Nonlinear Controller for Trajectory Tracking of a Continuous Stirred Tank Reactor

Romina Suvire*, Mario Serrano*, Jorge Vega, Oscar Ortiz*, Gustavo Scaglia***

**Instituto de Ingeniería Química, Universidad Nacional de San Juan, Argentina (phone: +549 264 4211700; e-mail: (rsuvire, rortiz, gscaglia)@unsj.edu.ar, serranoemanuel84@gmail.com)*
*** INTEC (Universidad Nacional del Litoral-CONICET) and Facultad Regional de Santa Fe (UTN), Santa Fe, Argentina (e-mail: jvega@santafe-conicet.gov.ar)*

Abstract: This work presents a linear algebra based methodology for trajectory tracking of output variables in a Continuous Stirred Tank Reactor (CSTR), which exhibits a highly nonlinear dynamics. Its main advantage is that the condition for the tracking error tends to zero and the calculation of control actions are obtained by solving a system of linear equations. The Monte Carlo Randomized Method is used for tuning the controller parameters and testing the system behavior under modeling errors. The controller performance is evaluated through several tests and compared with other controller reported in the literature. In addition, proofs of convergence to zero of the tracking error are presented.

Keywords: CSTR; Tracking Control Design; Linear Algebra; Nonlinear Model; Monte Carlo Experiment.

1. INTRODUCTION

THE HEART of a chemical or biochemical process is the chemical reactor. The continuous stirred tank reactors (CSTR) are the most commonly used in industry, so they have been strongly studied from both dynamic analysis and control perspectives. From the point of view of process control, many linear and nonlinear control techniques have been implemented (Velazco-Perez et al., 2011) e.g., classic control and its derivations (Perez and Albertos, 2004; Prakash and Srinivasan, 2009), adaptive control (Pan et al., 2007), robust control (Fissore, 2008), among others (Abdul et al., 2007; Assandri et al., 2009). However the literature shows that the use of conventional control strategies may have a poor performance; for example, feedback control schemes in CSTR can lead to instabilities in closed loop processes (Perez and Albertos, 2004).

An important problem in the control of several CSTR consists in the trajectory tracking of some key variables, as for example the output concentration of a given component of the reactant mixture or the reactor operation temperature in the non-isothermal case, while the temperature or cooling medium flow are typical manipulated inputs. Several control strategies have been developed for trajectory tracking of CSTR systems. In (Ungureanu et al., 2001), a control method combining cascade and split range control is suggested in the polymerization of methyl methacrilate for controlling the temperaturte profile, which gives the desired final polymer properties. In (Abdul Wahab et al., 2007), the temperature control of a pilot plant reactor system using a genetic algorithm model-based control approach is proposed but shows no advantage when compared with a classical PID control. On the other hand, the recursive controller proposed in (Wang et al., 2013) for a bioprocess has significant tracking errors on the reference signal. Another alternative adaptive control method, that combines Lyapunov's stability design, adaptive backstepping, and neural network approximation to derive a control algorithm and an adaptive learning law, is developed to achieve an asymptotic tracking control of the output composition in a CSTR plant (Zhang and Guay, 2005). Similarly, in (Kalhoodashti, 2011) a neural network called approximate generalized predictive control (NNAPC) has been proposed for concentration tracking in a CSTR. This algorithm basically tries to minimize the prediction error over the training data set. In both cases, good neural network training is only reachable when the collected data include a huge amount of information on the system dynamics (Zhang and Guay, 2005; Kalhoodashti, 2011).

In general, the design of an effective control law for the trajectory tracking problem in a CSTR is a difficult task. In (Zhang and Guay, 2005) the corresponding manipulated variable presents a very undesirable behavior. In other works of literature, highly complex calculations are required as in (Monroy-Loperena et al., 2004; Velazco-Perez et al., 2011), where to carry out the controller synthesis, the transfer function must be factored and an intermediate control input, obtained from a balance in the use of the control inputs via an optimization problem, must be added. In addition, some authors obtain the transfer function of the control law for each input in a parallel control architecture, which represents an advantage in process with many input variables (Alvarez-Ramirez et al., 2004).

In (Prakash and Srinivasan, 2009) the authors have proposed a procedure for designing a scheme of Nonlinear PID control (N-PID) and a scheme of Nonlinear Model Predictive Control

(F-NMPC) using a set of five local PID controllers based on linear models for control of trajectory tracking in CSTR processes. This is, five different linear controllers must be designed and then combined to generate the most suitable control action. Similarly in Rao and Chidambaram (2008) the proposed method requires the design of three controllers. This also implies the choice of several tuning parameters to get a good performance.

In this work, the control technique previously presented in (Serrano et al., 2013), now aims at providing a simple and effective solution for tracking the desired concentration profiles in a jacked CSTR. The specific problem consists in obtaining a given concentration profile of the final CSTR product by properly manipulating the coolant flow rate. To this effect, a control methodology based on a linear algebra approach is proposed (Serrano et al., 2014; Scaglia et al., 2014; Romoli et al., 2014; Gandolfo et al., 2014). The trajectory tracking controller structure arises naturally derived from a novel and simple procedure that is inferred by analyzing the mathematical model of the process. The main advantage of this approach is the simplicity of the controller, and the use of discrete-time equations, whose implementation on a computer system becomes natural. The controller parameters are adjusted through Monte Carlo Experiment (MCE) minimizing a given cost function. Also, by using the MCE, the effect of modeling errors on the system behavior is analyzed. The proposed methodology is validated through computer simulations that show the effectiveness of the proposed controller. In addition, the proof of the convergence to zero of the tracking error is included in this paper.

The paper is organized as follows: Section 2 describes the design technique of a control system using a simple linear algebra strategy. In Section 3, the CSTR model is presented. Section 4 develops the methodology for the controller design in the proposed chemical reactor. In Section 5, the theoretical results are validated through numerical simulations. Finally, Section 6 presents the main conclusions and some topics that will be addressed in future contributions.

2. NOMENCLATURE AND METHODOLOGY FOR CONTROLLER DESIGN

Let us consider the first-order differential equation,

$$\frac{dy}{dt} = \dot{y} = f\left(y,\, t,\, u\right) \qquad y\left(0\right) = y_0 \tag{1}$$

where y represents the system output to be controlled, u is the control action, and t is the time. If the equation is discretized, the values of $y(t)$ at discrete time $t=nT_0$, where T_0 is the sampling period and $n \in \{0, 1, 2, \ldots\}$, will be denoted as $y(n)$. Thus, for computing $y(n+1)$ from the knowledge of $y(n)$, (1) must be integrated over the time interval $nT_0 \le t < (n+1)T_0$. Among several numerical integration methods able to solve (1), the Euler method provides:

$$y_{(n+1)} \cong y_{(n)} + T_0\, f\left(y_{(n)}, t_{(n)}, u_{(n)}\right) \tag{2}$$

where u remains constant in the time interval $nT_0 \le t < (n+1)T_0$. Thus, if one knows beforehand the reference trajectory $y_{ref}(t)$ to be followed by $y(t)$, then $y(n+1)$ can be substituted by a function of $y_{ref}(n+1)$, $y_{ref}(n)$ and $y(n)$, with all values known at the time nT_0 (Scaglia et al., 2014):

$$y_{ref\,(n+1)} - k\left(y_{ref\,(n)} - y_{(n)}\right) \cong y_{(n)} + T_0\, f\left(y_{(n)}, t_{(n)}, u_{(n)}\right) \tag{3}$$

where, $0 < k < 1$ is a design parameter that regulates the convergence speed to zero of tracking errors. Then, the control action, $u(n)$ required to gradually carry the system from its current state ($y(n)$) to the desired one, can be calculated.

The system can be rearranged in matrix form as follows:

$$f\left(y_{(n)}, t_{(n)}, u_{(n)}\right) \cong \frac{y_{ref\,(n+1)} - k\left(y_{ref\,(n)} - y_{(n)}\right) - y_{(n)}}{T_0} \tag{4}$$

If the system is affine in the control, i.e.,

$$f\left(y_{(n)}, t_{(n)}, u_{(n)}\right) = f_1\left(y_{(n)}, t_{(n)}\right) + f_2\left(y_{(n)}, t_{(n)}\right) u_{(n)} \tag{5}$$

where,

$$y \in \Re^m,\ u \in \Re^p,\ f_1 \in \Re^m,\ f_2 \in \Re^{mxp} \tag{6}$$

therefore,

$$\underbrace{f_2(y_{(n)}, t_{(n)})}_{\mathbf{A}_{(n)}} u_{(n)} = \underbrace{\frac{y_{ref\,(n+1)} - k\left(y_{ref\,(n)} - y_{(n)}\right) - y_{(n)}}{T_0} - f_1(y_{(n)}, t_{(n)})}_{\mathbf{b}_{(n)}} \tag{7}$$

$$\mathbf{A}_{(n)}\, u_{(n)} \cong \mathbf{b}_{(n)} \tag{8}$$

In system (8) the matrices **A** and **b** define the relationship between the control action, the state variables of the system and the reference signal at time n. Thus,

$$\begin{aligned} &\mathbf{A}_{(n)} \text{ is a matrix that is a function of } \left\{y_{(n)}, t_{(n)}\right\} \\ &\mathbf{b}_{(n)} \text{ is a matrix that is a function of } \left\{y_{(n)}, t_{(n)}, y_{ref\,(n)}, y_{ref\,(n+1)}\right\} \end{aligned} \tag{9}$$

The control action ($u(n)$) that forces the system output (y) to follow the reference signal (y_{ref}) is calculated by solving the system of linear equations (8) at each sampling period. This method exhibits the advantage that for complex (linear or nonlinear) systems, the equations can be solved by using iterative algorithms for solving linear systems; and the initial value required to start the iteration can be chosen as the solution calculated at the previous sampling instant.

To sum up, the strategy used to find the control action consists in approximating the system model through numerical methods. Consequently, the control problem is reduced to the solving of a system of linear equations. The

key to the proposed method is to find the conditions under which the linear equation system has an exact solution.

3. CSTR MODEL

The considered model of the CSTR system was previously described by (Zhang and Guay, 2005). This system is schematically represented in Fig. 1. It consists of a constant volume reactor cooled by a single coolant stream flowing in a co-current mode. An irreversible exothermic reaction, $A \rightarrow B$, occurs in the tank. Since the reaction is exothermic, the coolant flow rate q_c allows a temperature control and hence the product concentration is also controlled.

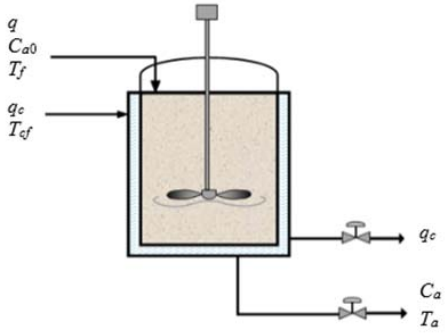

Fig. 1. Schematic representation of the continuous stirred tank reactor with a cooling jacket.

The following modeling assumptions are commonly made: i) the reactor and jacket volumes are constant, and ii) the mixing of both is perfect. Due to assumption ii), the reactant mixture properties are considered uniform anywhere within the vessel and thus are identical to the properties of the output current. On the basis of these hypotheses, the process is described by the following continuous-time, nonlinear, simultaneous, differential equations:

$$\dot{C}_a = \frac{q}{V}\left(C_{a0} - C_a\right) - a_0 C_a e^{-\frac{E}{RT_a}} \tag{10}$$

$$\dot{T}_a = \frac{q}{V}\left(T_f - T_a\right) + a_1 C_a e^{-\frac{E}{RT_a}} + a_3 X_c \left(T_{cf} - T_a\right) \tag{11}$$

where the variable X_c is defined as:

$$X_c = q_c\left[1 - e^{-a_2/q_c}\right] \tag{12}$$

The state variables, C_a and T_a, are the concentration and temperature in the tank, respectively; the coolant flow rate q_c is assumed to be the control input, as typically adopted in the literature (Zhang and Guay, 2005). Note that the auxiliary variable X_c, is directly related to the sought control variable, q_c. Initial conditions for the state variables and the system parameters are listed in Table 1.

Table 1. Initial conditions and parameters of the CSTR system.

Parameter	Description	Nominal Value
q	Process flowrate	100 l/min
C_{a0}	Inlet concentration of component A	1 mol/l
T_f	Feed temperature	350 K
T_{cf}	Inlet coolant temperature	350K
V	Volume of tank	100 l
h_a	Heat transfer coefficient	$7\text{x}10^5$ cal/min K
a_0	Preexponential factor	$7.2\text{x}10^{10}$ min^{-1}
E/R	Term of activation energy	$1\text{x}10^4$ K
$(-\Delta H)$	Heat of reaction	$-2\text{x}10^5$ cal/mol
ρ_l, ρ_c	Liquid densities	$1\text{x}10^3$ g/l
C_p, C_{pc}	Specific heats	1 cal/g K
$a_1 = \frac{(-\Delta H)a_0}{\rho_l C_p}$	Dimensionless model parameters	$1.44\text{x}10^{13}$
$a_2 = \frac{h_a}{\rho_c C_{pc}}$		$6.987\text{x}10^2$
$a_3 = \frac{\rho_c C_{pc}}{\rho_l C_p V}$		0.01

4. CONTROLLER DESIGN

The control objective is described as follows: given a pre-specified trajectory C_{aref} for the reactor effluent concentration, find the control action (i.e., the coolant flow rate, q_c) so that the reactor effluent concentration (C_a) can follow the reference trajectory. If (10) and (11) are discretized and integrated,

$$C_{a(n+1)} = C_{a(n)} + \int_{nT_0}^{(n+1)T_0} \left(\frac{q}{V}\left(C_{a0} - C_a\right) - a_0 C_a e^{-\frac{E}{RT_a}} \right) dt \tag{13}$$

$$T_{a(n+1)} = T_{a(n)} + \int_{nT_0}^{(n+1)T_0} \left(\begin{array}{l} \frac{q}{V}\left(T_f - T_a\right) + a_1 C_a e^{-\frac{E}{RT_a}} \\ + a_3 X_c \left(T_{cf} - T_a\right) \end{array} \right) dt \tag{14}$$

After applying the Euler approximation,

$$C_{a(n+1)} = C_{a(n)} + T_0\left(\frac{q}{V}\left(C_{a0} - C_{a(n)}\right) - a_0 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}}\right) \tag{15}$$

$$T_{a(n+1)} = T_{a(n)} + T_0\left(\begin{array}{l}\frac{q}{V}\left(T_f - T_{a(n)}\right) + a_1 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}} \\ +a_3 X_{c(n)}\left(T_{cf} - T_{a(n)}\right)\end{array}\right) \tag{16}$$

Equations (15) and (16) can be expressed in the following matrix form:

$$\begin{bmatrix}0\\ \\1\end{bmatrix} X_{c(n)} = \begin{bmatrix}\left(\frac{C_{a(n+1)} - C_{a(n)}}{T_0}\right) - \frac{q}{V}\left(C_{a0} - C_{a(n)}\right) + a_0 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}} \\ \frac{\left(\frac{T_{a(n+1)} - T_{a(n)}}{T_0}\right) - \frac{q}{V}\left(T_f - T_{a(n)}\right) - a_1 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}}}{a_3\left(T_{cf} - T_{a(n)}\right)}\end{bmatrix} \tag{17}$$

Then, $C_{a(n+1)}$ and $T_{a(n+1)}$ are replaced assuming proportional approximation to the error and considering a smooth trajectory tracking,

$$C_{a(n+1)} = C_{aref(n+1)} - k_1\left(C_{aref(n)} - C_{a(n)}\right) \tag{18}$$

$$T_{a(n+1)} = T_{aez(n+1)} - k_2\left(T_{aez(n)} - T_{a(n)}\right) \tag{19}$$

where the variable T_{aez} is the reactor temperature necessary to ensure that the tracking error tends to zero and its expression will be obtained later in this section, from the analysis of (21).

Remark 1: The tracking error is defined as the difference between the reference and the real trajectory. Then, $eC_{a(n)} = C_{aref(n)} - C_{a(n)}$ and $eT_{a(n)} = T_{aez(n)} - T_{a(n)}$. Besides, the norm of the total tracking error is represented by $\left\|e_{(n)}\right\| = \sqrt{\left(eT_{a(n)}\right)^2 + \left(eC_{a(n)}\right)^2}$.

In (18) and (19) the controller parameters fulfill $0< k_1, k_2 <1$ so that the tracking error tends to zero when $n \to \infty$ (see Proof of Theorem 1).

Remark 2: In (18) and (19) note that:

- If $k_1 = k_2 = 0$, the reference trajectory is reached in only one step.
- If $0< k_1, k_2 <1$, the system will slowly reach the reference profiles after several steps.

By replacing (18) and (19) into (17), the following equation is obtained:

$$\begin{bmatrix}0\\ \\1\end{bmatrix} X_{c(n)} = \begin{bmatrix}\left(\frac{C_{aref(n+1)} - k_1\left(C_{aref(n)} - C_{a(n)}\right) - C_{a(n)}}{T_0}\right) - \frac{q}{V}\left(C_{a0} - C_{a(n)}\right) + a_0 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}} \\ \frac{\left(\frac{T_{aez(n+1)} - k_2\left(T_{aez(n)} - T_{a(n)}\right) - T_{a(n)}}{T_0}\right) - \frac{q}{V}\left(T_f - T_{a(n)}\right) - a_1 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}}}{a_3\left(T_{cf} - T_{a(n)}\right)}\end{bmatrix} \tag{20}$$

Consider now the design of a control law capable of generating the signal $X_{c(n)}$ (and therefore $q_{c(n)}$, see Eq. (12)), with the aim of forcing the reactor effluent concentration, C_a, to track the reference trajectory (C_{aref}). To calculate $X_{c(n)}$, the Eq. (20) must have exact solution. To fulfill that goal, the reactor temperature ($T_{a(n)}$) is calculated as:

$$T_{a(n)} = \frac{-E/R}{\ln\left[-\left(\frac{C_{aref(n+1)} - k_1\left(C_{aref(n)} - C_{a(n)}\right) - C_{a(n)}}{T_0}\right) + \frac{q}{V}\left(C_{a0} - C_{a(n)}\right)\right] - \ln\left(a_0 C_{a(n)}\right)} \tag{21}$$

This particular value of $T_{a(n)}$ in (21) will be called $T_{aez(n)}$; i.e., $T_{aez(n)}$ is a key variable that forces the system (20) to have exact solution and ensures that the tracking errors tend to zero, thus enabling the reactor to follow the reference trajectory.

From the second row of system (20) and taking into account the expression (21) for $T_{aez(n)}$, the following control law is proposed:

$$X_{c(n)} = \frac{\frac{T_{aez(n+1)} - k_2\left(T_{aez(n)} - T_{a(n)}\right) - T_{a(n)}}{T_0} - \frac{q}{V}\left(T_f - T_{a(n)}\right) - a_1 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}}}{a_3\left(T_{cf} - T_{a(n)}\right)} \tag{22}$$

Finally, replacing (22) into (12), the coolant flow rate, $q_{c(n)}$, can be obtained. This is the control action which makes the tracking errors tend to zero in every sampling time (see Proof of Theorem 1).

Theorem 1: If the reactor behavior is governed by Eq. (17) and the controller is designed by Eq. (22) then, the tracking error $\left\|e_{(n)}\right\| = \sqrt{\left(eT_{a(n)}\right)^2 + \left(eC_{a(n)}\right)^2} \to 0, n \to \infty$ when the trajectory tracking problem is considered.

Proof of Theorem 1 and the convergence to zero of tracking errors can be seen in Appendix.

Figure 2 shows the architecture of the control strategy and in Fig. 3 a flowchart explaining how the controller is applied in the CSTR control is presented.

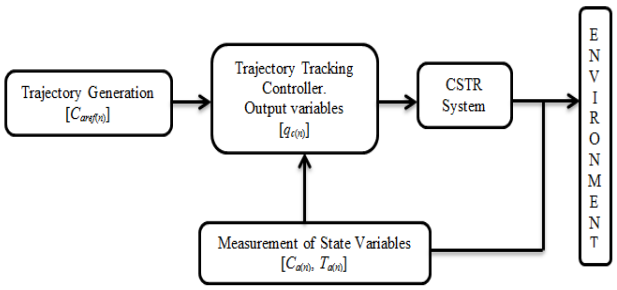

Fig. 2. Architecture of the trajectory tracking controller.

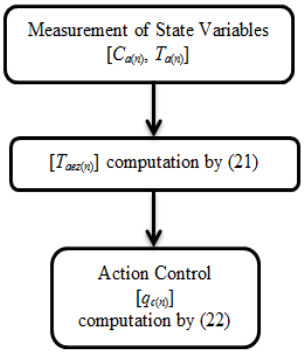

Fig. 3. Flowchart of the proposed strategy.

5. RESULTS AND DISCUSSION

In this Section, the effectiveness of the proposed control law will be verified through simulation examples. The control goal is that the reactor outlet concentration, C_a, follows a variable reference trajectory over time, C_{aref}. This path is composed of multiple step changes covering an operating range from 0.08 to 0.12 mol/l around a steady-state nominal concentration of 0.1 mol/l (Zhang and Guay, 2005). Two different tests are implemented over this reference: in the first one, the optimal controller parameters are synthesized through the Monte Carlo Experiment (Auat Cheein et al., 2013); secondly, the MCE is applied in order to verify the performance of the proposed controller under modeling errors. Finally, as a third and definitive test, the reactor is forced to follow a different concentration profile and compare the actual performance with the results obtained by another control strategy proposed in the literature by (Prakash and Srinivasan, 2009). All simulations were performed on the basis of the Matlab™ software.

5.1 Monte Carlo Experiments for Choosing the Controller Parameters

The Monte Carlo method is used because it is a simple tool that allows estimating a suboptimal solution to the problem of finding the optimal parameters of the controller. As it is necessary to find several parameters at once, raised as an optimization problem would be very complex to solve (non-linear programming with restrictions). It is then chosen, by a statistical method that provides a suboptimal solution within a confidence interval and a precision, fixed in advance (Tempo and Ishii, 2007).

In order to design the MCE, a widely-used strategy consists in minimizing a defined cost function (Auat Cheein and Scaglia, 2014). Let C_{aref} be a desired trajectory, where $\#C_{aref}$ represents the number of points of such trajectory. Then, the cost function ($\Phi_{C_{aref}}$) can be represented by the quadratic error in the reactor effluent concentration C_a, as follows:

$$\Phi_{C_{aref}} = \sum_{i=0}^{\#C_{aref}} \frac{1}{2}\left(C_{aref(i)} - C_{a(i)}\right)^2 \tag{23}$$

The objective is to find the values of the controller parameters, k_1 and k_2, that minimize $\Phi_{C_{aref}}$. To this effect, N = 1000 simulations were performed. In each of them, it is assumed a random value for the controller parameters taken from a uniform distribution (Auat Cheein et al., 2013). The sampling time was set to $T_0 = 0.1$ min. The initial conditions were $T_{aez(0)} = 400$K; $C_{aref(0)} = 0.1$mol/l.

Remark 3: In (18) and (19) even though $k_1 \neq k_2$ can be chosen, a relevant benefit was not obtained in the simulated examples. Therefore, we consider:

$$k_1 = k_2 = rand\,(a,b) \tag{24}$$

where $rand(a,b)$ is a random number taken from a uniform distribution in the interval (a,b). For the current CSTR system, $b < 1$ should be adopted to ensure system stability (i.e., error convergence) and $a > 0$ for proper reactor response. In this case, $a = 0.2$ and $b = 0.8$ were selected. These values were empirically chosen by considering a tradeoff between the speed of convergence to zero of tracking errors and mild reactor responses.

From Fig. 4 to Fig. 10 the results for the 1000 trials are shown. The $C_a(t)$ trajectories and their respective reference values ($C_{aref}(t)$) are shown in Fig. 4. As it can be seen, the reactor effluent concentration tends quickly to the reference trajectory without exhibiting undesirable oscillations. Figure 5 shows that the tracking error tends to zero thus emphasizing the good performance of the proposed control law. The reaction temperature ($T_a(t)$) and the control action ($q_c(t)$) are plotted in Fig. 6 and 7, respectively. It can be seen an adequate performance within a small range of variation, which is associated with the many step changes in the desired set point ($C_{aref}(t)$). Figure 8 shows the values taken by the cost function in each trial, where the scattered values are due to the random choice of the parameters. In addition, Fig. 9 shows the random values assigned to the controller parameters in each trial.

The minimum value of the cost function can be determined by inspection of Fig. 8 and Fig. 9: $\Phi_{C_{aref}} = 0.0019$. The corresponding parameters of the controller are:

$$k_1 = k_2 = 0.336 \tag{25}$$

This value will be used in the simulations of the next section (see Remark 3).

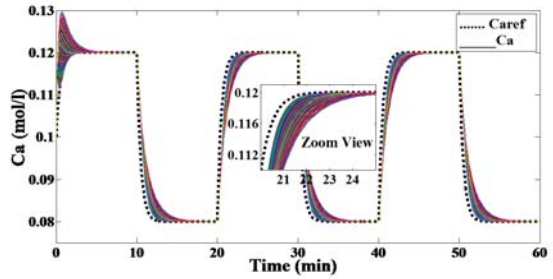

Fig. 4. Trajectory tracking for reactor effluent concentration. The state variable ($C_a(t)$) follows the reference value ($C_{aref}(t)$) for 1000 trials.

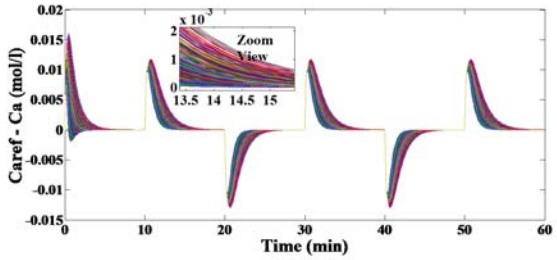

Fig. 5. Tracking error for 1000 trials.

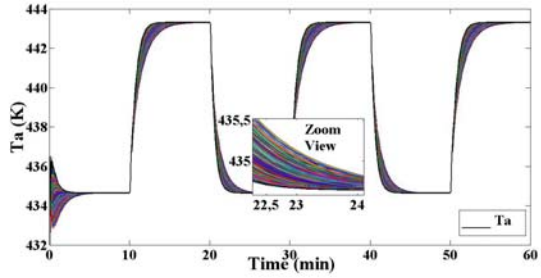

Fig. 6. Temperature profiles ($T_a(t)$) for 1000 trials.

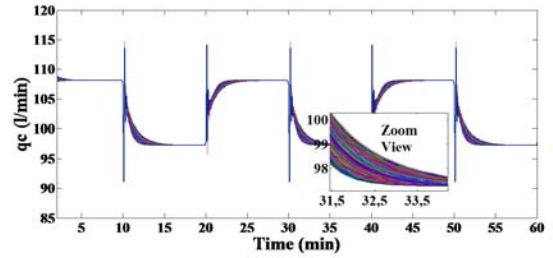

Fig. 7. Control action ($q_c(t)$) for 1000 trials.

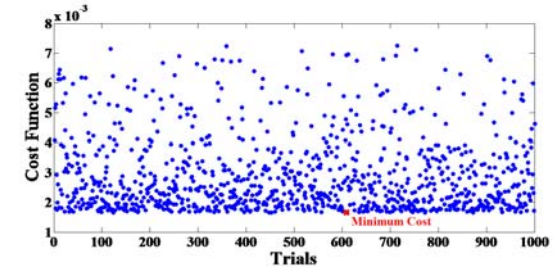

Fig. 8. Cost function for 1000 values of the parameters (Eq. (23)).

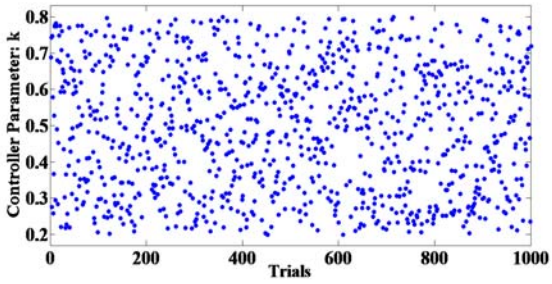

Fig. 9. Random values assigned to the controller parameters for 1000 trials (Eq. (24)).

This algorithm to tune the controller parameter, is another important contribution of this work. Noteworthy that it is a very effective technique for controller tuning because of its simplicity and its capability of being implemented online.

5.2 Monte Carlo Experiments for Analyzing Modeling Errors

When a system or process is modeled, it is very common that some parameters are not known, or they are known in a certain range of variation, this is named "parameter uncertainty". In this section, the MCE it used to analyze the system performance when some modeling errors are introduced in the model parameters. The considering parameters are a_0, a_1, a_2 and a_3 because they involve relations between all parameters of the mathematical and kinetic model of the CSTR system (see Eqs. 10-12 and Table 1). In this analysis, N = 100 simulations are performed, with random choices of above parameters (either above or below their nominal values) (Auat Cheein et al., 2013). The controller parameters (k_1 and k_2) are selected according to (25). The initial conditions and the sampling time are the same ones used in the previous Section. The studied variation ranges are the following,

$$\begin{aligned} a_0 &= \left[7.2x10^{10} - 3.6x10^{9}, 7.2x10^{10} + 3.6x10^{9}\right]; \\ a_1 &= \left[1.44x10^{13} - 7.2x10^{11}, 1.44x10^{13} + 7.2x10^{11}\right]; \\ a_2 &= \left[6.987x10^{2} - 1.4x10^{2}, 6.987x10^{2} + 1.4x10^{2}\right]; \\ a_3 &= \left[0.01 - 5x10^{-4}, 0.01 + 5x10^{-4}\right] \end{aligned} \quad (26)$$

The system response ($C_a(t)$) and its respective reference trajectory ($C_{aref}(t)$) are plotted in Fig. 10. According to such Figure, the reactor effluent concentration (in its multiple simulations) reaches the reference trajectory and then follows it along the time without meaningful errors. In fact, Fig. 11 shows how the tracking error tends to almost negligible values in spite of the simulated modeling errors. The output temperature ($T_a(t)$) and the control action ($q_c(t)$) are shown in Fig. 12 and Fig. 13, respectively. Both signals are within the acceptable ranges and have no significant fluctuations, similarly to the analyzed in the previous section. The random values taken by the a_0, a_1, a_2 and a_3 parameters in each simulation are shown in Fig. 13(a), 13(b), 13(c) and 13(d).

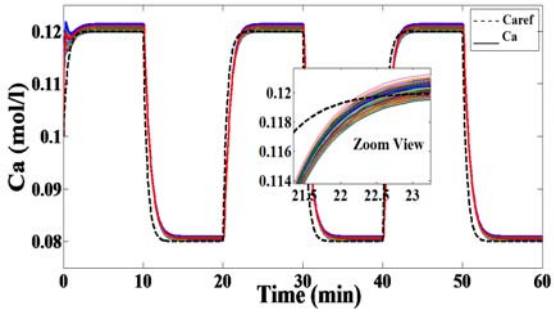

Fig. 10. Trajectory tracking for reactor effluent concentration. The state variable ($C_a(t)$) follows the reference value ($C_{aref}(t)$) for 100 trials.

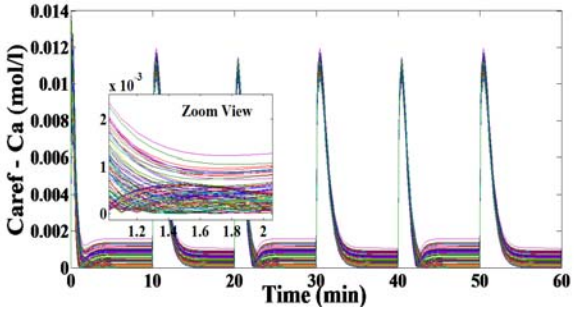

Fig. 11. Tracking error for 100 trials.

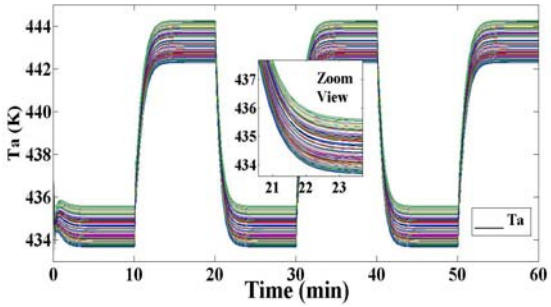

Fig. 12. Temperature profiles ($T_a(t)$) for 100 trials.

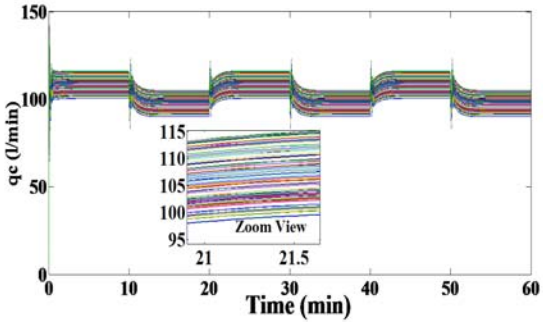

Fig. 13. Control action ($q_c(t)$) for 100 trials.

By comparison of Fig. 5 and Fig. 11, it can be seen that the error tends to zero (Fig. 5), or remains small even in the presence of parameter uncertainty (Fig. 11). All simulations suggest that the performance of the designed controller with the proposed technique is very satisfactory.

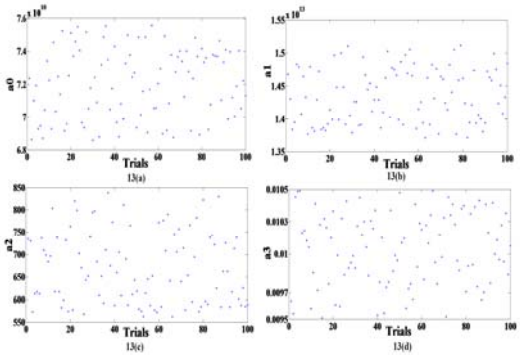

Fig. 14. Simulation results for 100 trials: random values of the model parameters in each trial of Monte Carlo Experiment: (a) a_0; (b) a_1; (c) a_2; (d) a_3.

5.3 Change of the Reference Trajectory

A final evaluation of simulation was carried out with the aim of testing the advantages of our proposal. In order to do so, a controller previously published in the scientific literature by (Prakash and Srinivasan, 2009) was implemented for comparison on the CSTR system. The authors in (Prakash and Srinivasan, 2009) applied a scheme of Nonlinear Model Predictive Control (F-NMPC) that requires the interpolation of a set of local PID controllers for control of trajectory tracking in CSTR process.

In this definitive proof, a new reference profile of the outlet concentration of component A (C_{aref}) is tested. Figure 15 again shows the tracking capability of the proposed methodology even in presence of a new reference, which is characterized by several irregular step jumps over time. This Figure illustrates that $C_a(t)$ quickly tends to the reference trajectory ($C_{aref}(t)$) without exhibiting undesirable oscillations and without meaningful errors. In addition, it is compared the operation of our controller with the obtained results by F-NMPC control strategy suggested by Prakash and Srinivasan (2009). It is evidenced in the graph, that the proposed tracking controller has a better performance. Besides, compared to (Prakash and Srinivasan, 2009) the adopted control technique here is easier to implement and does not need to interpolate the operation of a family of controllers to generate the control signal. However, Fig. 16 shows the principal advantage of the actual technique because of it has improved the results of (Prakash and Srinivasan, 2009) since the maximum tracking errors have been highly decreased.

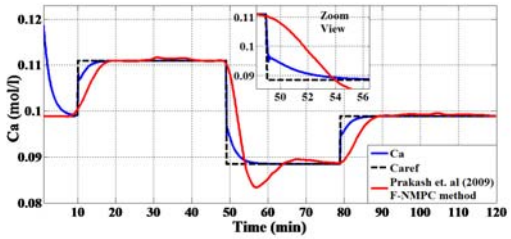

Fig. 15. Simulation results for a change in the reference trajectory (C_{aref}). Performance comparison of the proposed controller with the one given by Prakash and Srinivasan (2009).

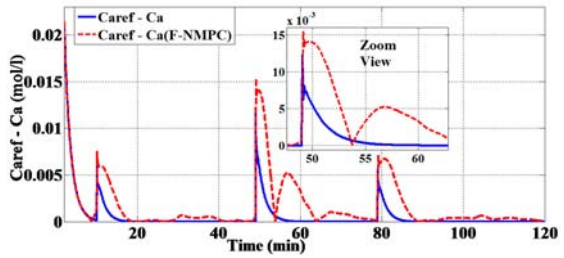

Fig. 16. Simulation results for a change in the reference trajectory (C_{aref}). Comparison of tracking errors of the proposed controller with the one given by Prakash and Srinivasan (2009).

6. CONCLUSIONS

A novel controller for trajectory tracking in a CSTR has been presented, evaluated and compared with others previously proposals in literature. The controller design was derived from a rather simple strategy based on a linear algebra approach. The main advantage of the proposed methodology is that the control action is easily obtained by solving a system of linear equations, and only the knowledge of the system model and the reference trajectory are necessary.

Contrary to what happens in other suggested previous methodologies (Zhang and Guay, 2005; Kalhoodashti, 2011), the proposed technique in this work only requires a few information on the system dynamics. In addition, this technique does not need to solve any optimization problem (Monroy-Loperena et al., 2004; Velazco-Perez et al., 2011) or represent the system by a parallel control architecture (Alvarez-Ramirez et al., 2004). In some cases it is necessary to design several controllers (Rao and Chidambaram, 2008; Prakash and Srinivasan, 2009), while in this approach only one controller is needed to successfully achieve a trajectory tracking. The proposed tracking controller exhibits the advantage of easy design and implementation, which favors the application of the algorithms on dedicated hardware to process control in a real system, because the use of discrete equations allows direct adaptation to any computer system or programmable device running sequential instructions at an adjustable clock speed.

Several simulations studies were carried out to show the effectiveness of the proposed controller. The optimal controller parameters were chosen according to the Monte Carlo sampling experiment. When the system behavior was tested under modeling errors and simulated through Monte Carlo Experiments, the performance of the designed controller proved to be very satisfactory. Besides, if the (Prakash and Srinivasan, 2009) controller is considered, it can be seen that the proposed controller in this paper has better performance and lower tracking errors. From a theoretical point of view, the proof of convergence to zero of tracking errors developed in the Appendix demonstrates the effectiveness of the presented methodology. The possibility to include the saturation of the control signals and observer-controller schemes in the controller design, as is shown in (Wondergem et al., 2011) will be addressed in future contributions.

ACKNOWLEDGMENT

The authors are grateful for the financial support received from CONICET, Universidad Nacional de San Juan, Universidad Nacional del Litoral, and Universidad Tecnológica Nacional.

REFERENCES

Alvarez-Ramirez, J., Velasco, A., and Fernandez-Anaya, G. (2004). A note on the stability of habituating process control. *Journal of Process Control*, *14*(8), 939-945.

Assandri, A., de Prada, C., Cristea, S., and Martínez, E. (2009). Nonlinear parametric predictive control. Application to a continuous stirred tank reactor. *Asia-Pacific Journal of Chemical Engineering*, *4*(6), 858-869.

Auat Cheein, F., Pereira, F. M. L., Di Sciascio, F., and Carelli, R. (2013). Autonomous Simultaneous Localization and Mapping driven by Monte Carlo uncertainty maps-based navigation. *The Knowledge Engineering Review*, *28*(01), 35-57.

Auat Cheein, F., and Scaglia, G. (2014). Trajectory tracking controller design for unmanned vehicles: a new methodology. *Journal of Field Robotics*, *31*(6), 861-887.

Ebadi Kalhoodashti, H. (2011). Concentration Control of CSTR using NNAPC. *International Journal of Computer Applications*, *26*(6), 34-38.

Fissore, D. (2008). Robust control in presence of parametric uncertainties: observer-based feedback controller design. *Chemical Engineering Science*, *63*(7), 1890-1900.

Gandolfo, D., Rosales, C., Patiño, D., Scaglia, G., and Jordan, M. (2014). Trajectory tracking control of a PVTOL aircraft based on linear algebra theory. *Asian Journal of Control*, *16*(6), 1849-1858.

Khairi Abdul Wahab, A., Azlan Hussain, M., and Omar, R. (2007). Temperature control of a pilot plant reactor system using a genetic algorithm model-based control approach. *Asia-Pacific Journal of Chemical Engineering*, *2*(6), 526-535.

Monroy-Loperena, R., Solar, R., and Alvarez-Ramirez, J. (2004). Balanced control scheme for reactor/separator processes with material recycle. *Industrial & Engineering Chemistry Research*, *43*(8), 1853-1862.

Pan, T., Li, S., and Cai, W. J. (2007). Lazy learning-based online identification and adaptive PID control: a case study for CSTR process. *Industrial & Engineering Chemistry Research*, *46*(2), 472-480.

Pérez, M., and Albertos, P. (2004). Self-oscillating and chaotic behaviour of a PI-controlled CSTR with control valve saturation. *Journal of Process Control*, *14*(1), 51-59.

Prakash, J., and Srinivasan, K. (2009). Design of nonlinear PID controller and nonlinear model predictive controller for a continuous stirred tank reactor. *ISA transactions*, *48*(3), 273-282.

Rao, A. S., and Chidambaram, M. (2008). Analytical design of modified Smith predictor in a two-degrees-of-freedom control scheme for second order unstable processes with time delay. *ISA transactions*, *47*(4), 407-419.

Romoli, S., Scaglia, G., Serrano, M. E., Godoy, S. A., Ortiz, O. A., and Vega, J. R. (2014). Control of a Fed-Batch Fermenter Based on a Linear Algebra Strategy. *Latin America Transactions, IEEE (Revista IEEE America Latina)*, *12*(7), 1206-1213.

Serrano, M. E., Scaglia, G., Mut, V., Ortiz, O. A., and Jordan, M. (2013). Tracking trajectory of underactuated surface vessels: a numerical method approach. *Control Engineering and Applied Informatics*, *15*(4), 15-25.

Serrano, M. E., Scaglia, G., Auat Cheein, F., Mut, V., and Ortiz, O. A. (2014). Trajectory-tracking controller design with constraints in the control signals: a case study in mobile robots. *Robotica*, doi: 10.1017/S0263574714001325, 1-18.

Scaglia, G., Aballay, P. M., Serrano, M. E., Ortiz, O. A., Jordan, M., and Vallejo, M. D. (2014). Linear algebra based controller design applied to a bench-scale oenological alcoholic fermentation. *Control Engineering Practice*, *25*, 66-74.

Tempo, R., & Ishii, H. (2007). Monte carlo and las vegas randomized algorithms for systems and control*: An introduction. *European journal of control*, *13*(2-3), 189-203.

Ungureanu, S., Petrila, C., and Curteanu, S. (2001). Sensitivity Analysis Of Non-Isothermal Free Radical Polymerization With Respect To Control Variables. *Journal of Control Engineering and Applied Informatics*, *3*(4), 51-60.

Velasco-Perez, A., Alvarez-Ramirez, J., and Solar-Gonzalez, R. (2011). Multiple input-single output (MISO) control of a CSTR. *Revista Mexicana de Ingeniería Química*, *10*(2), 321-331.

Wang, H. P., Tian, Y., Kalchev, B., Simeonov, I., and Christov, N. (2013). Pilot-scale Biogas Plant: Description, Modelling and Composed Recursive Control. *Journal of Control Engineering and Applied Informatics*, *15*(2), 38-45.

Wondergem, M., Lefeber, E., Pettersen, K. Y., and Nijmeijer, H. (2011). Output feedback tracking of ships. *Control Systems Technology, IEEE Transactions on*, *19*(2), 442-448.

Zhang, T., and Guay, M. (2005). Adaptive control of uncertain continuously stirred tank reactors with unknown actuator nonlinearities. *ISA transactions*, *44*(1), 55-68.

APPENDIX

PROOF OF THEOREM 1

If the reactor behavior is governed by Eq. (17) and the controller is designed by Eq. (22) then, the tracking error $\left\|e_{(n)}\right\| = \sqrt{\left(eT_{a(n)}\right)^2 + \left(eC_{a(n)}\right)^2} \to 0, n \to \infty$ when the trajectory tracking problem is considered.

The proof of convergence to zero of the tracking errors is started with the variable T_a. By replacing the control action $X_{c(n)}$ given by Eq. (22) in the second Equation of system (17), the following expression is found:

$$\frac{\dfrac{T_{aez(n+1)} - k_2\left(T_{aez(n)} - T_{a(n)}\right) - T_{a(n)}}{T_0} - \dfrac{q}{V}\left(T_f - T_{a(n)}\right) - a_1 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}}}{a_3\left(T_{cf} - T_{a(n)}\right)} = \frac{\dfrac{T_{a(n+1)} - T_{a(n)}}{T_0} - \dfrac{q}{V}\left(T_f - T_{a(n)}\right) - a_1 C_{a(n)} e^{-\frac{E}{RT_{a(n)}}}}{a_3\left(T_{cf} - T_{a(n)}\right)} \tag{A.1}$$

Then,

$$\frac{T_{aez(n+1)} - k_2\left(T_{aez(n)} - T_{a(n)}\right) - T_{a(n)}}{T_0} = \frac{T_{a(n+1)} - T_{a(n)}}{T_0} \tag{A.2}$$

$$T_{aez(n+1)} - T_{a(n+1)} = k_2\left(T_{aez(n)} - T_{a(n)}\right) \tag{A.3}$$

From Eq. (A.3),

$$eT_{a(n+1)} = k_2\, eT_{a(n)} \tag{A.4}$$

Finally, if $0 < k_2 < 1$, then $eT_{a(n)} \to 0$ when $n \to \infty$.

The analysis of the variable C_a is developed as follows. By considering the first equation of system (17),

$$C_{a(n+1)} = C_{a(n)} + T_0\left(\frac{q}{V}\left(C_{a0} - C_{a(n)}\right) - a_0 C_{a(n)} e^{-\frac{E/R}{T_{a(n)}}}\right) \tag{A.5}$$

Taking into account Eq. (18), we have,

$$C_{aref(n+1)} - k_1\, eC_{a(n)} = C_{a(n)} + T_0\left(\frac{q}{V}\left(C_{a0} - C_{a(n)}\right) - a_0 C_{a(n)} e^{-\frac{E/R}{T_{aez(n)}}}\right) \tag{A.6}$$

By applying the Taylor's formula to the here-defined $g(T_{a(n)})$ function, we have:

$$g(T_{a(n)}) = e^{-\frac{E/R}{Ta(n)}} = g\left(T_{aez(n)}\right) + \left.\frac{dg(T_{a(n)})}{dT_{a(n)}}\right|_{T_{a(n)} = \underbrace{T_{aez(n)} + \lambda\left(T_{a(n)} - T_{aez(n)}\right)}_{T_{a\lambda}}} \left(T_{a(n)} - T_{aez(n)}\right) \tag{A.7}$$

The first derivative is:

$$\frac{d\left(e^{-\frac{E/R}{Ta(n)}}\right)}{dT_{a(n)}} = e^{-\frac{E/R}{Ta(n)}} \frac{E/R}{T_{a(n)}^2} \tag{A.8}$$

Then,

$$e^{-\frac{E/R}{T_{a(n)}}} = e^{-\frac{E/R}{T_{aez(n)}}} + \underbrace{e^{-\frac{E/R}{T_{aez(n)}+\lambda\left(T_{a(n)}-T_{aez(n)}\right)}} \frac{E/R}{\left[T_{aez(n)}+\lambda\left(T_{a(n)}-T_{aez(n)}\right)\right]^2}}_{h(T_{a\lambda})}\left(T_{a(n)}-T_{aez(n)}\right);$$

$0< \lambda <1$

(A.9)

$$e^{-\frac{E/R}{T_{a(n)}}} = e^{-\frac{E/R}{T_{aez(n)}}} + h(T_{a\lambda})\underbrace{\left(T_{a(n)}-T_{aez(n)}\right)}_{-eT_{a(n)}}$$

(A.10)

By replacing into Eq. (A.5),

$$C_{a(n+1)} = C_{a(n)} + T_0\left[\begin{array}{l}\frac{q}{V}\left(C_{a0}-C_{a(n)}\right)- \\ a_0 C_{a(n)}\left(e^{-\frac{E/R}{T_{aez(n)}}} - h(T_{a\lambda})\, eT_{a(n)}\right)\end{array}\right] \quad \text{(A.11)}$$

$$C_{a(n+1)} = C_{a(n)} + T_0\left[\frac{q}{V}\left(C_{a0}-C_{a(n)}\right) - a_0\, C_{a(n)}\, e^{-\frac{E/R}{T_{aez(n)}}}\right] + \quad \text{(A.12)}$$

$$T_0\, a_0\, C_{a(n)}\, h(T_{a\lambda})\, eT_{a(n)}$$

Taking into account Eq. (A.6),

$$C_{a(n+1)} = C_{aref(n+1)} - k_1\, eC_{a(n)} + T_0\, a_0\, C_{a(n)}\, h(T_{a\lambda})\, eT_{a(n)} \quad \text{(A.13)}$$

$$\underbrace{C_{aref(n+1)} - C_{a(n+1)}}_{eC_{a(n+1)}} = k_1\, eC_{a(n)} - T_0\, a_0\, C_{a(n)}\, h(T_{a\lambda})\, eT_{a(n)} \quad \text{(A.14)}$$

From Equations (A.4) and (A.14) we can write,

$$\left[\begin{array}{c} eC_{a(n+1)} \\ eT_{a(n+1)}\end{array}\right] = \underbrace{\left[\begin{array}{cc} k_1 & 0 \\ 0 & k_2\end{array}\right]\left[\begin{array}{c} eC_{a(n)} \\ eT_{a(n)}\end{array}\right]}_{\textit{Linear System}} + \underbrace{\left[\begin{array}{c} -T_0\, a_0\, C_{a(n)}\, h(T_{a\lambda}) \\ 0\end{array}\right] eT_{a(n)}}_{\text{Nonlinearity}} \quad \text{(A.15)}$$

This expression shows that the model can be represented as a linear equation system plus a nonlinearity that tends to zero because according to Eq. (A.4), $eT_{a(n)} \rightarrow 0$ when $n \rightarrow \infty$. Finally, it is demonstrated that for $0 < k_1, k_2 < 1$, $eC_{a(n)}$ and $eT_{a(n)} \rightarrow 0$ when $n \rightarrow \infty$, and therefore the tracking error $\left\|e_{(n)}\right\| = \sqrt{\left(eT_{a(n)}\right)^2 + \left(eC_{a(n)}\right)^2} \rightarrow 0,\ n \rightarrow \infty$.

A complete demonstration can be found in Scaglia et al. (2014).