AN APPLICATION OF THE MAXIMUM ENTROPY METHOD WITH SMOOTHING TO INDUSTRIAL TOMOGRAPHY

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***Abstract*— The maximum entropy method combined with a smoothing restriction was applied to the tomography of a BNC connector using five noisy radiographies from different rotating views. A heuristic control of the filter parameter is constructed by combining the projection error and the global smoothness. The proposed algorithm can be used in cases where the homogeneity of internal material is a priori known from the design and manufacturing processes.**

***Keywords*— Maximum entropy, tomography, image processing, reconstruction, noise reduction.**

I. INTRODUCTION

Tomography is the construction of the internal image of an object taking as input data a finite set of radiographies taken at different projection angles. Tomography has many well-known applications in a variety of problems including medical diagnostics and nondestructive testing. Formally, the mathematical problem is to find the solution of the Radon anti-transformation of a spatial scalar field (Herman, 1980; Herman and Kuba, 1999), which falls into the domain of inverse problems.

In industrial applications as manufacture quality control, time schedules restrict the number of radiographies per tested object. Therefore the problem is underdetermined. However, in these cases a priori information (e.g. internal shapes, porosity, and smoothness) is often available from the design process (Barbuzza *et al.*, 2007b; Barbuzza and Clausse, 2011; Silva *et al.*, 2011; Rodríguez, 2010), which can be used to guide the image reconstruction. The maximum entropy method (MEM) was proposed to solve tomography problems by several authors (Dusaussoy and Abdou, 1991; Minerbo, 1979; Mohammad- Djafari *et al.*, 2002; Nguyen and Mohammad- Djafari, 1994), being an alternative to other methodologies as the algebraic reconstruction and back projection (Herman, 1980; Herman and Kuba, 1999). An advantage of MEM is the flexibility to incorporate prior information available in addition to that contained in the projection data.

Several proposals were presented to use a priori information in tomography (Bouman and Sauer, 1993; Frese *et al.*, 2002; Jaynes, 1957). The MEM principle was used in maximum a posteriori algorithms (MAP) (Herman and Kuba, 1999; Nguyen and Mohammad-Djafari, 1994), in the form of the penalized maximum log-likelihood (PML) and Gibbs priors (Bouman and Sauer, 1993), or with Penalized Least Squares (PLS) with priors functions like the Shannon entropy (Mohammad- Djafari *et al.*, 2002; Nguyen and Mohammad-Djafari, 1994). In PML, the Poisson nature of noise in the data explicitly enters in the algorithm whereas it is indirect in PLS (Mohammad- Djafari *et al.*, 2002; Nguyen and Mohammad- Djafari, 1994; Bouman and Sauer, 1993; Liao *et al*. 2010; Liao *et al*. 2011).

In a previous study, the MEM method was combined with smoothness restrictions accounting for a priori information available about the internal materials of the tested objects (Barbuzza *et al.*, 2010). The method showed promising results in synthetic cases. In the present work, the method is applied to the tomographic reconstruction of a real case. In order to deal with the noisy input data, a quality indicator was designed combining the projection error with the global smoothness of the solution.

II. MATERIALS AND METHODS

Let f denote an unknown image to be reconstructed, which will be here represented for convenience by an n-dimensional column vector. Let R be an $m\times n$ projection matrix, whose r_{ij} element denotes the coefficient of the contribution of the j-pixel to the i-ray (Fig. 1). Then, the discrete Radon transform of f is defined as:

$$g = R\, f \qquad (1)$$

where g is an m-dimensional projection vector (i.e., m rays are projected through f). In general, $m<n$. The CT problem consists of finding f given R and g.

In practice, the input g is incomplete and noisy, so there are many solutions f with equivalent degree of consistency with Eq. (1). The MEM reconstruction method consists of determining a suitable solution of f whose Shannon entropy:

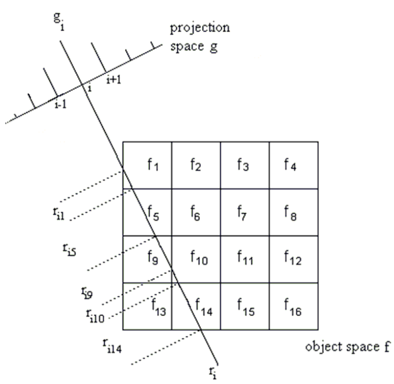

Figure 1. Path of the i-ray through a discretized object f.

$$H(f) = -\sum_{j=1}^{n} f_j \log f_j \qquad (2)$$

is maximum among all solutions approaching Eq. (1). In (Barbuzza *et al.*, 2010) proposed a method to apply MEM combined with a prior smoothness filter, $U(f)$, that is:

$$\min_f \; -H(f) + \beta U(f)$$
$$\begin{cases} f \geq 0 \\ R\,f - g \approx 0 \end{cases} \qquad (3)$$

The parameter β is a calibration constant. $U(f)$ is modeled by means of interactions among neighbor pixels as:

$$U(f) = \sum_{j}^{n} E(N_j) \qquad (4)$$

$$E(N_j) = \sum_{v \in N_j} (f_v - f_j)^2 \qquad (5)$$

where N_j are the nearest 3×3 neighbours of the j-pixel. $E(N_j)$ is a suitable indicator of the local "noise" (which vanishing in the extreme case of homogeneous clusters). $U(f)$ can be written in matrix form as:

$$U(f) = f^T M\, f \qquad (6)$$

Thus, the element m_{jv} of the matrix M of $n \times n$ is:

$$m_{jv} = \begin{cases} 2\,|N_j| & if \;\; v = j \\ -2 & if \;\; v \in N_j \\ 0 & elsewhere \end{cases} \qquad (7)$$

where $|N_j|$ is the cardinality of N_j cluster. The matrix M can also be thought as a regularization operator (Bovik, 2005; Cidade *et al.*, 2000; Fletcher, 1980; Katssangelos and Galatsanos, 1998; Katssangelos *et al.*, 1991; Luo *et al.*, 2011).

Equation (3) has an exact solution assuming $R\,f = g$, which is not a convenient restriction when the input g is noisy. The way this inconvenient is solved in Barbuzza and Clausse (2011) is by forcing the f and g to take integer values (i.e., the gray 0 to 255 scale). This heuristic procedure showed good results in synthetic reconstruction tests.

III. RESULTS

Figure 2 shows the reconstruction of a synthetic image (i.e., the solution f^* is known) using 16 projections contaminated by 2% white noise. The parameter β is used to control the smoothing filter. It can be seen that the reconstructed image is noisy for β=0 and blurry for β=10^5. In between there is an optimum range of β values that maximize the quality of the resulting image, which can be quantified as:

$$\sigma = \sum_j (f_j - f_j^*)^2 \qquad (8)$$

Figure 3 shows the variation of the output quality σ as β changes. The optimum value of β is around 10^3.

The synthetic case shown in Fig. 2 shows the feasibility of the method, but it is important to show the performance in a real case. The case studied is a tomographic reconstruction of the internal radiographic image of a BNC elbow connector stainless steel cased, used to joint coaxial cables (Fig. 4). A set of 5 radiographies obtained at rotation angles 0º, 30º, 60º, 90º and

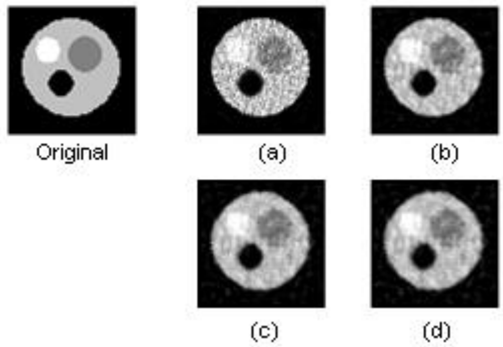

Figure 2. Reconstruction of a synthetic image using 16 projections contaminated with 2% white noise. (a) β=0 , (b) β=10^3 , (c) β=10^4 and (d) β=10^5.

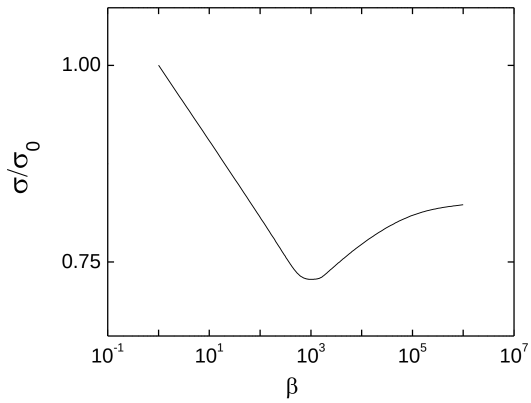

Figure 3. Quality of the synthetic image reconstruction shown in Fig. 2 (σ_0 is the initial values of σ).

Figure 4. Photograph of the BNC connector used as sample for the radiographs shown below.

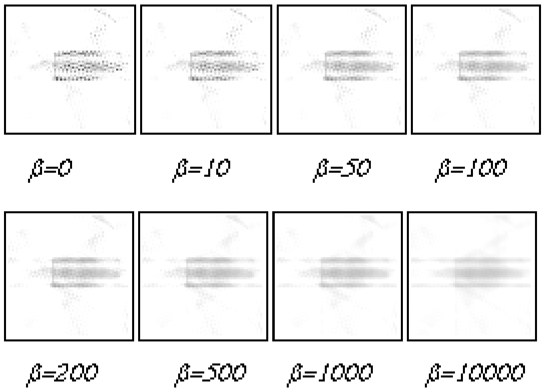

Figure 5. Internal cuts along the axis of the side branch of the connector shown in Fig. 4 varying the filter parameter β.

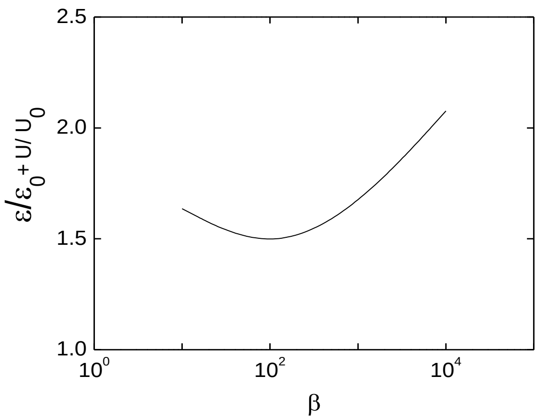

Figure 6. Combined projection-smoothness quality indicator (Eqs. 6 and 9) of the reconstruction of the object shown in Fig. 4 (ε_0 and U_0 are the initial values of ε and U).

120º around the major axis of the object (Barbuzza and Clausse, 2011; Barbuzza *et al.*, 2007a; Vénere *et al.*; 2001) were used as input data g to solve Eqs. 3 to 7 applying the Newton method (Barbuzza *et al.*, 2010; Fletcher, 1980). The resolution of the radiographs is 0.08 mm/pixel.

The main sources of noise in the radiographs are spatial intensity variations and axis alignment inaccuracies. The radiographs were pre-processed to normalize the total intensity. The reconstruction was done in separate slices perpendicular to the rotation axis. Figure 5 shows axial cuts of the side branch of the connector obtained varying the filter parameter β. As in the synthetic case (Fig. 2), it can be seen that for the limits of small and large β the output goes from noisy to blur. However, unlike in the synthetic case, Eq. 8 cannot be used here to determine a convenient value of β for the exact solution f* is not known a priori. Alternatively, an appropriate quality metric can be constructed using the quadratic deviation between the input radiographs g_i* and the corresponding projections g_i of the obtained solution, that is

$$\varepsilon = \sum_i (g_i - g_i^*)^2 \tag{9}$$

combined with the smoothness function U defined in Eq. 6. Figure 6 shows the resulting normalized quality indicator of the reconstructions varying β. A similar trend as the one found in the synthetic case (Fig. 3) can be observed. In Fig. 7 two axial cuts of the final tomography obtained with the optimum β = 100 (see Fig. 6) are shown.

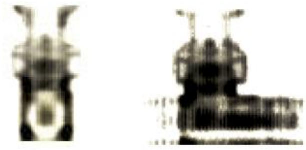

Figure 7. Axial cuts of the tomography of the object shown in Fig. 4 obtained with the β=100.

IV. CONCLUSIONS

An extended MEM algorithm combined with a smoothness filter was applied to the reconstruction of a real object from five radiographs at different rotation angles. The method had been already tested in synthetic cases showing good results, although an appropriate procedure to determine the optimum filter parameter remained an open issue. In the present work, an alternative quality indicator combining the projection error and the smoothness was proposed and tested. The proposed algorithm can be a useful tool in industrial tomography, where the homogeneity of internal material is usually known from the design and manufacturing processes. The method presented here can be compared with other approaches for tomographic reconstruction with limited data (Liao, 2007).

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