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Fwd: IGPL Submission 10-28

Martin Moguillansky <mom@cs.uns.edu.ar> Wed, Apr 20, 2011 at 10:52 AM
To: "Marcelo A. Falappa" <mfalappa@cs.uns.edu.ar>, Renata Wassermann <renata@ime.usp.br>

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From: Jane Spurr <jane.spurr@kcl.ac.uk>
Date: Wed, Apr 20, 2011 at 6:24 AM
Subject: IGPL Submission 10-28
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Dear Martin Moguillansky

I am writing regarding the revised version of your submission to the Logic Journal of the IGPL, 'Inconsistent-tolerant Base Revision through Argument Theory Change', IGPL 10-28.

I am pleased to tell you that both referees are happy with the revisions you've made, and that therefore your paper is accepted for publication.

Please will you send me the source file(s) and corresponding pdf, which I shall forward to the publisher who prepare all papers for publication.

Sincerely
Jane Spurr

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Inconsistent-tolerant Base Revision through Argument Theory Change

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Abstract

Reasoning and change over inconsistent knowledge bases (KBs) is of utmost relevance in areas like medicine and law. Argumentation may bring the possibility to cope with both problems. Firstly, by constructing an argumentation framework (AF) from the inconsistent KB, we can decide whether to accept or reject a certain claim through the interplay among arguments and counterarguments. Secondly, by handling dynamics of arguments of the AF, we might deal with the dynamics of knowledge of the underlying inconsistent KB.

Dynamics of arguments has recently attracted attention and although some approaches have been proposed, a full axiomatization within the theory of belief revision was still missing. A revision arises when we want the argumentation semantics to accept an argument. Argument Theory Change (ATC) encloses the revision operators that modify the AF by analyzing dialectical trees –arguments as nodes and attacks as edges– as the adopted argumentation semantics.

In this article, we present a simple approach to ATC based on propositional KBs. This allows to manage change of inconsistent KBs by relying upon classical belief revision, although contrary to it, consistency restoration of the KB is avoided. Subsequently, a set of rationality postulates adapted to argumentation is given, and finally, the proposed model of change is related to the postulates through the corresponding representation theorem. Though we focus on propositional logic, the results can be easily extended to more expressive formalisms such as first-order logic and description logics, to handle evolution of ontologies.

Keywords: Knowledge Representation and Reasoning, Argumentation, Belief Base Revision, Argumentation Dynamics, Reasoning over Inconsistency.

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1. Introduction

Dealing with inconsistencies is of utmost importance in areas like medicine and law. For instance, in law trials, two parties to a dispute present contradictory information in a tribunal, standing in favor or against the dispute (in criminal trials this is normally the presumption of innocence). The tribunal decision resolves afterwards the dispute upon presented evidence. This shows the need to consider some kind of paraconsistent semantics in order to appropriately reason over knowledge bases (KB) containing contradictory information.

For some settings, it will be also necessary to provide services for handling dynamics of knowledge with capabilities to tolerate inconsistencies from the KB considered. An interesting one arises in promulgation of laws. This usually involves a long process in which articles and principles from previous laws, and even evidence taken from the current state of affairs, may enter in conflict with articles composing the new law. Imagine a base containing knowledge about the complete legal system of a nation, including the National Constitution, the international law, and other political fundamental principles –such as the civil and penal codes, and other minor local laws. Such KB is required to evolve in a way that it incorporates the information conforming the new law, ensuring it to be constitutional. To this end, it is necessary to identify a set of articles and/or principles to be derogated, or amended, as part of the process of promulgation.

As an example, we will refer in a very brief manner to the Argentinean broadcasting media law reformed during 2009. The previous media law, promulgated by the latter *de facto* regime, empowered the government to regulate the different media allowing total control of news. When democracy was restituted, the regulation of media was extended to private investment groups. As years went by, these groups took over majorities of types of media, conforming monopolies in some cases. This brought excessive power to groups with partial interests, allowing them to manipulate the social opinion about the actual government, and even to condition politicians, thus striking to national sovereignty. Article 161 of the new media law became one of the most controversial points, since it forces monopolistic enterprises to get rid of part of their assets in a maximun period of one year. Some enterprises warned that they would be forced to sell off their assets at very low prices. This violates article 17 of the National Constitution which speaks about private property rights. Moreover, some members of the Supreme Court think that article 161 recalls the control over the media exercised by totalitarian regimes, which would violate article 1 of the National Constitution. In fact, such situation could evolve to a distrust state on the principle of legal security. These are just some of the controversial points for which the new media law keeps being studied by the Argentinean parliament at the time of this submission.

Belief revision studies the dynamics of knowledge, coping with the problem of how to change the beliefs standing for the conceptualization of a modeled world, to reflect its evolution. *Revisions*, as the most important change operations, concentrate on the incorporation of new beliefs in a way that the resulting base ends up consistent. As a simple example of a revision problem, consider the following: for many years it was believed that the boiling point of water was 100℃. However, later research on the matter proved that this holds only at standard pressure conditions. Moreover, it was shown that the boiling point decreases 1℃ every 285 m of elevation. This new evidence forces the beliefs to be revised in order to keep it up to date and consistent. Observe however, that for the aforementioned case of promulgation of laws, it is mandatory to keep most inconsistencies from the original KB to make it evolve appropriately. This led us to investigate new approaches of belief revision which operate over paraconsistent semantics in order to avoid consistency restoration.

Argumentation theory allows to reason over potentially inconsistent KBs. In general, this is done by replacing the usual meaning of inference from classical logic by the notion of *warrant* in argumentation: the process of warrant evaluates conflicting pieces of knowledge deciding which ones prevail despite the existence of beliefs in opposition. The notion of warrant is also identified as the argument's acceptance criterion corresponding to the adopted argumentation semantics. Thus, from an argumentation framework (AF), a belief α is warranted if there is an argument supporting α which is accepted by the adopted argumentation semantics. Among the most influential works on argumentation semantics we may refer to those over graphs of arguments such as [17, 4]. However, the semantics we adopt follows the idea of dialectical argumentation [29, 13]: *dialectical trees* are arguments trees built from the AF with arguments as nodes and attacks as edges. A pair of nodes connected through an edge stands for a source of inconsistency obtained from the KB considered. The use of dialectical trees allows to concentrate only on a specific query to build "on demand" those arguments that are somehow related to the query. Such sort of semantics allows to construct practical approaches avoiding the analysis of the complete graph of arguments.

Argument Theory Change (ATC) [30] applies notions from the classical theory of change, and particularly from the Alchourrón, Gärdenfors, and Makinson's well known AGM model of change [1] and Hansson's Kernel Contractions [20], to the field of *abstract argumentation* [17] by relying upon dialectical trees as the adopted argumentation semantics.[1] An argument revision *à la* ATC revises an AF by an argument seeking for its warrant. To such end, the AF –and thus the set of arguments obtained from it– is modified in order for the argumentation semantics to accept the new argument. This is the *success* condition adopted by ATC approaches. In classical belief revision, a basic set of postulates is usually specified to characterize a rational behavior of the utilized change operations. Among them, success and *minimal change* have concentrated much research. Success specifies the main objective of the change operation, usually, the acceptance (inference) of the new belief to incorporate. On the other hand, minimal change ensures as little as possible information to be modified in order to achieve success. Consequently, for ATC approaches, different criteria of min-

[1] In abstract argumentation, both the logic for arguments and their inner structure are abstracted away.

imal change arise depending on the desired standpoint: the amount of change can be analyzed with regards to (1) the set of arguments, (2) the dialectical tree rooted in the revising argument, (3) the set of accepted (warranted) arguments, or (4) any combination of them.

Among the most relevant uses of ATC, we may refer to hypothetical reasoning, dynamics in negotiation, persuasion, dialogues, strategies, planning, and more. For instance, in scheduling, consider the development of an enterprise's task scheduler. Assume employee assignments are managed by an agent interpreting a KB. The central authority incorporates new tasks to the KB. An agent uses this information to decide to which employees should the new requirements be assigned. Argumentation could deal with such a problem since it would be necessary to reason over inconsistency, given that conflicts will appear between the assigned measures of relevance of the different tasks and the current availability of employees. A new task with a high level of relevance could be sent to a specific employee for a matter of confidence, provoking the reassignment of his previous tasks to other employees. ATC can be useful in the definition of the re-scheduling process by recognizing which assignments enter in conflict with the new ones for specific employees.

The main objective we pursue in this article is to provide a methodology capable of changing potentially inconsistent propositional KBs without requiring consistency restoration. To this end, we will rely upon an AF with a concrete language for arguments: classical propositional logic. Afterwards, we propose a novel approach to ATC under the name of *dialectic-global model*, that revises the KB (by analyzing the AF built from it), by an argument $\mathcal{R}$, which contains a minimal set of propositional rules inferring its claim α. The outcome of this revision is a new (potentially inconsistent) propositional KB which contains the complete information within $\mathcal{R}$. The new revised KB determines a new AF whose argumentation semantics accepts α by rendering $\mathcal{R}$ undefeated from the dialectical tree rooted in it. This is achieved by identifying a set of arguments that should be removed from the AF to render $\mathcal{R}$ undefeated. Thus, pursuing α's acceptance may involve not only the addition of $\mathcal{R}$ to the original AF, but also the removal of other arguments from it. However, the removal of arguments from the AF cannot be done straightforwardly, but as a consequence of the removal of beliefs from the underlying KB from where the AF is built. This gives rise to the criterion of minimal change adopted in this work: as little as possible information *from the KB* should be modified in order to obtain the revised AF. Observe that this criterion is the usual one used in classical belief revision, but a new one regarding ATC, since it only arises when applying ATC to perform change to an inconsistent KB. This is the unique minimal change criterion we will adopt to guarantee a rational behavior of the operator proposed, and it is the main reason by which the dialectic-global model introduced here, constitutes the simplest and most practical approach to ATC defined so far.

It is important to mention that, unlike typical KB revision models in which a base is revised by a sentence, in this paper we are concerned with the operation of revising a base by a giving argument. However, this can be seen as a specialized sort of base revision in which the KB is revised by a (propositional) sentence α along with some (minimal and consistent) explanation set for it, *i.e.*, an argument in support of α. For instance, for the example given before on the boiling point of water, one's beliefs may need to be revised by an assertion α : *Boiling point of water on top of Mount Everest is about 69℃.* However, accepting such an assertion (for one knowing only that δ:*Boiling point of water is 100℃*) should presumably require further justification. Thus, one may provide the set of knowledge $\Gamma = \{\beta, \gamma, \delta\}$, where β : *Mount Everest is 8,848 m high* and γ : *The boiling point decreases 1℃ every 285 m of elevation.* Clearly, the set Γ conforms an explanation set for α. The use of explanation sets for revising KBs has been proposed in [18], although with a quite different orientation.

On the other hand, the use of arguments may bring a better alternative to revise a KB by a piece of information of higher level of conceptual complexity and structuring. For instance, considering the example given before on promulgation of laws, the dialectic-global model could revise the legal system by including an argument $\mathcal{R}$ standing for the new law to be promulgated. In this case, arguments to be removed from the original AF would contain different articles from other already promulgated laws. Thus, the new law is ensured to be constitutional by proposing to derogate other laws or amend them by removing specific articles which are part of arguments to be removed from the AF. Naturally, removals from the base are expected to be of less importance than the one for the new law. That is, laws to be amended should never correspond to the National Constitution unless the promulgation of the new law is expected to reform it. In case no minor laws arise to be amended, it is clear that the new one, $\mathcal{R}$ cannot be included as it is and thus, it necessarily needs to be modified to be accepted by the legal system.

In addition to the model of KB revision we also provide the definition of an AF revision, a second operation based on the same intuitive foundations and functional components that define the argument revision. The AF revision operation revises a given propositional AF by an argument in order to obtain a new AF warranting the new argument. Afterwards, some properties are studied with the intention to interrelate both revision operations. Since the main objective of this article is the proposal of the argument revision (for KB revision), in this article we do not provide the full axiomatization of the AF revision operation. Nevertheless, we do provide an analysis of the AF revision in terms of the postulates studied for the argument revision. The objective of the proposal of the AF revision is to give a first standpoint on the revision of propositional AFs (disregarding the existence of an underlying KB) in accordance to ATC.

This article is organized as follows: Section 2 briefly introduces the AF upon which we rely to define the dialectic-global model in Section 3. Section 4 introduces a basic set of argumentation postulates which serve to rationalize afterwards the proposed argumentation model of change through the corresponding representation theorem. Section 5 introduces the AF revision operation analyzing its rationality according to the results exposed in Section 4. Section 6 discusses related work and Section 7 points out the concluding remarks. Proofs and additional theoretical elements are provided in the Appendix.

2. Argumentation Overview

Intuitively, an *argument* may be seen as a *set of interrelated pieces of knowledge providing support to a claim*. We rely on a structure $\langle body,\ claim\rangle$ to represent arguments, where the body is a consistent and minimal set of beliefs inferring the claim. The symbol $\models$ will be written to identify the semantic entailment used to obtain inferences from knowledge inside a language $\mathcal{L}$. In this work, $\mathcal{L}$ will be assumed to identify the classical propositional logic. Arguments will be identified through caligraphic letters $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, and $\mathcal{R}$; where $\mathcal{R}$ will in general stand for the root argument.

Definition 2.1 (Argument). *An* ***argument*** $\mathcal{B}$ *is a structure* $\langle \Delta, \beta\rangle$*, where* $\Delta \subseteq \mathcal{L}$ *is the* ***body***, $\beta \in \mathcal{L}$ *the* ***claim***, *and it holds* (1) $\Delta \models \beta$, (2) $\Delta \not\models \bot$, *and* (3) $\nexists X \subset \Delta : X \models \beta$. *We say that* $\mathcal{B}$ ***supports*** β.

The idea we follow from argumentation theory is to give support to queries presented to a (potentially inconsistent) KB through the claim of arguments, and therefore the information inside those arguments might be interpreted as a justification for such queries. Hence, we assume the knowledge within the body of arguments to be included in a propositional KB $\Sigma \subseteq \mathcal{L}$. For more details on argumentation based on propositional logic refer to [7].

Observe that claims conform to $\mathcal{L}$ since they are not necessarily contained in Σ but implied by Σ. The *domain of arguments from* Σ is identified through the set $\mathbb{A}_\Sigma$. Given a query $\alpha \in \mathcal{L}$, an argument $\mathcal{B} \in \mathbb{A}_\Sigma$ is a *query supporter* if $\mathcal{B}$ supports α. For instance, assuming $\{p, p \rightarrow q\} \subseteq \Sigma$, the *primitive* arguments $\langle\{p\}, p\rangle$ and $\langle\{p \rightarrow q\}, p \rightarrow q\rangle$, and more complex arguments like $\langle\{p, p \rightarrow q\}, q\rangle$ and $\langle\{p \rightarrow q\}, \neg p \vee q\rangle$, appear (among others) within $\mathbb{A}_\Sigma$. By means of $\mathfrak{bd} : \mathbb{A}_\Sigma \longrightarrow 2^{\mathcal{L}}$ and $\mathfrak{cl} : \mathbb{A}_\Sigma \longrightarrow \mathcal{L}$, the body $\mathfrak{bd}(\mathcal{B})$ and claim $\mathfrak{cl}(\mathcal{B})$ of an argument $\mathcal{B} \in \mathbb{A}_\Sigma$ are identified. Like logically equivalent $\mathcal{L}$-formulae, arguments from $\mathbb{A}_\Sigma$ can be also associated through an equivalence relation: two arguments $\mathcal{R}_1 \in \mathbb{A}_\Sigma$ and $\mathcal{R}_2 \in \mathbb{A}_\Sigma$ are said to be *equivalent*, written $\mathcal{R}_1 \equiv \mathcal{R}_2$, when (1) $\mathfrak{cl}(\mathcal{R}_1)$ is logically equivalent to $\mathfrak{cl}(\mathcal{R}_2)$, and (2) $\mathfrak{bd}(\mathcal{R}_1)$ is logically equivalent to $\mathfrak{bd}(\mathcal{R}_2)$.

A *counterargument* is an argument whose claim poses a justification to disbelieve in another argument. Counterarguments bring about sources of KB-inconsistency determined by their bodies: considering two arguments $\mathcal{B}_1$ and $\mathcal{B}_2$, argument $\mathcal{B}_2$ counterargues $\mathcal{B}_1$ *iff* $\{\mathfrak{cl}(\mathcal{B}_2)\} \cup \mathfrak{bd}(\mathcal{B}_1) \models \bot$. In this case, $\mathcal{B}_1$ and $\mathcal{B}_2$ are referred as a *conflictive pair*. *Attacks* (or *defeats*) between arguments from $\mathbb{A}_\Sigma$ are finally adjudicated from conflictive pairs $\mathcal{B}_1$ and $\mathcal{B}_2$: $\mathcal{B}_2$ *defeats* $\mathcal{B}_1$ *iff* $\mathcal{B}_2$ counterargues $\mathcal{B}_1$. Argument $\mathcal{B}_2$ is said a *defeater* of $\mathcal{B}_1$ (or $\mathcal{B}_1$ is defeated by $\mathcal{B}_2$), noted as $\mathcal{B}_2 \hookrightarrow \mathcal{B}_1$. Infinite defeaters (common bodies and logically equivalent claims) of an argument may appear. Thus, we will point to a special counterargument as representative of all defeaters of an argument, named *canonical undercut* [7]: $\mathcal{B}_2$ is a canonical undercut of $\mathcal{B}_1$ *iff* $\mathfrak{bd}(\mathcal{B}_1) = \{\beta_1, \ldots, \beta_n\}$ and $\mathfrak{cl}(\mathcal{B}_2) = \neg(\beta_1 \wedge \ldots \wedge \beta_n)$, or equivalently $\mathfrak{cl}(\mathcal{B}_2) = \neg(\bigwedge \mathfrak{bd}(\mathcal{B}_1))$.

Example 1. *Assume a KB* $\Sigma \subseteq \mathcal{L}$ *and a query* p. $\mathcal{R} = \langle\{q, (q \rightarrow p)\}, p\rangle$ *is a* p*-supporter and* $\mathcal{B}_1 = \langle\{p_1, (p_1 \rightarrow p_2), (\neg p_2 \vee p_3), (p_3 \rightarrow \neg p)\}, \neg(q \wedge (q \rightarrow p))\rangle$ *is a* *canonical undercut of $\mathcal{R}$ given that $\mathfrak{cl}(\mathcal{B}_1) = \neg(\bigwedge \mathfrak{bd}(\mathcal{R}))$. (Observe that, $\mathfrak{bd}(\mathcal{B}_1)$ infers $\neg\mathfrak{cl}(\mathcal{R})$ (i.e., $\neg p$), and then we can ensure that $\mathfrak{bd}(\mathcal{B}_1)$ infers $\neg(\bigwedge \mathfrak{bd}(\mathcal{R})$.) The complete KB Σ and the rest of the arguments built from it that we will consider are:*

$$\Sigma = \left\{ \begin{array}{l} q,\ p_1,\ q_1,\ q_2,\ \neg q_3,\ \neg p_4, \\ (q \to p), (p_1 \to p_2), \\ (\neg p_2 \vee p_3), (p_3 \to \neg p), \\ (q_1 \to \neg p_3), (q_2 \to \neg q), \\ (q_2 \to \neg p_2), (\neg p_2 \to p_4), \\ (\neg p_2 \vee q_3) \end{array} \right\} \quad \begin{array}{l} \mathcal{B}_2 = \langle \{q_1, (q_1 \to \neg p_3)\}, \neg(\bigwedge \mathfrak{bd}(\mathcal{B}_1))\rangle, \\ \mathcal{B}_3 = \langle \{q_2, (q_2 \to \neg q)\}, \neg(\bigwedge \mathfrak{bd}(\mathcal{R}))\rangle, \\ \mathcal{B}_4 = \langle \{(q_2 \to \neg p_2), (\neg p_2 \to p_4), \neg p_4\}, \\ \qquad \neg(\bigwedge \mathfrak{bd}(\mathcal{B}_3))\rangle, \\ \mathcal{B}_5 = \langle \{(\neg p_2 \vee q_3), \neg q_3\}, \neg(\bigwedge \mathfrak{bd}(\mathcal{B}_4))\rangle, \textit{ and} \\ \mathcal{B}_6 = \langle \{(\neg p_2 \vee q_3), \neg q_3\}, \neg(\bigwedge \mathfrak{bd}(\mathcal{B}_1))\rangle. \end{array}$$

Every argument given (except for $\mathcal{R}$) is a canonical undercut, but the list is not exhaustive. Moreover, arguments with the same body can defeat different arguments depending on their claims, as is the case of $\mathcal{B}_5$ and $\mathcal{B}_6$. The set of attacks obtained is: $\mathcal{B}_1 \hookrightarrow \mathcal{R}$, $\mathcal{B}_2 \hookrightarrow \mathcal{B}_1$, $\mathcal{B}_3 \hookrightarrow \mathcal{R}$, $\mathcal{B}_4 \hookrightarrow \mathcal{B}_3$, $\mathcal{B}_5 \hookrightarrow \mathcal{B}_4$, and $\mathcal{B}_6 \hookrightarrow \mathcal{B}_1$.

The reasoning methodology is based on the acceptability of some query supporter obtained from $\mathbb{A}_\Sigma$. To this end, the notion of *argumentation line* [14] identifies a repeated interchange of arguments and counterarguments. An argumentation line λ is a non-empty sequence $[\mathcal{B}_1 \ldots, \mathcal{B}_n]$ of arguments from $\mathbb{A}_\Sigma$, where $\mathcal{B}_i \hookrightarrow \mathcal{B}_{i-1}$ $(1 < i \leq n)$. Argument $\mathcal{B}_1$ is identified as λ's *root*, and $\mathcal{B}_n$, as λ's *leaf*. An argumentation line could be seen as *two parties engaged in a discussion*: one standing by the root argument and the other arguing against it. Consequently, given a line λ, we identify the *set of pro (resp., con) arguments* containing all arguments placed on odd (resp., even) positions in λ, noted as λ^+ (resp., λ^-). We abuse notation writing $\mathcal{B} \in \lambda$ to identify $\mathcal{B}$ from λ.

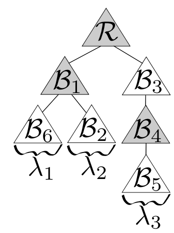
Acceptability conditions are used to build lines free of *fallacies*, namely *acceptable argumentation lines*: (1) the defeated part of an argument cannot appear twice in the same line, and (2) the set of pro (resp., con) arguments in a line has no conflicting pairs. For instance, in Example 1, $\mathcal{B}_7 = \langle \{p_1, (p_1 \to p_2), (\neg p_2 \vee p_3)\}, \neg(q_1 \wedge (q_1 \to \neg p_3))\rangle$ cannot defeat $\mathcal{B}_2$ since it would determine a cyclic line, violating condition 1). This is so, given that $\mathcal{B}_7$'s body was already defeated by $\mathcal{B}_2$ in the line $[\mathcal{R}, \mathcal{B}_1, \mathcal{B}_2]$ (note that $\mathfrak{bd}(\mathcal{B}_7) \subseteq \mathfrak{bd}(\mathcal{B}_1)$). For details on acceptability conditions, refer to [19]. In addition, *canonical lines* are acceptable lines built with a root argument and a sequence of canonical undercuts. We define the set $\mathbf{L}_\Sigma$ identifying the domain of all acceptable and canonical lines built with arguments from $\mathbb{A}_\Sigma$. Furthermore, an acceptable line is *exhaustive* if it cannot be extended with any defeater of its leaf without compromising its status of acceptability. Hence, we refer to the subset $\mathbb{L}_\Sigma \subseteq \mathbf{L}_\Sigma$ containing only acceptable, canonical, and exhaustive lines. An initial sequence of arguments in a line $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$ is identified through its *upper segment* $\lambda^\uparrow[\mathcal{B}_i] = [\mathcal{B}_1, \ldots, \mathcal{B}_i]$, with $1 \leq i \leq n$. Besides, the *proper upper segment* of λ wrt. $\mathcal{B}_i$ $(i \neq 1)$ is defined as $\lambda^\uparrow(\mathcal{B}_i) = [\mathcal{B}_1, \ldots, \mathcal{B}_{i-1}]$. We refer as "upper segments" to both proper and non-proper ones and will be distinguishable through the notation (round or square brackets respectively). Note that any upper segment of a line constitutes a (possibly non-exhaustive) line by itself, *i.e.*, for any $\lambda \in \mathbf{L}_\Sigma$ and any $\mathcal{B} \in \lambda$, $\lambda^\uparrow[\mathcal{B}] \in \mathbf{L}_\Sigma$.

A dialectical tree allows to determine whether its root node is to be accepted or rejected as a rationally justified belief. Such tree is built from a set of argumentation lines rooted in a common argument.

Definition 2.2 (Dialectical Tree). *Given $\Sigma \subseteq \mathcal{L}$, a **dialectical tree** $\mathcal{T}(\mathcal{R})$ rooted in an argument $\mathcal{R} \in \mathbb{A}_\Sigma$ is determined by a set $X \subseteq \mathbf{L}_\Sigma$ of lines rooted in $\mathcal{R}$ such that an argument $\mathcal{C}$ in $\mathcal{T}(\mathcal{R})$ is: (1) a **node** iff $\mathcal{C} \in \lambda$, for any $\lambda \in X$; (2) a **child** of a node $\mathcal{B}$ in $\mathcal{T}(\mathcal{R})$ iff $\mathcal{C} \in \lambda$, $\mathcal{B} \in \lambda'$, for any $\{\lambda, \lambda'\} \subseteq X$, and $\lambda'^\uparrow[\mathcal{B}] = \lambda^\uparrow(\mathcal{C})$. The **leaves** in $\mathcal{T}(\mathcal{R})$ are the leaves of each line in X. The set $\mathbf{T}_\Sigma$ identifies the domain of dialectical trees built with lines from $\mathbf{L}_\Sigma$.*

Example 2 (Continued from Example 1).

Given the set $X \subseteq \mathbf{L}_\Sigma$ of argumentation lines rooted in $\mathcal{R}$ determining the tree $\mathcal{T}(\mathcal{R}) \in \mathbf{T}_\Sigma$ (depicted on the right) such that $X = \{\lambda_1, \lambda_2, \lambda_3\}$, where $\lambda_1 = [\mathcal{R}, \mathcal{B}_1, \mathcal{B}_6]$, $\lambda_2 = [\mathcal{R}, \mathcal{B}_1, \mathcal{B}_2]$, and $\lambda_3 = [\mathcal{R}, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5]$, are three acceptable canonical lines. (Arguments taken from Example 1.) Observe that argument $\mathcal{B}_2$ is a child of $\mathcal{B}_1$ in $\mathcal{T}(\mathcal{R})$ given that $\lambda_1^\uparrow[\mathcal{B}_1] = [\mathcal{R}, \mathcal{B}_1] = \lambda_2^\uparrow(\mathcal{B}_2)$ (see Definition 2.2).



Due to canonical lines, a set $X \subseteq \mathbf{L}_\Sigma$ of lines rooted in a common argument is finite, and so is any tree built from X. However, the acceptability condition of dialectical trees also requires lines to be exhaustive. The *bundle set for $\mathcal{R}$* is the set $\mathcal{S}(\mathcal{R}) \subseteq \mathbb{L}_\Sigma$ containing all the acceptable, canonical, and exhaustive lines from $\mathbb{L}_\Sigma$ rooted in $\mathcal{R}$. We say that a dialectical tree $\mathcal{T}(\mathcal{R})$ is *acceptable* if and only if $\mathcal{T}(\mathcal{R})$ is built from a set $X \subseteq \mathbf{L}_\Sigma$ (according to Definition 2.2) such that $X = \mathcal{S}(\mathcal{R})$. We identify the domain of all acceptable dialectical trees from Σ as $\mathbb{T}_\Sigma \subseteq \mathbf{T}_\Sigma$. Note that given $\mathcal{R} \in \mathbb{A}_\Sigma$, the bundle set $\mathcal{S}(\mathcal{R})$ is unique, and so is its corresponding acceptable dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$.

Observation 2.3. $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$ *is the unique dialectical tree in* $\mathbb{T}_\Sigma$ *rooted in* $\mathcal{R}$.

We will slightly abuse notation writing $\lambda \in \mathcal{T}(\mathcal{R})$ to identify from any dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbf{T}_\Sigma$ a complete line λ, where λ's root is $\mathcal{T}(\mathcal{R})$'s root and λ's leaf is a leaf in $\mathcal{T}(\mathcal{R})$. Functions over lines and trees will be generalized (when necessary) by defining them over $\mathbf{L}_\Sigma$ and $\mathbf{T}_\Sigma$. This is done disregarding their general usage over the domains $\mathbb{L}_\Sigma \subseteq \mathbf{L}_\Sigma$ and $\mathbb{T}_\Sigma \subseteq \mathbf{T}_\Sigma$.

Given a KB $\Sigma \subseteq \mathcal{L}$, a query supporter $\mathcal{R} \in \mathbb{A}_\Sigma$ is finally accepted (warranted) by the argumentation semantics, by analizing the tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$. This evaluation is obtained by weighting all the information in the tree through the *marking function* $\mathfrak{mark} : \mathbb{A}_\Sigma \times \mathbf{L}_\Sigma \times \mathbf{T}_\Sigma \longrightarrow \mathbb{M}$, which defines an acceptance criterion by assigning to each argument in $\mathcal{T}(\mathcal{R})$ a marking value from the domain $\mathbb{M} = \{D, U\}$, where D/U means defeated/undefeated. The mark of an inner node in the tree is obtained from those of its children (*i.e.*, its defeaters) by following some specific *marking criterion*. We adopt a skeptical marking criterion defined as: (1) all leaves are marked U and (2) every inner node $\mathcal{B}$ is marked U *iff* every child of $\mathcal{B}$ is marked D, otherwise, $\mathcal{B}$ is marked D. This marking is used to implement the DeLP (*Defeasible Logic Programming*) argumentative machinery [19].[2] The *warranting function* $\mathfrak{warrant} : \mathbf{T}_\Sigma \longrightarrow \{true, false\}$ determines the root's acceptance verifying $\mathfrak{warrant}(\mathcal{T}(\mathcal{R})) = true$ *iff* $\mathfrak{mark}(\mathcal{R}, \lambda, \mathcal{T}(\mathcal{R})) = U$. Hence, $\mathcal{R}$ is *warranted* from $\mathcal{T}(\mathcal{R})$ *iff* $\mathfrak{warrant}(\mathcal{T}(\mathcal{R})) = true$. In such a case, $\mathcal{T}(\mathcal{R})$ is referred as *warranting tree*. These notions are graphically represented with arguments painted in grey/white standing for D/U marks. For instance, in Example 2, $\mathcal{R}$ is defeated and thus $\mathcal{T}(\mathcal{R})$ is non-warranting.

The following theorem shows that dialectical trees determine a conflict-free set of warranted arguments. This implies that the adopted argumentation semantics is consistent regarding the claims of warranted arguments. This property is developed in detail in Appendix A.

Theorem 2.4. *Given $\Sigma \subseteq \mathcal{L}$, for any pair of arguments $\mathcal{R} \in \mathbb{A}_\Sigma$ and $\mathcal{R}' \in \mathbb{A}_\Sigma$, if both $\mathcal{R}$ and $\mathcal{R}'$ are warranted then they are not conflicting.*

3. A Dialectic-global Approach to ATC

As already mentioned, ATC defines a revision operator that revises an AF by an argument, making the necessary modifications to warrant that argument by analyzing the dialectical tree rooted in it. The core of the change machinery involves the *alteration* of some lines in such dialectical tree when it happens to be non-warranting. Therefore, the objective of altering lines is to change the morphology of the tree containing them in order to turn it to warranting. In this article, alteration of lines comes by removing arguments from the AF. However, arguments cannot be simply removed from $\mathbb{A}_\Sigma$. Instead, they disappear as a result of removing beliefs from the KB Σ from which arguments are built. The argumentation lines from a tree to be altered by ATC are identified as *attacking* [31]: lines which are somehow responsible for a non-warranting tree.

Definition 3.1 (Attacking Line). *A line $\lambda \in \mathcal{T}(\mathcal{R})$, with $\mathcal{T}(\mathcal{R}) \in \mathbf{T}_\Sigma$, is **attacking** iff for every $\mathcal{B} \in \lambda$ it holds:*

$$\mathfrak{mark}(\mathcal{B}, \lambda, \mathcal{T}(\mathcal{R})) = \begin{cases} D & \text{if } \mathcal{B} \in \lambda^+ \text{ or} \\ U & \text{if } \mathcal{B} \in \lambda^- \end{cases}$$

For instance in Example 2, λ_3 is attacking. When an altered line turns to non-attacking, the alteration is called *effective*. Finally, effectively altering each attacking line from a tree allows to obtain a new tree –from the revised KB– warranting its root argument.

[2] Refer to [31] for details on this and other marking criteria.

Theorem 3.2. *Given the dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbf{T}_\Sigma$, there is no attacking line $\lambda \in \mathcal{T}(\mathcal{R})$ iff $\mathcal{T}(\mathcal{R})$ is a warranting tree.*

The alteration of a line λ involves the removal of some argument $\mathcal{B} \in \lambda$, which prunes the subtree rooted in $\mathcal{B}$ out of the dialectical tree, leaving the upper segment $\lambda^{\uparrow}(\mathcal{B})$. Only cutting a con argument $\mathcal{B} \in \lambda^{-}$ off the line provokes an effective alteration.

Proposition 3.3. *If $\lambda \in \mathcal{T}(\mathcal{R})$ is attacking then for any $\mathcal{B} \in \lambda^{-}$, the upper segment $\lambda^{\uparrow}(\mathcal{B})$ turns into non-attacking.*

In classical belief base revision, a revision operator "$*$" is expected to add to the KB Σ a given belief α ensuring it to be consistently inferred from the revised KB $\Sigma * \alpha$. Analogously, in argumentation, a revision operator "$*^{\omega}$" should aim at incorporating a new argument $\mathcal{R}$ (possibly out of the domain $\mathbb{A}_\Sigma$) supporting α, ensuring it to be accepted by the argumentation semantics from the revised KB, *i.e.*, $\mathcal{R}$ should end up warranted from $\Sigma *^{\omega} \mathcal{R}$. This is seen later as the success condition. For this matter, we rely upon the notion of *external argument* which is an argument $\mathcal{B} \in \mathbb{A}_{\Sigma'}$ such that $\Sigma' = \Sigma \cup \mathfrak{bd}(\mathcal{B})$ and $\mathcal{B} \notin \mathbb{A}_\Sigma$. The *domain of external arguments* is identified through the set $\mathbb{E}_\Sigma$, moreover we refer to the set $\mathbb{A}_\mathcal{L}$ to identify the full domain of $\mathcal{L}$-arguments, that is, arguments from $\mathbb{A}_\Sigma \cup \mathbb{E}_\Sigma$. Thus, we construct an argument revision operation which includes to the given KB $\Sigma \subseteq \mathcal{L}$ the body of the argument $\mathcal{R} \in \mathbb{A}_\mathcal{L}$ (included either in $\mathbb{A}_\Sigma$ or $\mathbb{E}_\Sigma$). Afterwards, the acceptable tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$ is altered by removing beliefs from Σ. This determines a revised KB $\Sigma *^{\omega} \mathcal{R}$ from which the tree $\mathcal{T}'(\mathcal{R}) \in \mathbb{T}_{(\Sigma *^{\omega} \mathcal{R})}$ ends up warranting.

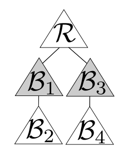
An incision function [20] constructed "globally" to the dialectical tree defines a *global incision* (Definition 3.5) which determines a set of beliefs to be removed from Σ. These removals allow to drop arguments from the tree in order to effectively alter all the necessary lines at once. However, other arguments containing beliefs to be removed will also disappear. Hence, a line considering some of those disappearing arguments will be *collaterally altered* by some *collateral incision*. Removing a con argument from an attacking line results in a non-attacking uppersegment (see Proposition 3.3). Nonetheless, deleting a pro argument in an argumentation line might lead to a flaw: while attacking lines keep their attacking condition, non-attacking lines might turn to attacking. Hence, the revision process should overcome such involuntary alterations by altering not only attacking lines, but also other lines that may turn to attacking from collateral incisions.

Given $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$, we need to effectively alter it to obtain a warranting tree. However, since collateral incisions may appear, it makes it difficult to decide which subset of lines from $\mathcal{S}(\mathcal{R})$ should be affected beyond attacking lines. Hence, the configuration of the global incision will rely on the analysis of *hypothetical trees*. These trees are deemed as hypothetical since they would appear as acceptable dialectical trees only after removing some beliefs from Σ.

Definition 3.4 (Hypothetical Tree). *Given $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$ and a subset $\Psi \subseteq \Sigma$, the **hypothetical tree** $\mathcal{H}(\mathcal{R}, \Psi) \in \mathbf{T}_\Sigma$ is the (possibly non-acceptable) tree rooted in $\mathcal{R}$ built from the set $X_1 \cup X_2$ of lines:*

$$
\begin{aligned}
X_1 &= \{\lambda \in \mathcal{T}(\mathcal{R}) \mid \forall \mathcal{B} \in \lambda : \Psi \cap \mathfrak{bd}(\mathcal{B}) = \emptyset\} \\
X_2 &= \{\lambda^{\uparrow}(\mathcal{B}) \mid \lambda \in \mathcal{T}(\mathcal{R}) \text{ such that } (\exists \mathcal{B} \in \lambda : \Psi \cap \mathfrak{bd}(\mathcal{B}) \neq \emptyset) \\
&\qquad \text{and } (\forall \mathcal{B}' \in \lambda^{\uparrow}(\mathcal{B}) : \Psi \cap \mathfrak{bd}(\mathcal{B}') = \emptyset)\}
\end{aligned}
$$

Example 3 (Continued from Example 2). *With the intention to alter λ_3, $\mathcal{B}_5$ may be removed from $\mathcal{T}(\mathcal{R})$ by removing $\neg q_3$ from Σ. Nonetheless, $\mathcal{B}_5$ and $\mathcal{B}_6$ are removed since both contain $\neg q_3$. The set $\{\lambda_1^{\uparrow}(\mathcal{B}_6), \lambda_2, \lambda_3^{\uparrow}(\mathcal{B}_5)\}$ resulting from Definition 3.4, is used for building the hypothetical tree $\mathcal{H}(\mathcal{R}, \{\neg q_3\})$ depicted on the right. Note however, that $\lambda_1^{\uparrow}(\mathcal{B}_6) \notin \mathcal{H}(\mathcal{R}, \{\neg q_3\})$ since $\lambda_1^{\uparrow}(\mathcal{B}_6) = [\mathcal{R}, \mathcal{B}_1]$ which is part of $\lambda_2 = [\mathcal{R}, \mathcal{B}_1, \mathcal{B}_2]$.*



The usage of hypothetical trees allows to analyze how the incisions would affect the original tree, recognizing new attacking lines that could arise from collateral incisions. Consequently, the global incision will progressively include beliefs in order to effectively alter not only attacking lines, but also other lines that could turn to attacking by looking into hypothetical trees.

Definition 3.5 (Global Incision). *Given the tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$, a function $\sigma : \mathbb{T}_\Sigma \longrightarrow 2^{\mathcal{L}}$ is **a global incision** iff it holds:*

1. *$\sigma(\mathcal{T}(\mathcal{R})) = \emptyset$ iff $\mathcal{T}(\mathcal{R})$ has no attacking line.*
2. *For any $\lambda \in \mathcal{T}(\mathcal{R})$, if any of the following are verified,*
   - (a) *λ is attacking, or*
   - (b) *there exists $\mathcal{C} \in \lambda$ such that $\emptyset \neq \sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{C}) = \Psi$, and $\lambda^{\uparrow}(\mathcal{C})$ is an attacking line in $\mathcal{H}(\mathcal{R}, \Psi)$,*

   then there is $\mathcal{B} \in \lambda^-$ such that $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{B}) \neq \emptyset$ and for any other $\mathcal{B}' \in \lambda^{\uparrow}(\mathcal{B})$ it holds $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{B}') = \emptyset$.
3. *For any $\beta \in \sigma(\mathcal{T}(\mathcal{R}))$, there is some $\lambda \in \mathcal{T}(\mathcal{R})$ verifying either 2a or 2b, and $\beta \in \mathfrak{bd}(\mathcal{B})$, for some $\mathcal{B} \in \lambda$.*

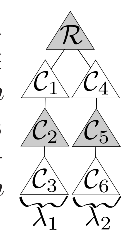
As stated before, the incision must provide a set of beliefs to remove from the KB in order to turn a non-warranting tree $\mathcal{T}(\mathcal{R})$ into warranting. Condition 1 in Definition 3.5 ensures the incision to be empty only in the case that $\mathcal{T}(\mathcal{R})$ warrants its root $\mathcal{R}$. This comes from Theorem 3.2, which relates the appearance of attacking lines to the non-warranting condition of a tree. Consequently, every attacking line should be altered (cond. 2a), as well as any collaterally incised line which could be turned to attacking (cond. 2b). This latter condition is checked upon hypothetical trees. From Proposition 3.3, we know that any upper segment $\lambda^{\uparrow}(\mathcal{B})$ is not attacking if it is the case that $\mathcal{B} \in \lambda^-$. Therefore, the consequent of cond. 2 ensures the incision to contain (at least) one belief from $\mathfrak{bd}(\mathcal{B})$ in order to effectively alter λ (turning it to non-attacking). Besides, the alteration of λ from such $\mathcal{B}$ is required to be the *uppermost alteration*, *i.e.*, no other argument $\mathcal{B}'$ placed above $\mathcal{B}$ in λ should be affected by the incision since this could compromise the attacking status of the line. Cond. 3 requires the incision to take beliefs only from arguments in lines that necessarily need to be altered (by either verifying 2a or 2b).

Notice that the global incision function is not always unique as can be seen in Example 5. On the other hand, there always exists, at least, one global incision function as is shown next.

Proposition 3.6. *Given the dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$, there is always a global incision function $\sigma(\mathcal{T}(\mathcal{R})) \subseteq \mathcal{L}$.*

Example 4.

Given $\mathcal{T}(\mathcal{R})$ depicted on the right, assume $\beta \in (\mathfrak{bd}(\mathcal{C}_3) \cap \mathfrak{bd}(\mathcal{C}_5))$. Suppose $\sigma(\mathcal{T}(\mathcal{R}))$ takes beliefs from $\mathcal{C}_3$, and in particular $\beta \in \sigma(\mathcal{T}(\mathcal{R}))$. In such a case, $\sigma(\mathcal{T}(\mathcal{R}))$ could not consider beliefs from $\mathcal{C}_6$ since it would not be the uppermost incision in λ_2 ($\mathcal{C}_5$ is over $\mathcal{C}_6$ and $\mathcal{C}_5$ is collateraly incised, i.e., $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{C}_5) \neq \emptyset$). Therefore, in order to alter λ_2, $\sigma(\mathcal{T}(\mathcal{R}))$ might also consider beliefs from $\mathcal{C}_4$ (since it is the only alternative in λ_2).



By removing the set of beliefs mapped by a global incision over $\mathcal{T}(\mathcal{R})$, we obtain a KB determining a tree rooted in $\mathcal{R}$ free of attacking lines, and thus, from Theorem 3.2, $\mathcal{R}$ ends up warranted. Next we define the argument revision upon the global incision function, allowing to revise an $\mathcal{L}$-KB by an argument from $\mathbb{A}_\mathcal{L}$.

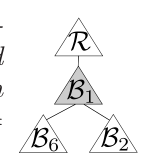
Definition 3.7 (Argument Revision). *Given $\Sigma \subseteq \mathcal{L}$ and $\mathcal{R} \in \mathbb{A}_\mathcal{L}$, an operator $*^\omega$ is an **argument revision** iff the operation $\Sigma *^\omega \mathcal{R}$ is:*

$$\Sigma *^\omega \mathcal{R} = (\Sigma \cup \mathfrak{bd}(\mathcal{R})) \setminus \sigma(\mathcal{T}(\mathcal{R})), \text{ where } \mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$$

When necessary we will write $\Sigma *^\omega_\sigma \mathcal{R}$ to specify that the revision $\Sigma *^\omega \mathcal{R}$ is obtained by effect of a specific global incision function σ.

Example 5. *As seen in Example 3, if we assume $\sigma(\mathcal{T}(\mathcal{R})) = \{\neg q_3\}$, Definition 3.5 would be verified. Hence, the hypothetical tree depicted in Example 3 ends up being the resulting altered tree from $\mathbb{T}_{(\Sigma *^\omega \mathcal{R})}$ which warrants the root argument $\mathcal{R}$. Note that the incision could also be $\sigma(\mathcal{T}(\mathcal{R})) = \{\neg p_2 \vee q_3\}$ with the same resulting altered tree, depicted in Example 3.*

A different alternative would be to assume another incision function such that $\sigma(\mathcal{T}(\mathcal{R})) = \{q_2\}$. In this case, argument $\mathcal{B}_3$ would be removed. The resulting altered tree (depicted on the right) from $\mathbb{T}_{(\Sigma *^\omega \mathcal{R})}$ warrants $\mathcal{R}$. Note that the incision could also be $\sigma(\mathcal{T}(\mathcal{R})) = \{q_2 \rightarrow \neg q\}$ with the same resulting altered tree.



Lemma 3.8. *Given $\Sigma \subseteq \mathcal{L}$, if $*^\omega$ is an argument revision then $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma *^\omega \mathcal{R})}$ has no attacking lines, for any $\mathcal{R} \in \mathbb{A}_\mathcal{L}$.*

4. Rationality Analysis

Revisions and contractions are usually defined independently with the intention to interrelate them afterwards by setting up a duality. A philosophic discussion is sustained on the matter of the nature of such independency. Some researchers assert that there is really no contraction whose existence could be justified without a revision. In fact, they state that a contraction conforms an intermediate state towards the full specification of the revision. This is the case of the ATC revision model. As a consequence, the axiomatization of the revision presented here was achieved by analyzing the different characters of both revision and contraction operations from the classical belief revision literature [1, 20, 21, 22]. The postulates discussed there are studied as a motivation to propose a basic set of postulates for base revision in argumentation, composed by the argumentation postulates of *success*, *consistency*, *inclusion*, *vacuity*, *core-retainment*, and *uniformity*. Next we introduce them one by one, analyzing their corresponding intuitions. For such matter, we assume revision operators from classical belief revision "$*$" and argumentation "$*^\omega$", an $\mathcal{L}$-KB Σ, a belief $\alpha \in \mathcal{L}$, and an argument $\mathcal{R} \in \mathbb{A}_\mathcal{L}$ such that $\mathfrak{cl}(\mathcal{R}) = \alpha$, pursuing to characterize the operation $\Sigma *^\omega \mathcal{R}$ by analyzing $\Sigma * \alpha$.

Success states that the new information α should be satisfied by the revised KB. This is usually written as $\Sigma * \alpha$ *implies* α. From the argumentation standpoint, it may be interpreted as the requirement of warranting the new argument $\mathcal{R}$ which in turn supports α (its claim).

(success) $\mathcal{R}$ is warranted from $\Sigma *^\omega \mathcal{R}$

Through *consistency*, a classical revision operation ensures the revised base to be consistent always that the new belief to be incorporated is so. That is, $\Sigma * \alpha$ *is consistent if* α *is consistent.* Argumentation theory gives the opportunity to reason consistently over a potentially inconsistent KB. Hence, there is no need to restore consistency to the revised KB, indeed this is our main objective: to manage dynamics of knowledge over inconsistencies. In turn, consistency in argumentation refers to the adopted semantics, *i.e.*, it should only ensure the set of warranted arguments to end up free of conflict.

(consistency) For any $\{\mathcal{B}, \mathcal{C}\} \subseteq \mathbb{A}_{(\Sigma *^\omega \mathcal{R})}$, if $\mathcal{B}$ and $\mathcal{C}$ are warranted from $\Sigma *^\omega \mathcal{R}$ then $\mathcal{B}$ and $\mathcal{C}$ are not conflicting.

Consistency is not needed for the representation theorem given that the models of change in ATC are dependent on the marking criterion which leads to a consistent argumentation semantics (see Appendix). Hence, any model of change proposed over such skeptical criterion is ensured to guarantee consistency.

Inclusion aims at guaranteeing that no other new information beyond α will be incorporated. That is, $\Sigma * \alpha \subseteq \Sigma \cup \alpha$. Its restatement to argumentation is given by including no more than $\mathcal{R}$'s body.

(inclusion) $\Sigma *^\omega \mathcal{R} \subseteq \Sigma \cup \mathfrak{bd}(\mathcal{R})$

Inclusion only refers to beliefs in the KB. From the standpoint of the set of arguments the relation is quite different: new information is added to the KB with the intention to build $\mathcal{R}$, but in consequence other arguments may appear: $\mathbb{A}_{(\Sigma *^{\omega} \mathcal{R})} \not\subseteq \mathbb{A}_{\Sigma} \cup \{\mathcal{R}\}$.

Vacuity captures the conditions under which the revision operation has nothing to do but the sole incorporation of the new information α. This is usually written as *If* Σ *does not imply* $\neg\alpha$ *then* $\Sigma \cup \alpha \subseteq \Sigma * \alpha$. Its restatement to argumentation may be seen as the fact of $\mathcal{R}$ being warranted straightforwardly, with no need to remove any belief. Hence, only the addition of $\mathcal{R}$ is required to obtain a warranting tree rooted in it.

(vacuity) If $\mathcal{R}$ is warranted from $\Sigma \cup \mathfrak{bd}(\mathcal{R})$ then $\Sigma \cup \mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^{\omega} \mathcal{R}$

Through *core-retainment* [22], the amount of change is controled by avoiding removals that are not related to the revision, *i.e.*, every belief that is lost serves to make room for α. This can be written as *If* $\beta \in \Sigma \setminus (\Sigma * \alpha)$*, then there is some* $\Sigma' \subseteq \Sigma$ *such that* $\Sigma' \cup \{\alpha\}$ *is consistent but* $\Sigma' \cup \{\alpha, \beta\}$ *is not.* In argumentation, we care not on consistency, but on the warrant condition of $\mathcal{R}$. Hence, any belief β that is removed should be needed to achieve an effective alteration.

(core-retainment) If $\beta \in \Sigma \setminus (\Sigma *^{\omega} \mathcal{R})$ then there is some $\Sigma' \subseteq \Sigma$ such that $\mathcal{R}$ is warranted from $\Sigma' \cup \mathfrak{bd}(\mathcal{R})$ but not from $\Sigma' \cup \mathfrak{bd}(\mathcal{R}) \cup \{\beta\}$

As usual in belief revision, it is natural to assume that revisions applied to Σ by either of α or β (logically equivalent $\mathcal{L}$-formulae) have necessarily identical outcomes. For instance, $\Sigma * (\neg p \vee q)$ should be equal to $\Sigma * (p \rightarrow q)$. This is captured by the *extensionality* postulate: *If* α *iff* β *then* $\Sigma * \alpha = \Sigma * \beta$. However, extensionality allows the following revisions: (1) $\{r, r \rightarrow p\} * \neg p = \{r \rightarrow p, \neg p\}$ and (2) $\{r, r \rightarrow p\} * \neg(p \vee q) = \{r, \neg(p \vee q)\}$. Although $\neg p$ and $\neg(p \vee q)$ are not logically equivalent, they are "equivalent" by considering that their complements are equally implied by $\{r, r \rightarrow p\}$ and its subsets (see [21]). The choice of which elements of the KB to retain should depend on their logical relations to the new information. Therefore, if two sentences are inconsistent with the same subsets of Σ, they should push out the same elements from Σ. This is known as *uniformity*: *For all* $\Sigma' \subseteq \Sigma$*, if* $\Sigma' \cup \{\alpha\}$ *is inconsistent iff* $\Sigma' \cup \{\beta\}$ *is inconsistent, then* $\Sigma \cap (\Sigma * \alpha) = \Sigma \cap (\Sigma * \beta)$. Uniformity is used for belief bases as a stricter version of extensionality, *i.e.*, it implies extensionality, but is not implied by it.

The question now is how can this be adapted to deal with the argumentation theory. To that end, we need to specify some relation $\tau \subseteq \mathbb{A}_{\mathcal{L}} \times \mathbb{A}_{\mathcal{L}}$ between pairs of arguments $\mathcal{R}_1 \in \mathbb{A}_{\mathcal{L}}$ and $\mathcal{R}_2 \in \mathbb{A}_{\mathcal{L}}$ such that if $\tau(\mathcal{R}_1, \mathcal{R}_2)$ then two equivalent revised KBs $\Sigma *^{\omega} \mathcal{R}_1$ and $\Sigma *^{\omega} \mathcal{R}_2$ arise. Since the argumentation model of change that we study is based on the analysis of dialectical trees, we need to ensure that the addition of either of such arguments determines dialectical trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$, such that $\mathcal{T}(\mathcal{R}_1)$ is warranting *iff* $\mathcal{T}(\mathcal{R}_2)$ is warranting. For that matter, both trees should be *morphologically identical*: two trees rooted in $\mathcal{R}_1$ and $\mathcal{R}_2$ such that either none of both have defeaters, or $\mathcal{R}_1$ has children (direct root defeaters) $\{\mathcal{B}_1, \ldots, \mathcal{B}_n\}$, and $\mathcal{R}_2$ has children $\{\mathcal{C}_1, \ldots, \mathcal{C}_m\}$ such that (1) $n = m$; and (2) for every $1 \leq i \leq n$, the subtrees rooted in $\mathcal{B}_i$ and $\mathcal{C}_i$ are morphologically identical trees. Afterwards, the alteration of any of both trees should end up equivalently in order for both revisions to behave in the same manner. That is, the same beliefs should be dropped from Σ by any of both revisions. In order to pursue equivalent alterations of morphologically identical trees, we also need to ensure that for any pair of lines $\lambda_1 \in \mathcal{T}(\mathcal{R}_1)$ and $\lambda_2 \in \mathcal{T}(\mathcal{R}_2)$, such that $\lambda_1 = [\mathcal{R}_1, \mathcal{B}_1, \ldots, \mathcal{B}_n]$ and $\lambda_2 = [\mathcal{R}_2, \mathcal{C}_1, \ldots, \mathcal{C}_n]$, if $\tau(\mathcal{R}_1, \mathcal{R}_2)$ then $\tau(\mathcal{B}_i, \mathcal{C}_i)$ holds for any $1 \leq i \leq n$. For instance, considering "τ" as the equality "$=$" between arguments would require the same root argument for both trees, thus determining a unique acceptable tree. On the other hand, considering "τ" as the equivalence "$\equiv$" seems to be too permissive since their respective trees may not end up morphologically identical.

Example 6. *Given a KB* $\Sigma = \{q \to \neg r,\ q \to r\}$*, and two external arguments* $\mathcal{R}_1 \in \mathbb{E}_\Sigma$ *and* $\mathcal{R}_2 \in \mathbb{E}_\Sigma$*, such that* $\mathcal{R}_1 \equiv \mathcal{R}_2$*, where* $\mathcal{R}_1 = \langle\{q, r, (r \wedge q \to p)\}, p \wedge q \wedge r\rangle$*, and* $\mathcal{R}_2 = \langle\{r, (r \to p \wedge q)\}, p \wedge q \wedge r\rangle$*. Arguments* $\mathcal{B}_1 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ *and* $\mathcal{B}_2 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ *are equivalent (*$\mathcal{B}_1 \equiv \mathcal{B}_2$*) with identical bodies* $\{q \to \neg r\}$*. Besides,* $\mathcal{B}_1$ *and* $\mathcal{B}_2$ *are canonical undercuts of* $\mathcal{R}_1$ *and* $\mathcal{R}_2$*, respectively. However, argument* $\mathcal{C}_1 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$*, such that* $\mathcal{C}_1 = \langle\{q, q \to r\}, \neg(q \to \neg r)\rangle$*, is a canonical undercut of* $\mathcal{B}_1$*, but has no equivalent canonical undercut of* $\mathcal{B}_2$ *in* $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$*.*

Considering equivalent root arguments, it is clear that we cannot always obtain morphologically identical trees. To this end, we strengthen the equivalence relation "$\equiv$" between arguments, and define a *strict equivalence* "$\dashv\Vdash$" standing for the relation "τ".

Definition 4.1 (Strictly Equivalent Arguments). *Arguments* $\mathcal{R}_1 \in \mathbb{A}_\mathcal{L}$ *and* $\mathcal{R}_2 \in \mathbb{A}_\mathcal{L}$ *are* ***strictly equivalent****, written* $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ *iff* $\mathcal{R}_1 \equiv \mathcal{R}_2$ *and for any subset* $\Psi_1 \subseteq \mathfrak{bd}(\mathcal{R}_1)$ *there is a subset* $\Psi_2 \subseteq \mathfrak{bd}(\mathcal{R}_2)$ *such that* Ψ_1 *iff* Ψ_2*.*

Example 7. *Given a KB* $\Sigma = \{q \to \neg r\}$*, and two external arguments* $\mathcal{R}_1 \in \mathbb{E}_\Sigma$ *and* $\mathcal{R}_2 \in \mathbb{E}_\Sigma$*, such that* $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$*, where* $\mathcal{R}_1 = \langle\{q, (q \to p), (q \to r)\}, p \wedge q \wedge r\rangle$*, and* $\mathcal{R}_2 = \langle\{q, (\neg q \vee p), (\neg q \vee r)\}, p \wedge q \wedge r\rangle$*. Argument* $\mathcal{B}_1 = \langle\{q \to \neg r\}, \neg(q \wedge (q \to p) \wedge (q \to r))\rangle \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ *is a new canonical undercut of* $\mathcal{R}_1$*, and* $\mathcal{B}_2 = \langle\{q \to \neg r\}, \neg(q \wedge (\neg q \vee p) \wedge (\neg q \vee r))\rangle \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$*, of* $\mathcal{R}_2$*. Observe that* $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$ *holds, and moreover, such condition also holds for the canonical undercuts of* $\mathcal{B}_1$ *and* $\mathcal{B}_2$ *with bodies* $\{q, q \to r\}$ *and* $\{q, \neg q \vee r\}$*, respectively.*

Given any pair of external arguments $\mathcal{R}_1$ and $\mathcal{R}_2$, if they are strictly equivalent, their dialectical trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ would end up being not only morphologically identical, but also *strictly equivalent*.

Definition 4.2 (Strictly Equivalent Lines and Trees). *Two argumentation lines* $\lambda_1 \in \mathbb{L}_\Sigma$ *and* $\lambda_2 \in \mathbb{L}_{\Sigma'}$*, where* $\lambda_1 = [\mathcal{B}_1 \ldots, \mathcal{B}_n]$ *and* $\lambda_2 = [\mathcal{C}_1 \ldots, \mathcal{C}_n]$*, are* ***strictly equivalent*** *iff* $\mathcal{B}_i \dashv\Vdash \mathcal{C}_i$ *for* $1 \leq i \leq n$*. Two dialectical trees* $\mathcal{T}(\mathcal{B}_1) \in \mathbb{T}_\Sigma$ *and* $\mathcal{T}(\mathcal{C}_1) \in \mathbb{T}_{\Sigma'}$ *are* ***strictly equivalent*** *iff for any* $\lambda \in \mathcal{T}(\mathcal{B}_1)$ *(resp.,* $\lambda \in \mathcal{T}(\mathcal{C}_1)$*) there is a strictly equivalent* $\lambda' \in \mathcal{T}(\mathcal{C}_1)$ *(resp.,* $\lambda' \in \mathcal{T}(\mathcal{B}_1)$*).*

Lemma 4.3. *Given a KB $\Sigma \subseteq \mathcal{L}$, two external arguments $\mathcal{R}_1 \in \mathbb{E}_\Sigma$ and $\mathcal{R}_2 \in \mathbb{E}_\Sigma$, and their dialectical trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma\cup\mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma\cup\mathfrak{bd}(\mathcal{R}_2))}$; if $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ then $\mathcal{T}(\mathcal{R}_1)$ and $\mathcal{T}(\mathcal{R}_2)$ are strictly equivalent trees.*

A KB to be revised by an argument $\mathcal{R}$ will possibly need to give up some knowledge. However, the beliefs included in $\mathcal{R}$ should not be compromised. This is necessary to ensure $\mathcal{R}$ to belong to the resulting set of arguments $\mathbb{A}_{(\Sigma *^\omega \mathcal{R})}$, and also to allow $\mathcal{R}$ to end up warranted. Therefore, only old knowledge is to be affected. While considering to revise Σ by any of two (or more) strictly equivalent external arguments, the sets of beliefs to remove from Σ will end up being identical given that, from Lemma 4.3, both (or all) trees are strictly equivalent, and that no new information is allowed to be affected. Uniformity is finally restated to argumentation as follows:

(uniformity) If $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ then $\Sigma \cap \Sigma *^\omega \mathcal{R}_1 = \Sigma \cap \Sigma *^\omega \mathcal{R}_2$

The construction of an argument revision is required to alter in the same manner different strictly equivalent dialectical trees. To this matter, we introduce an additional condition on incision functions to guarantee this property. Intuitions followed here are inspired by those of smooth incisions in Hansson's Smooth Kernel Contractions [20].

Definition 4.4 (Smooth Argument Incision). *An incision σ is* ***smooth*** *iff for any $\mathcal{L}$-KB Σ, and any $\{\mathcal{R}_1, \mathcal{R}_2\} \subseteq \mathbb{E}_\Sigma$; if $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ then for any $\mathcal{B}_1 \in \mathbb{A}_{(\Sigma\cup\mathfrak{bd}(\mathcal{R}_1))}$ there exists $\mathcal{B}_2 \in \mathbb{A}_{(\Sigma\cup\mathfrak{bd}(\mathcal{R}_2))}$ such that $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$ and $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) = \sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2)$.*

Proposition 4.5. *Given a KB $\Sigma \subseteq \mathcal{L}$, and two arguments $\mathcal{R}_1 \in \mathbb{E}_\Sigma$ and $\mathcal{R}_2 \in \mathbb{E}_\Sigma$ such that $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$, there always exists an argument incision function σ which is smooth.*

An argument revision will be referred as smooth if and only if it is defined by means of a smooth argument incision. Next we present the representation theorem for the dialectic-global model. We claim that, though in classical belief revision, guaranteeing success, inclusion, and core-retainment implies vacuity, in argumentation this is not possible due to the non-monotonicity nature of the argumentation semantics.

Representation Theorem 4.6. *Given $\Sigma \subseteq \mathcal{L}$ and $\mathcal{R} \in \mathbb{A}_\mathcal{L}$, $\Sigma *^\omega \mathcal{R}$ is a smooth argument revision iff it guarantees success, inclusion, vacuity, core-retainment, and uniformity.*

5. AF Revision Operation

In this section we will propose a revision operator for a propositional argumentation framework or AF, by relying upon an incision function according to

Definition 3.5. The new AF revision operator "$\circledast^{\omega}$", determines a revision of an AF ϕ by an argument $\mathcal{R}$ in order to obtain a new AF $(\phi \circledast^{\omega} \mathcal{R})$ warranting $\mathcal{R}$.

Usually, an AF is given through a pair *(set of arguments)–(attack relation)*. However, since we are working with propositional AFs, the attack relation can be obtained as a result of an inference process. Thus, an AF for us will identify only a subset of $\mathbb{A}_{\mathcal{L}}$, containing (propositional) $\mathcal{L}$-arguments (recall that $\mathbb{A}_{\mathcal{L}}$ was defined on page 10 as the set of all arguments over $\mathcal{L}$). This means that, for an AF ϕ which is obtained from an underlying KB $\Sigma \subseteq \mathcal{L}$, it is easy to see that $\phi = \mathbb{A}_{\Sigma}$ holds.

We will assume the existence of a specialized *closure condition for propositional AFs* in order to obtain a correct propositional AF. For instance, the propositional AF $\phi = \{\mathcal{B}\}$, where $\mathfrak{bd}(\mathcal{B}) = \{\alpha, \beta\}$ would not be correct given that $\alpha \in \mathcal{L}$ and $\beta \in \mathcal{L}$, and therefore, the primitive arguments for each of them, *i.e.*, $\langle \{\alpha\}, \alpha \rangle$ and $\langle \{\beta\}, \beta \rangle$, should also be included in the AF. Observe that a propositional AF will be in general infinite, if we admit arguments with claims of unrestricted propositional patterns (mostly because of the consideration of disjunctions). This matters can be treated with the aid of some of the elements proposed in [7] such as canonical undercuts and so; and exceeds the scope of this article. We will not step further into such subject.

In accordance to the aforementioned, it is clear that the formulation of a pure AF revision in the ATC sense can be useful only for theoretical matters. This posts a new fundamental justifying the proposal of the argument revision proposed in Section 3, as a practical approach to ATC for base revision, in which only the necessary portion of the AF is built "on demand" according to the dialectical tree rooted in the new argument to be included in the KB.

Before we can specify the closure condition for propositional AFs, we will define the obtention of the underlying KB of a given AF as follows.

Definition 5.1 (Underlying KB of an AF). *Given a propositional AF* $\phi \subseteq \mathbb{A}_{\mathcal{L}}$*, an operator* $\mathbb{K}$ *determines the* ***underlying KB of the AF*** ϕ *iff* $\mathbb{K}(\phi) = \bigcup_{\mathcal{B} \in \phi} \mathfrak{bd}(\mathcal{B})$.

We specify the closure condition for AFs by relying upon the operator $\mathbb{K}$.

Definition 5.2 (Propositional AF Closure). *Given a propositional AF* $\phi \subseteq \mathbb{A}_{\mathcal{L}}$*, an operator* $\mathbb{C}$ *determines the* ***propositional AF closure*** *of* ϕ *iff* $\mathbb{C}(\phi) = \mathbb{A}_{\Sigma}$*, where* $\Sigma = \mathbb{K}(\phi)$.

Consequently, we will say that a propositional AF $\phi \subseteq \mathbb{A}_{\mathcal{L}}$ is closed by "$\mathbb{C}$" (or simply, *closed*) if it holds $\phi = \mathbb{C}(\phi)$. From now on we will rely only upon closed propositional AFs. This condition is necessary for a correct application of the change methodologies proposed here.

Observation 5.3. $\mathbb{K}(\mathbb{A}_{\Sigma}) \subseteq \Sigma$.

The previous observation shows an elementary correspondence between $\mathbb{A}_{\Sigma}$ and Σ when working with the operator $\mathbb{K}$. It is important to mention that if it is the case that there is a sentence $\beta \in \Sigma$ such that $\beta \notin \mathbb{K}(\mathbb{A}_\Sigma)$ then the only alternative we have is $\beta \models \bot$, otherwise β would be part of (at least) a primitive argument in $\mathbb{A}_\Sigma$, and therefore $\beta \in \mathbb{K}(\mathbb{A}_\Sigma)$ contrary to the hypothesis.

Observation 5.4. $\mathbb{K}(\mathbb{A}_\Sigma) = \Sigma$ *iff for every* $\beta \in \Sigma$, $\beta \not\models \bot$.

The AF revision operator "$\circledast^\omega$" is finally formalized for a closed propositional AF by relying upon a global incision function, according to Definition 3.5.

Definition 5.5 (AF Revision). *Given a closed AF* $\phi \subseteq \mathbb{A}_\mathcal{L}$ *and an argument* $\mathcal{R} \in \mathbb{A}_\mathcal{L}$*, an operator* $\circledast^\omega$ *is an* ***AF revision*** *iff the operation* $\phi \circledast^\omega \mathcal{R}$ *is:*

$$\phi \circledast^\omega \mathcal{R} = \{\mathcal{B} \in \mathbb{A}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))} | \mathfrak{bd}(\mathcal{B}) \cap \sigma(\mathcal{T}(\mathcal{R})) = \emptyset, \text{ where } \mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))}\}$$

When necessary we will write $\phi \circledast^\omega_\sigma \mathcal{R}$ to specify that the revision $\phi \circledast^\omega \mathcal{R}$ is obtained by effect of a specific global incision function σ.

Note that an AF revision of a closed AF always results in a new closed AF.

Observation 5.6. *Given a closed AF* $\phi \subseteq \mathbb{A}_\mathcal{L}$ *and an argument* $\mathcal{R} \in \mathbb{A}_\mathcal{L}$*, the AF revision* $\phi \circledast^\omega \mathcal{R}$ *determines a new AF which is also closed.*

As shown before, in Section 3, the global incision function is not always unique, however, it will be shown to be a function as part of the proof of Theorem 4.6 (see (postulates $\Rightarrow$ construction) condition 1, in page 32). This means that, when applied over the same inputs (*i.e.*, over the same dialectical tree), it will certainly map to equal values from $\mathcal{L}$. In consequence, the following observation follows trivially from Definition 5.5 and Definition 3.7.

Observation 5.7. *Given a closed AF* $\phi \subseteq \mathbb{A}_\mathcal{L}$ *and an argument* $\mathcal{R} \in \mathbb{A}_\mathcal{L}$*, the AF revision* $\phi \circledast^\omega_\sigma \mathcal{R}$ *is obtained by effect of the global incision function* σ *iff the argument revision* $\mathbb{K}(\phi) *^\omega_\sigma \mathcal{R}$ *is obtained by effect of* σ.

For the previous observation, notice that for both revision operations, $\phi \circledast^\omega_\sigma \mathcal{R}$ and $\mathbb{K}(\phi) *^\omega_\sigma \mathcal{R}$, σ is applied over the same dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))}$. In addition, through Observation 5.7, we can ensure that for the following results (in which both revisions operations are interrelated), there will always apply a fixed global incision function.

Lemma 5.8. $\phi \circledast^\omega \mathcal{R} = \mathbb{A}_{(\mathbb{K}(\phi) *^\omega \mathcal{R})}$.

Lemma 5.9. $\mathbb{K}(\phi \circledast^\omega \mathcal{R}) = \mathbb{K}(\phi) *^\omega \mathcal{R}$.

For the full rationality of the AF revision operation we would need to propose a specialized set of postulates for this new sort of revision. This exceeds the scope of this work, however, we can show the rationality of the AF revision in terms of the rationality of the argument revision operation shown in Theorem 4.6, by following the results obtained in Lemma 5.8 and Lemma 5.9.

Theorem 5.10. *Given a closed AF* $\phi \subseteq \mathbb{A}_\mathcal{L}$ *and* $\mathcal{R} \in \mathbb{A}_\mathcal{L}$*, if* $\mathbb{K}(\phi) *^\omega \mathcal{R}$ *is a smooth argument revision then* $\phi \circledast^\omega \mathcal{R}$ *is a rational AF revision wrt. the postulates in Theorem 4.6, taking* $\Sigma = \mathbb{K}(\phi)$ *and* $\Sigma *^\omega \mathcal{R} = \mathbb{K}(\phi \circledast^\omega \mathcal{R})$.

6. Related Work

In this section, we present some of the existing work on belief change and argumentation. Some of them, like [12, 11, 9, 10, 3], do not really propose argumentative models of change but more likely analyze the impact of adding a new argument to an AF. We firstly describe such works and relate them to ATC. Afterwards, we give a brief summary of other articles working the relation between belief revision and other theories that might somehow be similar to our argumentative model of change towards revision of knowledge bases. Among those works, only [28, 27] use argumentation, however, rather differently to the way ATC approaches do: the former focusses on dynamics in epistemology, and the latter on a particular agents' belief base, relying on a model of belief dynamics which is alternative to AGM's. Afterwards, a particular work, [2], is analyzed. There, argumentation semantics are proposed to handle KB inconsistencies with a different orientation in comparison to our theories. Finally, we describe the relation between the dialectic-global model presented in this article and other existing ATC approaches.

Argumentation and Belief Revision

[12] proposed a revision theory upon Dung-style abstract argumentation systems. The main issue of any argumentation system is the selection of acceptable sets of arguments. An argumentation semantics defines the properties required for a set of arguments to be acceptable. The selected sets of arguments under a given semantics are called *extensions* of that semantics. Then, by considering how the set of extensions is modified under the revision process, they propose a typology of different revisions: decisive revision and expansive revision. A strong restriction is posed: the newly added argument must have at most one interaction (via attack) with an argument in the system. This restriction greatly simplifies the revision problem of AFs, as multiple interactions with the original system are more common to occur, and could become difficult to handle. Moreover, regarding inconsistent-tolerant revision of KBs, this restriction would make impossible to apply such model of change to a KB from where arguments and attacks are to be constructed. In ATC, this is addressed with the inclusion of subarguments and through the handle of collateralities. In addition, the objective of [12] differs from ours in that we apply (assuming it is allowed) additional change to the original AF (and consequently, to the KB) pursuing warrant of a single argument through the analysis of dialectical trees, whereas they study how the addition of a given argument would affect the set of extensions (without performing any alteration besides the addition of the new argument), by looking at an arguments graph. Working with dialectical trees seems to be a better alternative for the revision operator to handle a controlled (and thus tractable) set of arguments.

Another similar approach studying dynamics in argumentation was presented in [11]. There, the *abstraction* of a framework, *i.e.*, removal of a set of arguments or attacks, is considered, and a series of principles is proposed to establish under which conditions the semantics remains unchanged. This approach avoids recomputation of semantics. [9] discusses the existing relation between argumentation and belief revision. It considers argumentation as persuasion to believe and that persuasion should be related to belief revision. More recently, [10] presented the interrelation between argumentation and belief revision on multi-agent systems. When an agent uses an argument to persuade another one, he must consider not only the proposition supported by the argument, but also the overall impact of the argument on the beliefs of the addressee.

Revision over an argumentation-based decision making system was applied in [3] through a generalization of the revision technique from [12], which evaluates the warrant status of a newly inserted argument. A complementary approach could be followed by using ATC when the new argument is not initially warranted; in this way, we would know which arguments should be removed for the new one to end up warranted.

There is a main difference between these approaches and ATC: they assume that the framework is only changed as a reflection of the evolution of the modeled world, and thus their theory analyzes the impact of change on the semantics. On the other hand, ATC assumes certain external mechanism with capabilities to affect the world and the way it evolves. Thus, ATC allows to decide which changes to apply in order to make the modeled world evolve in the desired direction. Nonetheless, this is not intended to lead to dishonest use, by "hiding" neither arguments nor evidence from the argumentative machinery. To the contrary, it provides a powerful tool for research purposes. For instance, consider a legal process in which currently available evidence yields a dialectical tree conforming an unexpected scenario. Performing hypothetical (or abductive) reasoning by considering yet unavailable arguments may give intuitions about missing evidence. This can be used to lead further investigations towards the defense of the case in a trial.

Belief Revision and Knowledge Bases

Regarding ideas from classic belief revision applied to non-monotonic theories, in [8], the authors study the dynamics of a simple variant of *defeasible logic* through the definition of expansion, revision and contraction operators. Here, a *defeasible theory* contains facts, defeasible rules and *defeaters*. The first two elements are similar to those in DeLP, whereas defeaters are rules that, instead of being used to draw claims, they prevent their achievement. The focus of the paper is to provide a full account of postulates which are closely related to those from the AGM model. The intuitions behind each operator do not need any special consideration, and each one of them is formally checked to comply with the corresponding set of postulates. As revision of defeasible logics, this work is similar to [23], which proposes revision of DeLP programs by relying on an ATC approach (see next subsection for more details). However, contrary to ATC approaches –including the dialectic-global model– the work done in [8] requires consistency restoration.

The work in [5] is primarily oriented towards the treatment of inconsistency caused by the use of multiple sources of information. Knowledge bases are stratified, namely each formula in the knowledge base is associated with its level of certainty corresponding to the layer to which it belongs. They suggest that it is not necessary to restore consistency in order to make sensible inferences from an inconsistent knowledge base. Likewise, argumentation-based inference can derive conclusions supported by reasons to believe in them, independently of the consistency of the knowledge base.

[28] studied the dynamic of a belief revision system considering relations among beliefs in a "derivational approach" trying to obtain a theory of belief revision from a more concrete epistemological theory. According to them, one of the goals of belief revision is to generate a knowledge base in which each piece of information is justified (by perception) or warranted by arguments containing previously held beliefs. The difficulty is that the set of justified beliefs can exhibit all kinds of logical incoherences because it represents an intermediate stage in reasoning. Therefore, they propose a theory of belief revision concerned with warrant rather than justification.

[18] proposed a kind of non-prioritized revision operator based on the use of explanations. The idea is that an agent, before incorporating information that is inconsistent with its knowledge, requests an explanation supporting it. The framework used is oriented to defeasible reasoning. One of the most interesting ideas of this work is the generation of defeasible conditionals from a revision process. This approach preserves consistency in the strict knowledge and it provides a mechanism to dynamically qualify the beliefs as strict or defeasible.

[27] joined argumentation and belief revision in the same conceptual framework, highlighting the important role played by Toulmin's layout of argument in fostering such integration. They consider argumentation as "persuasion to believe" and this restriction is useful to make more explicit the connection with belief revision. They propose a model of belief dynamics alternative to the AGM approach: *data-oriented belief revision* (DBR). Two basic informational categories (data and beliefs) are put forward in their model, to account for the distinction between pieces of information that are simply gathered and stored by the agent (*data*), and pieces of information that the agent considers (possibly up to a certain degree) truthful representations of states of the world (*beliefs*). Whenever a new piece of evidence is acquired through perception or communication, it affects directly the agent's data structure and only indirectly his beliefs. Belief revision is often triggered by information update either on a fact or on a source: the agent receives a new piece of information, rearranges his data structure accordingly, and possibly changes his beliefs.

[16] addressed the problem of belief revision in (non-monotonic) logic programming under answer set semantics: given two logic programs P and Q, the goal is to determine a program R that corresponds to the revision of P by Q, denoted $P * Q$. They proposed formal techniques analogous to those of distance-based belief revision [15, 32] in propositional logic. They investigate two specific operators: (logic program) expansion and a revision operator based on the distance between the strong equivalence models (SE-models) of logic programs. However, our approach is very different. First, we use propositional KBs instead of logic programs, making our approach more expressive and general. Second, since we want an external argument $\mathcal{R}$ to end up undefeated after the

revision, we must modify the KB to accept (warrant) $\mathcal{R}$'s claim, thus defining a revision operator in a prioritized fashion. Third, and most important, our work does not pursue a consistent outcome from the revision operation, contrasting the objective in [16, 15, 32].

Argumentation and Knowledge Bases

The recognition of consistent subsets of (potentially inconsistent) KBs is studied in [2] by relying upon extension-based argumentation semantics *à la* Dung. Authors recognize arguments from the KB in a similar way to what we have done in this article. Then, they characterize the different semantics in terms of the subsets of the KB that are returned by each extension. A full correspondence between maximal consistent sub-bases of a KB and maximal conflict-free sets of arguments is shown afterwards.

In the present article, we define a single attack relation upon propositional inconsistency. This simplifies the recognition of attacks, allowing us to concentrate on knowledge dynamics matters and on the study of models of change. On the other hand, [2] elaborates AFs considering three different sorts of attack relations: rebut, strong-rebut, and undercut. In this way, they obtain three argumentation systems, generalizing them from propositional logic (which is the original context of definition of the three attack relations) to general Tarskian logics. They finally analyze such AFs and the relation between their extensions and the sub-bases of the KB, upon some characterization properties.

[2] proposes an interesting base from which KBs' inconsistencies are studied and handled through argumentation techniques. The difference with our work is the argumentation semantics in which we base our theories: they work upon complete graphs of arguments and over extension-based semantics, and we work upon partial graphs, *i.e.*, dialectical trees, and skeptical semantics defined exclusively for such argument structures. Although the advantages of working with dialectical trees bring us the opportunity to start designing prototypes, we believe it is necessary to bridge the gap between extension-based and dialectical trees semantics in order to study their results upon our models.

The Dialectic-Global Model and other ATC approaches

As mentioned before, the ATC model presented in [23], reifies the abstract ATC approach presented in [30] to DeLP aiming at revising DeLP programs. Similar to the case of [16], we use propositional KBs instead of (defeasible) logic programs, which makes our approach more expressive and general. Moreover, in contrast to the dialectic-global model, the alteration of dialectical trees in [23] is achieved differently: incisions are applied in composition with a *selection function*, which determines the precise argument from each argumentation line to which the incision is applied. The usage of selection functions in such approach, allows to specify different criteria of minimal change: removing as few beliefs as possible from the KB, altering as few argumentation lines as possible from the tree, and preserving the tree structure as much as possible by removing arguments placed as low as possible in each line, getting closer to the leaves. On the other hand, the dialectic-global model follows an alternative but more general viewpoint to alterate dialectical trees: we rely on an incision function that is defined "globally" to the dialectical tree. Therefore, a *global incision* function determines a possible set of beliefs to be removed in order to effectively alter all the necessary lines at once.

In addition, the objective of the model presented in [23] differs from the dialectic-global's in that the latter does not pursue such an extensive variety of minimal change criteria, but only avoids to lose beliefs that are not related to the revision through the postulate of core-retainment. Pursuing different criteria of minimal change makes models much more intrincate both to define and axiomatize. Moreover, it is important to remark that the notion of minimality is usually subjective: most approaches in classic belief revision do not obtain real minimality, but approximations to it by specifying different criteria interpreting the meaning of minimal change.

Regarding other ATC approaches like [30] and more recently, [24], the focus is stressed on changing abstract AFs rather than KBs. Besides, the alteration of dialectical trees in such works, is done in the same manner explained before, used for the model presented in [23].

7. Conclusions and Future Work

This article presents a novel model of change to handle both (1) dynamics of knowledge in inconsistent propositional KBs, and (2) dynamics of argumentation frameworks (AFs). This so called, *dialectic-global model*, takes inspiration from the classical belief revision literature to axiomatically characterize the proposed revision operator. However, contrary to classical aproaches in belief revision, it does not rely on consistency restoration. Thus, change is provoked providing a consistent outcome, not in a standard (Tarskian semantics) sense, but relying on argumentation semantics. From the standpoint of (1), the revision operator receives a minimal set $\mathcal{R}$ of propositional beliefs inferring a claim α and includes it to the (inconsistent propositional) KB in a way that α is finally accepted by the adopted inconsistent-tolerant semantics. On the other hand, for (2), the revision includes the argument $\mathcal{R}$ to the AF ensuring it to end up warranted and thus accepting α.

The article is centered on Argument Theory Change (ATC), which studies certain aspects of belief revision in order to make them suitable for abstract argumentation systems. However, the dialectic-global model provides a new approach to ATC by rendering concrete the language for arguments to classical propositional logic.

To our knowledge, the dialectic-global model is the unique argumentation model of change which has been completely axiomatized (according to belief revision). Firstly, a set of basic postulates was adapted from belief revision to argumentation, and afterwards, the proposed model is shown to be rational through the corresponding representation theorem. Its full rationalization has been inspired by Hansson's work on contractions [20]. There, kernel sets –

minimal sets inferring the sentence to be contracted– are "broken" similarly to the effective alteration of *attacking lines* as is done in ATC.

Attacking lines are recognized as the argumentation lines (a repeated interchange of arguments and counterarguments) that are somehow responsible for the non-warrant status of a *dialectical tree*. These trees conform the structure followed by the adopted argumentation semantics to decide whether to accept (warrant) or not the argument placed on the root of a particular tree. Thus, altering the composition of attacking lines in a dialectical tree renders a new dialectical tree free of attacking lines, which consequently ensures its root argument to be warranted. Additionally, the *alteration* of argumentation lines is performed by removing an argument in it. However, since the AF is constructed from the underlying propositional KB, *removal of arguments* is finally achieved by deleting beliefs from the KB. To this end, minimal change is ensured by deleting beliefs that are related to the revision, *i.e.*, beliefs that are part of arguments in lines that render non-warranting the dialectical tree rooted in $\mathcal{R}$.

As an ATC approach, the model of change proposed in this article applies exclusively upon dialectical trees as the adopted argumentation semantics. This choice is made aiming at introducing practical approaches to base revision with tolerancy to inconsistency. Thus, our claim supports the existing relation between the use of dialectical trees and practical argumentation: arguments are built "on demand" from the underlying KB only for constructing a needed dialectical tree rooted in the argument by which the KB requires to be revised. The use of complete graphs of arguments as the structure upon other argumentation semantics rely, represent a different approach to the study of argumentative reasoning. For theoretical reasons, we believe it would be interesting to study the relations between the warrants (according to dialectical trees) and the extensions (obtained from graph-based semantics, such as the ground, stable, and others). Moreover, this will provide the necessary foundamentals in order to apply the techniques used in this article for managing dynamics of knowledge (mainly, the AF revision, which operates upon complete AFs) to some well-known extension-based argumentation semantics. These subjects are left as future work.

Preference relations between arguments are usually used to decide whether a counterargument finally prevails to determine an attack. For simplicity we abstracted away from such relations, and left attacks to be determined just by logic contradiction. However, in a revision process, preference relations are necessary to decide if an argument could be dropped in benefit of the warrant pursual of the new argument to include. For instance, when promulgating a law, it should be inadmissible to consider the removal of articles corresponding to laws of higher importance than that of the new one: the National Constitution should never be affected unless the inclusion of the new law is intended to reform it. These considerations are part of the ongoing work.

The dialectic-global model can be easily extended to first-order logic (FOL). In fact, only the argumentation machinery needs to be adapted to handle first-order KBs, whereas the model of change remains unaffected. Nonetheless, a first-order argumentation machinery is certainly much more complex than the simple one adopted here for propositional logic. For details on first-order argumentation the reader is referred to [6, 7, 25].

A FOL-argumentation machinery may provide an interesting non-standard reasoning methodology for inconsistent ontologies. Reasoning and change applied to inconsistent ontologies is of utmost importance in fields like medical sciences and legal procedure. A preliminary description logics-based argumentation system was presented in [26]. The study of the dialectic-global model applied to it is underway towards a novel approach to ontology evolution. Such model would be capable of revising ontologies without requiring consistency restoration, which means that the ontologies would not need to be repaired.

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Appendix

This appendix provides the proofs corresponding to properties stated in the main article. Additional definitions, propositions, lemmata, and theorems, identified as A.XX, B.XX, C.XX, or D.XX, are introduced in the corresponding appendix for complementing the proofs provided there, but not for a right understanding of the theory proposed so far.

A. Proofs Corresponding to Section 2 – Consistency of the Adopted Argumentation Semantics

In this section we concentrate on showing that the adopted argumentative semantics based on dialectical trees always determines a consistent set of warranted arguments. To such end, we will rely on the *composition* operation "$\circ$" defined between lines and arguments as $\circ : \mathbb{A}_\Sigma \times \mathbf{L}_\Sigma \rightarrow \mathbf{L}_\Sigma$ and overloaded as $\circ : \mathbf{L}_\Sigma \times \mathbb{A}_\Sigma \rightarrow \mathbf{L}_\Sigma$, such that for any $\lambda \in \mathbf{L}_\Sigma$, if $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$ then $\lambda = \mathcal{B}_1 \circ [\mathcal{B}_2, \ldots, \mathcal{B}_n] = [\mathcal{B}_1, \ldots, \mathcal{B}_{n-1}] \circ \mathcal{B}_n$. Observe that any subsequence of an acceptable line is also acceptable, hence, since $\lambda \in \mathbf{L}_\Sigma$, both $[\mathcal{B}_2, \ldots, \mathcal{B}_n]$ and $[\mathcal{B}_1, \ldots, \mathcal{B}_{n-1}]$, are acceptable lines and thus both are contained in $\mathbf{L}_\Sigma$.

Proposition A.1. *Given $\Sigma \subseteq \mathcal{L}$ and a warranting dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$. For every $\lambda \in \mathcal{T}(\mathcal{R})$ such that λ is alternating, if $\lambda = \mathcal{R} \circ \lambda'$ and $\mathcal{B}$ is λ''s root then λ' is a line of $\mathcal{T}(\mathcal{B}) \in \mathbb{T}_\Sigma$ and $\lambda' \in \mathcal{T}(\mathcal{B})$ is attacking line.*

Proof: Firstly, we will show that for any undefeated argument $\mathcal{C} \in \lambda$ such that $\mathcal{C} \neq \mathcal{R}$, $\mathcal{C} \in \lambda'$ cannot be defeated in the context of $\mathcal{T}(\mathcal{B})$. Note that, since λ is a warranting alternating line, $\mathcal{C} \in \lambda^+$ holds.

Assume there is some $\mathcal{D} \in \mathbb{A}_\Sigma$ defeating $\mathcal{C}$, such that for any $\lambda'' \in \mathcal{T}(\mathcal{R})$, if $\mathcal{D} \in \lambda''$ then $\lambda''^{\uparrow}[\mathcal{D}] \neq \lambda^{\uparrow}[\mathcal{C}]\circ\mathcal{D}$. This means that $\mathcal{D}$ is not a child of $\mathcal{C} \in \lambda$ in $\mathcal{T}(\mathcal{R})$ given that either such $\mathcal{D}$ does not exist or $\lambda^{\uparrow}[\mathcal{C}]\circ\mathcal{D}$ is not acceptable. Assuming the latter option (the former leaves $\mathcal{C}$ undefeated in $\mathcal{T}(\mathcal{B})$), we know $\lambda_{\mathcal{R}} = \lambda^{\uparrow}[\mathcal{C}]\circ\mathcal{D}$ violates either acceptability condition (1) or (2) (as seen in page 7). We will show that $\lambda_{\mathcal{B}} = \lambda'^{\uparrow}[\mathcal{C}]\circ\mathcal{D}$ is not acceptable. Note that $\mathcal{B} \in \lambda^+_{\mathcal{B}}$ and since $\mathcal{C} \in \lambda^-_{\mathcal{B}}$, we also know that $\mathcal{D} \in \lambda^+_{\mathcal{B}}$. Assuming (1) is violated for $\lambda_{\mathcal{R}}$, we have that $\mathcal{D}$ includes the subset of $\mathcal{R}$ defeated by $\mathcal{B}$, and therefore $\mathcal{B}$ counterargues $\mathcal{D}$. This violates acceptability condition (2) on $\lambda_{\mathcal{B}}$ given that $\{\mathcal{B}, \mathcal{D}\} \subseteq \lambda^+_{\mathcal{B}}$. On the other hand, assuming (2) is violated for $\lambda_{\mathcal{R}}$, we have that $\mathcal{D}$ counterargues some con argument in $\lambda^{\uparrow}[\mathcal{C}]$ (note that $\mathcal{D} \in \lambda^-_{\mathcal{R}}$), and since $\lambda^-_{\mathcal{R}} = \lambda^+_{\mathcal{B}}$, we also have that $\lambda_{\mathcal{B}}$ violates (2). Hence, $\lambda_{\mathcal{B}}$ is not acceptable.

Clearly, every undefeated argument $\mathcal{C} \in \lambda$ ends up also undefeated in $\mathcal{C} \in \lambda'$. And moreover, this includes λ''s leaf, which is also undefeated. Afterwards, it is easy to see that $\lambda' \in \mathcal{T}(\mathcal{B})$, given that λ' is exhaustive (since λ is known to be acceptable and exhaustive, it is easy to see that line λ' is also acceptable given that the acceptability conditions are harder on λ than on λ'). Finally, since λ is warranting alternating, λ' ends up as attacking line from $\mathcal{T}(\mathcal{B})$. □

Definition A.2 (Alternating Lines). *An argumentation line $\lambda \in \mathcal{T}(\mathcal{R})$, with $\mathcal{T}(\mathcal{R}) \in \mathbf{T}_\Sigma$, is called* ***alternating*** *if for every pair $\mathcal{B} \in \lambda$ and $\mathcal{C} \in \lambda$ it holds $\mathfrak{mark}(\mathcal{B}, \lambda, \mathcal{T}(\mathcal{R})) = \mathfrak{mark}(\mathcal{C}, \lambda, \mathcal{T}(\mathcal{R}))$ iff either $\{\mathcal{B}, \mathcal{C}\} \subseteq \lambda^-$ or $\{\mathcal{B}, \mathcal{C}\} \subseteq \lambda^+$. When $\mathcal{T}(\mathcal{R})$ is warranting, λ is referred as* ***warranting alternating line****.*

Observation A.3. *A line is attacking iff it is non-warranting alternating.*

Proposition A.4. *Given $\Sigma \subseteq \mathcal{L}$, and the non-warranting tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$; there is always a non-warranting alternating line $\lambda \in \mathcal{T}(\mathcal{R})$.*

Proof: Assume a non-warranting dialectical tree $\mathcal{T}(\mathcal{R})$ rooted in an argument $\mathcal{R} \in \mathbb{A}_\Sigma$, we have that $\mathcal{R}$ is marked as D. From the adopted marking criterion defined in page 9, we know that (*) any argument marked as D has at least one child which is undefeated. This is the case of $\mathcal{R}$ which we assume to have a child $\mathcal{B}$ marked as U. Let us refer as λ to the line rooted in $\mathcal{R}$ followed by $\mathcal{B}$. If $\mathcal{B}$ is λ's leaf then we have an alternating line, whereas if $\mathcal{B}$ is an inner node, the only option for its child $\mathcal{C}$ is to be marked as D, which leaves us in the same situation of the root argument $\mathcal{R}$. Hence, we know there is an undefeated argument which is $\mathcal{C}$'s child. Finally, by following the same construction from (*), we have that there always exists a non-warranting alternating line $\lambda \in \mathcal{T}(\mathcal{R})$. □

Lemma A.5. *Given $\Sigma \subseteq \mathcal{L}$, for any warranting dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$ and any argument $\mathcal{B} \in \mathbb{A}_\Sigma$, if $\mathcal{B}$ defeats $\mathcal{R}$ then $\mathcal{T}(\mathcal{B}) \in \mathbb{T}_\Sigma$ is non-warranting.*

Proof: Straightforward from Proposition A.4, Proposition A.1, and Theorem 3.2. □

Lemma A.6. *Given two arguments $\mathcal{B} \in \mathbb{A}_\Sigma$ and $\mathcal{C} \in \mathbb{A}_\Sigma$, if $\mathcal{C}$ is a counterargument of $\mathcal{B}$ then there exists a subargument $\mathcal{D} \in \mathbb{A}_\Sigma$ of $\mathcal{C}$ such that $\mathcal{D}$ is a canonical undercut of $\mathcal{B}$.*

Proof: Since $\mathcal{C}$ is a counterargument of $\mathcal{B}$, by definition we have $\{\mathfrak{cl}(\mathcal{C})\} \cup \mathfrak{bd}(\mathcal{B}) \models \bot$, and from Definition 2.1, since $\mathfrak{bd}(\mathcal{C}) \models \mathfrak{cl}(\mathcal{C})$, we have $\mathfrak{bd}(\mathcal{B}) \cup \mathfrak{bd}(\mathcal{C}) \models \bot$. It is easy to see that $\mathfrak{bd}(\mathcal{C}) \models \bigvee_{\beta \in \mathfrak{bd}(\mathcal{B})} \neg\beta$ holds. Thus, assuming $\mathfrak{bd}(\mathcal{B}) = \{\beta_1 \ldots, \beta_n\}$, we have $\mathfrak{bd}(\mathcal{C}) \models \neg(\beta_1 \wedge \ldots \wedge \beta_n)$. Let φ be $\neg(\beta_1 \wedge \ldots \wedge \beta_n)$. Hence, there exists a minimal subset $\Psi \subseteq \mathfrak{bd}(\mathcal{C})$ such that $\Psi \models \varphi$. In addition, given Definition 2.1, and $\Psi \subseteq \mathfrak{bd}(\mathcal{C})$, we know that $\Psi \not\models \bot$; and since Ψ is minimal for φ, there exists an argument $\mathcal{D} \in \mathbb{A}_\Sigma$ such that $\mathfrak{bd}(\mathcal{D}) = \Psi$ and $\mathfrak{cl}(\mathcal{D}) = \varphi$. Finally, it is easy to see that $\mathcal{D}$ –which is a subargument of $\mathcal{C}$– is a canonical undercut of $\mathcal{B}$. □

Theorem 2.4 *Given $\Sigma \subseteq \mathcal{L}$, for any pair of arguments $\mathcal{R} \in \mathbb{A}_\Sigma$ and $\mathcal{R}' \in \mathbb{A}_\Sigma$, if both $\mathcal{R}$ and $\mathcal{R}'$ are warranted then they are not conflicting.*

Proof: By *reductio ad absurdum*, suppose $\mathcal{R}$ and $\mathcal{R}'$ are both warranted and conflicting. Assume without loss of generality that $\mathcal{R}$ defeats $\mathcal{R}'$, and from Lemma A.6, let us suppose $\mathcal{R}''$ is a canonical undercut of $\mathcal{R}'$ such that $\mathcal{R}'' \subseteq \mathcal{R}$. It is clear that, if $\mathcal{R}$ is warranted, so is $\mathcal{R}''$. Afterwards, $[\mathcal{R}', \mathcal{R}'', \ldots] \in \mathcal{T}(\mathcal{R}')$ holds. Since $\mathcal{R}'$ is warranted, from Lemma A.5 we know $\mathcal{T}(\mathcal{R}'')$ is non-warranting. Afterwards, we know $\mathcal{R}$ is not warranted, which is absurd. □

B. Proofs Corresponding to Section 3 – ATC and the Dialectic-global Model

Theorem 3.2 *Given the dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbf{T}_\Sigma$, there is no attacking line $\lambda \in \mathcal{T}(\mathcal{R})$ iff $\mathcal{T}(\mathcal{R})$ is a warranting tree.*

Proof: Considering a dialectical tree, we have two options: either it is warranting or not. If the tree is warranting, its root argument is known to be undefeated. Since the root is a pro argument, from Definition 3.1 we know there is no attacking line. On the other hand, if the tree is non-warranting, it is clear that its root argument is marked as D. In such a case, we know that there exists at least one of its children which is a con argument marked as U. Besides, such con argument ends up marked as U given that either (1) it is the leaf of a line λ, or (2) it necessarily has no undefeated defeater. For 1), it is clear that λ is an attacking line. For 2), each one of its children is known to be a pro argument marked as D. Since this is the case of the root argument, the proof necessarily ends in 1). Hence, if the tree is non-warranting we know there is always an attacking line.

For the other way around, assuming we have no attaking lines, two options arise, either the root argument is marked as D, or U. For the latter case, it is clear that the tree is warranting. For the former case, where the root is marked as D, ends in an absurd given that we have shown that in this case it always appears an attacking line. On the other hand, assuming we have some attacking line, the only alternative for the root argument is to be marked as D, and therefore the tree is non-warranting. $\square$

Proposition 3.3 *If $\lambda \in \mathcal{T}(\mathcal{R})$ is attacking then for any $\mathcal{B} \in \lambda^-$, the upper segment $\lambda^{\uparrow}(\mathcal{B})$ turns into non-attacking.*

Proof: When we cut the line by removing a con argument, the upper segment that is left has odd length. Finally, from Proposition B.1, only argumentation lines of even length may be attacking. $\square$

Proposition B.1. *Given the dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$, if $\lambda \in \mathcal{T}(\mathcal{R})$ is attacking then λ has even length.*

Proof: By *reductio ad absurdum*, if we assume an attacking line λ to be of odd length, its leaf argument is known to be a pro argument. From the marking criterion adopted, we know that every leaf is marked as U since it has no defeater. From Definition 3.1, we know that an attacking line has its pro arguments marked as D. This means that λ is not attacking, contrary to the hypothesis. $\square$

The root argument is preserved from being incised since we look for its warrant.

Proposition B.2. *Given a KB $\Sigma \subseteq \mathcal{L}$, if "$\sigma$" is a global incision then $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{R}) = \emptyset$ holds for any $\mathcal{R} \in \mathbb{A}_\Sigma$.*

Proof: From Definition 3.5 we know that the upmost argument placed in any line $\lambda \in \mathcal{T}(\mathcal{R})$ that can be incised is a con argument. Since $\mathcal{R}$ is the root argument, it follows that $\mathcal{R}$ is a pro argument. Hence, $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{R}) = \emptyset$ holds. $\square$

Proposition B.3. *Given a tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$ and a line $\lambda \in \mathcal{T}(\mathcal{R})$, if $\lambda = [\ldots, \mathcal{B}_1, \mathcal{B}_2, \ldots]$ then $\mathfrak{bd}(\mathcal{B}_2) \setminus \mathfrak{bd}(\mathcal{B}_1) \neq \emptyset$.*

Proof: Since $\mathcal{B}_2$ counterargues $\mathcal{B}_1$, we know $\{\mathfrak{cl}(\mathcal{B}_2)\} \cup \mathfrak{bd}(\mathcal{B}_1) \models \bot$. By *reductio ad absurdum*, if we assume that $\mathfrak{bd}(\mathcal{B}_2) \setminus \mathfrak{bd}(\mathcal{B}_1) = \emptyset$ holds, two options arise: either $\mathfrak{bd}(\mathcal{B}_1) = \mathfrak{bd}(\mathcal{B}_2)$ or $\mathfrak{bd}(\mathcal{B}_2) = \emptyset$. For the former case, it is clear that $\mathcal{B}_2$'s claim is a tautology, *i.e.*, $\top \models \mathfrak{cl}(\mathcal{B}_2)$ (see Definition 2.1). This means that, $\mathfrak{bd}(\mathcal{B}_1) \models \bot$ given that $\{\mathfrak{cl}(\mathcal{B}_2)\} \cup \mathfrak{bd}(\mathcal{B}_1) \models \bot$ holds as seen before. Hence, we reach an absurd given that Definition 2.1 has been contradicted in its condition (2). Afterwards, we necessarily have that $\mathfrak{bd}(\mathcal{B}_1) = \mathfrak{bd}(\mathcal{B}_2)$. But then, we have that $\{\mathfrak{cl}(\mathcal{B}_2)\} \cup \mathfrak{bd}(\mathcal{B}_2) \models \bot$ which is again an absurd by contradiction of the same condition from Definition 2.1. Finally, $\mathfrak{bd}(\mathcal{B}_2) \setminus \mathfrak{bd}(\mathcal{B}_1) \neq \emptyset$ holds. $\square$

Proposition 3.6 *Given the dialectical tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$, there is always a global incision function $\sigma(\mathcal{T}(\mathcal{R})) \subseteq \mathcal{L}$.*

Proof: Two options arise, either $\mathcal{T}(\mathcal{R})$ is warranting or not. For the fomer case, from Theorem 3.2 we know there is no attacking lines. Afterwards, from Definition 3.5, condition 1, we know $\sigma(\mathcal{T}(\mathcal{R})) = \emptyset$.

On the other hand, assuming $\mathcal{T}(\mathcal{R})$ is non-warranting, from Theorem 3.2 we know there is at least an attacking line $\lambda \in \mathcal{T}(\mathcal{R})$. From Definition 3.5, condition 2a, we know there is $\mathcal{B} \in \lambda^-$ such that $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{B}) \neq \emptyset$ and for any other $\mathcal{B}' \in \lambda^\uparrow(\mathcal{B})$ it holds $\sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{B}') = \emptyset$. In particular, if we assume $\mathcal{B}$ to be $\mathcal{R}$'s defeater in $\mathcal{T}(\mathcal{R})$, it is clear that $\mathcal{B}' = \mathcal{R}$. From Proposition B.3, we know there is some sentence $\beta \in \mathfrak{bd}(\mathcal{B})$ such that $\beta \notin \mathfrak{bd}(\mathcal{R})$. Hence, it is always possible to effectively alter an attacking line by incising over the root's direct defeater. Afterwards, if another line in the tree is collaterally altered and turned into attacking, by following the same prionciple we can ensure that such a line will be effectively altered by incising over its root's direct defeater. Clearly, this process reaches an end, since the bundle set used to build the dialectical tree is finite.

Note finally that there is always a global incision function $\sigma(\mathcal{T}(\mathcal{R})) \subseteq \mathcal{L}$. □

Lemma 3.8 *Given $\Sigma \subseteq \mathcal{L}$, if $*^\omega$ is an argument revision then $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma *^\omega \mathcal{R})}$ has no attacking lines, for any $\mathcal{R} \in \mathbb{A}_\mathcal{L}$.*
Proof: From condition 1 in Definition 3.5, we know that the incision σ is non-empty if there are attacking lines in $\mathcal{T}'(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$. Besides, from condition 2 (Definition 3.5), the incision function takes a belief from a con argument from each attacking line (or any other that may turn to attacking) ensuring such belief to be the uppermost taken from such line. From Proposition 3.3, λ is turned to non-attacking. Finally, from Definition 3.7, $\mathcal{T}(\mathcal{R})$ has no attacking lines. □

C. Proofs Corresponding to Section 4 – Rationality of the Dialectic-global Model

Proposition C.1. *If $*^\omega$ satisfies success then $\mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^\omega \mathcal{R}$.*

Proof: By *reductio ad absurdum*, if $\mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^\omega \mathcal{R}$ does not hold, then $\mathcal{R}$ could not be formed from $\Sigma *^\omega \mathcal{R}$ and hence $\mathcal{R}$ could not be warranted from $\Sigma *^\omega \mathcal{R}$, contrary to success. □

We extend the notion of strict equivalence to sets of arguments.

Definition C.2 (Strictly Equivalent Sets of Arguments). *Given two KBs $\Sigma \subseteq \mathcal{L}$ and $\Sigma' \subseteq \mathcal{L}$, the sets of arguments $\mathbb{A}_\Sigma$ and $\mathbb{A}_{\Sigma'}$ are **strictly equivalent sets**, written $\mathbb{A}_\Sigma \dashv\Vdash \mathbb{A}_{\Sigma'}$ iff for any $\mathcal{B} \in \mathbb{A}_\Sigma$ (resp., $\mathcal{B} \in \mathbb{A}_{\Sigma'}$) there is $\mathcal{C} \in \mathbb{A}_{\Sigma'}$ (resp., $\mathcal{C} \in \mathbb{A}_\Sigma$) such that $\mathcal{B} \dashv\Vdash \mathcal{C}$.*

Observe that the symbol "$\dashv\Vdash$" has been overloaded to identify strictly equivalent arguments, and strictly equivalent sets of arguments, given that its usage will be rather explicit: $\mathcal{B} \dashv\Vdash \mathcal{B}'$ and $\mathbb{A}_\Sigma \dashv\Vdash \mathbb{A}_{\Sigma'}$, respectively.

Proposition C.3. *Given $\Sigma \subseteq \mathcal{L}$, if $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ then $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$, for any $\{\mathcal{R}_1, \mathcal{R}_2\} \subseteq \mathbb{E}_\Sigma$.*

Proof: We need to show that for any $\mathcal{B} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ there is $\mathcal{C} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ such that $\mathcal{B} \dashv\Vdash \mathcal{C}$. It is easy to see that $\mathbb{A}_\Sigma \subseteq (\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \cap \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))})$, hence the problem is reduced to proving that for every argument $\mathcal{B} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ such that $\mathcal{B} \notin \mathbb{A}_\Sigma$ there is $\mathcal{C} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ such that $\mathcal{C} \notin \mathbb{A}_\Sigma$ and $\mathcal{B} \dashv\Vdash \mathcal{C}$. Assuming $\mathcal{B} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{B} \notin \mathbb{A}_\Sigma$, $\mathcal{B}$ does necessarily turn out to contain part of $\mathcal{R}_1$. Therefore, if $\mathfrak{bd}(\mathcal{B}) \cap \mathfrak{bd}(\mathcal{R}_1) = X$, since $X \subseteq \mathfrak{bd}(\mathcal{R}_1)$ and $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$, there is $Y \subseteq \mathfrak{bd}(\mathcal{R}_2)$ such that X *iff* Y. Then there is an argument $\mathcal{C} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ with $\mathfrak{bd}(\mathcal{C}) = (\mathfrak{bd}(\mathcal{B}) \setminus X) \cup Y$ such that $\mathfrak{cl}(\mathcal{B}) = \mathfrak{cl}(\mathcal{C})$ and $\mathcal{B} \dashv\Vdash \mathcal{C}$. Finally, $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ holds. □

The previous proposition shows that given two strictly equivalent external arguments, $\mathcal{R}_1$ and $\mathcal{R}_2$, by introducing one or the other to the KB we generate strictly equivalent sets of arguments, that is $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$. Next we show that the set of defeaters for any pair of strictly equivalent arguments from $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$, are also strictly equivalent.

Proposition C.4. *Given two sets of arguments $\mathbb{A}_\Sigma$ and $\mathbb{A}_{\Sigma'}$, and two arguments $\mathcal{B} \in \mathbb{A}_\Sigma$ and $\mathcal{C} \in \mathbb{A}_{\Sigma'}$, if $\mathbb{A}_\Sigma \dashv\Vdash \mathbb{A}_{\Sigma'}$ and $\mathcal{B} \dashv\Vdash \mathcal{C}$ then for every defeater $\mathcal{D} \in \mathbb{A}_\Sigma$ of $\mathcal{B}$ there exists a defeater $\mathcal{D}' \in \mathbb{A}_{\Sigma'}$ of $\mathcal{C}$ such that $\mathcal{D} \dashv\Vdash \mathcal{D}'$.*

Proof: Since $\mathcal{D} \hookrightarrow \mathcal{B}$ and $\mathcal{B} \dashv\Vdash \mathcal{C}$ we know that $\mathcal{D} \hookrightarrow \mathcal{C}$. From Definition C.2 and $\mathbb{A}_\Sigma \dashv\Vdash \mathbb{A}_{\Sigma'}$ we know that there exists an argument $\mathcal{D}' \in \mathbb{A}_{\Sigma'}$ such that $\mathcal{D} \dashv\Vdash \mathcal{D}'$. Thus, $\mathcal{D}' \hookrightarrow \mathcal{C}$ holds, or equivalently, $\mathcal{D}'$ defeats $\mathcal{C}$. □

Definition C.5 (Class of Strictly Equivalent Arguments). *Given $\Sigma \subseteq \mathcal{L}$, a **class of strictly equivalent arguments**, for short **cse**, is any set of arguments $\Psi \subseteq \mathbb{A}_\Sigma$ such that (1) for any pair $\mathcal{B}_1 \in \Psi$ and $\mathcal{B}_2 \in \Psi$, $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$, and (2) for any $\mathcal{B}_1 \in \Psi$ there is no $\mathcal{B}_2 \notin \Psi$ such that $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$.*

Subsets Ψ of strictly equivalent arguments might be recognized from a set of arguments. This is important to show that if an argument inside Ψ is a node in a dialectical tree $\mathcal{T}(\mathcal{R})$, then every argument within Ψ is a node in $\mathcal{T}(\mathcal{R})$, and moreover, the use of either of them determines strictly equivalent lines.

Proposition C.6. *For any cse $\Psi \subseteq \mathbb{A}_\Sigma$, any argument $\mathcal{C} \in \Psi$, $\mathcal{B} \in \mathbb{A}_\Sigma$, and $\mathcal{R} \in \mathbb{A}_\Sigma$, and a tree $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_\Sigma$; if there is a line $\lambda \in \mathcal{T}(\mathcal{R})$ such that $\lambda = [\mathcal{R}, \ldots, \mathcal{B}, \mathcal{C}, \ldots]$ then for every $\mathcal{C}' \in \Psi$ there exists a line $\lambda' \in \mathcal{T}(\mathcal{R})$ such that $\lambda' = [\mathcal{R}, \ldots, \mathcal{B}, \mathcal{C}', \ldots]$ and both λ and λ' are strictly equivalent.*

Proof: Given $\lambda = [\mathcal{R}, \ldots, \mathcal{B}, \mathcal{C}, \ldots]$ where $\mathcal{C} \in \Psi$, assume by *reductio ad absurdum* that there is no $\lambda' = [\mathcal{R}, \ldots, \mathcal{B}, \mathcal{C}', \ldots]$ for some other $\mathcal{C}' \in \Psi$. Since $\lambda^\uparrow(\mathcal{C}) = \lambda'^\uparrow(\mathcal{C}')$ holds, the only option we have is that $\mathcal{C}'$ does not counterargue $\mathcal{B}$. From Definition C.5 we know that $\mathcal{C} \dashv\Vdash \mathcal{C}'$ and from Definition 4.1 it is clear that $\mathcal{C}'$ counterargues $\mathcal{B}$ given that $\mathcal{C}$ counterargues $\mathcal{B}$. If λ is acceptable so is λ'. Thus, from Definition 2.2, λ' does exist and it is a line of $\mathcal{T}(\mathcal{R})$. Finally, since $\mathcal{C} \dashv\Vdash \mathcal{C}'$, from Proposition C.4 we know that for any defeater $\mathcal{D}$ of $\mathcal{C}$ there is a strictly equivalent defeater $\mathcal{D}'$ of $\mathcal{C}'$ (*i.e.*, $\mathcal{D} \dashv\Vdash \mathcal{D}'$), and therefore it is easy to see that both λ and λ' are strictly equivalent lines. □

Lemma 4.3 *Given a KB $\Sigma \subseteq \mathcal{L}$, two external arguments $\mathcal{R}_1 \in \mathbb{E}_\Sigma$ and $\mathcal{R}_2 \in \mathbb{E}_\Sigma$, and their dialectical trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$; if $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ then $\mathcal{T}(\mathcal{R}_1)$ and $\mathcal{T}(\mathcal{R}_2)$ are strictly equivalent trees.*

Proof: Assuming $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ holds, we have $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ follows from Proposition C.3. Afterwards, from Proposition C.4 we know that for every defeater $\mathcal{B} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ of $\mathcal{R}_1$, there is a defeater $\mathcal{C} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ of $\mathcal{R}_2$ such that $\mathcal{C} \dashv\Vdash \mathcal{B}$. Two alternatives arise: if $\mathfrak{bd}(\mathcal{B}) \cap \mathfrak{bd}(\mathcal{R}_1) = \emptyset$ then we know that there is a cse $\Psi \subseteq \mathbb{A}_\Sigma$ such that $\{\mathcal{B}, \mathcal{C}\} \subseteq \Psi$, and from Proposition C.6, the line $[\mathcal{R}_1, \mathcal{B}, \ldots] \in \mathcal{T}(\mathcal{R}_1)$ is strictly equivalent to $[\mathcal{R}_1, \mathcal{C}, \ldots] \in \mathcal{T}(\mathcal{R}_1)$. Analogously, the line $[\mathcal{R}_2, \mathcal{B}, \ldots] \in \mathcal{T}(\mathcal{R}_2)$ is strictly equivalent to $[\mathcal{R}_2, \mathcal{C}, \ldots] \in \mathcal{T}(\mathcal{R}_2)$. For the other way around, if it is the case that $\mathfrak{bd}(\mathcal{B}) \cap \mathfrak{bd}(\mathcal{R}_1) \neq \emptyset$, we know that $\mathfrak{bd}(\mathcal{C}) \cap \mathfrak{bd}(\mathcal{R}_2) \neq \emptyset$, and since $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ and $\mathcal{C} \dashv\Vdash \mathcal{B}$, from Proposition C.4 we have that for every defeater $\mathcal{D} \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ of $\mathcal{B}$ there is a defeater $\mathcal{D}' \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ of $\mathcal{C}$ such that $\mathcal{D} \dashv\Vdash \mathcal{D}'$. Finally, for every line in $\mathcal{T}(\mathcal{R}_1)$ there is a strictly equivalent line in $\mathcal{T}(\mathcal{R}_2)$ and therefore both $\mathcal{T}(\mathcal{R}_1)$ and $\mathcal{T}(\mathcal{R}_2)$ are striclty equivalent trees. □

Proposition 4.5 *Given a KB $\Sigma \subseteq \mathcal{L}$, and two arguments $\mathcal{R}_1 \in \mathbb{E}_\Sigma$ and $\mathcal{R}_2 \in \mathbb{E}_\Sigma$ such that $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$, there always exists an argument incision function "σ" which is smooth.*

Proof: Given $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ holds, $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ follows from Proposition C.3, hence for any argument $\mathcal{B}_1 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ we know there is an argument $\mathcal{B}_2 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$, such that $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$. Moreover, from Lemma 4.3 we also know that $\mathcal{T}(\mathcal{R}_1) \dashv\Vdash \mathcal{T}(\mathcal{R}_2)$, where $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$. Thus, two options appear: either $\mathcal{B}_1 \in \mathcal{T}(\mathcal{R}_1)$ or not. For the former case, since $\mathcal{T}(\mathcal{R}_1) \dashv\Vdash \mathcal{T}(\mathcal{R}_2)$, we know that $\mathcal{B}_2 \in \mathcal{T}(\mathcal{R}_2)$, and moreover from Definition 4.2 assuming $\mathcal{B}_1 \in \lambda_1$ where $\lambda_1 \in \mathcal{T}(\mathcal{R}_1)$, we know that there is a line $\lambda_2 \in \mathcal{T}(\mathcal{R}_2)$ which is strictly equivalent to λ_1 and therefore $\mathcal{B}_2 \in \lambda_2$. Assuming "$\sigma$" is an argument incision function, from Proposition B.2, we know that both $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) \subseteq \mathfrak{bd}(\mathcal{B}_1) \setminus \mathfrak{bd}(\mathcal{R}_1)$ and $\sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2) \subseteq \mathfrak{bd}(\mathcal{B}_2) \setminus \mathfrak{bd}(\mathcal{R}_2)$ hold. And moreover, it is easy to show that $\mathfrak{bd}(\mathcal{B}_1) \setminus \mathfrak{bd}(\mathcal{R}_1) = \mathfrak{bd}(\mathcal{B}_2) \setminus \mathfrak{bd}(\mathcal{R}_2)$, where $\mathcal{B}_2$ is some argument belonging to a cse $\Psi \subseteq \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ –from Proposition C.6 we know that every argument in Ψ is included by some line $\lambda_2' \in \mathcal{T}(\mathcal{R}_2)$ which is strictly equivalent to λ_2. Hence, $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) = \sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2)$.

On the other hand, if it holds that $\mathcal{B}_1 \notin \mathcal{T}(\mathcal{R}_1)$, since $\mathcal{T}(\mathcal{R}_1) \dashv\Vdash \mathcal{T}(\mathcal{R}_2)$ we also know that $\mathcal{B}_2 \notin \mathcal{T}(\mathcal{R}_2)$. Finally from Definition 3.5, $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) = \emptyset$ and $\sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2) = \emptyset$ hold. □

Representation Theorem 4.6 *Given $\Sigma \subseteq \mathcal{L}$ and $\mathcal{R} \in \mathbb{A}_\mathcal{L}$, $\Sigma *^\omega \mathcal{R}$ is a smooth argument revision iff it guarantees success, inclusion, vacuity, core-retainment, and uniformity.*

Proof: **(construction ⇒ postulates)** The proof for **success** is trivial from Lemma 3.8 and Theorem 3.2. **Inclusion** is trivially implied from Definition 3.7 and Definition 3.5. For **vacuity**, by *reductio ad absurdum*, if we assume that $\Sigma \cup \mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^\omega \mathcal{R}$ does not hold, it means that the incision is non-empty, implying there is an attacking line in $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$. Hence, from Theorem 3.2

we know that $\mathcal{T}(\mathcal{R})$ is non-warranting, and therefore $\mathcal{R}$ is not warranted from $\Sigma \cup \mathfrak{bd}(\mathcal{R})$ contradicting the hypothesis.

For **core-retainment**, if there is some $\beta \in \Sigma \setminus (\Sigma *^{\omega} \mathcal{R})$ then from the equivalence for σ adopted as hypothesis in this theorem, we know $\beta \in \sigma(\mathcal{T}(\mathcal{R}))$. From Definition 3.5, condition 3, we know $\beta \in \mathfrak{bd}(\mathcal{B})$, for some $\mathcal{B} \in \lambda$, $\lambda \in \mathcal{T}(\mathcal{R})$. Besides, we know that such λ needs necessarily to be either attacking (cond. 2a) or turned to attacking from a collateral incision (cond. 2b). Moreover, from the consequent of cond. 2 in Definition 3.5, we know that $\mathcal{B} \in \lambda^{-}$. Hence, from Proposition 3.3, we know that λ is effectively altered by eliminating β. From inclusion we know $\Sigma *^{\omega} \mathcal{R} \subseteq \Sigma \cup \mathfrak{bd}(\mathcal{R})$, and from success we also know $\mathcal{R}$ is warranted from $\Sigma *^{\omega} \mathcal{R}$, and along with Proposition C.1, we have that $\mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^{\omega} \mathcal{R}$. Let $\Sigma' = (\Sigma *^{\omega} \mathcal{R}) \setminus \mathfrak{bd}(\mathcal{R})$, then $\Sigma' \subseteq \Sigma$. Finally, it is easy to see that $\mathcal{R}$ is warranted from $\Sigma' \cup \mathfrak{bd}(\mathcal{R})$ (given that it equals $\Sigma *^{\omega} \mathcal{R}$), but $\mathcal{R}$ is not warranted from $\Sigma' \cup \mathfrak{bd}(\mathcal{R}) \cup \{\beta\}$, given that including β, λ is not effectively altered. Hence, core-retainment is verified.

Finally, for **uniformity**, if $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$ holds, from Lemma 4.3 both trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ are known to be strictly equivalent, therefore for any line $\lambda \in \mathcal{T}(\mathcal{R}_1)$ there is a strictly equivalent line $\lambda' \in \mathcal{T}(\mathcal{R}_2)$. Moreover, for any argument $\mathcal{B}_1 \in \lambda$ we know that there exists an argument $\mathcal{B}_2 \in \lambda'$ such that $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$. From Definition 4.4, we know that $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) = \sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2)$, and since this is valid for every argument in any of both trees, we also know that $\sigma(\mathcal{T}(\mathcal{R}_1)) = \sigma(\mathcal{T}(\mathcal{R}_2))$. From Definition 3.5, the uppermost incision in any line λ appears over an argument in λ^{-} (which excludes the root argument), therefore we know that $\sigma(\mathcal{T}(\mathcal{R}_1)) \subseteq \Sigma$ and $\sigma(\mathcal{T}(\mathcal{R}_2)) \subseteq \Sigma$. Finally, from Definition 3.7, we have $\Sigma *^{\omega} \mathcal{R}_1 = (\Sigma \cup \mathfrak{bd}(\mathcal{R}_1)) \setminus \sigma(\mathcal{T}(\mathcal{R}_1))$, and $\Sigma *^{\omega} \mathcal{R}_2 = (\Sigma \cup \mathfrak{bd}(\mathcal{R}_2)) \setminus \sigma(\mathcal{T}(\mathcal{R}_2))$. And since we know that both incisions take the same beliefs from Σ, then we have $\Sigma \cap \Sigma *^{\omega} \mathcal{R}_1 = \Sigma \cap \Sigma *^{\omega} \mathcal{R}_2$ holds.

(postulates $\Rightarrow$ construction) Suppose that we have an operation $*^{\omega}$ satisfying the four postulates for argument revision. We need to show that there is a smooth incision function σ such that $\Sigma \cup \mathfrak{bd}(\mathcal{R}) \setminus \sigma(\mathcal{T}(\mathcal{R})) = \Sigma *^{\omega} \mathcal{R}$.

First, we define σ as: $\sigma(\mathcal{T}(\mathcal{R})) = \Sigma \setminus \Sigma *^{\omega} \mathcal{R}$, where $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$. Now, we need to show that σ is an incision function according to Definition 3.5 which is smooth. Hence, we need to show:

1. σ is a function from trees to sets of formulae,
2. The conditions in Definition 3.5 hold for σ, and
3. σ is a smooth incision function according to Definition 4.4.

For 1, we assume two dialectical trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$. If $\mathcal{T}(\mathcal{R}_1) = \mathcal{T}(\mathcal{R}_2)$ then we need to show that $\sigma(\mathcal{T}(\mathcal{R}_1)) = \sigma(\mathcal{T}(\mathcal{R}_2))$. Since both trees are equal that means that they have exactly the same argumentation lines. Since all the lines begin at the root, this implies that $\mathcal{R}_1 = \mathcal{R}_2$ and therefore $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$. From **uniformity** we have $\Sigma \cap \Sigma *^{\omega} \mathcal{R}_1 = \Sigma \cap \Sigma *^{\omega} \mathcal{R}_2$, and thus $\Sigma \setminus \Sigma *^{\omega} \mathcal{R}_1 = \Sigma \setminus \Sigma *^{\omega} \mathcal{R}_2$. Finally, from the definition of σ we have that $\sigma(\mathcal{T}(\mathcal{R}_1)) = \sigma(\mathcal{T}(\mathcal{R}_2))$.

For 2 we need to show conditions 1, 2, and 3, from Definition 3.5. For condition 1, if $\sigma(\mathcal{T}(\mathcal{R})) = \emptyset$ then we need to show that there is no attacking line in $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$. From the definition of σ we have $\Sigma \setminus \Sigma *^{\omega} \mathcal{R} = \emptyset$, then it follows that $\Sigma \subseteq \Sigma *^{\omega} \mathcal{R}$. From **success** and Proposition C.1, we know that $\mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^{\omega} \mathcal{R}$, hence $\Sigma \cup \mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^{\omega} \mathcal{R}$. Thus, from **inclusion** we have that $\Sigma *^{\omega} \mathcal{R} = \Sigma \cup \mathfrak{bd}(\mathcal{R})$. This means that $\mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))} = \mathbb{T}_{(\Sigma *^{\omega} \mathcal{R})}$, and therefore $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma *^{\omega} \mathcal{R})}$ holds. Finally, from **success** we know that $\mathcal{R}$ is warranted from $\Sigma *^{\omega} \mathcal{R}$, and from Theorem 3.2 we conclude that $\mathcal{T}(\mathcal{R})$ has no attacking lines. For the other way around, if there is no attacking line in $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}))}$ then we need to show that $\sigma(\mathcal{T}(\mathcal{R})) = \emptyset$. It follows that no line in $\mathcal{T}(\mathcal{R})$ has its second argument undefeated, that is, every root's defeater is defeated. Hence, $\mathcal{R}$ is warranted from $\Sigma \cup \mathfrak{bd}(\mathcal{R})$, and from **vacuity** we have that $\Sigma \cup \mathfrak{bd}(\mathcal{R}) \subseteq \Sigma *^{\omega} \mathcal{R}$ holds. Finally, we have $\Sigma \subseteq \Sigma *^{\omega} \mathcal{R}$, and from the definition of σ adopted in the hypothesis, $\sigma(\mathcal{T}(\mathcal{R})) = \emptyset$ holds.

For cond. 2 in Definition 3.5, for any $\lambda \in \mathcal{T}(\mathcal{R})$ we only have to take into account two cases: either λ is attacking line (condition 2a), or λ might turn to attacking given the existence of collateral incisions $\Psi = \sigma(\mathcal{T}(\mathcal{R})) \cap \mathfrak{bd}(\mathcal{C}) \neq \emptyset$ over an argument $\mathcal{C} \in \lambda$, such that $\lambda^{\uparrow}(\mathcal{C})$ ends up attacking from the hypothetical tree $\mathcal{H}(\mathcal{R}, \Psi)$ (condition 2b). In both cases, it means that λ might threat the warrant status of the root $\mathcal{R}$ from the tree $\mathcal{T}(\mathcal{R})$. We need to show that there is an argument $\mathcal{B} \in \lambda$ such that $\mathcal{B} \in \lambda^{-}$ and $\mathcal{B}$ is the uppermost incised argument in λ. By *reductio ad absurdum*, it is easy to show that if $\mathcal{B} \notin \lambda^{-}$ is the uppermost incised argument, from Proposition 3.3, $\lambda^{\uparrow}(\mathcal{B})$ turns to attacking (only under conditions 2a or 2b), and since there is no other incision over another argument above $\mathcal{B}$ in λ (uppermost incised argument), then the resulting tree will have at least one attacking line. Afterwards, from **success** we know that $\mathcal{R}$ is warranted and from Theorem 3.2, we know that this is only possible from a dialectical tree with no attacking lines. Thus, we reach an absurd, and hence condition 2 in Definition 3.5 is verified.

For cond. 3 in Definition 3.5, let $\beta \in \sigma(\mathcal{T}(\mathcal{R}))$, then from the equivalence for σ adopted as hypothesis in this theorem, we know $\beta \in \Sigma \setminus (\Sigma *^{\omega} \mathcal{R})$. From **core-retainment** there is some $\Sigma' \subseteq \Sigma$ such that $\mathcal{R}$ is warranted from $\Sigma' \cup \mathfrak{bd}(\mathcal{R})$ but $\mathcal{R}$ is not warranted from $\Sigma' \cup \mathfrak{bd}(\mathcal{R}) \cup \{\beta\}$. Hence the tree rooted in $\mathcal{R}$ from $\mathbb{T}_{(\Sigma' \cup \mathfrak{bd}(\mathcal{R}) \cup \{\beta\})}$ includes some attacking line λ, and by removing β we know to provoke an effective alteration of such λ, given that $\mathcal{R}$ is warranted from $\Sigma' \cup \mathfrak{bd}(\mathcal{R})$. Thus, from Proposition 3.3, we know there is some $\mathcal{B} \in \lambda^{-}$, such that $\beta \in \mathfrak{bd}(\mathcal{B})$.

For 3, assume a pair of external arguments $\mathcal{R}_1 \in \mathbb{E}_{\Sigma}$, and $\mathcal{R}_2 \in \mathbb{E}_{\Sigma}$, such that $\mathcal{R}_1 \dashv\Vdash \mathcal{R}_2$. From Proposition C.3, we know that $\mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))} \dashv\Vdash \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$. That means that for any argument $\mathcal{B}_1 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$, we know there exists $\mathcal{B}_2 \in \mathbb{A}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ such that $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$. From Lemma 4.3 both trees $\mathcal{T}(\mathcal{R}_1) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_1))}$ and $\mathcal{T}(\mathcal{R}_2) \in \mathbb{T}_{(\Sigma \cup \mathfrak{bd}(\mathcal{R}_2))}$ are known to be strictly equivalent, therefore the incision function will work over strictly equivalent trees. Assume any node $\mathcal{B}_1$ in $\mathcal{T}(\mathcal{R}_1)$, if $\mathfrak{bd}(\mathcal{B}_1) \cap \mathfrak{bd}(\mathcal{R}_1) = \emptyset$, from Proposition C.6 we know that $\mathcal{B}_1$ is also placed at the same position in the context of $\mathcal{T}(\mathcal{R}_2)$. On the other hand, if $\mathfrak{bd}(\mathcal{B}_1) \cap \mathfrak{bd}(\mathcal{R}_1) \neq \emptyset$ then we know there exists a node $\mathcal{B}_2$ in $\mathcal{T}(\mathcal{R}_2)$ such that $\mathcal{B}_1 \dashv\Vdash \mathcal{B}_2$ holds, but $\mathcal{B}_1 \neq \mathcal{B}_2$. But also, $\mathfrak{bd}(\mathcal{B}_1) \setminus \mathfrak{bd}(\mathcal{R}_1) = \mathfrak{bd}(\mathcal{B}_2) \setminus \mathfrak{bd}(\mathcal{R}_2)$. From success and Proposition C.1, we have that $\mathfrak{bd}(\mathcal{R}_1) \subseteq \Sigma *^{\omega} \mathcal{R}_1$, and from the definition of σ (in the hypothesis) $\Sigma *^{\omega} \mathcal{R}_1 = \Sigma \cup \mathfrak{bd}(\mathcal{R}_1) \setminus \sigma(\mathcal{T}(\mathcal{R}_1))$, we know that $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{R}_1) = \emptyset = \sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{R}_2)$. Therefore, we have $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) \subseteq \Sigma$ and $\sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2) \subseteq \Sigma$. Finally, $\sigma(\mathcal{T}(\mathcal{R}_1)) \cap \mathfrak{bd}(\mathcal{B}_1) = \sigma(\mathcal{T}(\mathcal{R}_2)) \cap \mathfrak{bd}(\mathcal{B}_2)$ holds, for any such $\mathcal{B}_1$. □

D. Proofs Corresponding to Section 5 – AF Revision Operation

Lemma 5.8 $\phi \circledast^{\omega} \mathcal{R} = \mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})}$

Proof: From Observation 5.7, we know that the same incision function applies for both $\phi \circledast^{\omega} \mathcal{R}$ and $\mathbb{K}(\phi) *^{\omega} \mathcal{R}$. Afterwards, by double inclusion, we will show that 1) $\phi \circledast^{\omega} \mathcal{R} \subseteq \mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})}$ and 2) $\mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})} \subseteq \phi \circledast^{\omega} \mathcal{R}$.

1) For any $\mathcal{B} \in \phi \circledast^{\omega} \mathcal{R}$, from Definition 5.5, we know $\mathcal{B} \in \mathbb{A}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))}$ and $\mathfrak{bd}(\mathcal{B}) \cap \sigma(\mathcal{T}(\mathcal{R})) = \emptyset$, where $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))}$. Thus, it is easy to see that, $\mathcal{B} \in \mathbb{A}_{((\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R})) \setminus \sigma(\mathcal{T}(\mathcal{R})))}$ holds. Finally, from Definition 3.7, we know $\mathcal{B} \in \mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})}$ holds.

2) For any $\mathcal{B} \in \mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})}$, from Definition 3.7, $\mathcal{B} \in \mathbb{A}_{((\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R})) \setminus \sigma(\mathcal{T}(\mathcal{R})))}$ holds, where $\mathcal{T}(\mathcal{R}) \in \mathbb{T}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))}$. This means that, $\mathcal{B} \in \mathbb{A}_{(\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))}$ and $\mathfrak{bd}(\mathcal{B}) \cap \sigma(\mathcal{T}(\mathcal{R})) = \emptyset$. Finally, from Definition 5.5, it is clear that $\mathcal{B} \in \phi \circledast^{\omega} \mathcal{R}$.

From 1) and 2), we conclude that $\phi \circledast^{\omega} \mathcal{R} = \mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})}$ holds. □

Lemma 5.9 $\mathbb{K}(\phi \circledast^{\omega} \mathcal{R}) = \mathbb{K}(\phi) *^{\omega} \mathcal{R}$

Proof: From Observation 5.7, we know that the same incision function applies for both $\phi \circledast^{\omega} \mathcal{R}$ and $\mathbb{K}(\phi) *^{\omega} \mathcal{R}$.

It is easy to see that, $\mathbb{K}(\phi \circledast^{\omega} \mathcal{R}) = \mathbb{K}(\mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})})$ (see Lemma 5.8). Let us assume $\Sigma' = \mathbb{K}(\phi) *^{\omega} \mathcal{R}$. Afterwards, from Observation 5.4, we have that $\mathbb{K}(\mathbb{A}_{\Sigma'}) = \Sigma'$ *iff* for every $\beta \in \Sigma'$, $\beta \not\models \bot$. Afterwards, from Definition 3.7, we have that $\beta \in (\mathbb{K}(\phi) \cup \mathfrak{bd}(\mathcal{R}))$ which means that $\beta \in \mathbb{K}(\phi)$ or $\beta \in \mathfrak{bd}(\mathcal{R})$. Clearly, β is part either of $\mathcal{R}$ or of some argument in ϕ, which determines, from Definition 2.1, that $\beta \not\models \bot$. Hence, $\mathbb{K}(\mathbb{A}_{\Sigma'}) = \Sigma'$ holds, and therefore, by substituting Σ', we have that $\mathbb{K}(\mathbb{A}_{(\mathbb{K}(\phi) *^{\omega} \mathcal{R})}) = \mathbb{K}(\phi) *^{\omega} \mathcal{R}$ holds. □

Theorem 5.10 *Given a closed AF $\phi \subseteq \mathbb{A}_{\mathcal{L}}$ and $\mathcal{R} \in \mathbb{A}_{\mathcal{L}}$, if $\mathbb{K}(\phi) *^{\omega} \mathcal{R}$ is a smooth argument revision then $\phi \circledast^{\omega} \mathcal{R}$ is a rational AF revision wrt. the postulates in Theorem 4.6, taking $\Sigma = \mathbb{K}(\phi)$ and $\Sigma *^{\omega} \mathcal{R} = \mathbb{K}(\phi \circledast^{\omega} \mathcal{R})$.*

Proof: Assuming $\mathbb{K}(\phi) *^{\omega} \mathcal{R}$ is a smooth argument revision, from Definition 4.4 we know that there is a smooth global incision function σ such that $\mathbb{K}(\phi) *^{\omega}_{\sigma} \mathcal{R} = \mathbb{K}(\phi) *^{\omega} \mathcal{R}$. Besides, from Lemma 5.9, we have that $\mathbb{K}(\phi \circledast^{\omega} \mathcal{R}) = \mathbb{K}(\phi) *^{\omega} \mathcal{R}$, which means that (according to Observation 5.7) $\phi \circledast^{\omega}_{\sigma} \mathcal{R} = \phi \circledast^{\omega} \mathcal{R}$. Therefore, $\phi \circledast^{\omega} \mathcal{R}$ is a rational AF revision in accordance to Theorem 4.6, and the postulates for the argument revision operator "$*^{\omega}$", taking $\Sigma = \mathbb{K}(\phi)$ and $\Sigma *^{\omega} \mathcal{R} = \mathbb{K}(\phi \circledast^{\omega} \mathcal{R})$ (according to Lemma 5.9). □

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