Scaling Theory of the Anderson Transition in Random Graphs: Ergodicity and Universality

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We study the Anderson transition on a generic model of random graphs with a tunable branching parameter $1 < K < 2$, through large scale numerical simulations and finite-size scaling analysis. We find that a single transition separates a localized phase from an unusual delocalized phase that is ergodic at large scales but strongly nonergodic at smaller scales. In the critical regime, multifractal wave functions are located on a few branches of the graph. Different scaling laws apply on both sides of the transition: a scaling with the linear size of the system on the localized side, and an unusual volumic scaling on the delocalized side. The critical scalings and exponents are independent of the branching parameter, which strongly supports the universality of our results.

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Ergodicity properties of quantum states are crucial to assess transport properties and thermalization processes. They are at the heart of the eigenstate thermalization hypothesis that has attracted enormous attention lately [1]. A paramount example of nonergodicity is Anderson localization where the interplay between disorder and interference leads to exponentially localized states [2]. In 3D, a critical value of disorder separates a localized phase from an ergodic delocalized phase. At the critical point eigenfunctions are multifractal, another nontrivial example of nonergodicity [3,4]. Recently, those questions have been particularly highlighted in the problem of many-body localization [5–9]. Because Fock space has locally a treelike structure, the problem of Anderson localization on different types of graphs [10–15] has attracted renewed activity [16–26]. In particular, the existence of a delocalized phase with nonergodic (multifractal) eigenfunctions lying on an algebraically vanishing fraction of the system sites is debated [19–21,25,26].

The problem of nonergodicity also arises in another context corresponding to glassy physics [27]. For directed polymers on the Bethe lattice [28], a glass transition leads to a phase where a few branches are explored among the exponential number available. As there is a mapping to directed polymer models in the Anderson-localized phase [10,29–31], it has been recently proposed that this type of nonergodicity (where the volume occupied by the states scales logarithmically with system volume) could also be relevant in the delocalized phase [18]. Note however that it has been envisioned that this picture could be valid only up to a finite but very large length scale [32] so that the analogy with the usual physics of directed polymers may not be relevant in the delocalized phase.

In this Letter, we study the Anderson transition (AT) in a family of random graphs [33–35], where a tunable parameter p allows us to interpolate continuously between the 1D Anderson model and the random regular graph model of infinite dimensionality. Our main tool is the single parameter scaling theory of localization [36]. It has been used as a crucial tool to interpret the numerical simulations of Anderson localization in finite dimensions [3,37–39] and to achieve the first experimental measurement of the critical exponent of the AT in 3D [40]. In our case, the infinite dimensionality of the graphs leads to highly nontrivial finite-size scaling properties: unusually, we find different scaling laws on each side of the transition. Our detailed analysis of extensive numerical simulations leads to the following scenario. A single AT separates a localized phase from an ergodic delocalized phase. However, a characteristic nonergodicity volume (NEV) Λ emerges in the latter phase. For scales below Λ, states are nonergodic in the sense that they take significant values only on a few branches, on which they additionally display multifractal fluctuations. For scales above Λ, this structure repeats itself and leads to large scale ergodicity. At the threshold, Λ diverges, and the behavior below Λ extends to the whole system. The critical behaviors do not depend on the graph parameter p, which strongly supports the universality of this scenario.

We use two complementary approaches to describe localization properties. First, we derive recursive equations for the local Green function using a mapping to a tree [10], which we solve using the pool method from glassy physics [16,29], and analyze the critical behavior by finite-size scaling. Second, we perform exact diagonalization of very large system sizes, and extract the scaling properties of

