

Signatures of non-locality in the first-order coherence of the scattered light

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(Dated: February 1, 2008)

The spatial coherence of an atomic wavepacket can be detected in the scattered photons, even when the center-of-mass motion is in the quantum coherent superposition of two distant, non-overlapping wave packets. Spatial coherence manifests itself in the power spectrum of the emitted photons, whose spectral components can exhibit interference fringes as a function of the emission angle. The contrast and the phase of this interference pattern provide information about the quantum state of the center of mass of the scattering atom.

I. INTRODUCTION

Non-locality is a quantum mechanical property, which has no classical counterpart. Its disappearance in the classical world is related to the interaction with the surrounding environment, which in several cases can be identified with the electromagnetic field [1]. In fact, photon scattering may have the same effect as a projective measurement of the position of the system inducing the effective collapse of the quantum state into a position eigenstate, thus destroying spatial coherence [2]. Nevertheless, one can identify other physical situations where photon scattering enforces spatial coherence. Laser cooling is a suggestive example, where quantum states of the atomic center-of-mass motion are prepared at the steady state of a process based on photon scattering, achieving coherence lengths which can exceed the laser wavelength [3]. The emitted photons carry the information about the atomic state, whose dynamics can be characterized, for instance, in the correlation functions of the light [4, 5]. This observation leads to the natural expectation that non-locality can be measured by means of photon scattering. Even if in general the photons will destroy spatial coherence, a quantum scatterer will imprint features of its state in the photons emitted during the transient dynamics.

In this article we analyze light scattering by a particle prepared in the ground state of a double-well potential, and study the first-order coherence properties of the scattered photons as a function of the distance between the wells and of the size of the atomic wave packet, when the atomic transition is driven by a weak laser pulse. The setup is sketched in Fig. 1, and is reminiscent to a Young (double-slit) interference experiment. However, the slit is here a single atom, which is prepared in a coherent superposition of two locations. We show that in general non-locality can be measured when the particle can tunnel between the two wells, and it manifests itself in the spatial periodic modulation of the elastic peak and of the Stokes and anti-Stokes signal. Moreover, we predict that in certain situations it is possible to detect the phase of the quantum superposition between two spatially distant wave-packets. These results are discussed in connection

to recent theoretical works, which studied matter wave scattering by a quantum object [6, 7].

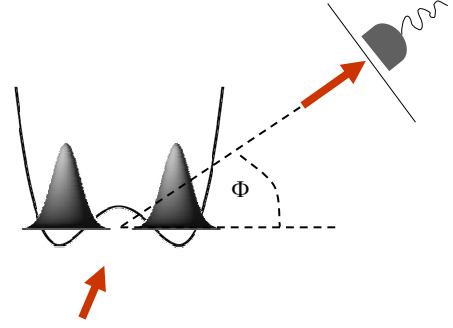


FIG. 1: An atom is confined by double-well potential and is driven by a laser. The intensity of the scattered light is measured in the far field by a detector which is sensitive to intensity gradients over the emission angle Φ . The spectral components of the resonance fluorescence, scattered by the atom in the ground state of the potential, exhibit interference fringes as a function of Φ , whose phase and contrast are determined by the coherence properties of the atomic wave packet.

II. COHERENCE OF LIGHT FROM A QUANTUM SCATTERER

We consider an atom of mass M , which is prepared in the stable electronic state $|g\rangle$ and whose center-of-mass is confined by a double-well potential $V_g(\mathbf{x})$. The center of mass is in a stable state $|\psi\rangle$, eigenstate of the free Hamiltonian

$$H_{\text{mec}}^{(g)} = \frac{\mathbf{p}^2}{2M} + V_g(\mathbf{x}) \quad (1)$$

where \mathbf{x}, \mathbf{p} are position and momentum of the atom. State $|\psi\rangle$ is described by the superposition of the ground states of each wells $|\psi_{\pm}\rangle$,

$$|\psi\rangle = \mathcal{N} (\cos\theta|\psi_{-}\rangle + e^{i\phi}\sin\theta|\psi_{+}\rangle) \quad (2)$$

where θ, ϕ are the azimuthal and polar angles and \mathcal{N} gives the proper normalization. The distance between

the wells centers is $d = |\mathbf{d}|$, and the potential of each well is approximated by two harmonic oscillators at frequency ν and ground-state wave packets

$$\langle \mathbf{x} | \psi_{\pm} \rangle = \frac{e^{-(\mathbf{x} \mp \mathbf{d}/2)^2 / (4a_0^2)}}{(2\pi a_0^2)^{3/4}} \quad (3)$$

with $a_0 = \sqrt{\hbar/2M\nu}$ size of ground state of each oscillator. A laser pulse, centered at frequency ω_L couples to the dipole transition $|g\rangle \rightarrow |e\rangle$ at frequency ω_0 , and the corresponding dynamics is described by the term

$$W_L = \hbar\Omega \int d\omega f(\omega) \left(\sigma^\dagger e^{-i(\omega t - \mathbf{k}(\omega) \cdot \mathbf{x})} + \text{H.c.} \right) \quad (4)$$

with $\sigma = |g\rangle\langle e|$ and σ^\dagger its adjoint, $f(\omega)$ is the normalized spectral distribution over the frequencies, centered in ω_L and with width $\Delta\omega$, $\mathbf{k}(\omega)$ the wave vector at frequency ω , and Ω is the Rabi frequency at frequency ω_L . The excited state couples to states $|g\rangle$ and $|g'\rangle$ with strength $g_{\mathbf{k}}$ and $g'_{\mathbf{k}}$, emitting a photon into the mode $\omega_{\mathbf{k}}$ and $\omega'_{\mathbf{k}}$ with wavevector \mathbf{k} and \mathbf{k}' , respectively. The scattered photons are collected in the far field by a detector, which is sensitive to the gradient of the scattered intensity over the angle of emission Φ and to the frequency ω , hence to the wavevector \mathbf{k} . In the far field the spherical waves of the scattered fields are well approximated by plane waves and the scattering cross section can be put in direct correspondence with the intensity measured at the detector [8]. We thus evaluate the scattering rate $R_{\mathbf{k}}$ of photons with wave vector \mathbf{k} , and thus into the solid angle defined by the direction of \mathbf{k} , and correspondingly the scattering rate $R'_{\mathbf{k}}$. These can be measured separately, for instance by using a polarizing filter before the detector.

Let us consider the rate of photon scattering along the transition $|g\rangle \rightarrow |e\rangle$. The scattering rate over a time T is $R_{\mathbf{k}} = P_{\mathbf{k}}/T$, where $P_{\mathbf{k}} = T g_{\mathbf{k}}^2 \Omega^2 \int d\omega \int d\omega' f(\omega) f(\omega') \mathcal{A}$ is the probability for a photon of being emitted in the mode with wave vector \mathbf{k} , and

$$\mathcal{A} = \frac{1}{T} \int_0^t d\tau \int_0^\tau d\tau' \int_0^t d\bar{\tau} \int_0^\tau d\bar{\tau}' e^{i(\omega_0 - \omega_{\mathbf{k}})\bar{\tau}} e^{-i(\omega_0 - \omega)\bar{\tau}'} \times e^{-i(\omega_0 - \omega_{\mathbf{k}})\tau} e^{i(\omega_0 - \omega')\tau'} \mathcal{W}(\tau, \tau', \bar{\tau}, \bar{\tau}') \quad (5)$$

where the initial state is the atom in $|g, \psi\rangle$, Eq. (2), and the electromagnetic field in the vacuum. In writing Eq. (5) we have introduced $\mathcal{W} = \langle \psi | \bar{U}^\dagger U | \psi \rangle$ with

$$U = e^{iH_{\text{mec}}^{(g)}\tau/\hbar} e^{-i\mathbf{k} \cdot \mathbf{x}} e^{-iH_{\text{mec}}^{(e)}(\tau - \tau')/\hbar} e^{i\mathbf{k}_L \cdot \mathbf{x}} e^{-iH_{\text{mec}}^{(g)}\tau'/\hbar} \quad (6)$$

and $\bar{U} = U|_{\tau \rightarrow \bar{\tau}; \tau' \rightarrow \bar{\tau}'}$, where $H_{\text{mec}}^{(e)}$ is the Hamiltonian for the atomic center of mass in state $|e\rangle$. The rate of Raman scattering, $R'_{\mathbf{k}}$, is analogously defined, where we assume that the Hamiltonian of the final states is $H_{\text{mec}}^{(g')} =$

$\mathbf{p}^2/2M$, namely the atom propagates freely. This situation may be realized with magnetic traps, where $|g\rangle$ is a weak-field seeker and $|g'\rangle$ is a state of the hyperfine multiplet with zero magnetic moment [9]. The dependence on the dipole pattern of emission $\mathcal{D}(\Phi)$ in the solid angle Φ enters in the rates through the factor $g_{\mathbf{k}}^2 \propto \gamma \mathcal{D}(\Phi)$, with γ linewidth of state $|e\rangle$. In the far field we neglect the variation of this factor over the angle.

Substituting the explicit form of $|\psi\rangle$ into \mathcal{W} , one can write the transition amplitude as the sum of four contributions,

$$\mathcal{W} = \mathcal{N}^2 [\sin^2 \theta \langle \psi_+ | \bar{U}^\dagger U | \psi_+ \rangle + \cos^2 \theta \langle \psi_- | \bar{U}^\dagger U | \psi_- \rangle + \sin \theta \cos \theta (e^{-i\phi} \langle \psi_+ | \bar{U}^\dagger U | \psi_- \rangle + e^{i\phi} \langle \psi_- | \bar{U}^\dagger U | \psi_+ \rangle)]$$

where the last two terms are due to the non-local properties of the initial state. Clearly, the contribution of the latter is maximal with respect to θ when $\theta = \pi/4$, i.e., when left and right states are initially equally populated. We thus restrict to this case, and identify situations where these terms do not vanish, allowing to observe features due to the non-local properties of the atomic wave packet in the scattered light.

In the following we evaluate scattering by an atom driven by a weak laser pulse, when the pulse duration is such that one can neglect the width $\Delta\omega$ about the mean value ω_L . We consider the case in which the laser is far-off resonance from the dipole transition $|g\rangle \rightarrow |e\rangle$, such that state $|e\rangle$ is practically empty. In this regime we can neglect the propagator in the excited state in Eq. (6), as one is on the flat tail of the Lorentz curve of the atomic resonance. We focus on the evaluation of the rates of Rayleigh scattering, when the final internal state of the atom is equal to the initial state $|g\rangle$, and of Raman scattering, when the atom is in the state $|g'\rangle$ after scattering one photon. We note that for large laser detuning and short interaction times we can approximate $U(\tau, \tau') \approx U(0, 0)$ in Eq. (6), and $\mathcal{W} \approx 1$: The scattering rate is the same as the atom was a point-like scatterer. In fact, for short interaction times the motion is essentially classical. We thus focus on long interaction times, so to be able to resolve the spectral components of the resonance fluorescence.

A. Rayleigh scattering

We now evaluate the rate of Rayleigh scattering $R_{\mathbf{k}}$ when the atom has been prepared in the symmetric state of the double-well potential, and consider the photons which are elastically scattered and inelastically scattered to the antisymmetric state. Be $\delta\nu$ the frequency splitting between the two states. From Eq. (3), term \mathcal{W} is explicitly evaluated using the relation $e^{i(\mathbf{k}_L - \mathbf{k}) \cdot \mathbf{x}} |\psi_{\pm}\rangle = |\beta_{\pm}\rangle e^{\pm i(\mathbf{k}_L - \mathbf{k}) \cdot \mathbf{d}/4}$ where $|\beta_{\pm}\rangle$ is a coherent state with amplitude $\beta_{\pm} = \pm \mathbf{d}/4a_0 + ia_0(\mathbf{k}_L - \mathbf{k})$. With these relations,

$$\mathcal{A} \approx \frac{1}{\Delta^2} e^{-|\Delta\mathbf{k}|^2 a_0^2} \left[\frac{1}{(1+\epsilon)^2} \left(\cos\left(\frac{\Delta\mathbf{k} \cdot \mathbf{d}}{2}\right) + \epsilon \right)^2 \delta^{(T)}(\omega_{\mathbf{k}} - \omega_L) + \frac{1}{1-\epsilon^2} \sin^2\left(\frac{\Delta\mathbf{k} \cdot \mathbf{d}}{2}\right) \delta^{(T)}(\omega_{\mathbf{k}} - \omega_L + \delta\nu) \right] \quad (8)$$

where $\Delta = \omega_L - \omega_0$, $\Delta\mathbf{k} = \mathbf{k}_L - \mathbf{k}$ is the difference between the wavevectors of the incoming and outgoing photons, $\delta^{(T)}(\omega)$ is the diffraction function [10], and

$$\epsilon = e^{-d^2/8a_0^2} \quad (9)$$

is the overlap between the two wavepackets. The finite width $\Delta\omega$ of the incident laser pulse can be neglected for $\Delta\omega \ll 2\pi c/d$. Moreover, if $\Delta\omega \ll \delta\nu$, then $R_{\mathbf{k}} \propto \mathcal{A}$. This result shows that the elastic peak, at $\omega_{\mathbf{k}} = \omega_L$, and the Stokes sideband, at $\omega_{\mathbf{k}} = \omega_L - \delta\nu$ exhibit interference fringes on the screen as a function of the emission angle, which are in opposition of phase and have periodicity determined by the distance d between the wells. When the antisymmetric state is initially occupied, one observes the signal at the anti-Stokes frequency, at $\omega_{\mathbf{k}} = \omega_L + \delta\nu$, which oscillates in phase with the Stokes sideband. These sinusoidal signals have Gaussian envelope with width $1/a_0$, thus determined by the size of the wave packets localized at each well. Hence, the number of observed fringes is small, since ideally $d \sim a_0$. In absence of a spectral filter, resolving the splitting between the doublet, the scattered intensity still depends on the solid angle, with a visibility determined by ϵ , which approaches unity as the spatial overlap between the two wave packets increases.

In a time-picture, the spectral resolution of elastic and Stokes components corresponds to selecting interaction times T which are larger than the tunnelling time $1/\delta\nu$, such that during the interaction with the photon the atom tunnels between the wells. Hence, the system is analogous to a Young's interference setup with one single slit, which is in a coherent superposition of two spatial locations. Remarkably, Eq. (8) also predicts an interference signal when the distance between the wells is very large, $\epsilon \rightarrow 0$, and the tunneling rate thus vanishes, provided that elastic and Stokes sideband are resolved in the spectrum of resonance fluorescence. However, in this regime the needed spectral resolution scales as $\delta\nu$, and thus is extremely small, requiring integration over diverg-

ing time intervals T .

Equation (8) takes into account the finite size of the atomic wave packet and the mechanical effect of the scattered photon on the atom. This result is the photonic counterpart of the scattering rate for matter waves by a quantum object derived in [6, 7]. The scattering rate in [6], which was obtained for a point-like scatterer, is recovered taking $\epsilon \rightarrow 0$. In Ref. [7] the interference pattern is explained in terms of entanglement swapping of the state of the scatterer with the state of the scattered matter wave. This interpretation applies also to photon scattering: the emerging photon is entangled with the scattering atom [11, 12], and interference between the paths of photon scattering through each well is hence possible when the two states, $|\psi_+\rangle$ and $|\psi_-\rangle$, have finite overlap. The interaction with the atom prepared in a superposition of two orthogonal wave-packets imprints a which-way information on the photon [13], and one can put the finite overlap between the wave-packets in direct connection with the fringe visibility at the screen [14].

B. Raman scattering

Is it then possible to detect by photon scattering whether the atom is in a non-local state, if there is no spatial overlap between the two wave packets in which the atom has been prepared? Let us assume the atom is initially in state $|g, \psi\rangle$ when $|\mathbf{d}| \gg a_0$, so that the tunneling rate vanishes, $\epsilon \rightarrow 0$. This state can be prepared, for instance, via adiabatic manipulation of the center-of-mass wave packet, as described in [15]. We assume now that the internal state is coupled by Raman scattering to the internal state $|g'\rangle$, which is not trapped. In this case, a photon of wave vector \mathbf{k}' is emitted in a spontaneous Raman process, which is detected at the screen. Accordingly, the atom in state $|g'\rangle$ propagates freely with momentum $\hbar(\mathbf{k}_L - \mathbf{k}')$. The corresponding signal at the detector is given by

$$\mathcal{A}' \propto \frac{1}{\Delta^2} e^{-2|\Delta\mathbf{k}|^2 a_0^2} [1 + \sin 2\theta \cos(\Delta\mathbf{k} \cdot \mathbf{d} + \phi)] \delta^{(T)}(\omega_L - \omega_{\mathbf{k}} + \delta\omega_{g'} + \hbar\Delta\mathbf{k}^2/2M) \quad (10)$$

where $\Delta\mathbf{k} = \mathbf{k}_L - \mathbf{k}'$ and $\delta\omega_{g'}$ accounts for the frequency difference between initial and final state. For $\Delta\omega \ll 2\pi c/d, \hbar\Delta\mathbf{k}^2/2M$, then $R'_{\mathbf{k}} \propto \mathcal{A}'$ and the intensity

at the detector exhibits an interference signal, provided that the frequency shift due to photon recoil is spectrally resolved. This signal has periodicity determined by d ,

it has maximum contrast when the occupation probability of the two wells is equal, $\theta = \pi/4$, and it depends on the phase ϕ of the initial state, Eq. (2). The situation here considered is similar to the experimental setup realized in Ref. [16], which was used for detecting the phase between two Bose-Einstein condensates by measuring interference in the flux of atoms outcoupled by stimulated Bragg scattering. Equation (10) shows that interference is already present at the single-particle level. As observed in [16], this effect is found whenever the final state has finite overlap with both initial wave packets. In analogy with interferometry, the final state is a quantum eraser, which projects the two orthogonal wave packets into the same state, thus restoring interference [17]. The detection efficiency of the scattered photons in this kind of experiment can be improved using stimulated Raman scattering or enhancing the rate of photon emission by means of a resonator, in a setup like the one discussed in Ref. [18]. We remark that a signal similar to the one in Eq. (10) is found also in the photons scattered by an atomic dipole, when the atomic wave packet in the excited state $|e\rangle$ has finite overlap with both wells and the vibrational excitations of $|e\rangle$ are spectrally resolved. In this case, propagation in the excited state leads to an overlap between the two initial wave packets and acts as a quantum eraser. Hence, inelastic scattering restores the visibility of the fringes. This is a remarkably different behaviour from the one studied in [8], where the coherence of light scattered by two atoms is destroyed by saturation effects.

III. CONCLUSIONS

Using simple, but experimentally plausible models we have shown that the features associated with non-locality can be measured in the scattered light. The interaction of a photon with an atom in a double-well potential can be thought of as a photon interferometer, and coherence between the two interaction paths appear through spatial modulation in the signals of the spectrum of resonance fluorescence. This setup can be generalized to the case of quantum transport of an atom in a periodic potential,

such as an optical lattice: Photon scattering will act as a multipath interferometer, providing information on the coherence properties of atomic motion. Such setup could be used in order to study the coherence length of a reservoir coupling to the atom, which could be, for instance, an atomic gas [19].

These results give a further example of how the spatial coherence of matter waves appear in the coherence properties of the scattered light, and may be of relevance for the realization of quantum light sources [18, 20].

Acknowledgments

Many of us have often heard Herbert Walther express his amusement of and quiet admiration for a close student-teacher relationship by quoting Friedrich Schiller's Wallenstein: "Wie er räuspert und wie er spuckt, das habt ihr ihm glücklich abgeguckt" (The way he clears his throat, the way he spits, you have emulated very well from him). Most likely Herbert Walther did not realize how much more he himself was a role model as a teacher for all of us. We were the students who tried to become a scientist like him. His excitement about a new discovery, his love of and dedication to science and most importantly his abundant energy has infected all of us. It is hard to believe that our hero has left us; he still could have taught us so much more. It has been a great privilege for us to have worked with him and we dedicate the present paper to his memory hoping that it would have found his interest and approval.

G.M. acknowledges discussions with E. Demler, J. Eschner, S. Rist, and U. Weiss, and the kind hospitality of the Los Alamos National Laboratory during completion of this work. Support by the European Commission (CONQUEST, MRTN-CT-2003-505089; EMALI, MRTN-CT-2006-035369), the scientific exchange programme Germany-Spain (HA2005-0001 and D/05/50582), the Spanish Ministerio de Educación y Ciencia (Ramon-y-Cajal; QLIQS, FIS2005-08257; Consolider Ingenio 2010 QOIT, CSD2006-00019), and the scientific exchange programme Spain-Argentina (AECI A/3448/05) are acknowledged.

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