called controller 2 (C2).

For a second-order representation of $cos(\psi)$ and $sin(\psi)$ it is valid, for instance,

$$\cos(\psi) = \cos(\psi_{ez}) - \sin(\psi_{ez})\, e_\psi - \cos(\psi_{ez}) \frac{e_\psi^2}{2} + \sin\left(\psi_{ez} + \zeta e_\psi\right) \frac{e_\psi^3}{6} \tag{14}$$

and hence

$$\begin{aligned} u &= \frac{\Delta_x \left[c(\psi_{ez}) - s(\psi_{ez}) e_\psi - c(\psi_{ez}) \frac{e_\psi^2}{2} \right] + \Delta_y \left[s(\psi_{ez}) - c(\psi_{ez}) e_\psi - s(\psi_{ez}) \frac{e_\psi^2}{2} \right]}{1 + \frac{e_\psi^4}{4}} \\ r &= \Delta_\psi \end{aligned} \tag{15}$$

with $c() = cos(.)$ and $s() = sin(.)$. The computation of ψ_{ez} is performed numerically by solving (16),

$$\frac{\sin(\psi_{ez}) + \cos(\psi_{ez})\, e_\psi - \sin(\psi_{ez}) \frac{e_\psi^2}{2}}{\cos(\psi_{ez}) - \sin(\psi_{ez})\, e_\psi - \cos(\psi_{ez}) \frac{e_\psi^2}{2}} = \frac{\Delta_y}{\Delta_x} \tag{16}$$

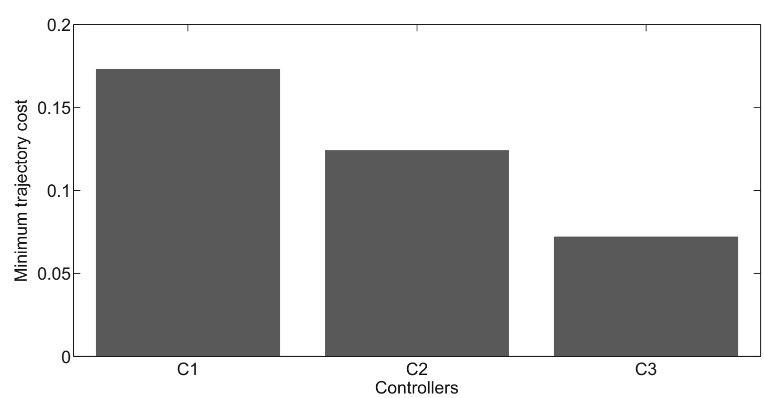


Fig. 5 Minimum cost obtained for each controller

Analogously, this controller with control action vector (15) is denoted by controller 3 (C3) in future.

3 Convergence Analysis

To prove convergence of the interpolation-based control systems, let us, for the sake of simplicity, take the case of controller 1 for analysis. Combining (1) and (10) one achieves the following tracking error system dynamics

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\psi \end{bmatrix} = \begin{bmatrix} -k_x & 0 & 0 \\ 0 & -k_y & 0 \\ 0 & 0 & -k_\psi \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\psi \end{bmatrix} - \delta_e \qquad (17)$$

$$\delta_e = \begin{bmatrix} -\left(\Delta_x \cos(\psi_{ez}) + \Delta_y \sin(\psi_{ez})\right) \sin(\psi_\xi) \\ -\left(\Delta_x \cos(\psi_{ez}) + \Delta_y \sin(\psi_{ez})\right) \cos(\psi_\xi) \\ 0 \end{bmatrix} e_\psi \qquad (18)$$

where δ_e represents a state-dependent perturbation originated in the Taylor approximation of bounded functions. Assuming the reference paths x_{ref} and y_{ref} are smooth, it is clear from Eq. 17 that for $k_\psi > 0$ it holds $\lim_{t\to\infty} e_\psi = 0$. However, the asymptotic behavior of the perturbation δ_e is not so evident. By introducing the reference variables of Eq. 1 in Eq. 18, one attains

$$\delta_e = e_\psi D \begin{bmatrix} e_x \\ e_y \\ e_\psi \end{bmatrix} + e_\psi F$$

$where,$

$$D = \begin{bmatrix} -k_x s(\psi_0) c(\psi_\xi) & -k_y s(\psi_0) s(\psi_\xi) & 0 \\ k_x c(\psi_0) c(\psi_\xi) & k_y s(\psi_0) c(\psi_\xi) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} -\dot{x}_r s(\psi_0) c(\psi_\xi) - \dot{y}_r s(\psi_0) s(\psi_\xi) \\ -\dot{x}_r c(\psi_0) c(\psi_\xi) + \dot{y}_r s(\psi_0) c(\psi_\xi) \\ 0 \end{bmatrix} \qquad (19)$$

with $c() = cos(.)$ and $s() = sin(.)$. Clearly, by uniformly bounded references and e_ψ tending asymptotically to zero, δ_e also vanishes in time asymptotically, and by selected gains $k_x > 0$ and $k_y > 0$, also the errors e_x and e_y tend exponentially to zero. The analysis of convergence with Controllers 2 and 3 is, though more complex, similar as previously. The error e_ψ appears in a form of higher powers in δ_e as long the order of the approximation increases. Consequently, the asymptotic convergence of δ_e occurs faster for this control systems than by the lowest-order controller, and so the exponential convergence of all tracking errors is ensured.

Table 1 Simulations summary of the Monte Carlo experiment

Controller	Minimum cost	Controller parameters
C1	$C^\Phi = 0.173$	$k_x = 0.99$
		$k_y = 1.04$
		$k_\psi = 1.98$
C2	$C^\Phi = 0.124$	$k_x = 1.18$
		$k_y = 0.99$
		$k_\psi = 2.11$
C3	$C^\Phi = 0.072$	$k_x = 1.06$
		$k_y = 1.12$
		$k_\psi = 1.95$

4 Simulation Results

The simulation results for the performance evaluation of the trajectory tracking controllers proposed in the previous section are presented in this section. The simulations are performed using MatLab® software platform and Mobile Sim program provided by the manufacturer Pioneer Mobile Robot.

The controlled system behavior depends on the parameters k_x , k_y and k_ψ. Thus, in this work, and in order to determine values of the controller's parameters,

Fig. 6 The pionner 3AT mobile robot and the laboratory facilities

Table 2 controladoresTrajectory tracking controllers

Controller	Formulation
Resende. C.Z. [28]	$\begin{cases} C = \begin{bmatrix} \cos\psi & -a\sin\psi \\ \sin\psi & a\cos\psi \end{bmatrix} \\ \begin{bmatrix} u \\ r \end{bmatrix} = C^{-1}\left(\begin{bmatrix} \dot{x}_{ref} \\ \dot{y}_{ref} \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right) \end{cases}$
Michalek, M. [21]	$\begin{cases} h_x = k_p e_x + \dot{x} \\ h_y = k_p e_y + \dot{y} \\ r = h_x \cos\psi + h_y \sin\psi \\ u = k_\psi\left(\psi_{ref} - \psi\right) + \dot{\psi}_{ref} \end{cases}$
Kanayama, Y. [12]	$\begin{cases} u = u_{ref}\cos\psi_e + k_x x_e \\ r = r_{ref} + u_{ref}\left(k_y y_e + k_\psi \sin\psi_e\right) \end{cases}$

the MC method used in Cheein and Scaglia [4], is applied.

4.1 Monte Carlo Randomized Algorithm

In the field of systems and control, probabilistic methods have been found useful especially for problems related to robustness of uncertain systems [35]. One of these methods, the Monte Carlo Randomized Algorithm, is widely used in many fields such as the radioactive decay, systems of interacting atoms, the traffic on roads, etc [2]. In the control area, Monte Carlo methods allow to estimate an expectation value and they provide effective tools for the analysis of probabilistically robust control schemes.

Because of its nature, these types of algorithms can give an erroneous result with a non zero probability. So, it could be posed the natural question of how many simulations must be performed to be sure of finding the correct answer. Under a sufficiently large sample size N, a probabilistic statement can be made as shown below:

Theorem 1 [35] *Let $\varepsilon, \delta \in (0, 1)$, where ε is an a priori specified accuracy, and δ, the confidence interval. If*

$$N \geq \left\lceil \frac{\log \frac{1}{\delta}}{\log \frac{1}{1-\varepsilon}} \right\rceil \tag{20}$$

then, the empirical maximum satisfies the following inequality with probability greater than $1-\delta$,

$$\mathrm{Prob}_{\Delta}\left\{J(\Delta) \leq \hat{J}_{\max}\right\} \geq 1-\varepsilon \tag{21}$$

That is,

$$\mathrm{Prob}_{\Delta(1,\ldots,N)}\left\{\mathrm{Prob}_{\Delta}\left\{J(\Delta) \leq \hat{J}_{\max}\right\} \geq 1-\varepsilon\right\} > 1-\delta \tag{22}$$

where J is the performance function and $\hat{J}_{\max}$, the *empirical maximun*. For further details, see Tempo and Ishii [35].

The theorem says that the empirical maximum is an estimate of the true value within an a priori specified accuracy ε with confidence, δ if the sample size N satisfies (20). The algorithm may not produce an approximately correct answer, but the probability of this event is no greater than δ. It is worthy to emphasize that, in Theorem 1, the sample size N is finite and moreover is not dependent on the size of the uncertain set **B**, the structured set of uncertainty matrices, and the probability density function $f_{\Delta}(\Delta)$, but only on ε and δ. In the next Section, Eq. 20 is used to estimate the number of simulations.

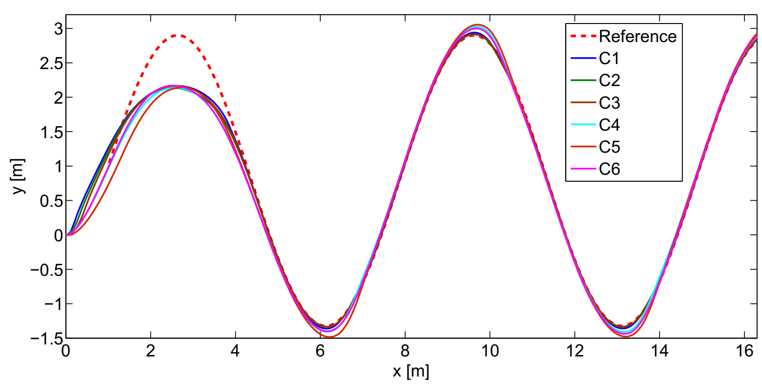


Fig. 7 Tracking trajectory of the mobile robot

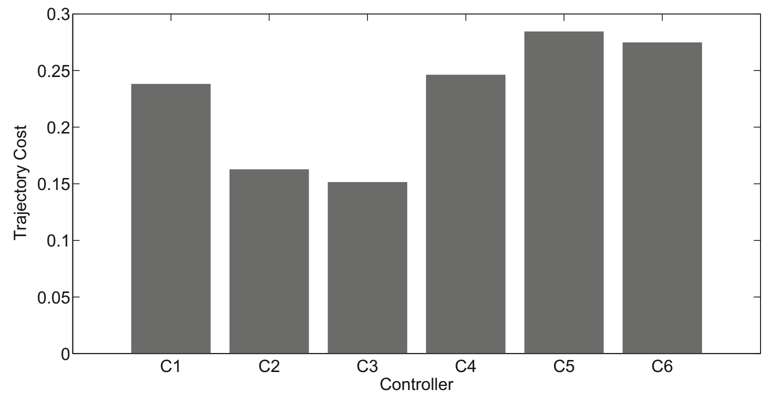


Fig. 8 Cost of the trajectory tracking

4.2 Monte Carlo Experiment

In this subsection, the Monte Carlo method is applied to select an optimal set of controller parameters. Although the optimum is not guaranteed, the Monte Carlo Experiment (MCE) provides an approximate solution based on a large number of trials (M). In this paper, it is adopted a confidence value (δ) of 0.01, and an accuracy of 0.007 (ε). Then, from Eq. 20, it is necessary to make 1,000 simulations. Hence, 1,000 values of each parameter ranging from 0 to 5 were simulated.

The aim of MCE is to find the parameter values (k_x , k_y and k_ψ) optimizing a defined cost function. An idea widely used in the literature is to consider the cost incurred by the error [4]. Let Φ be a desired trajectory, where t_f is the time duration of the trajectory. Let $C_x^{\Phi} = \frac{1}{2}\int_o^{t_f}\left(x_{ref}(t) - x(t)\right)^2 dt$ the quadratic error in the *x-coordinate*; and $C_y^{\Phi} = \frac{1}{2}\int_o^{t_f}\left(y_{ref}(t) - y(t)\right)^2 dt$ the quadratic error in the *y-coordinate*. Thus, the cost function can be represented by the combination of both quadratic errors,

$$C^{\Phi} = \frac{1}{2}\left(\int_o^{t_f}\left(x_{ref}(t) - x(t)\right)^2 dt + \int_o^{t_f}\left(y_{ref}(t) - y(t)\right)^2 dt\right) \tag{23}$$

Thus, the objective is to find k_x , k_y and k_ψ in such way that C^{Φ} is minimized. The MC experiment allows finding empirically the parameter values minimizing the cost function.

The MCE considerations:

– The simulations are performed with all controllers developed in Section 2, C1, C2 and C3, respectively. For each controller 1000 simulations are run.
– All simulations are implemented with the same desired trajectory Φ. In this section, a sinusoidal trajectory is considered.
– For each simulation, the controller parameters are chosen in a random way, such that $0 < k_x <$

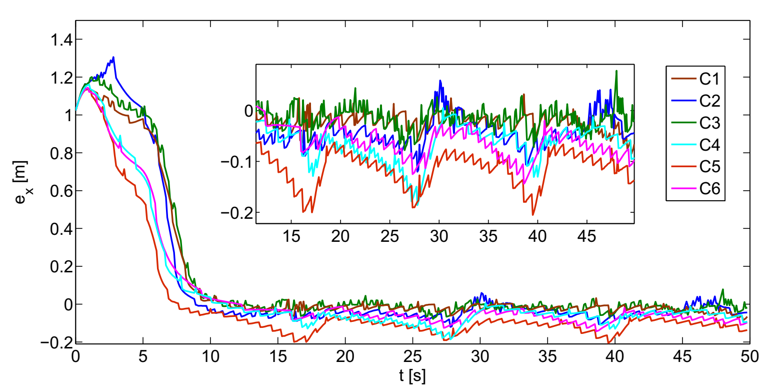


Fig. 9 Tracking error in x

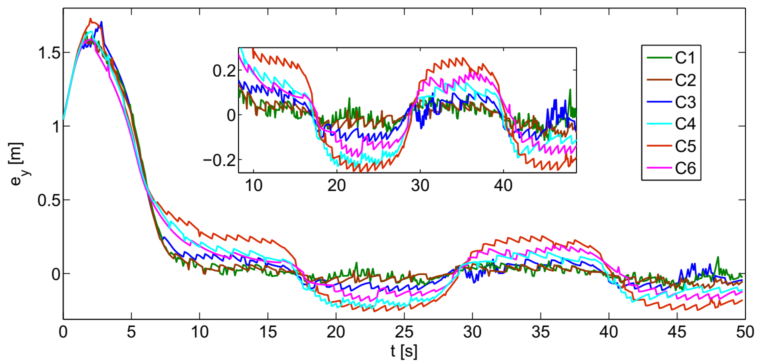


Fig. 10 Tracking error in y

5, $0 < k_y < 5$ and $0 < k_\psi < 5$. The upper bound is empirically chosen considering a trade off between the speed of convergence to zero of tracking errors and a *soft* robot response. The lower bound is chosen such that tracking errors tends to zero (see Convergence analysis in Section 3).

Figure 2 shows the results of the 1000 simulations for C1 controller. The results show the values taken by the cost function for each simulation; scattered values are obtained due to the randomness with which the parameters were chosen in each simulation. The minimum cost obtained for C1 is $C^\Phi = 0.173$. Figure 3 shows that the lowest cost obtained by C2 corresponds to $C^\Phi = 0.124$. The Fig. 4 shows the results of the cost function for 1000 trials when using the controller C3 proposed in this work. For this controller the lowest cost obtained is $C^\Phi = 0.072$. By inspection can be seen, in general all the cost obtained by C2 are under the minimum value obtained by C1. In addition, the cost obtained by C1 and C2 are over than those obtained by C3. The minimum cost obtained for each controller is resumed in Fig. 5.

The analysis of the results shows that the performance of the controller improves as the order of the representation of $cos(\psi)$ and $sin(\psi)$ increases. Thus, the results obtained by the MCE to choose the controller parameters verify the theoretical results obtained in the previous section. Table 1 shows the summary of the results obtained with each controller.

5 Experimental Results

To verify the performance of the proposed controller, we have carried out two experiments on a mobile robot (see Fig. 6). The experiments where performed using a PIONEER 3AT mobile robot. The PIONEER 3AT mobile robot includes an estimation system based on odometric-based positioning system. Updating through external sensors is necessary. This problem is separated from the strategy of trajectory tracking and it is not considered in this paper, [23, 24]. The PIONEER 3AT has a PID velocity controller used to maintain the velocities of the mobile robot at the desired value. In the implementation, the controllers are programmed in MatLab® environment.

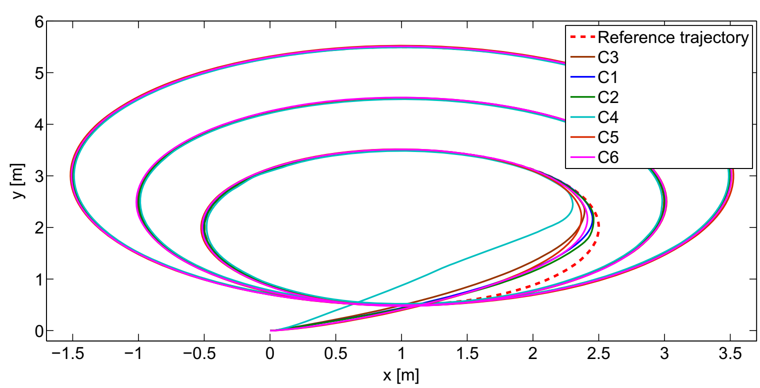
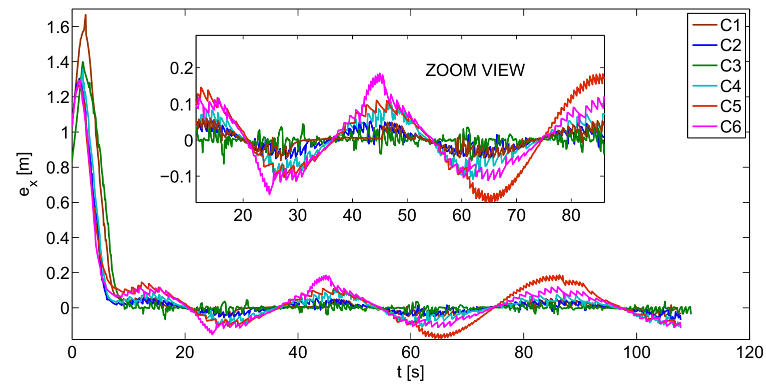


Fig. 11 Tracking trajectory of the mobile robot

Fig. 12 Tracking error in x



The control actions calculated are sent to the embedded computer of the PIONEER 3AT mobile robot using ethernet communication.

Figure 6 shows the PIONEER 3AT and the laboratory facilities where the experiments were carried out. The optimal controller parameters obtained in the previous section (Table 1) are used.

In order to compare the approaches proposed in this work (C1, C2 and C3) the controllers developed in Resende et al. [28], Michałek et al. [21] and Kanayama et al. [12] are also implemented in the PIONEER 3AT, these will be called C4, C5 and C6 respectively. The fuzzy controller proposed by Resende et al. [28], is designed through the application of the inverse kinematic of the mobile platform, guaranteeing the stability of the closed loop system. To reduce tracking errors, caused by the difference between the desired values of linear and angular velocities (system inputs) and the current velocity values assumed by the mobile platform, was used the heuristic knowledge. The controlled structure shown in Michalek et al. [21] results from simple geometrical interpretations related to the unicycle kinematics, from introduction of the so-called convergence vector field, and from decomposition of the control process into the orienting and pushing subprocesses. The Kanayamas controller [12] is designed for determining vehicle's linear and rotational velocities. The controller is designed linearizing the system's differential equation, and then is find a condition for critical dumping, which gives appropriate parameters for specilic control rules.

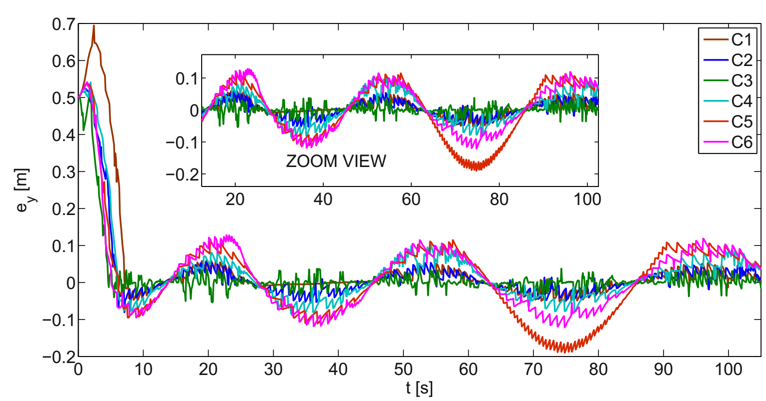
Table 2 summarizes the formulation of the controllers implemented for comparison (C4, C5 and C6).The designing details of the controller C4, C5 and C6 can be found in its respective references ([12, 21, 28]).

5.1 Sinusoidal Trajectory

In this subsection the robot should follow a sinusoidal trajectory following the guidelines previously published in [7, 17]. The initial conditions for the robot mobile position is the system origin and the trajectory begins in the position $(x_{ref}, y_{ref}) = (1\text{m},1\text{m})$.

Figure 7 shows the trajectory and the results obtained by implementing the controllers proposed in

Fig. 13 Tracking error in y



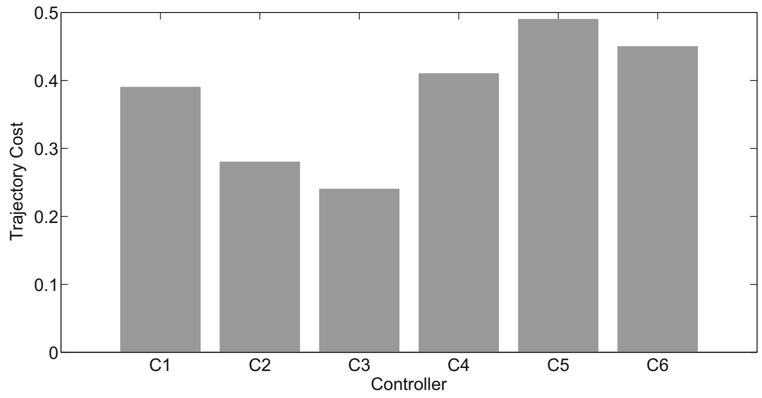


Fig. 14 Cost of the trajectory tracking

this paper (C1, C2, and C3) and controllers implemented for comparative purposes (C4, C5 and C6). As can be seen, all controllers reach and follow the desired trajectory without unexpected oscillations, however the interpolation based controllers show better performance. Figure 8 shows that C3 in the sequel has the lowest cost error when compared with the rest of the controller. Figures 9 and 10 show the tracking errors for the $x - coordinate$ and $y - coordinate$ respectively. By inspection of Figs. 9 and 10, when the same reference trajectory is given, the controllers proposed in this paper reduce the tracking errors.

5.2 Curvature Test

The second experimentation is a curvature test, in which the controllers performance using different circle-shaped trajectories are evaluated, as recommended in Batavia et al. [3]. Three circle-trajectories are used in this work, with different radius. The internal trajectory has a radius of r = 1.5m, the medium one r = 2.5m and the last one r = 3.5m. The initial robot position is the system origin and the reference trajectory begins in the position (x_{ref}, y_{ref}) = (1m, 0.5m).

The reference trajectory and the results of the controllers are shown in Fig. 11. As can be seen, all controllers reach and follow the desired trajectory. Figures 12 and 13 show the plots of the value of the tracking errors in the $x - coordinate$ and $y - coordinate$ according to each controller used in the test for the three curvatures shown in Fig. 11. Figures 12 and 13 shows that both errors (e_x and e_y) remain bounded and close to zero when the robot reaches the reference trajectory. It is interesting to note that during the trasient behavior all controllers have a similar performance. However, when mobile robot reaches the reference trajectory, the controllers proposed in this work (C1, C2, and C3) present a lower tracking error. The lowest cost is obtained by C3 as can be seen in Fig. 14. Compared with fuzzy-based controller (C4) the cost obtained by C3 is lower to 40 %. The well-know Kanayama controller (C6) has a cost upper to 45 % in the same control task in comparison to C3. For the experiment carried out, Michałek et al. [21] offered the worst results when compared with the other approaches.

6 Conclusion

A novel methodology for controller design based on the interpolation of the trigonometric functions of the kinematic model for a wheeled robot is proposed. The use of numerical methods to solve the equations between sample times, allows the use of the controller without computational issues. The main contribution of this work is that the methodology is simple and can be applied to the design of a large class of linear and nonlinear systems. In addition, the convergence of the controllers proposed was demonstrated showing that the asymptotic convergence of this controller occurs faster than the lowest order controller ensuring the exponential convergence of the tracking errors.

Different tests were carried out to demonstrate the effectiveness of the proposed methodology. A contribution of this work involves the application of a Monte Carlo method to controller tuning. These experiments show that the tracking error decreases when the order of Taylor's series increases. The decrease of tracking error also is observable during experimental tests using a PIONEER 3AT mobile robot. The performance of the proposed system is good, and the complexity of control algorithm does not increase in an excessive way. When the methodology proposed in this paper is compared to others from the literature the proposed method present better performance. The application results obtained in an experimental test have shown that the approach proposed have significantly improved the tracking errors up to 20 %.

Finally, the possibility to include the saturation of control signals and additive uncertainty, in the formulation of problem will be addressed in future contributions. The control actions constraint in the proposed methodology can be avoided with a low speed of convergence to zero of tracking errors. This convergence speed can be changed by modifying the values of the parameters of the controller. On the other hand, the effect of uncertainty in tracking errors can be reduced if a good estimate of the uncertainty is incorporated in the design methodology.

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