Universal decoherence induced by an environmental quantum phase transition

Fernando Martín Cucchietti,[1] Sonia Fernandez-Vidal,[2] and Juan Pablo Paz[1,3]

[1]*Theoretical Division, Los Alamos National Laboratory, MS B213, Los Alamos, New Mexico 87545, USA*

[2]*Departamento de Física, Universidad Autonoma de Barcelona, Bellaterra, Spain*

[3]*Departamento de Física, FCEyN, UBA, Pabellón 1 Ciudad Universitaria, 1428 Buenos Aires, Argentina*

(Received 18 April 2006; published 27 March 2007)

Decoherence induced by coupling a system with an environment may display universal features. We demonstrate that when the coupling to the system drives a quantum phase transition in the environment, the decay of quantum coherences in the system is Gaussian with a width independent of the system-environment coupling strength. We obtain analytical results for a class of solvable models, and present numerical evidence supporting the validity of our results in more general cases. This effect opens the way for a quantum simulation algorithm, where a single qubit is used to detect a quantum phase transition. We discuss possible implementations of such an algorithm and relate our results to available data on universal decoherence in NMR echo experiments.

DOI: 10.1103/PhysRevA.75.032337 PACS number(s): 03.67.Lx, 03.65.Yz, 05.70.Fh

I. INTRODUCTION

The coupling between a quantum system and its environment leads to decoherence, the process by which quantum information is degraded. Decoherence plays a crucial role in the understanding of the quantum to classical transition [1]. It also has practical importance: its understanding is essential in technologies that actively use quantum coherence, such as quantum information processing [2]. In general, the time scale t_{dec} of decoherence depends on the system-environment coupling strength, which we arbitrarily denote λ. For example, in the well-studied case of quantum Brownian motion (where the environment consists of a large number of noninteracting harmonic oscillators), quantum coherence generally decays exponentially with a rate $1/t_{dec}$ proportional to λ^2 [3]. In this paper we describe a class of systems with a drastically different behavior: The decay of quantum coherence is Gaussian with a rate which is independent of the system-environment coupling strength λ. Notice that there are many examples where Gaussian decoherence can be observed, however, in all these cases the rate depends on the coupling to the environment (see, e.g., Ref. [7] and references therein). This independence signals a type of universal behavior whose study is the aim of this work. In general, one should avoid building physical quantum information processing devices in the presence of this type of universal decoherence. However, we show that universality may also turn out to be a powerful property that could be used to our advantage: In fact, using a single qubit as a probe, by detecting decoherence in the universal regime we could extract valuable information about the environment.

Environment-independent decoherence rates are also found in other circumstances. For example, systems with a classically chaotic Hamiltonian display a "Lyapunov regime" where the decay is exponential and given by the Lyapunov exponent of the underlying classical dynamics [4,5]. These models are also often used to represent a *complex* environment. In particular, this was the motivation of Ref. [5] to explain the perturbation-independent decay of polarization detected in recent NMR echo experiments [6] (where, however, a nonexponential but Gaussian decay is actually observed). Our findings are different from the usual exponential Lyapunov regime: we discuss systems where the universal (independent of λ) decoherence is Gaussian. In our model, the complexity and sensitivity of the environment arise from the susceptibility of the environmental spectrum to the system's state. The relation between our results and the experiments of Ref. [6] will also be discussed below. The paper is organized as follows: In Sec. II we present the basic ingredients of our model (a central spin interacting with a spin bath). In Sec. III we discuss the conditions under which the environment induces a Gaussian decay of quantum coherence in the system and, more interestingly, the condition under which the decay width is independent of the system-environment coupling strength. We also show that such conditions are met in a model that can be exactly solved (the Ising chain with a transverse coupling to a central spin), and show some evidence supporting the validity of our results in more general models (the Bose-Hubbard model, in particular). These two models are considered paradigms of quantum phase transitions [11]. In Sec. IV we present our conclusions, and we show and discuss some of the implications of our work.

II. THE MODEL

Let us consider a spin 1/2 particle (a qubit) coupled to an environment that is "structurally unstable" with respect to the system state (in a sense that will be made clear below). The model we discuss is a generalization of the one studied by Quan *et al.* [8], who showed that an environment at the critical point of a quantum phase transition is highly efficient in producing decoherence. Below, we will not only generalize the results of [8] but also show that in these circumstances universal decoherence arises naturally. Afterwards, we will test the generality of our analytical results numerically in a system where our assumptions do not hold. We assume that the system and the environment evolve under the Hamiltonian

$$\mathcal{H}_{\mathcal{SE}} = \mathcal{I}_S \otimes \mathcal{H}_E + |0\rangle\langle 0| \otimes \mathcal{H}_{\lambda_0} + |1\rangle\langle 1| \otimes \mathcal{H}_{\lambda_1}. \quad (1)$$

Here, the operators $\mathcal{H}_E$, $\mathcal{H}_{\lambda_0}$, and $\mathcal{H}_{\lambda_1}$ act on the Hilbert space of the environment. If the system is in state $|j\rangle$ ($j=0,1$), the environment evolves with an effective Hamiltonian $\mathcal{H}_j = \mathcal{H}_E + \mathcal{H}_{\lambda_j}$ (λ_j is the system-environment coupling strength). Considering the initial state $|\Psi_{\mathcal{SE}}(0)\rangle = (a|0\rangle + b|1\rangle)|\mathcal{E}(0)\rangle$, the evolved reduced density matrix of the system is

$$\rho_{\mathcal{S}}(t) = \mathrm{Tr}_{\mathcal{E}}|\Psi_{\mathcal{SE}}(t)\rangle\langle\Psi_{\mathcal{SE}}(t)| = |a|^2|0\rangle\langle 0| + ab^* r(t)|0\rangle\langle 1| + a^* b r^*(t)|1\rangle\langle 0| + |b|^2|1\rangle\langle 1|. \quad (2)$$

The off-diagonal terms of this operator are modulated by the decoherence factor $r(t)$: the overlap between two states of the environment obtained by evolving the initial state $|\mathcal{E}(0)\rangle$ with two different Hamiltonians, i.e.,

$$r(t) = \langle \mathcal{E}(0)|e^{i\mathcal{H}_0 t}e^{-i\mathcal{H}_1 t}|\mathcal{E}(0)\rangle. \quad (3)$$

Notice that $r(t)$ has the form of a Loschmidt echo (or fidelity), which can show universal behavior (with an exponential decay) when $\mathcal{H}_i$ are classically chaotic Hamiltonians [5,7]. Assuming that the initial state of the environment is the ground state $|g_0\rangle$ of $\mathcal{H}_0$ [9], $r(t)$ is, up to an irrelevant phase factor, identical to the so-called survival probability amplitude

$$r(t) = \langle g_0|e^{-i\mathcal{H}_1 t}|g_0\rangle. \quad (4)$$

Let us first analyze models where both Hamiltonians $\mathcal{H}_j$ ($j=0,1$) can be diagonalized in terms of a suitable set of fermionic creation and annihilation operators $\gamma_k^{(j)}$:

$$\mathcal{H}_j = \sum_{k=1}^{N} \epsilon_k^{(j)}\left(\gamma_k^{(j)\dagger}\gamma_k^{(j)} - \frac{1}{2}\right). \quad (5)$$

Furthermore, we assume that the operators appearing in the two Hamiltonians $\mathcal{H}_j$ can be connected by a Bogoliubov transformation of the form

$$\gamma_k^{(1)} = \cos(\alpha_k)\gamma_k^{(0)} - i\sin(\alpha_k)\gamma_{-k}^{(0)\dagger}, \quad (6)$$

where the angles α_k define the Bogoliugov coefficients. Notice that this expression only includes mixing between modes with opposite values of the index k. Our treatment can be extended to more complicated situations, but we limit first to the simplest nontrivial case, where it is possible to relate the ground states $|g\rangle_j$ of $\mathcal{H}_j$ as

$$|g\rangle_0 = \prod_{k>0}[i\cos(\alpha_k) + \sin(\alpha_k)\gamma_k^{(1)\dagger}\gamma_{-k}^{(1)\dagger}]|g\rangle_1. \quad (7)$$

Under these assumptions the decoherence factor is

$$r(t) = \prod_{k>0}(\cos^2(\alpha_k)e^{it\epsilon_k^{(1)}} + \sin^2(\alpha_k)e^{-it\epsilon_k^{(1)}}). \quad (8)$$

The above expression will be the starting point of our calculation. We remind the reader that the assumptions under which it was derived are (a) the two effective Hamiltonians $\mathcal{H}_j$ can be diagonalized in terms of two sets of (fermionic) creation and annihilation operators $\gamma_k^{(j)}$ ($j=0,1$) that can be related by means of the Bogolubov transformation (6); and (b) The initial state of the environment is an eigenstate of $\mathcal{H}_0$. Below, we will show that under rather generic conditions for the coefficients of the Bogolubov transformation we can find a Gaussian decay for $r(t)$ and also discuss the conditions under which the decay displays universal features. Condition (a) restricts our results to a broad but specific class of systems. Below we will test numerically the validity of our results in a case where condition (a) is not valid.

III. CONDITIONS FOR A GAUSSIAN DECAY WITH UNIVERSAL WIDTH

It is worth noticing that the expression we obtained in Eq. (8) for $r(t)$ is completely analogous to the one found when studying decoherence on a qubit induced by noninteracting spin environments [10]. In that case, the index k labels the different environmental spins, the corresponding Bogoliubov coefficients define their initial states, and the energies $\epsilon_k^{(1)}$ are related with the coupling strengths between the central spin and the environmental ones.

Under reasonable assumptions on the angles α_k and the energies $\epsilon_k^{(1)}$, we can go further and—using the ideas developed in [10]—obtain a simple form for the temporal evolution of the overlap $r(t)$. To illustrate our procedure, let us analyze first an oversimplified case: suppose that the energies of all the modes are the same, i.e., $\epsilon_k^{(1)} = \epsilon$. In the simplest case $\alpha_k = \pi/4$, the overlap oscillates as $r(t) = (\cos\epsilon t)^{N/2}$. The same result is recovered as a consequence of the law of large numbers if the angles α_k are spread over the entire circle. In fact, $|r(t)|^2 \simeq |\cos\epsilon t|^N$ if the following Lindenberg conditions are satisfied:

$$\frac{1}{N}\sum_k \cos^2\alpha_k \simeq 1/2,$$

$$s_N^2 = \sum_k \sin^2 2\alpha_k(\epsilon_k^{(1)})^2 \gg \epsilon^2. \quad (9)$$

The first condition is satisfied when the angles are randomly distributed. The second one imposes a finite variance for the "random walk" in which a step of length $+\epsilon_k$ ($-\epsilon_k$) is taken with probability $\cos^2\alpha_k$ ($\sin^2\alpha_k$). When $\epsilon_k^{(1)} = \epsilon$, the condition takes the form $s_N^2 \gg 1$, and it is met when there is a sufficiently large number of modes for which $\sin 2\alpha_k$ does not vanish.

A more realistic situation is when the energies $\epsilon_k^{(1)}$ take values in a given spectral band. When the energies are distributed with a vanishing mean value, the decay of $r(t)$ is Gaussian with a width given by s_N^2 defined in Eq. (9) [10]. Consider the more general case where the energies are distributed about an arbitrary mean value, i.e., $\epsilon_k^{(1)} = \epsilon + \delta_k$ (where δ_k has zero mean). We now define the dispersion $\tilde{s}_N^2$ as the cumulative variance of the fluctuations of the energy, and we find that, in general, when

$$\frac{1}{N}\sum_k \cos^2 \alpha_k \simeq 1/2,$$

$$\tilde{s}_N^2 = \sum_k \sin^2(2\alpha_k)\delta_k^2 \gg \epsilon^2, \qquad (10)$$

$r(t)$ is described by a Gaussian envelope modulating an oscillating term,

$$|r(t)|^2 = \exp(-\tilde{s}_N^2 t^2)|\cos(\epsilon t)|^{N/2}. \qquad (11)$$

When the operators $\gamma_k^{(0)}$ and $\gamma_k^{(1)}$ are similar, the angles α_k are small and Eqs. (9) do not hold: Therefore in such a case there is almost no decoherence. However, a drastic difference in the nature of the eigenstates of $\mathcal{H}_0$ and $\mathcal{H}_1$ can only be accounted for with α_k varying in the full range $[0,2\pi)$. This occurs when the environment suffers a quantum phase transition when λ is varied. Thus denoting λ_c the critical point of the transition, for $\lambda_0 \ll \lambda_c \ll \lambda_1$ we expect the decoherence factor to behave as indicated in Eq. (11). When $\tilde{s}_N^2$ is only given by the properties of the environment Hamiltonian, the decay of $r(t)$ becomes *universal* (independent of λ).

A. Ising spin chain environment

An important model where the above assumptions (5) and (6) are satisfied is the following: We consider a system that consists of a central spin which interacts with an environment formed by an Ising chain. The system environment interaction is such that the two computational states of the system induce two different transverse magnetic fields in the environment. This model was previously studied in [8] and is such that the two effective Hamiltonians for the environment that are associated with the two computational states of the system are

$$\mathcal{H}_j = -J\left(\sum_{i=1}^{N} \sigma_i^z\sigma_{i+1}^z + \lambda_j \sum_{i=1}^{N} \sigma_i^x\right). \qquad (12)$$

Notice that the coupling of the system to all spins in the environment chain is rather unrealistic. However, numerical results coupling the system to only a few spins in the environment (local interaction) show behavior similar to our results [12]. Both the Bogoliubov coefficients and the energies can be analytically computed and turn out to be given by the following simple expressions [11]:

$$\epsilon_k^{(j)} = 2J\sqrt{1+\lambda_j^2 - 2\lambda_j\cos(2\pi k/N)}, \qquad (13)$$

$$2\alpha_k = [\theta_k(\lambda_1) - \theta_k(\lambda_0)]. \qquad (14)$$

In this case the critical value is $\lambda_c=1$ and the angles $\theta_k(\lambda)$ are defined as

$$\tan(\theta_k) = \sin(2\pi k/N)/[\lambda - \cos(2\pi k/N)].$$

When $\lambda_1 \gg 1$ and $\lambda_0 < 1$, the angles $\alpha_k(\lambda) \approx \pi k/N$ and the Bogoliubov coefficients satisfy conditions (9) [13]. Moreover, the energies $\epsilon_k^{(1)}$ are distributed between $|\lambda_1 - 1|$ and $|\lambda_1+1|$, which gives $\tilde{s}_N^2 \approx N$. As the dispersion $\tilde{s}_N^2$ turns out to be independent of λ_1, the width of the Gaussian envelope is

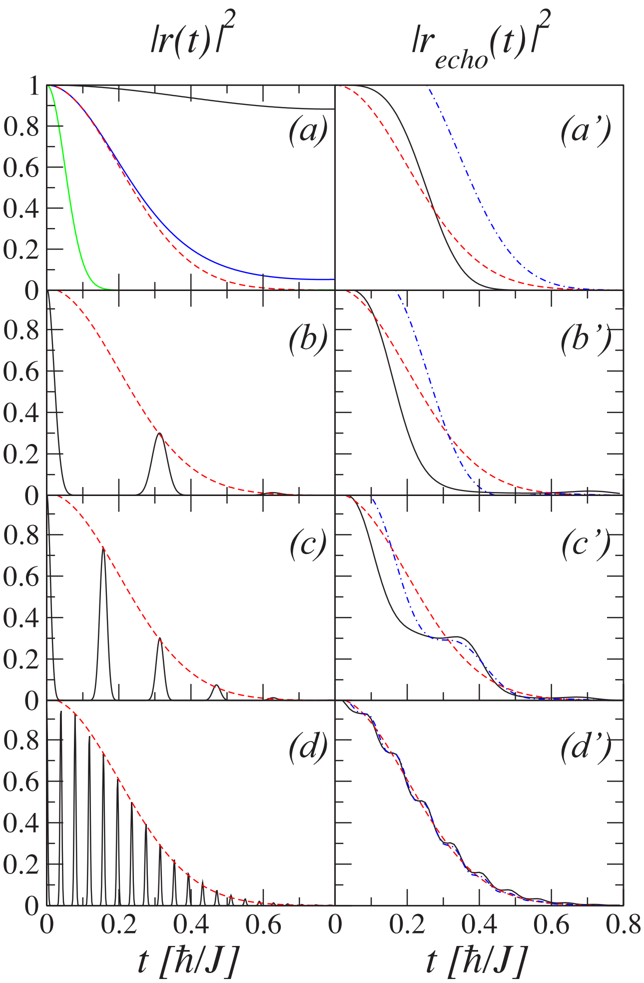

FIG. 1. (Color online) Left: Decoherence factor for a spin transversely coupled to an Ising chain with 50 spins for different coupling strengths λ_1. (a) $\lambda_1=0.1$, 0.5, and 2, (b) $\lambda_1=5$, (c) $\lambda_1=10$, and (d) $\lambda_1=40$. Right: Decoherence factor after the spin echo sequence, (a′) $\lambda_1=2$, (b′) $\lambda_1=5$, (c′) $\lambda_1=10$, and (d′) $\lambda_1=40$. The dot-dashed line is Eq. (15). In all plots $\lambda_0=0$, and the dashed line is the predicted universal Gaussian envelope.

independent of λ_1. Thus this simple model displays the two main features we are interested in discussing: A Gaussian decay with a width that turns out to be independent of the coupling strength between the system and the environment.

In Fig. 1 we display $r(t)$ for $\lambda_0=0$, showing the accuracy of Eq. (11). After the critical point of the quantum phase transition, the envelope of $r(t)$ has a *universal* envelope independent of λ_1. However, oscillations (whose frequency depends on λ_1) are not universal. Yet, it is possible to eliminate them by performing a spin-echo experiment: first, evolve the system coupled to the Ising chain environment for a time t. At this time, flip the environmental spins in the x direction (e.g., with a rf pulse that applies a π-rotation around the z-axis). Finally, evolve for another time t. The total evolution can be described using the Hamiltonian $\mathcal{H}_1=\mathcal{H}_E+\mathcal{H}_{\lambda_1}$ from time 0 to t, and $\mathcal{H}_{-1}=\mathcal{H}_E-\mathcal{H}_{\lambda_1}$ from time t to $2t$. Thus in this echo experiment the decoherence factor is

$$r_{echo}(2t) = \langle g|_0 e^{-i\mathcal{H}_{-1}t} e^{-i\mathcal{H}_1 t}|g\rangle_0. \tag{15}$$

This overlap is simply computed using the Bogoliubov transformation that connects the modes diagonalizing the Hamiltonians $\mathcal{H}_{-1}$ and $\mathcal{H}_0$. If we denote $\gamma_k^{(-1)}$ the modes of $\mathcal{H}_{-1}$, the Bogoliubov coefficients associated with the corresponding angles $\tilde{\alpha}_k$ are such that $\gamma_k^{(-1)} = \cos(\tilde{\alpha}_k)\gamma_k^{(0)} - i\sin(\tilde{\alpha}_k)\gamma_{-k}^{(0)\dagger}$. The analytic form for the overlap $r_{echo}(t)$ is simplified introducing the sum and difference of the energies, $\epsilon_k^{(\pm)} = \epsilon_k^{(1)} \pm \epsilon_k^{(-1)}$, and the Bogoliubov angles, $\alpha_k^{(\pm)} = \tilde{\alpha}_k \pm \alpha_k$. We obtain

$$\begin{aligned} r_{echo}(2t) = \prod_{k>0} [&\cos \epsilon_k^{(+)}t \cos^2 \alpha_k^{(-)} + \cos \epsilon_k^{(-)}t \sin^2 \alpha_k^{(-)} \\ &+ i \sin \epsilon_k^{(+)}t \cos \alpha_k^{(-)} \cos \alpha_k^{(+)} \\ &+ i \sin \epsilon_k^{(-)}t \sin \alpha_k^{(-)} \sin \alpha_k^{(+)}]. \end{aligned} \tag{16}$$

For the Ising model this can be evaluated explicitly. In the limit of large λ we obtain, using similar arguments as above,

$$r_{echo}(t) \approx \exp(-\tilde{s}_N^2 t^2)\left(1 - \frac{K(t)}{\lambda}\sin(\lambda t)\right), \tag{17}$$

where $K(t) = 2\Sigma_k \sin(\epsilon_k^{(-)}t)\cos(2\pi k/N)\sin^2(2\pi k/N)$. Figure 1 shows the echo dependence on λ.

B. Bose-Hubbard chain environment

One can, in fact, question the generality of our results which were obtained on the basis of a simple argument based on some hypothesis. To test the generality of our results against the restrictiveness and uncontrollability of assumptions (5) and (6), we studied a system which belongs to a completely different class of models: the Bose-Hubbard model (BHM) [11], with Hamiltonian

$$\mathcal{H}_{BH} = -g\sum_{\langle i,j\rangle} a_i^\dagger a_j + u\sum_n a_n^\dagger a_n (a_n^\dagger a_n - 1). \tag{18}$$

Here a_n are boson annihilation operators in site n of a discrete lattice. For $g \gg u$, the system behaves as a superfluid of noninteracting particles; but when $u \gg g$, the interaction term dominates and the ground state is Mott-insulator-like. This model cannot be cast in terms of fermionic operators as in Eq. (5), in fact, no analytic solution is known. The BHM has practical relevance because it can be experimentally simulated using cold atoms in optical lattices [14]. We calculate $r(t)$ numerically for a spin 1/2 coupled to the hopping term of H_{BH}, that is, we take $g \equiv \lambda$. In Fig. 2 we show the decoherence factor for several values of λ for a BHM with a fixed number of bosons. The same overall behavior of the Ising chain is observed: a universal Gaussian envelope (independent of λ) modulating an oscillation with frequency proportional to λ. The very different nature of the BHM hints at a more general validity of our results.

IV. CONCLUSIONS AND OUTLOOK

In this paper we demonstrated the existence of a class of models for which the decoherence induced by the coupling between a system and its environment displays a rather interesting feature: the decay of quantum coherence is Gaussian and independent of the system-environment coupling strength. There are many other cases where a Gaussian decay has been predicted (see [10] for example). The universal regime (characterized by the independence of the decay-width on the system-environment coupling strength) is a more interesting effect.

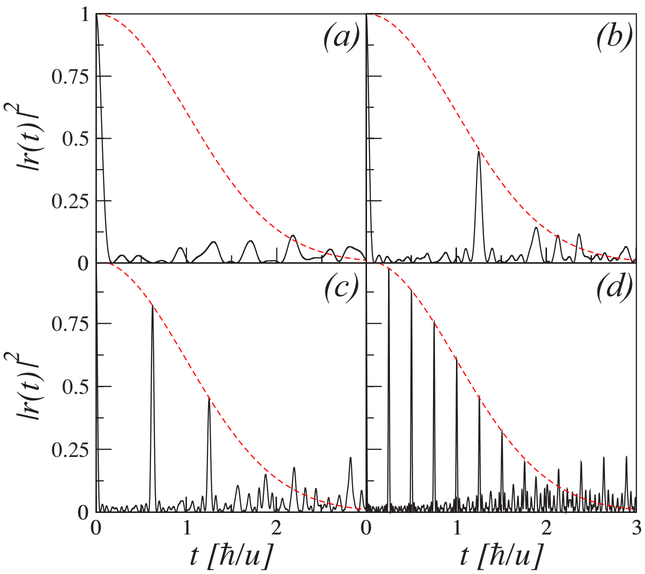

FIG. 2. (Color online) Decoherence factor for a spin 1/2 coupled to a Bose-Hubbard model through the hopping strength J (six bosons in six sites), for (a) $J=5$, (b) $J=10$, (c) $J=20$, and (d) $J=50$. In all plots the dashed line is the universal Gaussian envelope; the width was obtained numerically.

A Gaussian decay of coherence with a rate independent of the coupling to the environment was indeed observed in NMR polarization echo experiments [6]. Arguing on the complexity of the experimental many-body system, these results have been related to the environment-independent decoherence predicted in classically chaotic Hamiltonians [4,5,7]. The experimental situation is quite different from the one we considered here: the decoherence factor is measured after an echo created by a change of sign of the environment Hamiltonian, and not the system-bath interaction. Our model points to a different way of introducing complexity and sensitivity in the environment: a quantum phase transition. Further research using this approach might explore more realistic models that account for all the details of the experiments.

The universal decoherence regime of this work can also be understood using analogies to the regime of strong perturbations of the survival probability, Eq. (4), a particular case of a Loschmidt echo. Indeed, $r(t)$ is the Fourier transform of the strength function or local density of states (LDOS), $L(E) = \Sigma_n |\langle g_0|\phi_n\rangle|^2 \delta(E-E_n)$, where $|\phi_n\rangle$ are the eigenvectors of $\mathcal{H}_1$ and E_n its eigenenergies. In typical LDOS studies, $\mathcal{H}_1$ differs from $\mathcal{H}_0$ by a perturbation. In complex systems (e.g., random matrices, or classically chaotic Hamiltonians) for sufficiently strong perturbations $|g_0\rangle$ is a random superposition of the $|\phi_n\rangle$ states. Therefore the LDOS becomes independent of the perturbation: it equals the full density of states of $\mathcal{H}_1$. Chaotic systems, for instance, give an LDOS with a

Lorentzian shape [15] [leading to an exponential decay of $r(t)$], while spin systems lead to Gaussian LDOS [16]. In our model, the saturation of the LDOS when $\mathcal{H}_0$ and $\mathcal{H}_1$ are on both sides of the quantum phase transition occurs because of the radically different nature of the eigenstates.

Universal decoherence can be harmful for quantum information applications. However, it can be a useful tool to extract information about a critical system, e.g., its spectral structure or the critical point of its quantum phase transition. The latter example can be thought of as a "critical point finding" algorithm in a one-qubit quantum computer (or a quantum simulator): in systems where the spectrum is not shifted by the coupling [which gives the oscillatory $\cos(\lambda t)^N$ term], the critical point can be simply obtained as the λ value for which one observes the onset of universality. Otherwise, the oscillation term obscures the critical point. In these cases one can instead couple the system weakly to the environment and drive the transition with an external parameter (as in Ref. [8]). The critical point is then signaled by the λ value for which there is a maximum decoherence decay. A demonstration of this algorithm can be performed in an NMR setting simulating the Ising Hamiltonian studied above [17].

We have shown that when the coupling to the system drives a quantum phase transition in the environment, the decoherence factor decays as a Gaussian with an environment-independent width. We showed numerically that our findings are more general than what can be expected from the analytical approximations we used. Our results could lead to an alternative interpretation of hitherto unexplained NMR experimental results on environment independent decoherence rates. Finally, we discussed how the universal behavior of the decoherence factor can be used to study critical systems in a simulation algorithm for one-qubit quantum computers.

ACKNOWLEDGMENTS

We acknowledge fruitful discussions with W.H. Zurek. J.P.P acknowledges support from Conicet and was partially supported by grants from ANPCyT.

---

[1] J. P. Paz and W. H. Zurek, in *Coherent Matter Waves, Les Houches Session LXXII*, edited by R Kaiser, C. Westbrook, and F. David (Springer-Verlag, Berlin, 2001), pp. 533–614; W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, New York, 2000).

[3] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992).

[4] W. H. Zurek and J. P. Paz, Phys. Rev. Lett. **72**, 2508 (1994).

[5] R. A. Jalabert and H. M. Pastawski, Phys. Rev. Lett. **86**, 2490 (2001).

[6] H. M. Pastawski, P. R. Levstein, G. Usaj, J. Raya, and J. Hirschinger, Physica A **283**, 166 (2000).

[7] T. Gorin, T. Prosen, T. H. Seligman, and M. Znidaric, Phys. Rep. **435**, 33 (2006); T. Prosen and M. Znidaric, J. Phys. A **35**, 1455 (2002).

[8] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Phys. Rev. Lett. **96**, 140604 (2006).

[9] Any eigenstate of $\mathcal{H}_{\mathcal{E}}^0$ results in the same expression.

[10] F. M. Cucchietti, J. P. Paz, and W. H. Zurek, Phys. Rev. A **72**, 052113 (2005).

[11] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, England, 1999).

[12] D. Rossini, T. Calarco, V. Giovannetti, S. Montangero, and R. Fazio, Phys. Rev. A **75**, 032333 (2007).

[13] For $\lambda=\lambda_1-\delta=\lambda_0+\delta$, and $\delta\rightarrow 0$ [8], a sufficient number of modes satisfy conditions (9) only when the thermodynamic limit $N\propto 1/\delta$ is taken at $\lambda=1$.

[14] M. Greiner *et al.*, Nature (London) **419**, 51 (2002).

[15] E. P. Wigner, Ann. Math. **62**, 548 (1955); **65**, 203 (1957); Ph. Jacquod and D. L. Shepelyansky, Phys. Rev. Lett. **75**, 3501 (1995); D. Cohen and E. J. Heller, *ibid.* **84**, 2841 (2000).

[16] V. V. Flambaum and F. M. Izrailev, Phys. Rev. E **61**, 2539 (2000).

[17] R. Laflamme (private communication).