

The 1/2 BPS Wilson loop in ABJM theory at two loops

Marco S. Bianchi¹, Gaston Giribet², Matias Leoni², and Silvia Penati³

¹ *Institut für Physik, Humboldt-Universität zu Berlin, Newtonstraße 15, 12489 Berlin, Germany*

² *Physics Department, FCEyN-UBA & IFIBA-CONICET Ciudad Universitaria, Pabellón I, 1428, Buenos Aires, Argentina*

³ *Dipartimento di Fisica, Università degli studi di Milano-Bicocca and INFN,*

Sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy

marco.bianchi@physik.hu-berlin.de, gaston@df.uba.ar,

leoni@df.uba.ar, silvia.penati@mib.infn.it

We compute the expectation value of the 1/2 BPS circular Wilson loop in ABJM theory at two loops in perturbation theory. The result shows perfect agreement with the prediction from localization and the proposed framing factor.

I. INTRODUCTION

One of the most interesting instances of AdS/CFT correspondence is the duality between string theory on $\text{AdS}_4 \times \mathbb{CP}^3$ background and the $\mathcal{N} = 6$ superconformal extension of Chern-Simons theory (CS) constructed in [1, 2], hereafter referred to as ABJM theory. The precise statement of the duality is that type IIA string theory on $\text{AdS}_4 \times \mathbb{CP}^3$ is dual to the $\mathcal{N} = 6$ CS theory coupled to bifundamental matter, with gauge group $U(N) \times U(M)$, in the limit of large N , M and large CS level k , with $\lambda = N/k$ and $\hat{\lambda} = M/k$ fixed¹. In the last four years, this particular realization of the AdS/CFT correspondence has been extensively investigated, and different observables of the theory, such as scattering amplitudes [3]–[13] and Wilson loops (WL) [14]–[19] were studied. In this paper, we will be concerned with the computation of the expectation value of WL operators in ABJM theory.

Among the gauge theory observables WL are of particular importance. This is due to the fact that, on the one hand, WL encode important information about gauge theory, such as the interaction potential of colored particles and Schwinger pair production probability. On the other hand, in the context of holography, WL happen to be the gauge theory duals to fundamental string states in AdS spaces. In the special case of ABJM theory, it containing a CS contribution, WL are natural observables to look at. Moreover, the evaluation of WL allows to investigate the intriguing relation that seems to exist between perturbative results of the ABJM theory and the $\mathcal{N} = 4$ four-dimensional super Yang-Mills (SYM) (see for instance [14]).

Supersymmetric circular WL in ABJM theory were first discussed in Refs. [14–16], where operators preserving 1/6 of the supersymmetry were constructed as the holonomy of the connections $\mathcal{A} = A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| \mathcal{M}_J^I C_I \bar{C}^J$ and $\hat{\mathcal{A}} = \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| \hat{\mathcal{M}}_J^I \bar{C}^J C_I$, combining the gauge fields A and \hat{A} with the bifundamental scalars C and \bar{C} through proper matrices $\mathcal{M} = \hat{\mathcal{M}} = \text{diag}(-1, -1, 1, 1)$.

Such operators are direct generalizations of the configurations previously studied in $\mathcal{N} = 4$ SYM [20–22], and they also exist in CS theories with less supersymmetry. Of particular importance to our discussion are the 1/6 BPS operators studied in Ref. [14], where linear combinations of WL transforming oppositely under time-reversal were considered and perturbative computations for their expectation values were performed.

A year after 1/6 BPS WL operators were found, Kapustin et al. discussed in [23] the computation of WL expectation values in CS theories with matter by resorting to localization techniques [24]. In Ref. [25], the matrix model constructed in [23] was used to compute exact expressions for the expectation value of the 1/6 BPS WL operators in ABJM in the large N limit, providing in this way an interpolating function between the weak and the strong coupling regimes of the theory.

Simultaneously, in Ref. [17], a WL operator that preserves 1/2 of the supersymmetry in ABJM was found. As shown in [17], the different couplings of this 1/2 BPS operator to the matter fields of the theory can be accommodated in a single superconnection \mathcal{L} for the supergroup $U(N|M)$

$$\mathcal{L}(\tau) = \begin{pmatrix} \mathcal{A} & -i \sqrt{\frac{2\pi}{k}} |\dot{x}| \eta_I \bar{\psi}^I \\ -i \sqrt{\frac{2\pi}{k}} |\dot{x}| \psi_I \bar{\eta}^I & \hat{\mathcal{A}} \end{pmatrix}, \quad (1)$$

whose holonomy gives the WL

$$W_{1/2}[C] = \frac{1}{N+M} \text{Tr} P \exp \left(-i \int_C d\tau \mathcal{L}(\tau) \right) \quad (2)$$

\mathcal{A} and $\hat{\mathcal{A}}$ are the same combinations of gauge and scalar fields as for the 1/6 BPS case, although with a different matrix $\mathcal{M} = \text{diag}(-1, 1, 1, 1)$; η_I and $\bar{\eta}^I$ are commuting spinors controlling the coupling to the bifundamental fermions ψ and $\bar{\psi}$, whose supersymmetry preserving form for a circular WL is spelled out in [17].

It was observed in [17] that the 1/6 BPS and the 1/2 BPS circular WL operators happen to belong to the same cohomology class under the supercharge used for localization. This results in an equivalence between their corresponding expectation values. In particular, adapting the matrix model of [23] to the superconnection representation, this enables to derive a prediction for the expectation value of the 1/2 BPS operator. Here, we present a perturbative test of this prediction.

¹ Precisely, the theory is referred to as ABJM when $M = N$ and ABJ when $M \neq N$, but we will not make distinction here.

In Section II we review the matrix model calculation and the prediction for the weak coupling expectation value, particularly in the case of the 1/2 BPS WL operator. In Section III we present the explicit computation of the 1/2 BPS operator in planar perturbation theory, up to two loops. This includes the novel fermionic diagrams, which turn out to be the most technically involved contributions. Finally, in Section IV we compare the results we obtain in perturbation theory with those coming from localization. We find perfect matching, once the result from localization gets mapped to framing zero by suitably removing the framing phase.

II. LOCALIZATION

In Ref. [23], by means of localization techniques, it was shown that the path integral of the supersymmetric CS theory reduces to a non-Gaussian matrix model. This matrix model yields the following expression for the partition function

$$\mathcal{Z} = \int \prod_{a=1}^N d\lambda_a e^{i\pi k \lambda_a^2} \prod_{b=1}^M d\hat{\lambda}_b e^{-i\pi k \hat{\lambda}_b^2} \times \frac{\prod_{a < b}^N \sinh^2(\pi(\lambda_a - \lambda_b)) \prod_{a < b}^M \sinh^2(\pi(\hat{\lambda}_a - \hat{\lambda}_b))}{\prod_{a=1}^N \prod_{b=1}^M \cosh^2(\pi(\lambda_a - \hat{\lambda}_b))} \quad (3)$$

The evaluation of expectation values of $W_{1/6}[C]$ and $\hat{W}_{1/6}[C]$ amounts to inserting in (3) the following contributions

$$w_{1/6} = \frac{1}{N} \sum_{a=1}^N e^{2\pi \lambda_a} \quad \text{and} \quad \hat{w}_{1/6} = \frac{1}{M} \sum_{a=1}^M e^{2\pi \hat{\lambda}_a} \quad (4)$$

where $w_{1/6}$ and $\hat{w}_{1/6}$ correspond to the $U(N)$ and $U(M)$ pieces, respectively.

As said above, it was observed in [17] that the 1/6 BPS and the 1/2 BPS circular WL operators belong to the same cohomology class, and this yields an equivalence between the corresponding expectation values. In fact, computing the 1/2 BPS operator in terms of the matrix model amounts to plugging in (3) the operator

$$w_{1/2} = \frac{1}{N+M} \left(\sum_{a=1}^N e^{2\pi \lambda_a} + \sum_{a=1}^M e^{2\pi \hat{\lambda}_a} \right) \quad (5)$$

From equations (4) and (5), we observe the relation $(N+M)w_{1/2} = Nw_{1/6} + M\hat{w}_{1/6}$. Therefore, at the level of the expectation values, the relation between the 1/6 BPS and the 1/2 BPS operators can be expressed by [17]

$$\langle W_{1/2}[C] \rangle_{f=1} = \frac{N \langle W_{1/6}[C] \rangle_{f=1} + M \langle \hat{W}_{1/6}[C] \rangle_{f=1}}{N+M} \quad (6)$$

where $\langle W_{1/6}[C] \rangle_{f=1}$ and $\langle \hat{W}_{1/6}[C] \rangle_{f=1}$ represent the expectation values of the 1/6 BPS operators corresponding to the $U(N)$ and $U(M)$ pieces, respectively. Since

coming from localization, identity (6) has to be understood at the nontrivial framing one, and this is precisely what the subindices $f=1$ in (6) refer to. Framing [26–28] is the choice of a normal vector field along the contour C , defining a nearby path which allows for a point splitting regularization of the self-linking number of a knot in a topologically invariant manner. For pure Chern–Simons this procedure has been shown to provide sensible results [27], since the WL expectation values do not depend on the particular choice of the framing contour, but just on its linking number with the original path. In order for framing to be compatible with localization, it has to respect supersymmetry, and this in turn imposes that the computation be performed at framing one [23]. Then, using the matrix model one can compute the framing one expectation value of these operators at weak coupling in the planar limit and obtain [29]

$$\begin{aligned} \langle W_{1/2}[C] \rangle_{f=1} &= 1 + \frac{i\pi}{k}(N-M) \\ &\quad - \frac{\pi^2}{3k^2}(2N^2 + 2M^2 - 5NM) + \mathcal{O}(1/k^3) \end{aligned} \quad (7)$$

In order to compare with a perturbative computation performed at framing zero, one has to identify and remove the framing phase. This was carried out successfully for the 1/6 BPS WL's for which the prescriptions [23]

$$\begin{aligned} \langle W_{1/6}[C] \rangle_{f=0} &= e^{-\frac{i\pi}{k}N} \langle W_{1/6}[C] \rangle_{f=1} \\ \langle \hat{W}_{1/6}[C] \rangle_{f=0} &= e^{\frac{i\pi}{k}M} \langle \hat{W}_{1/6}[C] \rangle_{f=1} \end{aligned} \quad (8)$$

provide results which match the perturbative evaluation [14–16].

For the 1/2 BPS WL, the framing factor has been identified in (7) as [29]

$$\begin{aligned} \langle W_{1/2}[C] \rangle_{f=1} &= e^{\frac{i\pi}{k}(N-M)} \langle W_{1/2}[C] \rangle_{f=0} = e^{\frac{i\pi}{k}(N-M)} \\ &\quad \times \left[1 - \frac{\pi^2}{6k^2}(N^2 + M^2 - 4NM) + \mathcal{O}(1/k^3) \right] \end{aligned} \quad (9)$$

We will prove that the square bracket is indeed the perturbative result at framing zero.

III. PERTURBATIVE COMPUTATION

In this Section, we present the perturbative evaluation of the circular 1/2 BPS WL in ABJM, up to two loops in the planar limit. Since such a computation is quite involved, requiring the regularization and the calculation of intricate trigonometric integrals, here we simply quote the main results referring to [30] for details on the calculation and the techniques used.

We start from the expression (2) for the WL where C is a circle parametrized as $x_i^\mu = (0, \cos \tau_i, \sin \tau_i)$. In ordinary perturbation theory the evaluation is performed as usual by Taylor expanding the exponential of the superconnection (1) and taking the expectation value by Wick contracting the fields. Since we are interested

in the two-loop quantum corrections, it suffices to expand it up to the fourth order. In this process we get purely bosonic contributions from the diagonal part of the $U(N|M)$ super-matrix (1), purely fermionic contributions from the off-diagonal blocks and mixed contributions from the mixing of the two.

In order to deal with potentially divergent diagrams, we use dimensional regularization with dimensional reduction (DRED), which has been proven to preserve gauge invariance and supersymmetry of Chern–Simons theories up to two loops [31]. This amounts to perform all tensor manipulations in three dimensions before promoting loop integrals to $D = 3 - 2\epsilon$. In particular, care is required when contracting 3d metric tensors coming from Feynman rules with D-dimensional metric tensors coming from tensor integrals. Moreover, in order to avoid potential problems arising from contracting 3d epsilon tensors (coming from gauge propagators and cubic vertices) with tensorial loop integrals we use helpful identities for eliminating $\varepsilon_{\mu\nu\rho}$ tensors before analytically extending the integrals to D-dimensions.

The integrals generally converge in the complex half-plane defined by some critical value of the real part of the regularization parameter ϵ . Using techniques that will be presented in [30], we have been able to compute them analytically for any complex value of ϵ . They turn out to be expressible in terms of hypergeometric functions. In the regime where they converge we have successfully tested our results numerically. In a neighborhood of $\epsilon = 0$, we have evaluated them by analytically continuing the hypergeometric functions and expanding the result up to finite terms.

At one-loop, neglecting tadpoles and diagrams vanishing due to the antisymmetry of the ε tensor, the only non-trivial Feynman diagram comes from a fermion exchange. The corresponding integral can be easily evaluated and turns out to be subleading in ϵ . Thus, $\langle W_{1/2}[C] \rangle_{\epsilon=0}^{(1)} = 0$, in line with the prediction from localization when the framing factor (9) is removed.

At two loops, neglecting diagrams which vanish identically because of the antisymmetry of the ε tensor, we are left with purely bosonic diagrams coming from contracting the diagonal terms in (1) plus two extra diagrams where exchanges of fermions appear.

The diagrams arising from the contraction of the diagonal blocks are already present in the evaluation of 1/6 BPS WL. Although the definition of the \mathcal{M} matrix governing the coupling to the scalars is different in the two cases, only the trace of \mathcal{M}^2 enters at this order, which is exactly the same. Therefore, borrowing the results for the 1/6 BPS case [14–16], the sum of the relevant contributions from the upper diagonal block reads

$$\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} = \frac{N^2 M - \frac{1}{6} N^3}{M + N} \frac{\pi^2}{k^2} \quad (10)$$

Similarly, the contribution from the lower diagonal block is obtained upon exchanging $M \leftrightarrow N$.

Two additional contributions involving exchange of

fermions need be considered, one with a fermionic ladder and one with a gauge/fermion cubic interaction. Potentially, also a diagram with the exchange of a one-loop corrected fermion propagator should be considered, but it vanishes when taking the trace of the superconnection exponential.

The fermionic ladder is straightforwardly computed by calculating the relevant integrals over the WL parameters. There are two contributions depending on which pairs of fermions are contracted. Even though they are individually divergent, their sum is finite and up to order $\mathcal{O}(\epsilon)$ it is given by

$$\text{Diagram 4} + \text{Diagram 5} = \frac{3}{2} \pi^2 \frac{MN}{k^2} \quad (11)$$

The vertex graph is troublesome, since it involves both an integration over the WL parameters and a space-time integral from the internal interaction vertex. By carrying out the algebra of the diagram we end up with a linear combination of integrals of the form

$$\partial_1^\mu \partial_2^\nu \partial_3^\rho \int \frac{d^{3-2\epsilon} x}{[(x-x_1)^2]^{\frac{1}{2}-\epsilon} [(x-x_2)^2]^{\frac{1}{2}-\epsilon} [(x-x_3)^2]^{\frac{1}{2}-\epsilon}} \quad (12)$$

where $\partial_i^\mu \equiv \frac{\partial}{\partial x_{i\mu}}$. In some terms the indices carried by the derivatives are (partially) contracted among themselves, whereas in other terms they are contracted with external tensorial structures.

Whenever two of the indices are contracted the computation simplifies, since we can exploit the D-dimensional Green equation for the propagators and get rid of the space-time integral. After collecting all such pieces and performing the parametric integrals we find a divergent contribution

$$\text{Diagram 6} \Big|_c = \pi^{2+2\epsilon} \frac{MN}{2k^2} \left(-\frac{1}{2\epsilon} - 1 - \gamma_E - 4 \log 2 \right) \quad (13)$$

where $|_c$ refers to contributions that involve integral (12) with the indices contracted.

In the remaining terms the space-time integration is unavoidable. Still, we were able to carry it out and the final contribution reads

$$\text{Diagram 7} \Big|_u = \pi^{2+2\epsilon} \frac{MN}{2k^2} \left(\frac{1}{2\epsilon} - 3 + \gamma_E + 4 \log 2 \right) \quad (14)$$

where in this case $|_u$ stands for contributions that involve integral (12) with the indices uncontracted. Summing the contracted and uncontracted parts of the vertex diagram we can ascertain that the divergence cancels out, as well as all unpleasant constants. The final result from this diagram, up to $\mathcal{O}(\epsilon)$ terms, reads

$$\text{Diagram 8} = -2\pi^2 \frac{MN}{k^2} \quad (15)$$

Therefore, summing the contribution (10) plus its analogous from $M \leftrightarrow N$ with the contributions (11) and (15), the expectation value of the 1/2 BPS WL at two loops in the planar limit is

$$\langle W_{1/2}[C] \rangle_{f=0}^{(2)} = -\frac{\pi^2}{6k^2} [N^2 + M^2 - 4NM] \quad (16)$$

Notably, this equals the prediction (9) from localization once the phase factor $e^{\frac{i\pi}{k}(N-M)}$ has been removed.

IV. CONCLUSIONS

Up to two loops, we have determined the 1/2 BPS circular Wilson Loop in the ABJM theory. We have proved that the perturbative result perfectly matches the weak coupling prediction from localization, once the framing-one factor has been removed.

As already mentioned, in the $W_{1/6}[C]$ case the prescription for relating framing-one and framing-zero results by rescaling with a $e^{i\pi N/k}$ factor was supported by matching the localization result with known results from perturbation theory. In the $\langle W_{1/2}[C] \rangle$ case, while the localization result at framing one has been easily inferred using identity (6), the prescription (7) for removing the framing factor, even if natural, was lacking a direct confirmation from a perturbative result. Here, we have provided the perturbative confirmation.

This calculation is technically involved, and this is due to the fact that in contrast to the 1/6 BPS operators, which couple only to the gauge fields and the scalars of the theory, the 1/2 BPS operator also couples

to the fermions, which are in the bifundamental representation of $U(N) \times U(M)$. This introduces notable technical difficulties. In particular, individual contributions from fermionic diagrams are divergent at two loops and only in the end all singularities cancel out. Therefore one has to introduce a regularization for the integrals. This is a delicate point because finite terms are affected by the choice of regularization scheme. It turns out that a careful application of DRED yields the correct result. Dimensionally regulating integrals also sources technical issues, since it requires solving them analytically for any value of the parameter ϵ , which is a hard task. The details of such computations will be reported in a future publication [30].

The techniques we used may be applied to the perturbative analysis of other Wilson loop operators. In particular, it would be interesting to extend the present computation to other supersymmetric Wilson loops in ABJM, such as the generalizations proposed in [32, 33], for which an exact result from localization has not been derived yet.

V. ACKNOWLEDGEMENTS

We thank L. Griguolo, J. Maldacena and D. Seminara for very useful discussions. The work of MB has been supported by the Volkswagen-Foundation. The work of GG and ML has been supported by the research project CONICET PIP0396. The work of SP has been supported in part by INFN, MIUR-PRIN contract 2009-KHZKRX and MPNS-COST Action MP1210 "The String Theory Universe".

-
- [1] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, JHEP **0810** (2008) 091 [arXiv:0806.1218 [hep-th]].
 - [2] O. Aharony, O. Bergman and D. L. Jafferis, JHEP **0811** (2008) 043 [arXiv:0807.4924 [hep-th]].
 - [3] A. Agarwal, N. Beisert and T. McLoughlin, JHEP **0906** (2009) 045 [arXiv:0812.3367 [hep-th]].
 - [4] T. Bargheer, F. Loebbert, C. Meneghelli, Phys. Rev. **D82** (2010) 045016 [arXiv:1003.6120 [hep-th]].
 - [5] Y. -t. Huang, A. E. Lipstein, JHEP **1011** (2010) 076 [arXiv:1008.0041 [hep-th]].
 - [6] D. Gang, Y. -t. Huang, E. Koh, S. Lee and A. E. Lipstein, JHEP **1103** (2011) 116 [arXiv:1012.5032 [hep-th]].
 - [7] S. Lee, Phys. Rev. Lett. **105** (2010) 151603 [arXiv:1007.4772 [hep-th]].
 - [8] T. Bargheer, N. Beisert, F. Loebbert and T. McLoughlin, J. Phys. A **45** (2012) 475402 [arXiv:1204.4406 [hep-th]].
 - [9] M. S. Bianchi, M. Leoni, A. Mauri, S. Penati and A. Santambrogio, JHEP **1207** (2012) 029 [arXiv:1204.4407 [hep-th]]; JHEP **1201** (2012) 056 [arXiv:1107.3139 [hep-th]]; JHEP **1112** (2011) 073 [arXiv:1110.0738 [hep-th]].
 - [10] A. Brandhuber, G. Travaglini and C. Wen, JHEP **1207** (2012) 160 [arXiv:1205.6705 [hep-th]]; JHEP **1210** (2012) 145 [arXiv:1207.6908 [hep-th]].
 - [11] W. -M. Chen and Y. -t. Huang, JHEP **1111** (2011) 057 [arXiv:1107.2710 [hep-th]].
 - [12] M. S. Bianchi, M. Leoni and S. Penati, JHEP **1204** (2012) 045 [arXiv:1112.3649 [hep-th]].
 - [13] S. Caron-Huot and Y. -t. Huang, JHEP **1303** (2013) 075 [arXiv:1210.4226 [hep-th]].
 - [14] S. -J. Rey, T. Suyama and S. Yamaguchi, JHEP **0903** (2009) 127 [arXiv:0809.3786 [hep-th]].
 - [15] N. Drukker, J. Plefka and D. Young, JHEP **0811** (2008) 019 [arXiv:0809.2787 [hep-th]].
 - [16] B. Chen and J. -B. Wu, Nucl. Phys. B **825** (2010) 38 [arXiv:0809.2863 [hep-th]].
 - [17] N. Drukker and D. Trancanelli, JHEP **1002** (2010) 058 [arXiv:0912.3006 [hep-th]].
 - [18] J. M. Henn, J. Plefka, K. Wiegandt, JHEP **1008** (2010) 032 [arXiv:1004.0226 [hep-th]].
 - [19] M. S. Bianchi, M. Leoni, A. Mauri, S. Penati, C. Ratti, A. Santambrogio and , JHEP **1106** (2011) 118 [arXiv:1103.3675 [hep-th]].
 - [20] J. M. Maldacena, Phys. Rev. Lett. **80** (1998) 4859 [hep-th/9803002].
 - [21] S. -J. Rey and J. -T. Yee, Eur. Phys. J. C **22** (2001) 379 [hep-th/9803001].
 - [22] K. Zarembo, Nucl. Phys. B **643** (2002) 157 [hep-th/0205160].

- [23] A. Kapustin, B. Willett and I. Yaakov, JHEP **1003** (2010) 089 [arXiv:0909.4559 [hep-th]].
- [24] V. Pestun, Commun. Math. Phys. **313** (2012) 71 [arXiv:0712.2824 [hep-th]].
- [25] M. Marino and P. Putrov, JHEP **1006** (2010) 011 [arXiv:0912.3074 [hep-th]].
- [26] E. Witten, Commun. Math. Phys. **121** (1989) 351.
- [27] E. Guadagnini, M. Martellini and M. Mintchev, Nucl. Phys. B **330** (1990) 575.
- [28] M. Alvarez and J. M. F. Labastida, Nucl. Phys. B **395** (1993) 198 [hep-th/9110069].
- [29] N. Drukker, M. Marino and P. Putrov, Commun. Math. Phys. **306** (2011) 511 [arXiv:1007.3837 [hep-th]].
- [30] M. S. Bianchi, G. Giribet, M. Leoni and S. Penati, To appear.
- [31] W. Chen, G. W. Semenoff and Y. -S. Wu, Phys. Rev. D **46** (1992) 5521 [hep-th/9209005].
- [32] L. Griguolo, D. Marmiroli, G. Martelloni and D. Seminara, arXiv:1208.5766 [hep-th].
- [33] V. Cardinali, L. Griguolo, G. Martelloni and D. Seminara, arXiv:1209.4032 [hep-th].