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Deterministic optimization of the thermal Unit Commitment problem: A Branch and Cut search

Marian G. Marcovecchio[a,b,*], Augusto Q. Novais[b], Ignacio E. Grossmann[c]

[a] INGAR/CONICET, Consejo Nacional de Investigaciones Científicas y Técnicas, Santa Fe, Argentina UNL, Universidad Nacional del Litoral, Santa Fe, Argentina
[b] LNEG, Laboratório Nacional de Energia e Geologia, Lisbon, Portugal
[c] Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, USA

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ABSTRACT

This paper proposes a novel deterministic optimization approach for the Unit Commitment (UC) problem, involving thermal generating units. A mathematical programming model is first presented, which includes all the basic constraints and a set of binary variables for the on/off status of each generator at each time period, leading to a convex mixed-integer quadratic programming (MIQP) formulation. Then, an effective solution methodology based on valid integer cutting planes is proposed, and implemented through a Branch and Cut search for finding the global optimal solution. The application of the proposed approach is illustrated with several examples of different dimensions. Comparisons with other mathematical formulations are also presented.

1. Introduction

The increasing electricity demand motivates the need to study different operational alternatives for planning power generation by integrating conventional generation sources with renewables, while ensuring profitability (Verhaegen, Meeus, Delvaux, & Belmans, 2007; del Río, 2011; Ventosa, Baíllo, Ramos, & Rivier, 2005; Xiao, Hodge, Pekny, & Reklaitis, 2011); as well as, ways of improving the energy efficiency of existing power systems (Siirola & Edgar, 2012). Furthermore, multi-period and multi-paradigm models have also been proposed in order to plan and optimize the energy system and components for a long time planning horizon (Hodge, Huang, Siirola, Pekny, & Reklaitis, 2011; Zhang, Liu, Ma, Li, & Ni, 2012; Zhang, Liu, Ma, & Li, 2013). Recently, Soroush and Chmielewski (2013) have presented an overview of the state of the art and the current process systems opportunities in power generation, storage and distribution.

Planning the generation of electric power is based on three different classes of decisions defined according to the length of the planning time horizon: long-term decisions (capacity, type and number of power generators); medium term decisions (scheduling of the existing units); short-term decisions (programming of the power that each committed unit must produce to meet the real-time electricity demand). These three levels of decision are usually referred to as Power Expansion, Unit Commitment (UC) and Economic Dispatch, respectively. The UC problem has been more widely studied due to its practical importance (Yamin, 2004; Padhy, 2004). Moreover, this problem has diverse applications in the chemical engineering area, for example in Mitra, Grossmann, Pinto, and Arora (2012) the UC constraints were applied to air separation plants to decide when to turn on and off compressors and liquefiers.

The UC can be formulated as a mathematical programming problem using different alternative models. Implementing schedulings based on the optimal solutions of these models, may result in significant economic savings. However, solving the UC problem is very difficult. In fact, this problem is a mixed integer programming problem, linear or nonlinear, that is well known to be NP-hard due to the exponential computational time that may be required in the worst case (Nemhauser & Wolsey, 1988). A large effort has been spent over the last few decades to develop efficient methods capable of solving the UC problem for real industrial cases in practical computational times.

This paper focuses on the thermal UC problem. The solution methods proposed in the literature for solving this problem are either deterministic or heuristic. Approaches based on deterministic methods include: priority list (Senjyu, Shimabukuro, Uezato, & Funabashi, 2003), integer mathematical programming (linear

* Corresponding author at: INGAR/CONICET, Consejo Nacional de Investigaciones Científicas y Técnicas, Santa Fe, Argentina UNL, Universidad Nacional del Litoral, Santa Fe, Argentina. Tel.: +54 3424535568; fax: +54 3424553439.
E-mail address: mariangm@santafe-conicet.gov.ar (M.G. Marcovecchio).

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Nomenclature

Indexes

i	unit index
t	time period index

Constants

I	total number of thermal generating units
T	length of the planning time horizon
a_i, b_i, c_i	coefficients of the fuel cost function of unit i
D_t	power load demand for time period t
R_t	spinning reserve required at time period t
p_i^L	minimum power generation of unit i
p_i^U	maximum power generation of unit i
TU_i	minimum uptime of unit i
TD_i	minimum downtime of unit i
T_i^{ini}	initial status of unit i
DR_i	ramp-down limit of unit i
UR_i	ramp-up limit of unit i
SD_i	maximum shutdown rate of unit i
SU_i	maximum startup rate of unit i
Hsc_i	hot start cost of unit i
Csc_i	cold start costs of unit i
T_i^{cold}	cold start hours of unit i
Dc_i	shut-down cost of unit i
$cost^{UP}$	upper bound for the objective function
ε^{abs}	absolute tolerance for global optimality
ε^{rel}	relative tolerance for global optimality
A_t^{opt1}	objective value of the optimal solution of problem P1 for time period t
A_t^{opt2}	objective value of the optimal solution of problem P2 for time period t
A_t^{LO}	lower bound for the number of committed units at time period t
A_t^{UP}	upper bound for the number of committed units at time period t

Variables

$u_{i,t}$	binary variable representing the on/off status of unit i at period t
$p_{i,t}$	power output of unit i in period t
$cu_{i,t}$	start-up cost of unit i in period t
$cd_{i,t}$	shut-down cost of unit i in period t
A_t	auxiliary variable for computing integer cutting planes

and nonlinear) (Cohen & Yoshimura, 1983; Rajan & Takriti, 2005; Carrión & Arroyo, 2006; Frangioni, Gentile, & Lacalandra, 2009; Zondervan, Grossmann, & de Haan, 2010; Ostrowski, Anjos, & Vannelli, 2012), dynamic programming (Ouyang & Shahidehpour, 1991), Lagrangian relaxation (Ongsakul & Petcharaks, 2004; Frangioni, Gentile, & Lacalandra, 2011; Dieu & Ongsakul, 2011) and other decomposition techniques (Habibollahzadeh & Bubenko, 1986; Niknam, Khodaei, & Fallahi 2009). However, few of these proposed methods guarantee global optimality. As for heuristic approaches, the most widely used are: artificial neural networks (Sasaki, Watanabe, Kubokawa, Yorino, & Yokoyama, 1992), genetic algorithms (Kazarlis, Bakirtizis, & Petridis, 1996; Swarup & Yamashiro, 2002), evolutionary programming (Juste, Kita, Tanaka, & Hasegawa, 1999; Chen & Wang, 2002), simulated annealing (Simopoulos, Kavatza, & Vournas, 2006), fuzzy systems (El-Saadawi, Tantawi, & Tawfik, 2004), particle swarm optimization (Ting, Rao, & Loo, 2006; Oñate Yumbla, Ramirez, & Coello Coello, 2008), tabu search (Mantawy, Abdel-Magid, & Selim, 1998) and hybrid methods (Cheng, Liu, & Liu, 2000; Mantawy, Abdel-Magid, & Selim, 1999).

Yamin (2004), Padhy (2004) and Sen and Kothari (1998) give complete reviews for contributions on deterministic and heuristic methodologies for solving the UC problem. Nevertheless, the methods proposed so far are not always able to solve real world problems to optimality in acceptable computational times.

In this paper a new deterministic optimization approach is proposed for the thermal UC problem. The problem addressed can be stated as follows: given a number of thermal power generators (differing in their operating and production characteristics) and a specified time-variant demand over the planning time horizon, determine for each unit the start-up and shut-down schedules and the power production, in order to minimize the operational costs while meeting demand.

The mathematical model is a convex mixed-integer quadratic programming problem (MIQP) for which a Branch and Cut method is proposed that takes advantage of the characteristics of the UC problem.

The paper is organized as follows. Section 2 presents a detailed description of the mathematical formulation for the thermal UC problem. Section 3 describes the proposed deterministic optimization approach, outlining in Section 3.1 the steps to construct the proposed integer cutting planes. In Section 3.2 a particular implementation of the general Branch-and-Bound framework is described. Section 4 presents computational tests with the proposed optimization approach. In Section 4.1, the performance of the proposed integer cutting planes is illustrated. In Section 4.2 three application examples are presented with the proposed technique, compared with other deterministic methods, and alternative mathematical formulations. Finally, Section 5 draws the general conclusions.

2. Mathematical problem formulation

The thermal UC problem is formulated as the following MIQP model. Consider a set of I thermal generating units and a specified time-varying demand over T time periods defining the planning time horizon, with the units being indexed with $i = 1,\ldots,I$ and the time periods with $t = 1,\ldots,T$. The mathematical programming model involves $I \times T$ binary variables: $u_{i,t}$; and $3(I \times T)$ continuous variables: $p_{i,t}$, $cu_{i,t}$ and $cd_{i,t}$; for $i = 1,\ldots,I$ and $t = 1,\ldots,T$.

The objective function to be minimized is the operating cost, which includes fuel consumption calculated by a quadratic function with fixed charges, and fixed start-up and shut-down costs:

$$\min\ cost = \sum_{i=1}^{I}\sum_{t=1}^{T}[(a_i u_{i,t} + b_i p_{i,t} + c_i p_{i,t}^2) + cu_{i,i} + cd_{i,t}] \quad (1)$$

The constraints to be satisfied are given by (2)–(20).

Satisfying power demand for each time period:

$$D_t \le \sum_{i=1}^{I} p_{i,t} \quad t = 1, \ldots, T \quad (2)$$

Spinning reserve is guaranteed by the available capacity of active units:

$$D_t + R_t \le \sum_{i=1}^{I} p_i^U u_{i,t} \quad t = 1, \ldots, T \quad (3)$$

The generation power limits of each unit at each time period are given by:

$$u_{i,t} p_i^L \le p_{i,t} \le p_i^U u_{i,t} \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \quad (4)$$

Requirements of minimum up and down times are mathematically modeled by the sets of constraints (5)–(10).

The on-off status of a unit i in its earliest periods of operation are determined by its initial status and its minimum up and down times:

$$u_{i,t} = 0 \quad \forall i: \quad T_i^{ini} < 0; \quad t = 1, \ldots, (TD_i + T_i^{ini}) \tag{5}$$

$$u_{i,t} = 1 \quad \forall i: \ T_i^{ini} > 0; \quad t = 1, \ldots, (TU_i - T_i^{ini}) \tag{6}$$

where T_i^{ini} is the number of periods that unit i has been initially switched off ($T_i^{ini} < 0$) or turned on ($T_i^{ini} > 0$).

Constraints (7) and (8) model the unit minimum-up time for the general case and for the first time period, respectively.

$$u_{i,t} - u_{i,t-1} \leq u_{i,t+j} \quad i = 1, \ldots, I; \quad t = 2, \ldots, T; \quad j = 1, \ldots, (TU_i - 1) \tag{7}$$

$$u_{i,1} \leq u_{i,1+j} \quad \forall i: \quad T_i^{ini} < 0; \quad j = 1, \ldots, (TU_i - 1) \tag{8}$$

Similarly, constraints (9) and (10) model the minimum down-time for a unit.

$$u_{i,t+j} \leq u_{i,t} - u_{i,t-1} + 1 \quad i = 1, \ldots, I; \quad t = 2, \ldots, T; \quad j = 1, \ldots, (TD_i - 1) \tag{9}$$

$$u_{i,1+j} \leq u_{i,1} \quad \forall i: \quad T_i^{ini} > 0; \quad j = 1, \ldots, (TD_i - 1) \tag{10}$$

The ramp rate limits are modeled by (11) and (12).

$$p_{i,t-1} - DR_i u_{i,t-1} - SD_i(1 - u_{i,t}) \leq p_{i,t} \quad i = 1, \ldots, I; \quad t = 2, \ldots, T \tag{11}$$

$$p_{i,t} \leq p_{i,t-1} + UR_i u_{i,t-1} + SU_i(1 - u_{i,t-1}) \quad i = 1, \ldots, I; \quad t = 2, \ldots, T \tag{12}$$

Eqs. (11) and (12) ensure that the ramp rate limits are imposed over unit i at time period t only if the unit is online at that period and was online also at time period $(t-1)$.

The start-up cost function is defined as a hot start cost: $cu_{i,t} = Hsc_i$ if downtime $\leq (TD_i + T_i^{\text{cold}})$, and a cold start cost: $cu_{i,t} = Csc_i$ otherwise. This cost function can be modeled by Eqs. (13)–(17).

Eqs. (13) and (14) constrains variable $cu_{i,t}$ to be greater or equal to the hot start cost Hsc_i if unit i is started-up at time period t:

$$(u_{i,t} - u_{i,t-1})Hsc_i \leq cu_{i,t} \quad i = 1, \ldots, I; \quad t = 2, \ldots, T \tag{13}$$

$$u_{i,1} Hsc_i \leq cu_{i,1} \quad \forall i / T_i^{ini} < 0 \tag{14}$$

If instead, the unit i is turned on at the time period t and the downtime at that moment is greater than $(TD_i + T_i^{\text{cold}})$, (15) and (16) impose $cu_{i,t}$ to be greater or equal than the cold start cost Csc_i:

$$\left(u_{i,t} - \sum_{j \leq TD_i + T_i^{\text{cold}} + 1} u_{i,t-j} \right) Csc_i \leq cu_{i,t} \quad i = 1, \ldots, I; \quad (TD_i + T_i^{\text{cold}}) < t \leq T \tag{15}$$

$$\left(u_{i,t} - \sum_{j<t} u_{i,t-j} \right) Csc_i \leq cu_{i,t} \quad \forall i / T_i^{ini} < 0; \quad (TD_i + T_i^{\text{cold}} + 1) < t \leq (TD_i + T_i^{\text{cold}}) \tag{16}$$

Eq. (17) ensures that variable $cu_{i,t}$ takes value 0 when the unit i is not turned on at the time period t:

$$0 \leq cu_{i,t} \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{17}$$

Since the variables $cu_{i,t}$ are only involved in Eqs. (13)–(17), the optimization procedure will ensure that the cost assumed will be exactly zero, Hsc_i or Csc_i, for each case.

Some units can incur in a shut-down cost when they are turned off. This is modeled by constraints (18) and (19). Constraint (20) prevents variable $cd_{i,t}$ taking negative values if the unit is not shut down at that time period.

$$(u_{i,t-1} - u_{i,t})Dc_i \leq cd_{i,t} \quad i = 1, \ldots, I; \quad t = 2, \ldots, T \tag{18}$$

$$(1 - u_{i,1})Dc_i \leq cd_{i,1} \quad \forall i / T_i^{ini} > 0 \tag{19}$$

$$0 \leq cd_{i,t} \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{20}$$

Analogously to the start-up cost, after optimization $cd_{i,t}$ exactly takes either of the values 0 or Dc_i.

Finally, the specification on the variables is as follows:

$$0 \leq p_{i,t} \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{21}$$

$$u_{i,t} \in \{0, 1\} \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{22}$$

The thermal UC problem can be mathematically represented by the MIQP model given by Eqs. (1)–(22). All constraints are linear and the objective function is convex in the continuous space since it is a quadratic function with positive definite Hessian. Thus, the main difficulty for solving this problem is due to the presence of binary variables.

It should also be noted that other formulations different than the above and which are tighter have been reported in the literature (Rajan & Takriti, 2005; Ostrowski et al., 2012). They will be compared to the previous one in case study 3 at Section 4.

3. Deterministic optimization approach

The solution of the UC problem will be addressed through a deterministic optimization approach. It consists of a Branch and Cut implementation: appropriate integer cutting planes are defined and a Branch and Cut search is developed that exploits the characteristics of the proposed cuts. The global optimal solution is found within a specified tolerance for global optimality, i.e. lower and upper bound for the objective value of the global solution are found, the difference of which is below a specified value ε^{abs}.

3.1. Integer cutting planes

The proposed integer cutting planes are conceptually quite simple but highly effective. They are based on the idea of finding valid lower and upper bounds for the number of committed units at each time period. By implementing them, the relaxation gap of the MIQP is considerably reduced.

Next, the steps to construct the cuts are described. An upper bound for the objective function, $cost^{UP}$, is assumed to be available. If a feasible solution of the problem is known, then its objective value yields an upper bound; otherwise, a large enough value is chosen. The absolute and relative tolerances for global optimality are denoted ε^{abs} and ε^{rel}, respectively.

Step 1

For each time period t, the two following NLP problems are solved:

P1: *min* A_t – P2: *max* A_t

s.t.: Eqs. (2)–(21)

$$A_t = \sum_{i=1}^{I} u_{i,t} \tag{23}$$

$$cost = \sum_{i=1}^{I}\sum_{t=1}^{T}[(a_i u_{i,t} + b_i p_{i,t} + c_i p_{i,t}^2) + cu_{i,t} + cd_{i,t}] \le cost^{UP} - \varepsilon^{abs} \tag{24}$$

$$0 \le u_{i,t} \le 1 \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{25}$$

$$A_t \in [A_t^{LO}, A_t^{UP}] \quad t = 1, \ldots, T \tag{26}$$

Eq. (23) defines the variable A_t that is minimized (P1) and maximized (P2); it computes the sum of the states of the units for each time period. Eq. (24) imposes an upper bound, ($cost^{UP} - \varepsilon^{\text{abs}}$), for the operating cost. Eq. (25) relaxes the discrete requirements of the variables u.

Initially: $A_t^{LO} = 0$ and $A_t^{UP} = I$, both values being adjusted as the cuts are computed.

Step 2

For each time period t, set:

$A_t^{LO} = \lceil A_t^{opt1} \rceil$ and $A_t^{UP} = \lfloor A_t^{opt2} \rfloor$where A_t^{opt1} and A_t^{opt2} are the objective value of the optimal solutions for problems P1 and P2, respectively.

Note that the global optimal solutions can be found for P1 and P2 since they are convex NLP problems where the integer variables have been relaxed to be continuous.

Step 3

For each time period t, the inequalities:

$$A_t^{LO} \le \sum_{i=1}^{I} u_{i,t} \tag{27}$$

$$\sum_{i=1}^{I} u_{i,t} \le A_i^{UP} \tag{28}$$

are valid integer cutting planes for the relaxed problem, since they only exclude feasible points of the relaxed problem with some variables $u_{i,t}$ at non-integer values. At the same time, the cuts eliminate feasible solutions for the original MIQP that do not improve the upper bound $cost^{UP}$ at least by ε^{abs}.

If the binary variables were not relaxed to be continuous, the variable A_t would represent the number of units that are committed at time period t, and P1 and P2 would find the minimum and maximum number of units that are required at time period t to satisfy all the original constraints, while the cost is decreased at least by ε^{abs}. However, as the binary variables $u_{i,t}$ are treated as continuous, the optimal values of A_t, i.e. A_t^{opt1} and A_t^{opt2}, are not necessarily integer for the general case, but the real number of online units must indeed be integer. Therefore, A_t^{opt1} and A_t^{opt2} are bounds for A_t, but they are not as tight as possible. After the rounding in Step 2, A_t^{LO} and A_t^{UP} are tighter bounds, and (27) and (28) represent valid inequalities that eliminate non-integer solutions.

The proposed integer cutting planes yield 2 linear constraints for each time period: (27) and (28). However, for practical purposes, the cuts are implemented by adding (23) to the relaxed problem and adjusting the bounds A_t^{LO} and A_t^{UP} as the cuts are being computed and updated. In practice these are generated sequentially, i.e. they are calculated and the bounds A_t^{LO} and A_t^{UP} updated at the relaxed problem, as well as at problems P1 and P2. Therefore, they modify the feasible region for the computation of the subsequent cuts. In fact, the cuts become progressively tighter as the feasible region of the relaxed problem is being reduced.

Clearly, $A_t^{LO} \le A_t^{UP}$ will be true for each time period, since A_t^{LO} and A_t^{UP} are lower and upper bounds for the number of committed units, respectively. On the other hand, if the global optimal solution of the MIQP problem is found, and the integer cutting planes are sequentially updated using its objective value as upper bound $cost^{UP}$, then as long as problems P1 and P2 are feasible, the value for each pair of parameters, A_t^{LO} and A_t^{UP}, should become identical, i.e. $A_t^{LO} = A_t^{UP}$. Furthermore, once the global optimum is found and an improvement on the objective function of ε^{abs} is required, the MIQP will be infeasible, which will then provide the termination criterion. The same conclusion can also be achieved while sequentially updating the cuts, i.e. P1 and P2 will eventually become infeasible, since the relaxed problem and P1 and P2 have the same feasible region.

The proposed cuts are combined with a branch-and-bound search as described in the following section.

3.2. Branch and Cut search

The cuts proposed in the previous section have demonstrated to be highly efficient in reducing the relaxation gap, as will be shown in Section 4. To take advantage of this reduction technique, a Branch-and-Bound search is defined by incorporating at each node the cuts for improving the relaxed approximation. Note that the Branch-and-Bound search (Nemhauser & Wolsey, 1988) yields rigorous bounds on the objective, since the relaxed problem of the UC model is convex.

3.2.1. Integer cutting plane implementation

In order to implement the proposed cuts, (23) is added to the relaxed problem and so only T variables (A_t) and T linear constraints are required to apply the cuts. The bounds for the variables A_t are modified to tighten the cut approximation. Therefore, no new constraint or variable is needed to adjust the cuts. Problems P1 and P2 are solved as quadratically constraints problems (QP).

At the root node, the cuts will be initialized by analytically estimating the value of the bounds A_t^{LO} and A_t^{UP}. Specifically, A_t^{LO} is initialized as the minimum number of units that being committed and operating at their upper level, would be sufficient to satisfy the demand plus the spinning reserve at time period t when no other constraint is considered. That is, for each time period, the upper generation limits for each unit are considered, with the highest ones being chosen and added until the load and spinning reserve of the period are met. For this simple procedure the number of units that are added constitutes a valid lower bound for the committed unit at each time period. A similar procedure is followed to initialize the upper bounds A_t^{UP}, by considering the lower generation limits for each unit instead, and choosing the lowest among them. Valid bounds are thus obtained, although not as tight as they would be if they were calculated by solving problems P1 and P2, which would require solving *2 T* convex NLP optimization problems.

3.2.2. Update of integer cutting planes

Two cuts are updated at each node of the search tree. They are selected alternatively as those corresponding to the time period having the largest difference ($A_t^{UP} - A_t^{LO}$), since these two cuts offer the greatest potential to provide a more accurate approximation for the feasible region of the MIQP problem. Each new node inherits its parent's updated cuts. If either P1 or P2 become infeasible at a given node, then the node is fathomed since it will not contain any solution of the MIQP problem that improves the upper bound $cost^{UP}$ at least by ε^{abs}. On the other hand, each time that a better solution of the MIQP problem is found, all cuts are updated using the new upper bound.

3.2.3. Branching variable selection

The branching is performed over the binary variables and two child nodes are generated by fixing the selected branching variable at their integer values: 0 and 1. The new nodes are added to the waiting node list. The branching variable selection is carried out by following a priority order that is established for each node, according to the current values of the binary variables at the relaxed problem optimal solution and the fixed charge costs. Firstly, the time period to perform the branching is chosen as the first having at least one unit with a non-integer value. Next, the unit is selected among those having non-integer value for the chosen time period at the relaxed solution, as the one with largest fixed charge cost: a_i. The criterion adopted to select the time period of the branching variable among the first ones of the time horizon, is based on the observation that this choice could allow subsequent periods for the chosen unit to be determined. In fact, if the branching is performed on a variable whose value of the associated unit for the previous period was already fixed at the current node, then in one of the two new child nodes, not only the state of the unit in the branching period will be fixed, but also the same will apply to its states in as many subsequent periods as dictated by its minimum up or down time. On the other hand, the unit for the branching is chosen according to the values of the fixed charge cost. This is due to the significant influence this item exerts on the final operating cost and considering that the branching might determine the startup of the unit at the chosen period in one of the child nodes.

3.2.4. Lower bounding

The lower bound of the objective function can be computed at each node by solving the relaxed QP problem. However, for the proposed technique it is not necessary to solve the relaxed problem at each node. In fact, since the feasible region of the relaxed problem is the same as that of P1 and P2, the feasibility of the relaxed problem can be determined when the cuts are updated. The solution of the relaxed problem can thus be avoided. In this case, neither the lower bound for each node nor the global one would be known. This does not affect the stopping criterion of the algorithm nor its convergence. In effect, the tolerance for global optimality is achieved when there are no nodes on the waiting list, i.e. after all generated nodes have been discarded ensuring that the relaxed problem is infeasible to improve the upper bound at least by ε^{abs}.

3.2.5. Upper bounding

The initial upper bound is calculated after solving the relaxed problem. According to the obtained solution, the binary variables that are close to 0 or 1 are fixed at the corresponding values, thus reducing the number of binary variables to optimize. A local search is next carried out by optimizing the binary variables that are still free, together with the continuous ones. By implementing this procedure, good initial solutions can be obtained quickly.

To improve the upper bound in the branch-and-cut tree, a local search is carried out in those nodes whose depth is multiple of a prespecified integer number. In order to prevent spending too much computational time, a time limit is imposed for the local search. As was mentioned before, in case a better solution of the original MIQP problem is found, all cuts are updated using the new upper bound.

The general steps of the proposed branch are cut are shown in Fig. 1, where RP denotes the relaxed problem, and *sd* is the search depth, i.e. an integer number for choosing the nodes where a restricted local search will be performed.

4. Computational results

The algorithm and solution of the test problems and case studies were implemented in GAMS (Brooke, Kendrick, Meeraus, & Raman, 1998). CPLEX 12.2 solver was used to solve the original MIQP problem. Problems P1, P2, and the relaxed problem were also solved with CPLEX 12.2 solver as quadratically constrained problems (QP).

4.1. Performance of the proposed integer cutting planes

Firstly, in order to illustrate the performance of the proposed integer cutting planes, they are applied to reduce the relaxation gap on ten instances of a base problem that includes 10 thermal units over a scheduling time horizon of 24 h (Kazarlis et al., 1996). The operating unit characteristics and the hourly load distribution for the 10-units system are listed on Tables A.1 and A.2 of Appendix A, respectively. The ten instances are obtained by scaling the basic problem from 1 to 10, resulting in 10 systems with up to 100 units. For each instance, an initial solution is calculated with CPLEX, setting the relative tolerance for optimality as the default value of 10%.

By using these solutions to set the upper bound $cost^{UP}$, the integer cutting planes are computed by solving sequentially P1 and P2 for each time period in order to eliminate non integer solutions that do not improve the upper bound by at least 0.3%, i.e. $\varepsilon^{abs} = 0.003$ $cost^{UP}$ is set in Eq. (24).

In Table 1, the solution of the relaxed problem (lower bound) is compared for both cases: with and without the proposed integer cutting planes; and the percentage of gap reduced is reported. The lower bounds and gap reductions are also compared to the ones obtained by CPLEX at the root node after adding its own cuts. The results were obtained on a laptop with Intel Core i7 Q740 1.73 GHz and 8 GB RAM memory.

As shown in Table 1, the relaxation gap (difference between the upper and lower bounds) is considerably reduced by introducing the proposed cuts. Moreover, the gap reductions obtained with the proposed integer cutting planes for the systems with 10–100 units range from 1.3% to 58.4%, while those obtained with CPLEX after applying its cuts at the root node range between 0% and 16.6%. For the system with 100 units, the cuts that CPLEX applies at the root node could not improve the lower bound. In contrast, even in this case, the proposed cuts achieve a positive gap reduction.

Note, however, that the relaxation gap reduction decreases as the problem size increases. This behavior is due to the dependence of the tightness of the cuts on the quality of the upper bound. For larger problems, the objective values of the solutions of the MIQPs obtained by CPLEX with the default tolerance do not provide tight upper bounds, and the gap reduction worsens. Nevertheless, even for large upper bounds, by applying the proposed cuts a reduction in the gap is obtained.

4.2. Application examples

4.2.1. Case study 1

The proposed optimization approach described in Section 3, hereafter denoted by B&C, was applied to solve the base case and instances mentioned in the previous sub-section. The results were obtained on a laptop with Intel Core i7 Q740 1.73 GHz and 8 GB RAM memory. The spinning reserve requirement to be met is set as a 10% of the load demand for each time period.

Two variants of each instance are examined. The first does not consider ramp rate constraints, like the original problem. In the second instance, ramp rate constraints are included: ramp-up and ramp-down rates of each unit are equal and estimated at 20% of the unit maximum power output per time period, whereas the startup and shutdown ramp rates of each unit are set at its

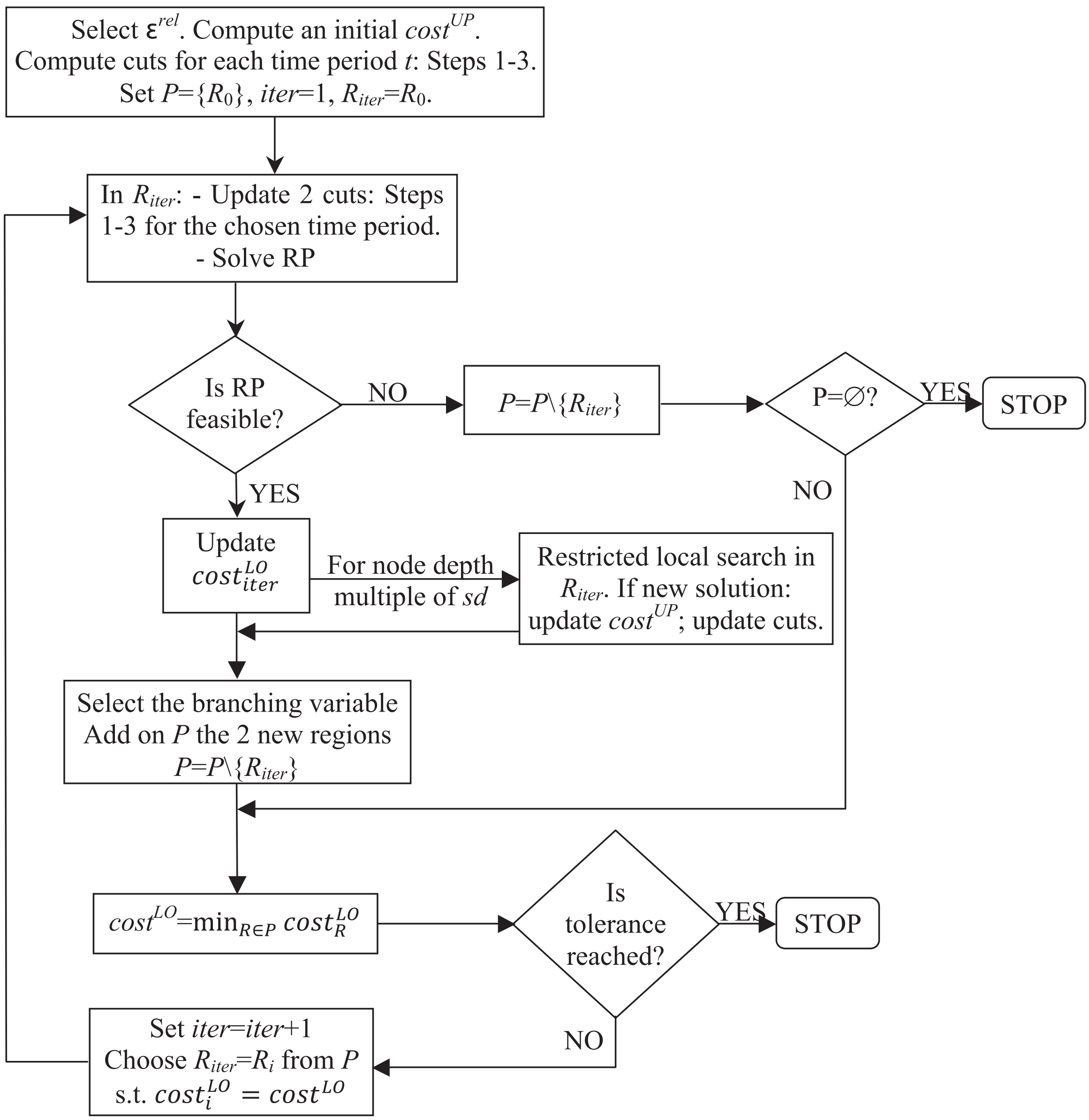

Fig. 1. Branch and Cut scheme.

maximum generation output (Dieu & Ongsakul, 2011; Simopoulos et al., 2006).

The 10 instances for both cases were solved for global optimality with the proposed optimization approach for three relative tolerances: 0.3, 0.2 and 0.1%. The MIQP problem for the base 10-unit system contains 240 binary and 480 continuous variables, while the largest system with 100 units involves 2400 binary and 4800 continuous variables. The number of constraints ranges from 2445 for the former up to 24,009 for the latter, when no ramp rate constraints are considered. If ramp rate constraints are included, the

Table 1
Performance of integer cutting planes. Comparison with the cuts at root node in CPLEX.

No. of units	Upper bound	Lower bound without cuts	Lower bound with CPLEX at root node, after adding its cuts	% Gap reduction	Lower bound with proposed cutting planes	% Gap reduction
10	565,303	558,128	559,319	16.6	562,316	58.4
20	1,126,936	1,116,256	1,116,898	6.0	1,119,493	30.3
30	1,687,841	1,674,383	1,674,862	3.6	1,677,407	22.5
40	2,248,242	2,232,511	2,233,149	4.1	2,235,365	18.1
50	2,812,193	2,790,639	2,790,705	0.3	2,791,250	2.8
60	3,372,820	3,348,767	3,348,993	0.9	3,349,599	3.5
70	3,933,142	3,906,894	3,907,280	1.5	3,909,173	8.7
80	4,493,726	4,465,022	4,465,567	1.9	4,467,456	8.5
90	5,054,788	5,023,150	5,023,855	2.2	5,025,844	8.5
100	5,621,543	5,581,278	5,581,278	0.0	5,581,784	1.3

number of constraints ranges respectively from 2905 up to 28,609. The computational results are reported in Tables 2 and 3, respectively, for the instances of the original system and for the instances including ramp rate constraints. Tables 2 and 3 report the total CPU time, the percentage of the total time spent for updating the cuts, the total number of nodes analyzed in the B&C tree, the maximum number of nodes stored in memory during the search and the objective value of the obtained optimal solution satisfying the specified relative tolerance. As it was explained in the previous section, solving the relaxed problem is avoided. Then, the remaining percentage to the one indicated in Tables 2 and 3 for the time spent for updating cuts corresponds to the upper bounding, i.e. local search aiming to find a better integer solution.

As it can be seen, instances with ramp rate constraints are computationally more demanding than those without them. In all cases solved, the required tolerance for global optimality was achieved with reasonable CPU times. The optimal production schedules for the most challenging case (100-unit system), with and without ramp rate constraints, are presented on Tables A.3 and A.4 of Appendix A, respectively.

This case study has been widely used in order to test different methodologies for solving the UC problem efficiently. Most of the proposed techniques are heuristic, and therefore, they cannot determine how close the solution is from the optimum. Niknam et al. (2009) reports the objective values of the solutions found for different authors.

The proposed approach was compared with four other deterministic methodologies. In particular, the 10 instances for both cases were solved with the same Branch and Bound scheme, but omitting the integer cutting planes (hereafter denoted by B&B); and with three of the solvers incorporated in the GAMS modeling system: SBB, DICOPT and CPLEX. For all cases, a time limit of 3600 s was imposed and a tolerance of 0.3% was required. SBB could not achieve the required tolerance within 3600 s in any of the instances. Therefore, computational results are reported in detail only for the other three methods. Since CPLEX and the proposed B&C show similar performance for that tolerance, and in order to establish a further comparison, CPLEX was additionally tested for 0.2% of tolerance for global optimality. Tables 4 and 5 show the computational performances for each instance solved with B&B, DICOPT and CPLEX, excluding and including ramp rate constraints, respectively. They include the total CPU time in seconds and also, depending on the solver, the total number of nodes analyzed, number of iterations or major iterations, and relative tolerance for global optimality achieved after finishing within the time limit imposed.

From the results presented, the superiority of the proposed B&C search is clear. In all instances, the proposed method was able to guarantee that the required optimality tolerances were reached within the time limit imposed. Furthermore, compared to CPLEX and B&B, the proposed algorithm examined a much smaller number of nodes. DICOPT has very good performances for small systems, but for systems with more than 50 units, the execution times reported by the proposed B&C are faster. Among the 4 methods tested for 0.3% of tolerance for global optimality, CPLEX showed the best performance. In this case, the execution times of CPLEX are comparable with those of the proposed B&C. However, for the systems with more than 50 and 70 units for the instances respectively with and without ramp rate constraints, CPLEX could not reach 0.2% of global optimality tolerance within the specified time limit. These results show that even if CPLEX is highly efficient in finding good feasible solutions, for nontrivial instances and tight tolerances, it is not able to prove optimality for the specified tolerance.

4.2.2. Case study 2

The proposed optimization approach was also applied to a test case study based in a Taipower 38-unit system, taken from Huang,

Table 2
Computational results of the proposed B&C: case study 1, without ramp rate constraints.

No. of units	Relative tolerance: 0.30%					Relative tolerance: 0.20%					Relative tolerance: 0.10%				
	CPU time, s	Time cuts, %	No. of nodes	Max. node	Operat. cost, $	CPU time, s	Time cuts, %	No. of nodes	Max. node	Operat. cost, $	CPU time, s	Time cuts, %	No. of nodes	Max. node	Operat. Cost, $
10	0.4	12	0	0	563,990	1.3	77	3	2	563,938	1.7	83	3	2	563,938
20	4.0	82	3	2	1,124,858	5.3	67	3	2	1,123,587	7.4	91	3	2	1,123,370
30	14.0	50	3	2	1,683,532	14.2	55	3	2	1,683,532	51.4	81	15	6	1,683,154
40	19.7	57	3	2	2,243,688	39.3	76	9	4	2,243,688	131.5	82	21	4	2,242,678
50	25.8	68	3	2	2,801,238	26.7	66	3	2	2,800,617	144.2	75	15	4	2,800,717
60	37.9	70	3	2	3,361,951	35.5	65	3	2	3,360,939	233.8	81	21	6	3,360,492
70	52.1	60	3	2	3,921,228	118.3	79	9	4	3,921,228	958.3	72	63	14	3,921,101
80	22.4	5	0	0	4,480,798	249.0	72	33	4	4,480,798	1555.5	66	105	12	4,480,798
90	23.4	5	0	0	5,040,234	378.6	85	51	4	5,039,429	1463.5	65	111	20	5,039,429
100	28.6	3	0	0	5,597,993	360.0	79	39	4	5,597,962	1723.2	67	117	20	5,597,770

Table 3
Computational results of the proposed B&C: case study 1, with ramp rate constraints.

No. of units	Relative tolerance: 0.30%					Relative tolerance: 0.20%					Relative tolerance: 0.10%				
	CPU time, s	Time cuts, %	No. of nodes	Max. node	Operat. cost, $	CPU time, s	Time cuts, %	No. of nodes	Max. node	Operat. cost, $	CPU time, s	Time cuts, %	No. of nodes	Max. node	Operat. cost, $
10	1.9	74	3	2	565,243	2.3	74	3	2	565,243	2.4	71	3	2	565,186
20	12.0	45	3	2	1,125,950	14.6	51	3	2	1,125,674	32.5	65	9	4	1,125,577
30	27.2	46	3	2	1,686,916	67.6	69	15	4	1,686,319	305.9	73	51	14	1,686,079
40	33.7	58	3	2	2,246,556	108.8	73	15	6	2,246,486	479.8	69	57	12	2,246,120
50	49.5	73	3	2	2,805,295	183.0	84	21	4	2,804,758	812.0	73	45	14	2,804,669
60	72.2	76	3	2	3,365,466	334.3	87	27	4	3,365,320	931.2	62	39	12	3,365,260
70	97.1	69	3	2	3,928,622	410.7	83	39	4	3,927,604	1391.4	70	45	10	3,927,602
80	102.4	64	3	2	4,488,591	324.0	78	21	8	4,487,787	1823.0	71	57	12	4,487,504
90	95.2	53	3	2	5,047,651	699.7	79	33	12	5,047,531	1917.2	69	51	14	5,047,461
100	91.0	56	3	2	5,606,181	507.1	85	27	8	5,606,181	2066.8	64	45	4	5,606,011

Table 4
Computational performance for B&B, DICOPT and CPLEX: case study 1, without ramp rate constraints.

No. of units	B&B, no cuts			DICOPT			CPLEX						
	Required tolerance: 0.3%			Required tolerance:0.3%			Required tolerance: 0.3%			Required tolerance: 0.2%			
	CPU time, s	Nodes	Reached relat. tol., %	CPU time, s	Maj. iter	Iters.	CPU time, s	Nodes	Iters.	CPU time, s	Nodes	Iters.	Reached Relat. tol. (%)
10	3600	23,045	0.38	0.8	2	604	0.2	0	469	0.2	0	565	0.20
20	3600	8343	0.44	6.3	2	3154	6.8	569	24,905	8.3	1026	45,387	0.20
30	3600	5251	0.41	9.7	2	8426	11.3	340	22,658	50.7	12,478	694,182	0.20
40	3600	3838	0.39	20.7	2	8712	17.6	869	51,252	291.1	44,993	3,214,228	0.20
50	3600	2464	0.33	27.5	2	8792	31.3	2878	110,876	1739.8	265,092	13,394,497	0.20
60	3600	1710	0.32	44.4	2	19,464	29.3	1535	68,216	184.7	22,695	1,048,141	0.20
70	3600	2177	0.36	63.6	2	20,280	56.5	4037	152,677	3600.0	285,112	17,227,997	0.22
80	3600	1971	0.31	103.4	2	32,515	44.9	2064	85,696	3600.0	275,143	14,755,943	0.22
90	3600	1666	0.31	103.6	2	17,427	76.9	3363	147,288	3600.0	241,514	12,819,135	0.21
100	3600	1107	0.31	163.5	2	36,754	97.3	4826	200,139	3600.0	289,794	11,516,833	0.22

Table 5
Computational performance for B&B, DICOPT and CPLEX: case study 1, with ramp rate constraints.

No. of units	B&B, no cuts			DICOPT			CPLEX						
	Required tolerance: 0.3%			Required tolerance:0.3%			Required tolerance: 0.3%			Required tolerance: 0.2%			
	CPU time, s	Nodes	Reached relat. tol.	CPU time, s	Maj. iter	Iters.	CPU time, s	Nodes	Iters.	CPU time, s	Nodes	Iters.	Reached Relat. tol. (%)
10	3600	16,228	0.48%	1.1	2	850	0.3	0	816	0.4	0	929	0.20
20	3600	7053	0.53%	6.7	2	7332	14.8	1582	86,859	28.4	5257	314,495	0.20
30	3600	3508	0.48%	16.7	2	8368	27.0	2579	121,244	840.8	132,087	7,757,818	0.20
40	3600	2242	0.44%	27.5	2	13,263	32.0	1447	77,729	2401.9	194,909	13,574,196	0.20
50	3600	1343	0.37%	47.9	2	18,514	58.3	3741	186,281	3600.0	253,103	16,069,490	0.22
60	3600	999	0.35%	68.9	2	23,390	76.0	3644	173,321	3600.0	173,429	13,005,408	0.22
70	3600	708	0.38%	105.6	2	23,049	83.7	3555	168,484	3600.0	216,789	9,859,178	0.25
80	3600	1524	0.37%	138.6	2	26,803	98.6	3666	168,269	3600.0	210,029	8,639,819	0.25
90	3600	1433	0.35%	209.1	2	34,231	128.7	4557	181,909	3600.0	164,886	7,031,254	0.24
100	3600	960	0.31%	242.1	2	30,474	149.7	4308	165,027	3600.0	163,063	6,014,342	0.24

Yang, and Yang (1998), implemented on a laptop with Intel Core i7 Q740 1.73 GHz and 8 GB RAM memory. Even though the number of units is smaller than the largest instance of case study 1, the problem is particularly hard to solve. The planning horizon is 24 h with time periods of 1 h. In order to reproduce the same conditions employed in other works, the spinning reserve requirement to be met is assumed to be 11% of the load demand for each time period. Hot and cold start up costs are equally set to a constant value (startup cost). The 38-unit data and the hourly load distribution are given in Tables B.1 and B.2 of the Appendix B, respectively.

Again, two instances of the problem are considered: with and without ramp rate constraints. As in case study 1, three relative tolerances for global optimality were applied: 0.3%, 0.2% and 0.1%. The MIQP problem contains 912 binary, 1824 continuous variables, and 13,633 constraints with no ramp rate constraints. When ramp rate constraints are taken into account, the number of constraints increases to 15,381. The computational results are reported in Table 6.

In all cases, the required tolerance for global optimality is achieved in modest computational times. The optimal production schedules of the best solutions for the cases without and with ramp rate constraints are presented in Tables B.3 and B.4 of Appendix B, respectively.

Different heuristic techniques have been proposed in the literature to address this case study. El-Saadawi, Tantawi, and Tawfik (2004) and Saber, Senjyu, Yona, and Funabashi (2007) propose and apply heuristic methodologies for this example, and also list and compare the objective values and computational requirements provided by other authors addressing the same problem with other approaches.

This case study was also solved with the same B&B search, omitting the integer cutting planes, and with SBB, DICOPT and CPLEX solvers in GAMS. For all cases, a time limit of 3600 s was imposed and a tolerance for global optimality of 0.3% was required. Unlike example 1, CPLEX could not converge even for 0.3% of optimality tolerance.

For this example, the advantage of applying the proposed approach against B&B and SBB, DICOPT and CPLEX solvers is even more evident. While the B&C could achieve for both instances global optimality tolerances within 0.1% in practical CPU times (see Table 6), none of the 4 methods tested could converge after 1 h for 0.3% of optimality. Here again, CPLEX showed better performance than B&B, SBB and DICOPT, even if it did not close the required optimality gap.

4.2.3. Case study 3

Finally, a comparison with other tighter mathematical formulations for the UC problem was carried out. Among the numerous mathematical formulations proposed in the literature for the UC problem, there is two approaches that have been alternatively preferred. One of these approaches employs three sets of binary variables representing the on-off status and the startup and shutdown of each unit at each time period (Rajan & Takriti, 2005; Ostrowski et al., 2012). On the other hand, other formulations use a single set of binary variables, only for the on-off state of each unit at each time period (Carrión & Arroyo, 2006; Frangioni et al., 2009). In Carrión and Arroyo (2006), a comparison with formulations based on three set of binary variables is carried out, and their results show that UC models with fewer binary variables perform better. Conversely, in Ostrowski et al. (2012), the authors perform a computational comparison and conclude that despite the fact of being larger, 3-binary variable formulations are easier to solve, probably due to their very tight relaxations.

Similarly to the model in Carrión and Arroyo (2006), the mathematical formulation implemented in the present paper employs a single set of binary variables. Then, comparison with two

Table 6
Computational results of the proposed B&C: case study 2.

Rel. tol. (%)	Without ramp rate constraints					With ramp rate constraints				
	CPU time, s	Time cuts, %	No. of nodes	Max. nodes	Operat. cost, $	CPU time, s	Time cuts, %	No. of nodes	Max. nodes	Operat. cost, $
0.3	30.2	74	3	2	203,346,218	57.6	90	7	4	203,402,782
0.2	108.5	83	15	6	203,323,396	154.4	93	21	8	203,402,782
0.1	311.8	92	49	16	203,321,193	572.3	93	77	20	203,353,299

Table 7
Computational results: case study 3, relative tolerance: 1%.

MIQP models. Relative tolerance: 1%							
Prob.	No. of units	F1 [8] with CPLEX	F1 [8] with B&C	F2 [12] with CPLEX	F2 [12] with B&C	F3: Sect. 2 with CPLEX	F3: Sect. 2 with B&C
		CPU time, s					
P11	132	87.1	3.2	299.6	36.8	4.8	3.2
P12	156	477.3	12.2	357.3	15.3	245.6	9.5
P13	156	327.2	15.3	533.6	59.1	182.9	5.7
P14	165	740.4	33.6	936.3	74.7	475.2	13.6
P15	167	448.0	261.9	549.7	254.3	324.7	39.2
P16	172	210.3	78.8	352.7	20.8	260.0	6.4
P17	182	381.9	144.2	907.7	192.2	471.4	126.0
P18	182	646.6	336.6	670.2	323.3	396.9	23.3
P19	183	741.9	68.5	702.3	71.7	434.6	18.8
P20	187	634.3	171.8	500.6	234.2	502.3	37.4

Table 8
Computational results: case study 3, relative tolerance: 0.5%.

MIQP models. Relative tolerance: 0.5%							
Prob.	No. of units	F1 [8] with CPLEX	F1 [8] with B&C	F2 [12] with CPLEX	F2 [12] with B&C	F3: Sect. 2 with CPLEX	F3: Sect. 2 with B&C
		CPU time, s					
P11	132	205.1	192.9	299.6	240.7	46.2	55.3
P12	156	482.6	309.1	780.1	235.1	363.8	199.4
P13	156	327.2	282.2	937.7	250.9	302.5	223.8
P14	165	739.9	525.3	935.5	617.4	475.0	375.5
P15	167	452.1	423.1	550.0	624.1	323.5	267.6
P16	172	210.4	273.2	930.8	364.0	423.5	253.4
P17	182	941.2	319.0	901.5	762.1	471.2	256.8
P18	182	646.4	447.4	668.1	503.4	396.3	374.7
P19	183	741.5	286.4	699.5	668.6	433.5	105.4
P20	187	632.0	415.9	496.9	509.3	659.6	396.6

models involving $3 \times I \times T$ binary variables is presented here. The first formulation F1, corresponds to the model reported in Rajan and Takriti (2005), including the "minimum up/down polytopes" proposed there, which are proved to be facets for the convex hull of that formulation. The second alternative formulation F2, reproduces the model proposed in Ostrowski et al. (2012), which is based in formulation 1, but including tighter inequalities that dominate the ramping constraints. With the aim of comparing the same kind of problems, MIQP models were adopted for formulations 1 and 2, despite of what is proposed in Rajan and Takriti (2005) and Ostrowski et al. (2012), respectively, where MILP problems are solved by implementing piece-wise linearization for the quadratic part of the objective function. The mathematical model described in Section 2 is referred to as formulation 3, i.e. F3, which is generally not as tight as F1 and F2.

Ten problems from Ostrowski et al. (2012) were solved with these three formulations for two relative optimality tolerances: 1 and 0.5%. The test problems are based on the same 10-units system addressed in case study 1 (Kazarlis et al., 1996; Carrión & Arroyo, 2006), but unlike these instances, the base generators are not replicated by identical numbers. Ostrowski et al. (2012) have recently generated 20 problems of varying sizes. Here, only the instances considering more than 100 units are addressed: Problems 11–20. The planning horizon is 24 h with time periods of 1 h. For the complete data of the problems, the reader is referred to Ostrowski et al. (2012) and Carrión and Arroyo (2006).

For Formulation 3, the MIQP problem for the smallest system contains 3168 binary and 6336 continuous variables, and for the largest one, it involves 4488 binary and 8976 continuous variables. The number of constraints ranges from 51,866 for the first problem up to 71,265 for the last one.

In Ostrowski et al. (2012), these problems were solved to 1% optimality. Here, both 1 and 0.5% relative tolerances for global optimality were used. The MIQP problems for formulations 1, 2 and 3 were solved both with CPLEX 12.2 and by applying the proposed B&C. Tables 7 and 8 report the time required to solve the 10 instances addressed, for 1 and 0.5% of optimality tolerance, respectively. The results were obtained with an Intel Core i7-3770 K 3.5 GHz and 16 GB RAM memory.

For the solutions requiring 1% of tolerance for global optimality, the execution times reported by the proposed B&C with F3 are significantly faster. For 0.5% of optimality, in 8 out of the 10 problems, the proposed B&C with F3 reached the required tolerance in smaller execution times. For problem 11, the best time is obtained with the formulation 3, when it is solved with CPLEX. And for problem 16, the formulation 1, solved with CPLEX is faster.

Regarding the different formulations solved with CPLEX, in 7 out of the 10 problems, the best times were obtained with F3, for 1%

of optimality, and in 8 out of the 10 for 0.5%, despite the tighter relaxations of F1 and F2. That is probably due to the fewer number of binary variables involved in F3. Comparing F1 (Rajan & Takriti, 2005) and F2 (Ostrowski et al., 2012), in 7 out of the 10 cases, F1 can be solved with smaller execution times for both global optimality tolerances.

Finally, Tables 7 and 8 show that the methodology proposed in this paper is also efficient for reducing the time required to solve formulations F1 and F2. In fact, for solutions requiring 0.5% of optimality tolerance, in 9 of the 10 problems, the proposed B&C solves F1 in smaller execution time than CPLEX. And for F2, the B&C is faster than CPLEX in 8 of the 10 cases. Moreover, when 1% of optimality is required, the proposed B&C achieves faster execution times for all the cases.

Summarizing, for most of the cases tested, the proposed optimization technique applied to solve F3 exhibits the best performance.

5. Conclusions

In this work, a deterministic optimization approach consisting of a Branch-and-Cut search has been proposed for solving the UC problem. It consists of integer cutting planes specific for the UC problem and a particular implementation of the general Branch-and-Bound framework.

The proposed approach proved to be highly effective, since it could achieve a tight tolerance for global optimality in all cases tested, within reasonable computational times. Comparisons with other deterministic methods clearly showed the superiority of the proposed technique.

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Appendix A.

A.1. Data for case study 1

The data for the 10-unit system of case study 1 addressed in Section 4.2 are provided in Tables A.1 and A.2 (Kazarlis et al., 1996).

A.2. Optimal production schedules for case study 1

The optimal production schedules for the 100-unit systems neglecting and including ramp rate constraints are presented in Tables A.3 and A.4, respectively.

Table A.1
10-unit system data for case study 1.

Units	p_i^L (MW)	p_i^U (MW)	a_i ($/h)	b_i ($/MWh)	c_i ($/MW^{2}h)	TU_i (h)	TD_i (h)	Hsc_i ($/h)	Csc_i ($/h)	T_i^{cold} (h)	T_i^{ini} (h)
1	150	455	1000	16.19	0.00048	8	8	4500	9000	5	8
2	150	455	970	17.26	0.00031	8	8	5000	10,000	5	8
3	20	130	700	16.60	0.00200	5	5	550	1100	4	−5
4	20	130	680	16.50	0.00211	5	5	560	1120	4	−5
5	25	162	450	19.70	0.00398	6	6	900	1800	4	−6
6	20	80	370	22.26	0.00712	3	3	170	340	2	−3
7	25	85	480	27.74	0.00079	3	3	260	520	2	−3
8	10	55	660	25.92	0.00413	1	1	30	60	0	−1
9	10	55	665	27.27	0.00222	1	1	30	60	0	−1
10	10	55	670	27.79	0.00173	1	1	30	60	0	−1

Table A.2
Load demand for case study 1.

Hour	1	2	3	4	5	6
Demand (MW)	700	750	850	950	1000	1100
Hour	7	8	9	10	11	12
Demand (MW)	1150	1200	1300	1400	1450	1500
Hour	13	14	15	16	17	18
Demand (MW)	1400	1300	1200	1050	1000	1100
Hour	19	20	21	22	23	24
Demand (MW)	1200	1400	1300	1100	900	800

Table A.3
Case study 1:Optimal production schedule (MW) for the 100-unit systems, without ramp rate constraints.

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1–10	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0
11–18	245.0	295.0	390.0	451.5	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	375.0	325.0	425.0	455.0	455.0	455.0	455.0	377.5	350.0
19–20	245.0	295.0	390.0	451.5	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	375.0	325.0	425.0	455.0	455.0	455.0	455.0	377.5	0.0
21	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
22–23	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
24–25	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
26–27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
28–30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
31	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
32	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
33–34	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
35	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
36–39	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
41–42	0.0	0.0	25.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.2	0.0	0.0
43	0.0	0.0	0.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.2	25.0	0.0
44–45	0.0	0.0	0.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.1	0.0	0.0
46–47	0.0	0.0	0.0	25.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	0.0	0.0	0.0
48–49	0.0	0.0	0.0	0.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	37.1	0.0	0.0
50	0.0	0.0	0.0	0.0	25.0	47.0	45.0	30.0	104.0	162.0	162.0	162.0	162.0	104.0	81.0	25.0	25.0	25.0	80.5	162.0	105.5	0.0	0.0	0.0
51–52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
53–55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
56	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	20.0	0.0	0.0	0.0	0.0	0.0	43.5	20.0	20.0	0.0	0.0
57–58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	35.0	75.0	80.0	35.0	0.0	0.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
59–60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	35.0	75.0	80.0	35.0	20.0	20.0	0.0	0.0	0.0	20.0	43.5	20.0	0.0	0.0	0.0
61–62	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
63–64	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0	0.0
66	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
67–70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
71–72	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0
73–77	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	45.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	45.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
81–88	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
89–90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
91–92	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0
93–98	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99–100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A.4
Case study 1: Optimal production schedule (MW) for the 100-unit systems, including ramp rate constraints.

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1–10	455.0	451.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0
11–18	245.0	299.0	390.0	451.5	455.0	455.0	455.0	442.2	451.2	455.0	455.0	455.0	455.0	439.2	455.0	364.0	325.0	402.9	455.0	455.0	455.0	452.9	377.5	350.0
19–20	245.0	299.0	390.0	451.5	455.0	455.0	455.0	442.2	451.2	455.0	455.0	455.0	455.0	439.2	455.0	364.0	325.0	402.9	455.0	455.0	455.0	452.9	377.5	0.0
21	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
22–23	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
24–25	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
27–28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	128.8	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0
31–32	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
33–34	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
35	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
36–38	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0
39–40	0.0	0.0	0.0	0.0	0.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	0.0	0.0	0.0
41	0.0	0.0	25.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	57.4	89.8	122.2	123.3	0.0	0.0	0.0
42	0.0	0.0	25.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	0.0	0.0
43–44	0.0	0.0	0.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	57.4	89.8	122.2	123.3	0.0	0.0	0.0
45	0.0	0.0	0.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	25.0	0.0
46–47	0.0	0.0	0.0	25.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	0.0	0.0
48	0.0	0.0	0.0	0.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	57.4	89.8	122.2	123.3	0.0	0.0	0.0
49–50	0.0	0.0	0.0	0.0	25.0	47.0	45.0	64.8	97.2	129.6	162.0	162.0	133.8	101.4	69.0	36.6	25.0	40.2	72.6	105.0	72.6	40.2	0.0	0.0
51–52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0	33.3	49.3	65.3	75.0	59.0	43.0	37.1	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
53	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	80.0	80.0	0.0	0.0	0.0	0.0	0.0	20.0	36.0	20.0	0.0	0.0	0.0
54–55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	75.0	59.0	43.0	37.1	0.0	0.0	0.0	20.0	36.0	20.0	0.0	0.0	0.0
56–57	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	75.0	59.0	43.0	37.1	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	49.3	65.3	75.0	59.0	43.0	0.0	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
59	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	80.0	80.0	80.0	80.0	0.0	0.0	0.0	0.0	0.0	20.0	36.0	20.0	0.0	0.0	0.0
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	80.0	80.0	75.0	59.0	43.0	37.1	0.0	0.0	0.0	0.0	52.0	36.0	20.0	0.0	0.0
61	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
62–64	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0	0.0
66–67	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
68–69	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	0.0	0.0	0.0
70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.0	25.0	25.0	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
71–72	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	37.0	44.0	55.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0
73–78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.0	10.0	21.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0
79–80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0
81–82	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	21.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	11.1	0.0	0.0	0.0	0.0
83–88	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	21.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.1	0.0	0.0	0.0	0.0
89	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	21.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.1	0.0	0.0	0.0	0.0
91	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
92–98	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99–100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Appendix B.

B.1. Data for case study 2

The data for the Taipower 38-unit system of case study 2 addressed in Section 4.2 are provided in Tables B.1 and B.2 (Saber et al., 2007).

B.2. Optimal production schedules for case study 2

The optimal production schedules for the Taipower 38-unit systems neglecting and including ramp rate constraints are presented in Tables B.3 and B.4, respectively.

Table B.1
Taipower 38-unit system data for case study 2.

Units	p_i^L (MW)	p_i^U (MW)	a_i ($/h)	b_i ($/MWh)	c_i ($/MW^{2}h)	TU_i (h)	TD_i (h)	Startup cost ($/h)	UR_i (MW/h)	DR_i (MW/h)
1	220	550	64,782	796.9	0.3133	18	8	805,000	92	138
2	220	550	64,782	796.9	0.3133	18	8	805,000	92	138
3	200	500	64,670	795.5	0.3127	18	8	805,000	84	120
4	200	500	64,670	795.5	0.3127	18	8	805,000	84	120
5	200	500	64,670	795.5	0.3127	18	8	805,000	84	120
6	200	500	64,670	795.5	0.3127	18	8	805,000	84	120
7	200	500	64,670	795.5	0.3127	18	8	805,000	84	120
8	200	500	64,670	795.5	0.3127	18	8	805,000	84	120
9	200	500	172,832	915.7	0.7075	7	7	402,500	128	256
10	114	500	172,832	915.7	0.7075	7	7	402,500	128	256
11	114	500	176,003	884.2	0.7515	7	7	402,500	128	256
12	114	500	173,028	884.2	0.7083	7	7	402,500	128	256
13	110	500	91,340	1250.1	0.4211	9	8	575,000	110	170
14	90	365	63,440	1298.6	0.5145	12	8	575,000	92	125
15	82	365	65,468	1298.6	0.5691	12	8	575,000	92	125
16	120	325	72,282	1290.8	0.5691	10	8	575,000	82	125
17	65	315	190,928	238.1	25.881	1	1	23,000	320	70
18	65	315	285,372	1149.5	38.734	1	1	23,000	320	70
19	65	315	271,376	1269.1	36.842	1	1	23,000	320	70
20	120	272	39,197	696.1	0.4921	9	8	575,000	55	91
21	120	272	45,576	660.2	0.5728	9	8	575,000	55	91
22	110	260	28,770	803.2	0.3572	11	8	460,000	53	132
23	80	190	36,902	818.2	0.9415	14	7	92,000	48	98
24	10	150	105,510	33.5	52.123	1	1	23,000	460	20
25	60	125	22,233	805.4	11.421	8	8	115,000	42	60
26	55	110	30,953	707.1	20.275	14	7	287,500	28	56
27	35	75	17,044	833.6	30.744	14	7	253,000	20	38
28	20	70	81,079	2188.7	16.765	1	1	5750	70	30
29	20	70	124,767	1024.4	26.355	1	1	5750	70	30
30	20	70	121,915	837.1	30.575	1	1	5750	70	30
31	20	70	120,780	1305.2	25.098	1	1	5750	75	30
32	20	60	104,441	716.6	33.722	1	1	7670	70	30
33	25	60	83,224	1633.9	23.915	1	1	7670	70	30
34	18	60	111,281	969.5	32.562	1	1	7670	70	20
35	8	60	64,142	2625.8	18.362	1	1	7670	70	20
36	25	60	103,519	1633.9	23.915	1	1	7670	75	30
37	20	38	13,547	694.7	8.482	11	8	69,000	10	20
38	20	38	13,518	655.9	9.693	11	8	69,000	10	20

Table B.2
Load demand for case study 2.

Hour	1	2	3	4	5	6
Demand (MW)	5700	5400	5150	4850	4950	4800
Hour	7	8	9	10	11	12
Demand (MW)	4850	5400	6700	7850	8000	8100
Hour	13	14	15	16	17	18
Demand (MW)	6900	8150	8250	8000	7800	7100
Hour	19	20	21	22	23	24
Demand (MW)	6800	7300	7100	6800	6550	6450

Table B.3
Case study 2: optimal production schedule (MW) for the system without ramp rate constraints.

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1–2	550.0	486.1	476.5	442.6	453.8	437.0	442.6	495.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0
3–8	500.0	489.3	479.6	445.7	456.9	440.0	445.7	498.2	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	279.3	422.2	433.2	405.5	401.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	279.3	422.2	433.2	405.5	401.0	353.3	341.2	385.2	353.3	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	386.1	402.7	413.8	283.9	418.4	428.8	402.7	398.5	353.6	342.2	383.6	353.6	363.1	359.4	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	411.8	409.7	427.3	439.0	301.3	443.9	454.9	427.3	422.8	375.2	363.1	407.0	375.2	385.3	381.3	388.3
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	258.2	254.6	284.2	303.9	110.0	312.2	330.7	284.2	276.7	196.6	176.3	250.2	196.6	213.6	207.0	218.7
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	90.0	164.2	161.2	185.5	201.6	90.0	208.4	223.5	185.5	179.4	113.8	97.1	157.6	113.8	127.7	122.3	131.9
15	82.0	82.0	82.0	82.0	82.0	82.0	82.0	82.0	148.4	145.8	167.7	182.3	82.0	188.4	202.1	167.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	184.1	166.8	0.0	0.0	0.0	0.0	0.0	0.0	237.5	236.9	241.8	245.0	0.0	246.3	249.3	241.8	240.5	227.5	0.0	236.2	227.5	230.3	0.0	231.1
18–19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20–21	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0
22	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
23	190.0	150.5	147.2	136.0	139.7	134.1	136.0	153.4	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0
24	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	125.0	125.0	125.0	117.7	120.8	116.1	117.7	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
26	110.0	97.3	95.8	90.5	92.3	89.7	90.5	98.6	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0
27	58.2	43.6	42.6	39.1	40.3	38.6	39.1	44.5	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
28–34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.0	38.0	38.0	38.0	36.3	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0
38	27.6	23.0	22.7	21.6	21.9	21.4	21.6	23.3	38.0	38.0	38.0	38.0	33.8	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0

Table B.4
Case study 2: optimal production schedule (MW) for the system with ramp rate constraints.

	Time period																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1–2	550.0	486.1	476.5	442.6	453.8	437.0	442.6	493.3	550.0	550.0	550.0	550.0	516.5	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0
3–8	500.0	489.3	479.6	445.7	456.9	440.0	445.7	496.5	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	302.1	430.1	433.2	405.5	401.0	353.3	341.2	385.2	353.3	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	387.9	405.5	417.2	302.1	430.1	433.2	405.5	401.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	386.1	402.7	413.8	301.6	429.6	428.8	402.7	398.5	353.6	342.2	383.6	353.6	363.1	359.4	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	411.9	409.7	427.3	439.0	323.9	451.9	454.9	427.3	422.8	375.2	363.1	407.0	375.2	385.3	381.3	388.3
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	258.3	254.6	284.2	303.9	163.0	273.0	330.7	284.2	276.7	196.6	176.3	250.2	196.6	213.6	207.0	218.7
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	90.0	164.3	161.2	185.5	201.6	90.0	182.0	223.5	185.5	179.4	113.8	97.1	157.6	113.8	127.7	122.3	131.9
15	82.0	82.0	82.0	82.0	82.0	82.0	82.0	82.0	148.5	145.8	167.7	182.3	82.0	174.0	202.1	167.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	184.1	166.8	0.0	0.0	0.0	0.0	0.0	0.0	237.5	236.9	241.8	245.0	0.0	291.5	249.3	241.8	240.5	227.5	0.0	236.2	227.5	230.3	0.0	231.1
18–19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20–21	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0	272.0
22	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
23	190.0	150.5	147.2	136.0	139.7	134.1	136.0	152.8	190.0	190.0	190.0	190.0	160.6	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0
24	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	125.0	125.0	125.0	117.7	120.8	116.1	117.7	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
26	110.0	97.3	95.8	90.5	92.3	89.7	90.5	98.4	110.0	110.0	110.0	110.0	102.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0
27	58.2	43.6	42.6	39.1	40.3	38.6	39.1	55.0	75.0	75.0	75.0	75.0	55.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
28–34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.0	38.0	38.0	38.0	28.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0
38	27.6	23.0	22.7	21.6	21.9	21.4	21.6	27.5	37.5	38.0	38.0	38.0	28.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0	38.0

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