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# ON INFLATION AND VARIATION OF THE STRONG COUPLING CONSTANT

N. CHAMOUN[*,†], S. J. LANDAU[‡,§,¶] and H. VUCETICH[*,‡]

[*]*Departamento de Física, Universidad Nacional de La Plata,*
*c.c. 67, 1900 La Plata, Argentina*

[†]*Department of Physics, Higher Institute for Applied Sciences and Technology,*
*P. O. Box 31983, Damascus, Syria*

[‡]*Observatorio Astronómico, Universidad Nacional de La Plata,*
*Paseo del Bosque S/N, CP 1900 La Plata, Argentina*

[§]*Departamento de Física, Universidad Nacional de Buenos Aires,*
*Ciudad Universitaria — Pab. 1, 1428 Buenos Aires, Argentina*

Variation of constants in the very early universe can generate inflation. We consider a scenario where the strong coupling constant was changing in time and where the gluon condensate underwent a phase transition ending the inflation.

## 1. Introduction

Alternatives to inflationary cosmology[1–4] include varying speed of light (VSL) theories.[5–8] Usually all inflationary models are based on using new fundamental scalar fields, the "inflatons," whose nature is still unknown. Some models change the matter content of the universe, while others give the inflaton geometrical interpretations within brane settings.[9,10] VSL scenarios may solve the cosmological problems usually tackled by inflation ("horizon," "flatness" and "structure formation" problems) without introducing inflatons, whereas many inflationary models lead to variation of other constants of nature.[11] In this paper, we will follow the reverse route that variation of constants in the very early universe can generate inflation. In an earlier work,[12] we considered a Bekenstein-like model for the QCD strong coupling constant $\alpha_S$ introducing a scalar field $\epsilon$ expressing the time variation of $\alpha_S$. We found that experimental constraints going backward till quasar formation

[¶]Member of CONICET.

times rule out $\alpha_S$ variability. However, when this model is implemented in the very early universe, the scalar field $\epsilon$ can play the role of an inflaton, and one can realize a consistent inflation scenario with a suitable value for the gluon condensate. We find that the "time varying" QCD Lagrangian leads naturally to a monomial quadratic potential like the chaotic scenario. However, while the large values of the inflaton matter field plague the latter scenario, they just amount in our model to a reduction of the strong charge by around ten times during the inflation. A way out can be achieved if the gluon condensate suffers a phase transition, reducing its value to its current value thus ending the inflation. We will not dwell on the possible mechanisms for such a phase transition to occur, but instead concentrate on the conditions our model should satisfy in order to present a consistent set up able to accommodate the recent measurements from the Cosmic Microwave Background (CMB)[13,14] and the WMAP results.[15,16]

## 2. Analysis

We follow the notations of Ref. 12. Our starting point is the "time varying" QCD Lagrangian:

$$L_{QCD} = L_\epsilon + L_g + L_m$$
$$= -\frac{1}{2l^2}\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2} - \frac{1}{2}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_f \bar{\psi}^{(f)}(i\gamma^\mu D_\mu - M_f)\psi^{(f)}, \qquad (1)$$

where $l$ is the Bekenstein scale length, $\epsilon(x)$ is a scalar gauge-invariant and dimensionless field, with the "variable" QCD coupling given by $g(x) = g_0\epsilon(x)$. The covariant derivative is $D_\mu = \partial_\mu - ig_0\epsilon(x)A_\mu$ and the gluon tensor field is $G^a_{\mu\nu} = \frac{1}{\epsilon}[\partial_\mu(\epsilon A^a_\nu) - \partial_\nu(\epsilon A^a_\mu) + g_0\epsilon^2 f^{abc}A^b_\mu A^c_\nu]$.

Assuming homogeneity and isotropy for an expanding universe, we consider only temporal variations for $\alpha_S \equiv \frac{g^2(t)}{4\pi} = \alpha_{S_0}\epsilon^2(t)$. One gets the following equations of motion

$$\left(\frac{G^{\mu\nu}_a}{\epsilon}\right)_{;\mu} - g_0 f^{abc} G^{\mu\nu}_b A^c_\mu + \sum_f g_0 \bar{\psi} t^a \gamma^\nu \psi = 0, \qquad (2)$$

$$\left(a^3 \frac{\dot{\epsilon}}{\epsilon}\right)^{\cdot} = \frac{a^3(t)l^2}{2}\langle G^2\rangle, \qquad (3)$$

where $a(t)$ is the expansion scale factor in the R-W metric.

Subtracting the total derivative $\Delta T^{\alpha\beta} = \partial_\nu\left(-\frac{G^{\alpha\nu}_a}{\epsilon}\epsilon A^{a\beta}\right)$ from the canonical energy–momentum tensor $\frac{\partial L}{\partial(\partial_\alpha\phi_i)}\partial^\beta\phi_i - g^{\alpha\beta}L$ we get the gauge-invariant energy–momentum tensor:

$$T^{\alpha\beta} = G^{\alpha\nu}_a G^{a\beta}_\nu + i\sum_f \bar{\psi}^{(f)}\gamma^{(\alpha}D^{\beta)})\psi^{(f)} - \frac{1}{l^2}\frac{\partial^\alpha\epsilon\partial^\beta\epsilon}{\epsilon^2}$$

$$- g^{\alpha\beta}\left[-\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} + \sum_f \bar{\psi}^{(f)}(i\gamma^\mu D_\mu - M_f)\psi^{(f)} - \frac{1}{2l^2}\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2}\right]. \qquad (4)$$

In the analysis below, we consider only the gauge fields and the scalar field, i.e. we drop the matter fields in the radiation dominated very early era. We decompose our energy–momentum tensor into two parts: the gauge part and the $\epsilon$-scalar field part:

$$T_{\alpha\beta} = T^g_{\alpha\beta} + T^\epsilon_{\alpha\beta}$$
$$= \left( G_{\alpha\nu} G^\nu_\beta - g_{\alpha\beta} \left[ -\frac{1}{4} G^{\mu\nu}_a G^a_{\mu\nu} \right] \right) + \left( -\frac{1}{l^2} \frac{\partial_\alpha \epsilon \partial_\beta \epsilon}{\epsilon^2} - g_{\alpha\beta} \left[ -\frac{1}{2l^2} \frac{\epsilon_{,\mu} \epsilon^{,\mu}}{\epsilon^2} \right] \right). \quad (5)$$

Here all the operators are supposed to be renormalized and it is essential in the inflationary paradigm that quantum effects are small in order to get small fluctuations in the CMB.

The contribution of the scalar field to the energy density $\rho_\epsilon = T^\epsilon_{00}$ and to the pressure $T^\epsilon_{ij} = g^{(3)}_{ij} p_\epsilon$ are

$$\rho_\epsilon = -\frac{1}{2l^2} \left( \frac{\dot{\epsilon}}{\epsilon} \right)^2 = p_\epsilon. \quad (6)$$

On the other hand, the gauge field contribution $T^g_{\alpha\beta}$ can be decomposed into traceless and trace parts:

$$\rho_g = \rho^r_g + \rho^T_g, \quad (7)$$
$$p_g = p^r_g + p^T_g, \quad (8)$$

where $\rho^r_g, p^r_g$ are the density and the pressure corresponding to the "traceless" part of the gauge field satisfying $\rho^r_g = 3p^r_g$, while the trace part of the gauge field energy–momentum tensor is proportional to $g_{\alpha\beta}$ and behaves like a "cosmological constant" term:

$$\rho^T_g = -p^T_g. \quad (9)$$

This equation is reminiscent of "ordinary" inflationary models. However, to compute the trace part of the density one needs a "trace anomaly" relation for our "time varying" QCD. Since the energy–momentum tensor $T^g_{\alpha\beta}$ is identical in form to "ordinary" QCD and since the trace anomaly which involves only gauge invariant quantities should, by dimensional analysis, be proportional to $G^2$, we take it to be the same as in "ordinary" QCD (we have checked that changing the numerical value of proportionality will not alter the conclusions). Thus we take, up to leading order in the time-varying coupling "constant" $\alpha_S = \alpha_{S_0} \epsilon^2$, the relation[17]:

$$T^{\mu g}_\mu = \rho_g - 3p_g = -\frac{9\alpha_{S_0} \epsilon^2}{8\pi} G^{\mu\nu}_a G^a_{\mu\nu}. \quad (10)$$

This leads to:

$$\rho^T_g = -\frac{9\alpha_{S_0}}{32\pi} \epsilon^2 \langle G^2 \rangle. \quad (11)$$

As we said before, Eq. (9) suggests, by analogy to ordinary inflationary models, that the QCD trace anomaly could generate the inflation. For this, let us assume

that the "trace-anomaly" energy mass density contribution is much larger than the other densities:

$$\rho_g^T \gg \rho_\epsilon, \rho_g^r \Rightarrow \rho \sim \rho_g^T. \tag{12}$$

Then, Eq. (11) tells us that the vacuum gluon condensate $\langle G^2 \rangle$ should have a negative value, which is not unreasonable since the inflationary vacuum has "strange" properties. In ordinary inflationary models, it is filled with repulsive-gravity matter turning gravity on its head.[18] This reversal of the vacuum properties is reflected, in our model, by a reversal of sign for the vacuum gluon condensate.

Now we seek a consistent inflationary solution to the FRW equations in a flat space–time:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_{\mathrm{N}}}{3} \rho, \tag{13}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\mathrm{N}}}{3} (\rho + 3p), \tag{14}$$

where $G_{\mathrm{N}}$ is Newton's constant. The first FRW equation with (11) will give

$$H \equiv \frac{\dot{a}}{a} = \epsilon \sqrt{\frac{3\alpha_{S_0}}{4} G_{\mathrm{N}} |\langle G^2 \rangle|}. \tag{15}$$

On the other hand, the equation of motion (3) of the scalar field can be expressed in the following way:

$$\frac{\ddot{\epsilon}}{\epsilon} + 3H\frac{\dot{\epsilon}}{\epsilon} - \left( \frac{\dot{\epsilon}}{\epsilon} \right)^2 = \frac{l^2 \langle G^2 \rangle}{2}. \tag{16}$$

This equation differs from the ordinary "matter" inflationary scenarios in the term $\left( \frac{\dot{\epsilon}}{\epsilon} \right)^2$. However, for "slow roll" solutions, we neglect the terms involving $\frac{\ddot{\epsilon}}{\epsilon}$ and $\left( \frac{\dot{\epsilon}}{\epsilon} \right)^2$ to get

$$3H\dot{\epsilon} = \frac{l^2 \langle G^2 \rangle}{2} \epsilon = -V'(\epsilon), \tag{17}$$

which is the same as the "slow roll" equation of motion of the inflaton in ordinary scenarios. In our model, the "slow roll" condition can be written as:

$$\delta \equiv \left| \frac{\dot{\epsilon}}{H\epsilon} \right| = \frac{2}{9\alpha_{S_0}} \left( \frac{l}{L_P} \right)^2 \frac{1}{\epsilon^2} \ll 1. \tag{18}$$

We set $\epsilon_f$, the value of $\epsilon$-field at the end of inflation $t_f$, to 1 so that the time evolution of the strong coupling terminates with the end of inflation and we expect for "slow roll" solutions that $\epsilon_i$, the value of $\epsilon$ at the initial time of inflation $t_i$ corresponding to when the CMB modes froze out, is of order 1. If the gluon condensate value $\langle G^2 \rangle$ stays approximately constant during much of the inflation, the changes of the

Hubble constant and the energy mass density are not very large. In this case, we have

$$\epsilon(t) = \epsilon_i - \frac{1}{3^{3/2}(\alpha_{S_0})^{1/2}} \left(\frac{l}{L_p}\right)^2 G_N^{1/2} |\langle G^2\rangle|^{1/2} (t - t_i) \tag{19}$$

and, as in chaotic scenarios, we get a simple quadratic potential:

$$V(\epsilon) = \frac{l^2 |\langle G^2\rangle|}{4} \epsilon^2. \tag{20}$$

One can make explicit the correspondence between our model with $\epsilon$-scalar field and the chaotic scenario with an $\phi$-inflaton matter field. Comparing Eqs. (15) and (17) with the corresponding relations in ordinary inflationary models:

$$H^2 = \frac{8\pi}{3M_{pl}^2} G_N \mathcal{V}(\phi), \tag{21}$$

$$3H\dot{\phi} = -\mathcal{V}'(\phi), \tag{22}$$

we find the relations between $(\epsilon, V(\epsilon))$ and $(\phi, \mathcal{V}(\phi))$:

$$\phi = \frac{\sqrt{y}}{l}\epsilon \quad \text{with} \quad y = \frac{9\alpha_{S_0}}{8\pi}, \tag{23}$$

$$\frac{y}{l^2} V(\epsilon) = \mathcal{V}(\phi) = \frac{l^2 |\langle G^2\rangle|}{4} \phi^2. \tag{24}$$

## 3. Results and Conclusion

We now check that our model is able to fix the usual problems of the standard (Big Bang) cosmology. First, in order to solve the "horizon" and "flatness" problems, we need an inflation $a(t_f)/a(t_i)$ of order $10^{28}$ implying an inflation period $\Delta t = t_f - t_i$ such that

$$H\Delta t \sim 65. \tag{25}$$

Furthermore, it should satisfy the constraint

$$10^{-40}s \leq \Delta t \ll 10^{-10}\,\text{s} \tag{26}$$

so as not to conflict with the explanation of the baryon number and not to create too large density fluctuations.[19,20] The bound $10^{-10}$ s corresponds to the time after the Big Bang when electroweak symmetry breaking took place. Presumably, our inflation should have ended far before this time. Thus, from Eqs. (25), (26) and (15), we get the following bounds on $|\langle G^2\rangle|$:

$$3 \times 10^7 \,\text{GeV}^2 \ll \epsilon |\langle G^2\rangle|^{1/2} \leq 3 \times 10^{37} \,\text{GeV}^2 \tag{27}$$

In order to determine $\epsilon_i$, we have

$$\frac{d \ln a}{d\epsilon} = \frac{H}{\dot{\epsilon}} \simeq -\frac{3H^2}{V'(\epsilon)} \simeq -\frac{8\pi \rho_g^T}{M_{Pl}^2 V'},$$

which gives, using Eqs. (11) and (17), the relation:

$$65 \sim \ln \left(\frac{a(t_f)}{a(t_i)}\right) = \left(\frac{L_P}{l}\right)^2 \frac{9\alpha_{S_0}}{4}(\epsilon_i^2 - 1). \tag{28}$$

Next comes the "formation of structure" problem and we require the fractional density fluctuations at the end of inflation to be of the order of $\frac{\delta M}{M}|_{t_f} \sim 10^{-5}$ so that quantum fluctuations in the de Sitter phase of the inflationary universe form the source of perturbations providing the seeds for galaxy formation and in order to agree with the CMB anisotropy limits. Within the relativistic theory of cosmological perturbations,[21,22] the above fractional density fluctuations represent (to linear order) a gauge-invariant quantity and satisfy the equation

$$\left.\frac{\delta M}{M}\right|_{t_f} = \left.\frac{\delta M}{M}\right|_{t_i} \left.\frac{1}{1 + \dfrac{p}{\rho}}\right|_{t_i}, \tag{29}$$

where $\delta M$ represents the mass perturbations and where the initial fluctuations are generated quantum mechanically and are given by[21−23]

$$\left.\frac{\delta M}{M}\right|_{t_i} = \frac{\mathcal{V}'(\Phi)H}{\rho} = \sqrt{y}\frac{V'(\epsilon)H}{l\rho}, \tag{30}$$

whence

$$10^{-5} \sim \left.\left|\frac{\delta M}{M}\right|\right|_{t_f} = \sqrt{y}\left.\left|\frac{V'H}{l}\right|\right|_{t_i} \frac{1}{|(\rho + p)|_{t_i}}. \tag{31}$$

In order to evaluate $(\rho + p)|_{t_i}$, we use the energy conservation equation:

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0, \tag{32}$$

and after substituting $\rho \sim \rho_g^T$, we get

$$|(\rho + p)|_{t_i} = \frac{1}{24\pi} \left(\frac{l}{L_P}\right)^2 |\langle G^2 \rangle|. \tag{33}$$

In fact, the energy conservation equation can be used to solve for $\rho_g^r$ and we can check that

$$\rho_g^r(\dot{\rho_g^r}) \sim \rho_\epsilon(\dot{\rho_\epsilon}) \sim \delta \times \rho_g^T(\delta \times \dot{\rho_g^T}), \tag{34}$$

where $\delta \equiv |\frac{\dot{\epsilon}}{H\epsilon}| \sim \frac{1}{\epsilon^2}(\frac{l}{L_P})^2$ and so, when the "slow roll" condition (18) is satisfied, our solution assuming the predominance of the "trace-anomaly" energy mass

density is self-consistent. Substituting Eq. (33) in (31) and using Eq. (17), we get

$$\frac{l}{L_P} \sim 9\sqrt{\frac{3\pi}{2}}\alpha_{S_0}10^5\epsilon^2|\langle G^2\rangle|^{\frac{1}{2}}G_N. \tag{35}$$

Hence, taking $G_N \sim 10^{-38}\,\mathrm{GeV}^{-2}$, we obtain

$$\frac{l}{L_P} \sim \frac{|\langle G^2\rangle|^{\frac{1}{2}}}{10^{34}\,\mathrm{GeV}^2}\epsilon^2, \tag{36}$$

and combining this last result with (27), we get

$$10^{-27} \ll \frac{1}{\epsilon}\frac{l}{L_p} \leq 10^3. \tag{37}$$

The "slow roll" condition (18) is consistent with the upper bound, while the lower bound restricts $\epsilon_i$ from being too large.

On the other hand, it is possible to calculate the spectral index of the primordial power spectrum for a quadratic potential as follows:

$$n - 1 = -4\eta \quad \text{where} \quad \eta = \frac{M_{Pl}^2}{8\pi}\frac{\mathcal{V}''}{\mathcal{V}} = \frac{2}{9\alpha_{S_0}}\left(\frac{l}{L_P}\right)^2\frac{1}{\epsilon^2} = \delta \tag{38}$$

and we find

$$n = 1 - \frac{1}{\pi y}\left(\frac{l}{L_p}\right)^2\frac{1}{\epsilon_i^2}. \tag{39}$$

The inflation would end ($\epsilon_f = 1$) when the "slow roll" parameter $\eta = \delta = 1$. We should evaluate the QCD coupling constant

$$\alpha_{S_0}(\mu) = \frac{4\pi}{\beta_0 \ln\left(\dfrac{\mu^2}{\Lambda_{\mathrm{QCD}}^2}\right)}$$

at an energy scale corresponding to the inflationary period. We take this to be around the GUT scale $\sim 10^{15}\,\mathrm{GeV}$ and $\beta_0 = 11 - \frac{2}{3}n_f = 7$ (the weak logarithmic dependence would assure the same order of magnitude for $\alpha_{S_0}$ calculated at other larger scales). With $\Lambda_{\mathrm{QCD}} \sim 0.2\,\mathrm{GeV}$ (Ref. 24), we estimate $\alpha_{S_0} \sim 0.025$, and so we get

$$\left(\frac{l}{L_P}\right)^2 \sim 10^{-1}. \tag{40}$$

This is in disagreement with Bekenstein assumption that $L_P$ is the shortest length scale in any physical theory. However, it should be noted that Beckenstein's framework is very similar to the dilatonic sector of string theory and it has been pointed out in the context of string theories[25,26] that there is no need for a universal relation between the Planck and the string scale. Furthermore, determining the order of magnitude of $\frac{l}{L_P}$ is interesting in the context of these theories.

From (28), we have $\epsilon_i \sim 11$, and then using (39), we have $n = 0.97$, which is within the range of WMAP results.[15,16]

The model reproduces the results of the chaotic inflationary scenario. However, the shape of the potential was not designed by hand, rather a gauge theory with a changing coupling constant led naturally to it. Moreover, in typical chaotic models, the inflaton field starts from very large values ($\phi_i \sim 15 M_{Pl}$) and ends at around $1 M_{Pl}$. One might suspect whether field theory is reliable at such high energies. Nonetheless, this problem is absent in our model since the large values have another meaning in that they just refer to a reduction of the strong coupling by around ten times during the inflation.

Furthermore, chaotic inflations get a typical reheating of order $T_{rh} \sim 10^{15}$ GeV, and one might need to worry about the relic problem. Similarly, Eq. (36) leads in our model to a gluon condensate $|\langle G^2 \rangle|_i \sim 10^{62}$ GeV$^4$ at the start of inflation. From Eq. (19), we see that this corresponds to an inflation time interval $\Delta t \sim 10^{-35}$ s satisfying the constraint (26). If the gluon condensate stays constant, as we assumed in our analysis, we will have the same reheating temperature as in chaotic models ($T_{rh} \sim \rho(t_f)^{1/4}$). However, we should compare this value for $\langle G^2 \rangle$ with its present value renormalized at the GUT scale $\sim 10^{15}$ GeV, which can be calculated knowing its value at the 1 GeV Ref. 12 and that the anomalous dimension of $\alpha_S G^2$ is identically zero. We get

$$\langle G^2(\text{now}, \mu \sim 10^{15}\,\text{GeV}) \rangle \sim 1\,\text{GeV}^4, \tag{41}$$

which represents a decrease of 62 orders of magnitude.

This can give a possible picture for an exit scenario. Lacking a clear theory for the non-perturbative dynamics of the gluon condensate, we consider its value $|\langle G^2 \rangle|$ depending on energy, and thus implicitly on cosmological time, as given by the standard RGE, which turns it off logarithmically at high energy. However, we can furthermore assume the condensate value to depend explicitly on time during inflation:

$$\langle G^2(E, t) \rangle = \langle G_0^2(E(t)) \rangle f(t),$$

where $\langle G_0^2(E) \rangle$ is the piece determined by the RGE; the unknown function $f(t)$ should be such that it varies slowly during most of the inflationary era, to conform with an approximately constant huge and negative value of $\langle G^2 \rangle$, while at the end of inflation it causes a drastic drop of the condensate value $\langle G^2 \rangle$ to around zero. The energy release of this helps in reheating the universe, while reaching the value 0 leads to a minute "trace-anomaly" energy mass density (Eq. 11) ending, thus, the inflation. The other types of energy density would contribute to give the gluon condensate its "small" positive value of (41), and the subsequent evolution is just the standard one given by RGE. Certainly, this phenomenological description needs to be tested and expanded into a theory where the concept of symmetry breaking of such a phase transition for the condensate $\langle G^2 \rangle$ provides the physical basis for ending the inflation. Nonetheless, with a test function of the form $f(t) = -\beta^2 \tanh^2(\epsilon - 1)$ with $\beta \sim 10^{31}$, one can integrate analytically the equation of motion, and in the "slow roll" regime we have $\epsilon = 1 + \text{arcsinh}(\exp[-\alpha\beta t])$ with
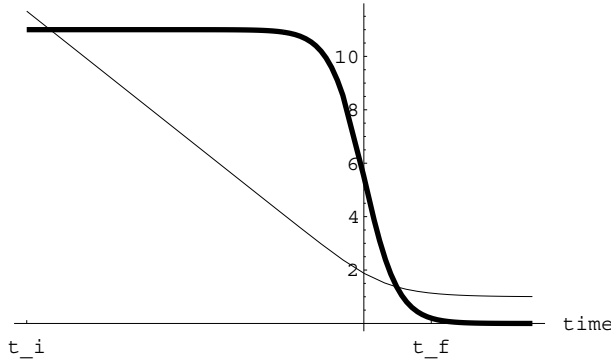
Fig. 1. Temporal evolution of the condensate $|\langle G^2 \rangle|$ (thick line) and the $\epsilon$-field (thin line), for the choice $f(t) = -\beta^2 \tanh^2(\epsilon - 1)$. The $\langle G^2 \rangle$ scale has been adapted so as to visualize both graphs together.

$\alpha\beta \sim 10^{12}$ GeV. The graph in Fig. 1 shows the time evolutions of the condensate and the $\epsilon$-field, which agree with the required features. This example is meant to be just a proof of existence of such functions, and the temporal dependence of the condensate could be of a completly different shape while the whole picture is still self-consistent. The issue demands a detailed study for the condensate within an underlying theory and we do not further it here. We hope this work will stimulate interest in the subject.

## Acknowledgments

## References

1. A. H. Guth, *Phys. Rev. D* **23**(2) (1981) 347.
2. A. Albrecht and P. Steindhardt, *Phys. Rev. Lett.* **48** (1982) 1220.
3. A. Linde, *Phys. Lett. B* **129** (1983) 177.
4. A. Linde, D. Linde and A. Mezhlumian, *Phys. Rev. D* **49**(4) (1994) 1783.
5. J. Maguejo, *Rep. Prog. Phys.* **66** (2003) 2025.
6. J. Moffat, *Int. J. Phys. D* **2**(3) (1993) 351.
7. J. Moffat, *Founda. Phys.* **23** (1993) 411.
8. A. Albrecht and J. Maguejo, *Phys. Rev. D* **59**(4) (1999) 3516.
9. J. Khoury, B. Ovrut, N. Seiberg, P. Steindhardt and N. Turok, *Phys. Rev. D* **65** (2002) 086007.
10. J. Khoury, B. Ovrut, P. Steindhardt and N. Turok, *Phys. Rev. D* **64** (2001) 123522.
11. V. Vanchurin and A. Vilenkin, *Phys. Rev. D* **61** (2000) 083507.
12. N. Chamoun, S. Landau and H. Vucetich, *Phys. Lett. B* **504**(1) (2001) [astro-ph/0008436].
13. P. de Bernardis *et al.*, *Nature* **404** (2000) 995.
14. S. Hanany *et al.*, *Astrophys. J.* **545** (2000) L5.

15. H. V. Peiris *et al.*, First year Wilkinson microwave anisotropy probe (WMAP) observations: Implications for inflation, *Astrophys. J. Suppl.* **148** (2003) 213 [astro-ph/0302225].
16. D. Spergel *et al.*, Wilkinson microwave anisotropy probe (WMAP) three year results: Implications for cosmology, astro-ph/0603449.
17. W. Greiner and A. Schafer, *Quantum Chromodynamics*, 2nd Edn. (Springer, 1995).
18. A. H. Guth, in *Proc. Cosmic Questions 1999* (The New York Academy of Sciences Press, 1999) [astro-ph/0101507].
19. S. Hawking, *Phys. Lett. B* **115**(4) (1982) 295.
20. S. Hawking, *Phys. Lett. B* **150**(5) (1985) 339.
21. R. Brandenberger, in *ICTP 1997 Summer School in High Energy Physics and Cosmology*, eds. E. Gava *et al.* (World Scientific, 1997).
22. R. Brandenberger, *Invited Lectures at TASI-94*, Colorado (June 1994) [astro-ph/9411049].
23. V. Mukhanov, H. Feldman and R. Brandenberger, *Phys. Rep.* **215** (1992) 203.
24. Particle Data Group, http://pdg.lbl.gov/ (14 June, 2000).
25. C. P. Bachas, *Class. Quant. Grav.* **17** (2000) 951 [hep-th/0001093].
26. I. Antoniadis and B. Pioline, *Nucl. Phys. B* **550** (1999) 41 [hep-th/9902055].