Magnetic catalysis and inverse magnetic catalysis in nonlocal chiral quark models

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Abstract

We study the behavior of strongly interacting matter under an external constant magnetic field in the context of nonlocal chiral quark models within the mean field approximation. We find that at zero temperature the behavior of the quark condensates shows the expected magnetic catalysis effect, our predictions being in good quantitative agreement with lattice QCD results. On the other hand, in contrast to what happens in the standard local Nambu−Jona-Lasinio model, when the analysis is extended to the case of finite temperature our results show that nonlocal models naturally lead to the Inverse Magnetic Catalysis effect.

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Over the last years the understanding of the behavior of strongly interacting matter under extremely intense magnetic fields has attracted increasing attention, due to its relevance for various subjects such as the physics of compact objects like magnetars [1], the analysis of heavy ion collisions at very high energies [2] or the study of the first phases of the Universe [3]. Consequently, considerable work has been devoted to study the structure of the QCD phase diagram in the presence of an external magnetic field (see Refs. [4–6] for recent reviews). On the basis of the results arising from most low-energy effective models of QCD it was generally expected that, at zero chemical potential, the magnetic field would lead to an enhancement of the chiral condensate ("magnetic catalysis"), independently of the temperature of the system. However, lattice QCD (LQCD) calculations carried out with physical pion masses [7, 8] show that, whereas at low temperatures one finds indeed such an enhancement, the situation is quite different close to the critical chiral restoration temperature: in that region light quark condensates exhibit a nonmonotonic behavior as functions of the external magnetic field, which results in a decrease of the transition temperature when the magnetic field is increased. This effect is known as inverse magnetic catalysis (IMC). Although many scenarios have been considered in the last few years to account for the IMC [9–28], the mechanism behind this effect is not yet fully understood. With this motivation, in this work we study the behavior of strongly interacting matter under an external magnetic field in the framework of nonlocal chiral quark models. These theories are proposed as a sort of nonlocal extensions of the well-known Nambu−Jona-Lasinio (NJL) model, intending to go a step further towards a more realistic effective approach to QCD. In fact, nonlocality arises naturally in the context of successful descriptions of low-energy quark dynamics [29, 30], and it has been shown [31] that nonlocal models can lead to a momentum dependence in quark propagators that is consistent with LQCD results. Another advantage of these models is that the effective interaction is finite to all orders in the loop expansion, and therefore there is no need to introduce extra cutoffs [32]. Moreover, in this framework it is possible to obtain an adequate description of the properties of light mesons at both zero and finite temperature/density [31, 33–42]. A previous attempt of considering the effect of an external magnetic field within these models was done in Ref. [43]. In that work the magnetic field was introduced by using a simplified extension of the method usually followed in the local NJL model, and no signal of IMC was found. In the present article we concentrate on the analysis of nonlocal quark models with separable interactions, including

a coupling to a uniform magnetic field. We address the problem by following a more rigorous procedure based on the Ritus eigenfuncion method [44], which allows us to properly obtain the corresponding mean field action and to derive the associated gap equation. Then we solve this equation numerically for different values of the external magnetic field, considering the case of systems at both zero and finite temperature. We find that at zero temperature the behavior of the quark condensates shows the expected magnetic catalysis effect, our predictions being in good quantitative agreement with LQCD results. On the other hand, in contrast to what happens in the standard local Nambu−Jona-Lasinio model, when the analysis is extended to the case of finite temperature our results show that nonlocal models naturally lead to the IMC effect already at the mean field level.

Theoretical formalism

We begin by stating the Euclidean action for a simple nonlocal chiral quark model that includes two light flavors,

$$S_E = \int d^4x \, \left\{ \bar{\psi}(x) \left(-i\slashed{\partial} + m_c \right) \psi(x) - \frac{G}{2} j_a(x) j_a(x) \right\} \; . \tag{1}$$

Here m_c is the current quark mass, which is assumed to be equal for u and d quarks. The nonlocal currents $j_a(x)$ are given by

$$j_a(x) \;=\; \int d^4z \, \mathcal{G}(z) \, \bar{\psi}(x + \frac{z}{2}) \, \Gamma_a \, \psi(x - \frac{z}{2}) \; , \tag{2}$$

where $\Gamma_a = (\mathbb{1}, i\gamma_5 \vec{\tau})$, and the function $\mathcal{G}(z)$ is a nonlocal form factor that characterizes the effective interaction. Since we are interested in studying the influence of a magnetic field, we introduce in the effective action Eq. (1) a coupling to an external electromagnetic gauge field $\mathcal{A}_\mu$. For a local theory this can be done by performing the replacement $\partial_\mu \to \partial_\mu - i\,\hat{Q}\mathcal{A}_\mu(x)$, where $\hat{Q} = \mathrm{diag}(q_u, q_d)$, with $q_u = 2e/3$, $q_d = -e/3$, is the electromagnetic quark charge operator. In the case of the nonlocal model under consideration the situation is more complicated since the inclusion of gauge interactions implies a change not only in the kinetic terms of the Lagrangian but also in the nonlocal currents in Eq. (2). One has

$$\psi(x - z/2) \to W\left(x, x - z/2\right) \, \psi(x - z/2) \; , \tag{3}$$

and a related change holds for $\bar{\psi}(x + z/2)$ [31, 39, 42]. Here the function $W(s, t)$ is defined by

$$W(s, t) \;=\; \mathrm{P} \, \exp\left[-i\hat{Q} \int_s^t dr_\mu \, \mathcal{A}_\mu(r) \right] \; , \tag{4}$$

where r runs over an arbitrary path connecting s with t. As it is usually done, we take it to be a straight line path.

To proceed we bosonize the fermionic theory, introducing scalar and pseudoscalar fields $\sigma(x)$ and $\vec{\pi}(x)$ and integrating out the fermion fields. The bosonized action can be written as [31, 42]

$$S_{\text{bos}} = -\ln \det \mathcal{D} + \frac{1}{2G} \int d^4x \left[\sigma(x)\sigma(x) + \vec{\pi}(x) \cdot \vec{\pi}(x) \right] , \tag{5}$$

where

$$\mathcal{D}\left(x + \frac{z}{2}, x - \frac{z}{2}\right) = \gamma_0 \, W\left(x + \frac{z}{2}, x\right) \gamma_0 \left[\delta^{(4)}(z) \left(-i\slashed{\partial} + m_c \right) + \right.$$
$$\left. \mathcal{G}(z) \left[\sigma(x) + i\vec{\tau} \cdot \vec{\pi}(x) \right] \right] W\left(x, x - \frac{z}{2}\right) . \tag{6}$$

Let us consider the particular case of a constant and homogenous magnetic field orientated along the 3-axis. Choosing the Landau gauge, the corresponding gauge field is given by $\mathcal{A}_\mu = B\,x_1\,\delta_{\mu 2}$. Next, we assume that the field σ has a nontrivial translational invariant mean field value $\bar{\sigma}$, while the mean field values of pseudoscalar fields π_i are zero. It should be stressed at this point that the assumption that $\bar{\sigma}$ is independent of x does not imply that the resulting quark propagator will be translational invariant. In fact, as discussed below, one can show that such an invariance is broken by the appearance of the usual Schwinger phase. Our assumption just states that the deviations from translational invariance that are inherent to the magnetic field are not affected by the dynamics of the theory. In this way, within the mean field approximation (MFA) we get

$$\mathcal{D}^{\text{MFA}}(x, x') = \delta^{(4)}(x - x') \left(-i\slashed{\partial} - \hat{Q}\,B\,x_1\,\gamma_2 + m_c \right) +$$
$$\bar{\sigma}\,\mathcal{G}(x - x')\,\exp\left[\frac{i}{2}\,\hat{Q}\,B\,(x_2 - x_2')\,(x_1 + x_1') \right] . \tag{7}$$

At this stage it is convenient to follow the Ritus eigenfuncion method [44]. Thus, we introduce the function

$$\mathcal{D}^{\text{MFA}}_{p,p'} = \int d^4x \, d^4x' \, \bar{\mathbb{E}}_p(x) \, \mathcal{D}^{\text{MFA}}(x, x') \, \mathbb{E}_{p'}(x') , \tag{8}$$

where $\mathbb{E}_p$ are the usual Ritus matrices, with $p = (k, p_2, p_3, p_4)$. The r.h.s. of Eq. (8) can be worked out, and after some calculation one arrives to a relatively compact expression for

$\mathcal{D}^{\mathrm{MFA}}_{p,p'}$, which is shown to be diagonal in flavor space. For each flavor $f = u, d$ one has

$$
\begin{aligned}
\mathcal{D}^{\mathrm{MFA},f}_{p,p'} = {} & (2\pi)^4 \, \delta_{kk'} \, \delta(p_2 - p_2') \, \delta(p_3 - p_3') \, \delta(p_4 - p_4') \times \\
& \left[\left[\mathbb{I} + \delta_{k0}(\Delta^{s_f} - \mathbb{I}) \right] \left(- s_f \sqrt{2k\,|q_f B|} \, \gamma_2 + p_3\, \gamma_3 + p_4\, \gamma_4 \right) + \sum_{\lambda=-1,1} \Delta^\lambda M^{\lambda,f}_{\bar{p},k} \right], \quad (9)
\end{aligned}
$$

where we have defined $s_f = \mathrm{sign}(q_f B)$ and $\Delta^\lambda = \mathrm{diag}(\delta_{1\lambda}, \delta_{-1\lambda}, \delta_{1\lambda}, \delta_{-1\lambda})$, whereas $M^{\lambda,f}_{\bar{p},k}$ is given by

$$
M^{\lambda,f}_{\bar{p},k} = (-1)^{k - \frac{1-\lambda\,s_f}{2}} \int_0^\infty dr\, r\, \exp(-r^2/2) \left[m_c + \bar{\sigma}\, g\left(\frac{|q_f B|}{2} r^2 + \bar{p}^2 \right) \right] L_{k - \frac{1-\lambda\,s_f}{2}}(r^2) \, . \quad (10)
$$

Here $g(p^2)$ stands for the Fourier transform of $\mathcal{G}(z)$, $\bar{p} = (p_3, p_4)$ is a two-dimensional vector and $L_n(x)$ are the Laguerre polynomials. We use the standard convention $L_{-1}(x) = 0$, hence $M^{-s_f,f}_{\bar{p},0} = 0$. From Eq. (9) we finally find that the MFA action per unit volume can be expressed as

$$
\begin{aligned}
\frac{S^{\mathrm{MFA}}_{\mathrm{bos}}}{V^{(4)}} = {} & \frac{\bar{\sigma}^2}{2G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \int \frac{d^2\bar{p}}{(2\pi)^2} \left\{ \ln\left[\bar{p}^2 + \left(M^{s_f,f}_{\bar{p},0} \right)^2 \right] + \right. \\
& \left. \sum_{k=1}^{\infty} \ln\left[\left(2k|q_f B| + \bar{p}^2 + M^{-1,f}_{\bar{p},k} M^{+1,f}_{\bar{p},k} \right)^2 + \bar{p}^2 \left(M^{+1,f}_{\bar{p},k} - M^{-1,f}_{\bar{p},k} \right)^2 \right] \right\} \, . \quad (11)
\end{aligned}
$$

The corresponding gap equation can be now easily found by minimizing this expression with respect to $\bar{\sigma}$. It is worth mentioning that this gap equation can be also obtained using the Schwinger-Dyson formalism for the quark propagator discussed in e.g. Refs. [45–47]. Actually, it turns out that the two point function in Eq. (9) can be casted into a form similar to that given in Ref. [46]. Thus, using the analysis discussed in that work, one can show that the associated quark propagator in coordinate space can be written as the product of the exponential of a Schwinger phase (which breaks translational invariance) times a translational invariant function.

The above results can be now extended to finite temperature using the Matsubara formalism. This amounts to perform the replacement

$$
\int \frac{d^2\bar{p}}{(2\pi)^2} \, F(\bar{p}^2) \; \to \; T \sum_{n=-\infty}^{\infty} \int \frac{dp_3}{2\pi} \, F(\bar{p}_n^2) \, , \quad (12)
$$

where $\bar{p}_n = (p_3, \omega_n)$, $\omega_n = (2n+1)\pi T$ being the Matsubara frequencies for fermionic modes. In this way one can easily obtain the MFA finite temperature thermodynamical potential Ω^{MFA}, as well as the related gap equation. Given Ω^{MFA}, the magnetic field dependent quark

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condensate for each flavor can be calculated by taking the derivative with respect to the corresponding current quark mass. This leads to

$$
\langle \bar{q}_f q_f \rangle_{B,T} = -\frac{N_c |q_f B| T}{\pi} \int \frac{dp_3}{2\pi} \sum_{k=0}^{\infty} \sum_{n=-\infty}^{\infty}
$$

$$
\frac{M_{\bar{p}_n,k}^{-1,f} \left[\bar{p}_n^2 + 2k|q_f B| + M_{\bar{p}_n,k}^{+1,f\,2} \right] \; + \; (+ \leftrightarrow -)}{\left(2k|q_f B| + \bar{p}_n^2 + M_{\bar{p}_n,k}^{-1,f} M_{\bar{p}_n,k}^{+1,f} \right)^2 + \bar{p}_n^2 \left(M_{\bar{p}_n,k}^{+1,f} - M_{\bar{p}_n,k}^{-1,f} \right)^2} \; . \qquad (13)
$$

As it is usually found in the context of nonlocal models [38], this expression turns out to be divergent beyond the chiral limit. We obtain a regularized condensate by subtracting the corresponding expression in the absence of quark-quark interactions and adding it in a regularized form. Thus we have

$$
\langle \bar{q}_f q_f \rangle_{B,T}^{\mathrm{reg}} = \langle \bar{q}_f q_f \rangle_{B,T} - \langle \bar{q}_f q_f \rangle_{B,T}^{\mathrm{free}} + \langle \bar{q}_f q_f \rangle_{B,T}^{\mathrm{free,reg}} \; . \qquad (14)
$$

Notice that "free" condensates are defined keeping the interaction with the magnetic field. In the case of $\langle \bar{q}_f q_f \rangle_{B,T}^{\mathrm{free,reg}}$ the Matsubara sum can be performed analitically, leading to

$$
\langle \bar{q}_f q_f \rangle_{B,T}^{\mathrm{free,reg}} = -\frac{N_c m_c^3}{4\pi^2} \left[\frac{\ln \Gamma(x_f)}{x_f} - \frac{\ln 2\pi}{2x_f} + 1 - \left(1 - \frac{1}{2x_f} \right) \ln x_f \right] +
$$

$$
\frac{N_c |q_f B|}{\pi} \sum_{k=0}^{\infty} \alpha_k \int \frac{dp}{2\pi} \frac{m_c}{\epsilon_k^f \left[1 + \exp(\epsilon_k^f / T) \right]} \; , \qquad (15)
$$

where $x_f = m_c^2 / (2|q_f B|)$, $\alpha_k = 2 - \delta_{k0}$ and $\epsilon_k^f = \sqrt{2k|q_f B| + p^2 + m_c^2}$. Finally, to make contact with the LQCD results quoted in Ref. [8] we define the quantity

$$
\Sigma_{B,T}^f = \frac{2\, m_c}{S^4} \left[\langle \bar{q}_f q_f \rangle_{B,T}^{\mathrm{reg}} - \langle \bar{q}_f q_f \rangle_{0,0}^{\mathrm{reg}} \right] + 1 \; , \qquad (16)
$$

where the scale S is given by $S = (135 \times 86)^{1/2}$ MeV. We also introduce the definitions $\Delta \Sigma_{B,T}^f = \Sigma_{B,T}^f - \Sigma_{0,T}^f$ and $\Delta \bar{\Sigma}_{B,T} = (\Delta \Sigma_{B,T}^u + \Delta \Sigma_{B,T}^d)/2$.

Numerical results

To obtain the numerical predictions that follow from the above formalism, it is necessary to specify the particular form of the nonlocal form factor. Here, for simplicity, we consider the often-used Gaussian form $g(p^2) = \exp(-p^2/\Lambda^2)$, where the effective scale Λ is an additional parameter of the model. This form factor has the particular advantage that the integral in

Eq. (10) can be performed analytically. **One gets**

$$M_{\bar{p},k}^{\lambda,f} = m_c + \bar{\sigma} \, \frac{(1 - |q_f B|/\Lambda^2)^{k - \frac{1 - \lambda \, s_f}{2}}}{(1 + |q_f B|/\Lambda^2)^{k - \frac{1 + \lambda \, s_f}{2}}} e^{-\bar{p}^2/\Lambda^2} \tag{17}$$

Numerical results at $T = 0$ are shown in Fig. 1. In the upper panel we quote the model predictions for $\Delta\bar{\Sigma}_{B,0}$ as function of eB for various model parametrizations, while in the lower panel we show the corresponding results for $\Sigma_{B,0}^u - \Sigma_{B,0}^d$. LQCD data from Ref. [8] are also displayed in both cases for comparison. Note that the nonlocal model has three parameters, namely, m_c, G and Λ. They have been fixed so as to reproduce the empirical values of the pion mass and decay constant, and to lead to a certain chosen value of the quark condensate at zero temperature and magnetic field that we identify by $\Phi_0 \equiv (-\langle \bar{q}_f q_f \rangle_{0,0}^{\text{reg}})^{1/3}$. Details of this parameter fixing procedure can be found in Ref. [39], where the explicit values of the parameters for $\Phi_0 = 220$ MeV and 240 MeV are given. As seen in Fig. 1, the predictions for $\Delta\bar{\Sigma}_{B,0}$ are very similar for all parameterizations considered, and show a very good agreement with LQCD results. In the case of $\Sigma_{B,0}^u - \Sigma_{B,0}^d$ we see that, although the overall agreement with LQCD calculations is still good, there is a somewhat larger dependence on the model parametrization.

We turn now to our numerical results for the case of finite temperature. In the left panel of Fig. 2 we quote the values obtained for $\Delta\bar{\Sigma}_{B,T}$ as a function of eB, for some representative values of the temperature, while in the right panel we show the results for $(\Sigma_{B,T}^u + \Sigma_{B,T}^d)/2$ as a function of T, for some selected values of eB. All these values correspond to the parametrization leading to $\Phi_0 = 230$ MeV, yet qualitatively similar results are found for the other parametrizations under consideration. The plots in the left panel clearly show that, in contrast to what happens at zero temperature, the quantity $\Delta\bar{\Sigma}_{B,T}$ does not display a monotonous increase with eB when one approaches the chiral transition temperature [for this parameter set one has $T_c(eB = 0) = 129.8$ MeV]. In fact, the curves reach a maximum after which $\Delta\bar{\Sigma}_{B,T}$ starts to decrease with increasing eB, implying that the present nonlocal model naturally exhibits the IMC effect found in LQCD. This feature can also be seen from the results displayed in the right panel of Fig. 2. As expected, all curves show a crossover transition from the chiral symmetry broken phase to the (partially) restored one as the temperature increases. However, contrary to what happens e.g. in the standard local NJL model [4–6], it is seen that within the present model the transition temperature decreases as the magnetic field increases. To be more specific, let us define the critical
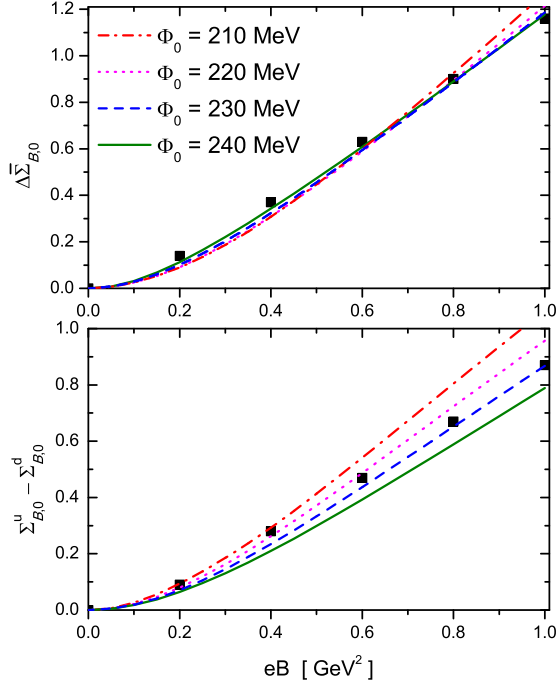
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Figure 1: Normalized condensates as functions of the magnetic field at $T = 0$. The curves correspond to different model parametrizations identified by $\Phi_0 = (-\langle \bar{q}_f q_f \rangle_{0,0}^{\mathrm{reg}})^{1/3}$. Full square symbols correspond to LQCD results of Ref. [8]. Upper panel: subtracted flavor average; lower panel: flavor difference [see Eq. (16) and text below].

transition temperature as the value of T at which the derivative $\partial[(\Sigma_{B,T}^u + \Sigma_{B,T}^d)/2]/\partial T$ reaches a maximum. Since, as known from previous analyses [36, 38, 40], the present model is too simple so as to provide realistic values for the critical temperatures even at vanishing external magnetic field, for comparison with LQCD calculations we consider the relative quantity $T_c(B)/T_c(0)$. The corresponding results for our four parameterizations are shown in Fig. 3, together with LQCD results from Ref. [8]. From the figure it is clearly seen that for magnetic fields beyond $eB \simeq 0.4$ GeV^2 all parameter sets considered here lead to a decrease of the critical temperature when eB gets increased, i.e. in all cases the IMC effect is observed. In fact, only for the case of $\Phi_0 = 240$ MeV a slightly opposite behavior is found for lower values of eB. On the other hand, the strength of the IMC effect is rather sensitive to the parametrization, the best agreement with LQCD corresponding to the parameter set associated with the lowest value of Φ_0 considered here.

To shed some light into the mechanism that produces the IMC effect in the

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present model it is important to note that the associated non-local form factor turns out to be magnetic field dependent. This can be clearly seen in Eq.(10). It is important to remind here that such form factors play the role of some finite range gluon-mediated effective interaction. Thus, in a way, this magnetic field dependence follows from the backreaction of the sea quarks on the gluon fields. Interestingly, as seen from Eq.(17), in the particular case of the gaussian interaction a clear separation between parallel and transverse components can be performed. It follows that while the $3, 4$ (parallel) components preserve the original exponential form (i.e. $\exp\left[-\bar{p}^2/\Lambda^2\right]$) the $1, 2$ (transverse) part gives rise to a polynomial dependence on B/Λ^2. It is tempting to interpret such transverse part as a sort of effective magnetic dependent coupling constant in the line of those considered in e.g. Ref.[16] in the framework of the local NJL model. Of course, one should proceed with some care since, contrary to what happens in the case of the local NJL, here such coupling is not the same for all the Landau modes. This important difference prevents a detailed comparison with the particular forms used in the local NJL calculations. In spite of this, the qualitative relevant feature is that for any value of k the effective strength decreases as eB increases in analogy to what happens with the B-dependent coupling constant used in e.g. Refs.[16, 17], thus leading to the IMC effect in a similar way.

Summary and outlook

In this work we have studied the behavior of strongly interacting matter under an external homogeneous magnetic field in the context of nonlocal chiral quark models. These theories are a sort of nonlocal extensions of the local NJL model, intending to represent a step further towards a more realistic modelling of QCD. We have found that at zero temperature the behavior of the quark condensates under the external field shows the expected magnetic catalysis effect, our predictions being in good quantitative agreement with LQCD results. On the other hand, in contrast to what happens in the standard local NJL model at the mean field level, when the analysis is extended to the case of finite temperature our results show that nonlocal models naturally lead to the Inverse Magnetic Catalysis effect. It is worth stressing that in these models the current-current couplings turn out to be dependent on the temperature and the magnetic field through the nonlocal form factors, which in principle
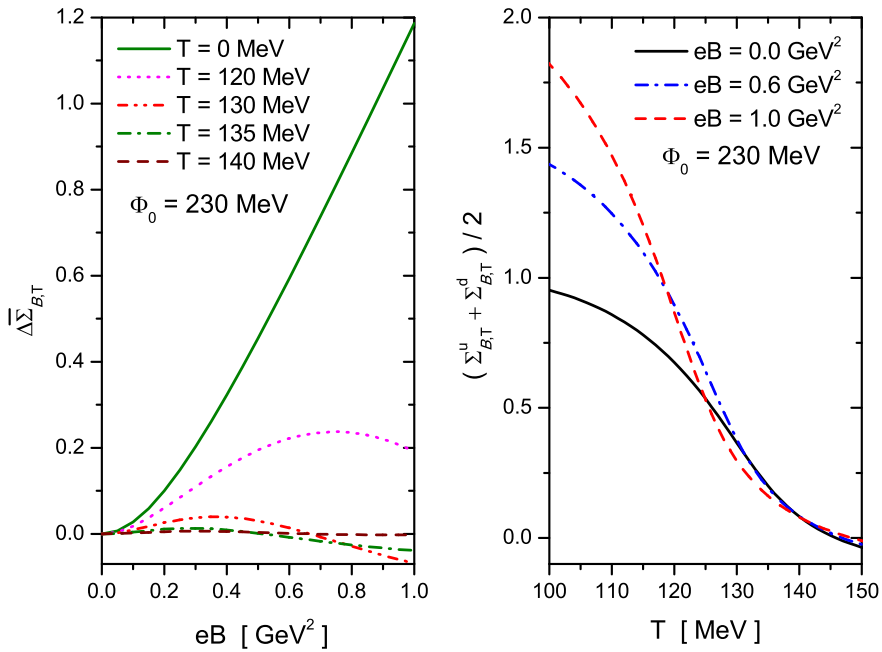
Figure 2: Left: subtracted normalized flavor average condensate as a function of eB for different representative temperatures. Right: normalized flavor average condensate as a function of the temperature for different representative values of eB. Results in both panels correspond to $\Phi_0 = 230$ MeV.

follow from some finite range gluon-mediated effective interaction. Our results indicate that this scheme seems to capture the main features of more sophisticated approaches to the QCD dynamics in the presence of external magnetic fields, in which IMC is observed. In this work we have just discussed the numerical results corresponding to a simple Gaussian nonlocal form factor. Although numerically more involved, the extension to other form factor shapes often considered in the literature (see e.g. Ref. [39]) is straightforward. A detailed comparison of the predictions arising from different form factors, together with a more extended presentation of the formalism, will be provided in a forthcoming article [48]. We point out, however, that the results we have already obtained using Lorenztian form factors indicate that the presence of the IMC effect at finite temperature appears to be a general feature of the present type of nonlocal models. It is also worth noticing that, as a first step in this research line, we have considered here a simple version of nonlocal models in which e.g. we have not incorporated interactions leading to quark wave function
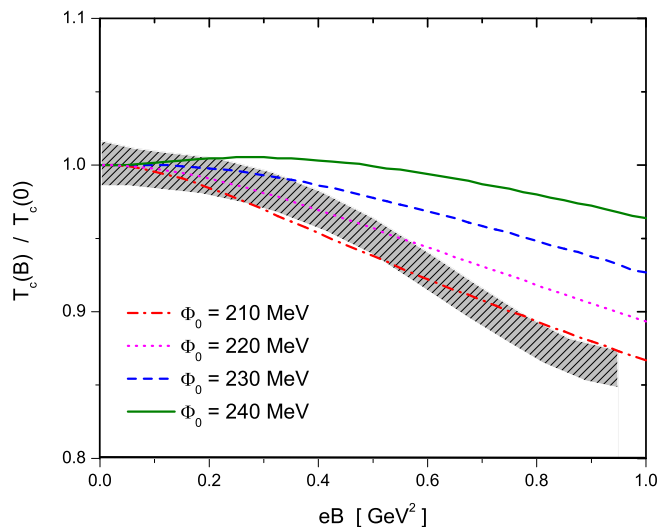
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Figure 3: Normalized chiral restoration temperatures as functions of eB for various model parametrizations. For comparison, LQCD results of Ref. [8] are indicated by the grey band.

renormalization, nor the coupling of fermions to the Polyakov loop. It is clear that the inclusion of these interactions is important to provide a more realistic description of strong interaction thermodynamics [40, 41]. We expect to report progresses in this direction in the near future.

Acknowledgements

[1] R. C. Duncan and C. Thompson, Astrophys. J. 392, L9 (1992); C. Kouveliotou et al., Nature **393**, 235 (1998).

[2] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A **803**, 227 (2008); V. Skokov, A. Y. Illarionov, and V. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009); V. Voronyuk,

V. Toneev, W. Cassing, E. Bratkovskaya, V. Konchakovski, and S. Voloshin, Phys. Rev. C **83**, 054911 (2011).

[3] T. Vachaspati, Phys. Lett. **B265**, 258 (1991); K. Enqvist and P. Olesen, Phys. Lett. **B319**, 178 (1993).

[4] D. E. Kharzeev, K. Landsteiner, A. Schmitt and H. U. Yee, Lect. Notes Phys. **871**, 1 (2013).

[5] J. O. Andersen, W. R. Naylor and A. Tranberg, Rev. Mod. Phys. **88**, 025001 (2016).

[6] V. A. Miransky and I. A. Shovkovy, Phys. Rept. **576**, 1 (2015).

[7] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP **1202**, 044 (2012).

[8] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502 (2012).

[9] E. S. Fraga, J. Noronha and L. F. Palhares, Phys. Rev. D **87** 114014 (2013).

[10] F. Bruckmann, G. Endrodi and T. G. Kovacs, JHEP **1304**, 112 (2013).

[11] G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013).

[12] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. **110**, 031601 (2013).

[13] J. Chao, P. Chu and M. Huang, Phys. Rev. D **88**, 054009 (2013).

[14] E. S. Fraga, B. W. Mintz and J. Schaffner-Bielich, Phys. Lett. B **731**, 154 (2014).

[15] M. Ferreira, P. Costa, D. P. Menezes, C. Providência and N. Scoccola, Phys. Rev. D **89**, 016002 (2014);

[16] M. Ferreira, P. Costa, O. Lourenço, T. Frederico and C. Providência, Phys. Rev. D **89**, 116011 (2014).

[17] R. L. S. Farias, K. P. Gomes, G. I. Krein and M. B. Pinto, Phys. Rev. C **90**, 025203 (2014).

[18] A. Ayala, M. Loewe, A. J. Mizher and R. Zamora, Phys. Rev. D **90**, 036001 (2014).

[19] A. Ayala, M. Loewe and R. Zamora, Phys. Rev. D **91**, 016002 (2015); A. Ayala, C. A. Dominguez, L. A. Hernandez, M. Loewe and R. Zamora, Phys. Rev. D **92**, 096011 (2015); Addendum: [Phys. Rev. D **92**, 119905 (2015)].

[20] S. Fayazbakhsh and N. Sadooghi, Phys. Rev. D **90**, 105030 (2014).

[21] J. O. Andersen, W. R. Naylor and A. Tranberg, JHEP **1502**, 042 (2015).

[22] N. Mueller and J. M. Pawlowski, Phys. Rev. D **91**, 116010 (2015).

[23] A. Ayala, J. J. Cobos-Martínez, M. Loewe, M. E. Tejeda-Yeomans and R. Zamora, Phys. Rev. D **91**, 016007 (2015).

[24] E. J. Ferrer, V. de la Incera and X. J. Wen, Phys. Rev. D **91**, 054006 (2015).

[25] J. Braun, W. A. Mian and S. Rechenberger, Phys. Lett. B **755**, 265 (2016).

[26] R. Rougemont, R. Critelli and J. Noronha, Phys. Rev. D **93**, 045013 (2016).

[27] A. Ayala, C. A. Dominguez, L. A. Hernandez, M. Loewe and R. Zamora, Phys. Lett. B **759**, 99 (2016).

[28] S. Mao, Phys. Lett. B **758**, 195 (2016).

[29] T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).

[30] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994); C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000).

[31] S. Noguera and N. N. Scoccola, Phys. Rev. D **78**, 114002 (2008).

[32] G. Ripka, *Quarks Bound by Chiral Fields*, (Oxford University, New York, 1997).

[33] R.D. Bowler and M.C. Birse, Nucl. Phys. A **582**, 655 (1995); R.S. Plant and M.C. Birse, Nucl. Phys. A **628**, 60 (1998).

[34] S. M. Schmidt, D. Blaschke and Y. L. Kalinovsky, Phys. Rev. C **50**, 435 (1994).

[35] B. Golli, W. Broniowski and G. Ripka, Phys. Lett. B **437**, 24 (1998); W. Broniowski, B. Golli and G. Ripka, Nucl. Phys. A **703**, 667 (2002).

[36] I. General, D. Gomez Dumm and N. N. Scoccola, Phys. Lett. B **506**, 267 (2001); D. Gomez Dumm and N.N. Scoccola, Phys. Rev. D **65**, 074021 (2002).

[37] A. Scarpettini, D. Gomez Dumm and N.N. Scoccola, Phys. Rev. D **69**, 114018 (2004).

[38] D. Gomez Dumm and N. N. Scoccola, Phys. Rev. C **72**, 014909 (2005).

[39] D. Gomez Dumm, A. G. Grunfeld and N.N. Scoccola, Phys. Rev. D **74**, 054026 (2006).

[40] G. A. Contrera, D. Gomez Dumm and N. N. Scoccola, Phys. Lett. B **661**, 113 (2008); G. A. Contrera, M. Orsaria and N. N. Scoccola, Phys. Rev. D **82**, 054026 (2010); J. P. Carlomagno, D. Gómez Dumm and N. N. Scoccola, Phys. Rev. D **88**, 074034 (2013).

[41] T. Hell, S. Roessner, M. Cristoforetti and W. Weise, Phys. Rev. D **79**, 014022 (2009); T. Hell, S. Rossner, M. Cristoforetti and W. Weise, Phys. Rev. D **81**, 074034 (2010).

[42] D. Gomez Dumm, S. Noguera and N.N. Scoccola, Phys. Lett. B **698**, 236 (2011); Phys. Rev. D **86**, 074020 (2012).

[43] K. Kashiwa, Phys. Rev. D **83**, 117901 (2011).

[44] V. I. Ritus, Sov. Phys. JETP **48**, 788 (1978).

[45] C. N. Leung, Y. J. Ng and A. W. Ackley, Phys. Rev. D **54**, 4181 (1996).

13

[46] P. Watson and H. Reinhardt, Phys. Rev. D **89**, 045008 (2014).

[47] N. Mueller, J. A. Bonnet and C. S. Fischer, Phys. Rev. D **89**, 094023 (2014).

[48] V.P. Pagura, D. Gomez Dumm, S. Noguera and N.N. Scoccola, in preparation.