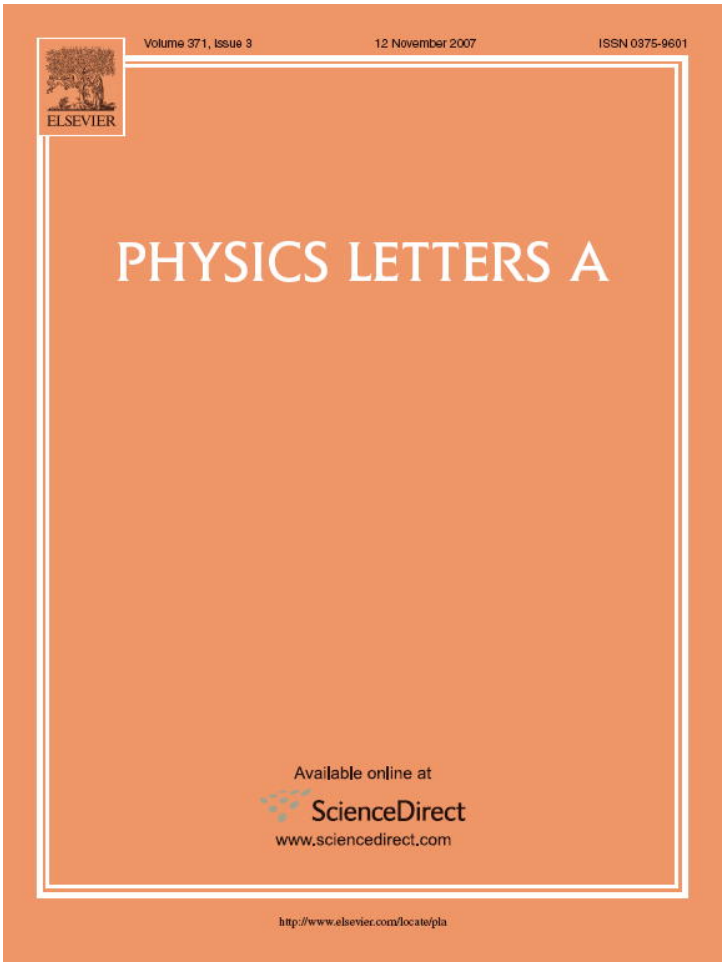

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Noise induced energy excitation by a general environment

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Abstract

We analyze the effects that general environments, namely ohmic and non-ohmic, at zero and high temperature induce over a quantum Brownian particle. We state that the evolution of the system can be summarized in terms of two main environmental induced physical phenomena: decoherence and energy activation. In this Letter we show that the latter is a post-decoherence phenomenon. As the energy is an observable, the excitation process is a direct indication of the system-environment entanglement particularly useful at zero temperature.

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1. Introduction

The open quantum system approach to the dynamic of a particle coupled to a reservoir provides a number of very interesting results. Some are the appearance of noise and dissipation, decoherence (for a complete overview see Ref. [1] and references therein), entanglement, and energy exitation [2]. Thus, a necessary but not suffice condition for decoherence is the entanglement between the main system and the bath. As long as the composed system is governed by a reversible time evolution, entanglement alone cannot generate decoherence. The non-unitary evolution that follows the tracing out of the environmental degrees of freedom provides the essential source of decoherence and dissipation. The entanglement assures that the system measures or saves the state of the environment or alternatively, there is information about the system state stored in the environment. In this framework, we stress that the system-environment interaction is more than just a matter of renormalization.

There are simple examples in which decoherence is generated by a reservoir even at zero temperature [3]. In general, a small system coupled to an environment fluctuates even in the zero-T limit. These fluctuations can take place without generating an energy trace in the bath. The fluctuations in energy of the small system are a peculiar fact of the entanglement with the quantum environment [4].

In Ref. [5] we have studied the evolution of a simple time-independent bistable system, by following the quantum evolution of a particle initially localized at one of the minima of a double-well potential, when coupled to an external environment at both zero and high temperature. The zero temperature case shows subtly different and, in some ways, unexpected properties. Tunneling is undoubtedly quickly suppressed, as can be seen by inspecting either the probability of the particle to remain on the original well or the evolution of its Wigner function. In both cases we observed typical classical features since very early times. At zero temperature, the quantum fluctuations of the environmental oscillators, absent in a purely classical evolution, lead to non-zero diffusive terms. Their effect is felt primarily through the anomalous diffusion coefficient $f(t)$ that can have a large magnitude. We have conjectured that these non-trivial diffusion effects induced by the quantum environment are large enough to excite the particle over the potential

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barrier. This is to be contrasted with the case where the environment is classical forbidding any kind of activation phenomenon. Though the late time evolution in the presence of a quantum vacuum is in nature very different from high-T thermal activation, we suggested that it could still be interpreted in terms of a purely classical setting, if the environment oscillators are described by a particular non-thermal statistical state [5].

The importance of general environments at zero temperature lies in its leeway to model various physical situations. The environments we shall study hereafter are known as supraohmic or subohmic depending on the form of the spectral density in the low frequency part of the spectrum. The ohmic environment is the most studied case in the literature and produces a dissipative force that in the limit of the frequency cutoff $\Lambda \to 0$ is proportional to the velocity. The supraohmic case, on the one hand, is generally used to model the interaction between defects and phonons in metals [6] and also to mimic the interaction between a charge and its own electromagnetic field (see for example [7]). In particular, the use of the supraohmic case will allow us to establish a close relation with the decoherence process in quantum field theory [8]. On the other hand, the quantum behaviour of "free" electrons in mesoscopic systems is affected by their interaction with the environment, which, for example in such cases, consist of other electrons, phonons, photons or scatterers. Which environment dominates the destruction of the interference phenomena generally depends on the temperature. For instance, the temperature dependence of the weak-localization correction to the conductivity reveals in metals that electron–electron interactions dominate over the phonon contribution to decoherence at the low temperature regime.

In [3] we analyzed the effect of quantum fluctuations of an ohmic environment as a source of decoherence. Therein, we presented the analytical expressions of the diffusion coefficients at zero temperature for different physical situations and showed that decoherence at zero temperature does occur contrary to what is most commonly believed. However, the suppression of the interferences is not as fast as it is at high temperature limit. In the latter case, it is expected to happen at times of $\mathcal{O}(1/2M\gamma_0 k_B T L_0^2)$ while we have shown that at zero temperature it is smaller than $\mathcal{O}(1/\gamma_0)$ [3].

In this Letter, we follow the investigation initiated in [5], in order to thoroughly analyze the effects that general non-ohmic environments (in Ref. [5] only the ohmic case was considered) at all temperatures induce over a quantum particle in a harmonic oscillator potential (Quantum Brownian Motion [QBM] model). We are interested in analyzing the energy excitation process for ohmic and non-ohmic environments at zero and high temperature. A system entangled with environmental states has a number of properties which distinguishes it from systems for which the ground state factorizes. In order to understand the physics underlying the entanglement process, it is important to consider the evolution of the energy of the main system. The energy is always an observable, and at zero temperature, fluctuations in the energy of the main system are a direct indication of the system-environment entanglement. If the system is isolated, it is in the lowest energy state. Quantum fluctuations are determined only by the diagonal elements of the density matrix. Therefore, it is not obvious that one can make any statement on entanglement. In general, additional information about the non-diagonal elements is needed. If we find the main system in an excited state, one can conclude that it is entangled [4]. Hence, in this Letter we shall analyze the evolution of the mean energy of a quantum system, coupled to an environment at zero and high temperature, and show that the system is energetically activated due to the coupling with the bath. In the zero-temperature case, we shall show that there is a quantum effect, analogous to the thermal activation process, by which diffusion produces an increase of the energy as a function of time; i.e., a noise induced activation. We shall pay attention to these effects in general environments. The isolated harmonic oscillator (in its ground state) obeys two important properties: minimum uncertainty and equipartition of energy between average kinetic and potential energies. When we study the dynamics of quantum open systems, the effect of the environment on the system is manifested through violations of these properties. As the energy of the subsystem is an observable, it illustrates the distinction between separable and entangled states. We shall show that the main system can be found in higher energy states, no matter how weakly coupled to an environment at zero temperature it might be.

Our main scope is to show the existence of "noise induced activation" phenomenon at zero temperature, and confirm it is a post-decoherence process. Its analogous in the high temperature limit is the "thermal activation" process. Even though this phenomenon is worldwide accepted, it has not been studied for non-ohmic environments nor has it been shown to be a post-decoherence process. Thus, we shall analyze two different thermal regimes and study whether a system coupled to an environment at arbitrary temperature, apart from suffering the destruction of its interferences and dissipation, can benefit in some way. We shall confirm the existence of "thermal activation" for non-ohmic "decoherent" environments. In the case of zero temperature, we are interested in answering if the system can increase its own energy by interacting with the environment in the same way that at high temperature it is "activated" by the presence of it. This process has been shown to exist in the case of a quantum particle localized in one minimum of a double-potential well in Ref. [5].

2. The model

Let us consider a quantum particle (characterized by its mass M and its bare frequency Ω) bi-linearly coupled to an environment composed of an infinite set of harmonic oscillators (of mass m_n and frequency ω_m) (for a general presentation of the model see, for example [9–11] and references therein).

The dynamics of the quantum Brownian particle can be obtained by tracing over the degrees of freedom of the environment and obtaining a master equation for the reduced density matrix of the system, $\rho_r(t)$. We shall assume that the initial states of the system and environment are uncorrelated, with the latter being in thermal equilibrium at temperature T (strictly zero temperature also allowed) for $t = 0$ (i.e., when the interaction between system and environment is switched on). At

the initial time, the state is a product of a given state of the system and a thermal state for the environment. Only when the interaction is turned on the system is allowed to evolve. The initial condition is not an equilibrium state of the complete action [5]. Under these assumptions, and using that the system-environment coupling is small, the reduced density matrix satisfies the following master equation (we set $\hbar = 1$)

$$\begin{aligned} i\frac{\partial}{\partial t}\rho_{\rm r}(x,x',t) = {} & \left[-\frac{1}{2M^2}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right)\right]\rho_{\rm r}(x,x',t) \\ & + \frac{1}{2}M\Omega^2\left(x^2-x'^2\right)\rho_{\rm r}(x,x',t) \\ & + \frac{1}{2}M\delta\Omega^2(t)\left(x^2-x'^2\right)\rho_{\rm r}(x,x',t) \\ & - i\gamma(t)(x-x')\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)\rho_{\rm r}(x,x',t) \\ & - iM\mathcal{D}(t)(x-x')^2\rho_{\rm r}(x,x',t) \\ & - f(t)(x-x')\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial x'}\right)\rho_{\rm r}(x,x',t). \quad (1) \end{aligned}$$

In this expression, $\delta\Omega^2(t)$ is the shift in the system frequency, which produces the renormalized one $\tilde{\Omega}^2 = \Omega^2 + \delta\Omega^2$, $\gamma(t)$ is the dissipation coefficient, and $\mathcal{D}(t)$ and $f(t)$ are the normal and anomalous diffusion coefficients respectively, which produce the decoherence effects. These coefficients have all been defined in [3,12]. The diffusion coefficients are deduced from the noise kernel, source of stochastic forces in the associated Langevin equation, whereas $\delta\Omega^2(t)$ and $\gamma(t)$ are related to the dissipation kernel. The dissipation $\eta(t)$ and noise $\nu(t)$ kernels are respectively defined as $\eta(t) = \int_0^\infty d\omega\, I(\omega)\sin\omega t$ and $\nu(t) = \int_0^\infty d\omega\, I(\omega)\coth\frac{\beta\omega}{2}\cos\omega t$. $I(w)$ is the spectral density of the environment defined as $I(\omega) = (2/\pi)M\gamma_0\omega(\omega/\Lambda)^{n-1}e^{-\omega^2/\Lambda^2}$ and Λ is the physical high-frequency cutoff, which represents the highest frequency present in the environment, and $\beta = 1/T$ its inverse temperature (with Boltzmann constant set to unity, $k_B = 1$). We shall consider the quantum system coupled to a general environment, namely ohmic ($n = 1$), supraohmic ($n = 3$), and subohmic ($n = 1/2$).

We numerically solve Eq. (1) considering its coefficients up to second order in the system-environment coupling, this fact has been taken into account in all the simulations we shall present. We will work in the under-damped case, which ensures the validity of the perturbative solutions up to the times we are interested in [12,13]. Hereafter, let us envisage the situation in which $\gamma_0 \ll \hbar$, which is called the weak-interaction situation and sets the temporal domain for perturbative solutions. All the results obtained below are for periods of the evolution well within the regime for which this approximation is valid. It is worth noting that Eq. (1) is valid at any temperature, and is local in time, despite the fact that no Markovian approximation was explicitly taken. In the following, we shall show how the general master equation simplifies in different regimes, making it more tractable for both analytical and numerical techniques.

In particular, in the high temperature limit, i.e., $\hbar\omega \ll k_B T$, of an ohmic environment, the coefficients of the master equation (Eq. (1)) are constant, and therefore, the expression is further simplified [5,12]. In such a case, the diffusion coefficient can be approximated by $\mathcal{D} \simeq 2\gamma_0 k_{\rm B} T M$, where γ_0 is the dissipation coefficient [12]. In this limit, while γ_0 is a constant and $\mathcal{D}(t) \propto T$, the coefficient $f \propto T^{-1}$ can be neglected. Therefore, the term proportional to $\mathcal{D}$ is the relevant one in the master equation at high temperatures in order to evaluate, for example, the decoherence time.

On the opposite thermal regime, i.e., strictly zero temperature, the master equation is much more complicated because the coefficients are not constant and depend explicitly upon the time. In [3], we have computed those coefficients for a quantum Brownian particle coupled to an ohmic environment at strictly zero temperature.

3. Decoherence

The decoherence process in the limit of high temperature has been extensively studied in the literature [1,12,14]. However, no much has been said about general environments, namely supraohmic and subohmic, particularly if the environment is at stricly zero temperature. Therefore, as we have done in [3] for an ohmic environment, in this Section we shall analyze the decoherence process for non-ohmic environments since it is necessary for the understanding of the excitation induced phenomena. We shall study the dynamic evolution of an initial superposition of two delocalized (separated a distance $2\,L_0$ in position) states when the system is coupled to an non-ohmic environment at zero temperature. We consider two wave packets symmetrically located in phase space as in [3,14] and evaluate the coeficient $A_{\rm int}$, which results crucial to estimate the decoherence time $t_{\mathcal{D}}$

$$\dot{A}_{\rm int} \approx 4L_0^2\mathcal{D}(t) - 2f(t), \quad (2)$$

where $\mathcal{D}(t)$ and $f(t)$ are the corresponding normal and anomalous diffusion coefficients, respectively for each environment considered.

The dynamics of the evolution of a system coupled to a supraohmic environment is quite peculiar for that everything that happens in the system is during the initial jolt timescale [12]. It is important to deepen the study of this kind of environment since it can be very useful, for example, as a toy model to understand the physics of interacting quantum fields. In Ref. [15] we have made analytical estimations of the decoherence time for this kind of environment in a simplified spin-boson model. Notably, timescales obtained therein coincide with the ones obtained numerically in the present Letter.

The analytical estimation of the fringe visibility factor $A_{\rm int}$ can be obtained for different physical limits. For times such that $\Omega t \ll 1$ (short times) it can be proved that $D(t)_{n=3} \sim (2M\gamma_0)/(\pi\Lambda^2)\Omega^4 t$ and $f(t)_{n=3} \sim -(2\gamma_0)/(\pi)\Omega t$. Using these expressions and Eq. (2) we obtain $\dot{\rm A}_{\rm int} \sim 4\gamma_0\Omega t(1 + 2L_0^2 M\Omega^3/\Lambda^2)$. Thus, it is possible to get

$$\mathrm{A}_{\rm int} \sim 2\frac{\gamma_0}{\Omega}\left(1 + 2L_0^2 M\frac{\Omega^3}{\Lambda^2}\right)\Omega^2 t^2, \quad (3)$$

which is smaller than unity (particularly in the case $\Lambda \gg \Omega$). This means that the decoherence factor $\Gamma(t) = \exp(-A_{\rm int})$ [16]

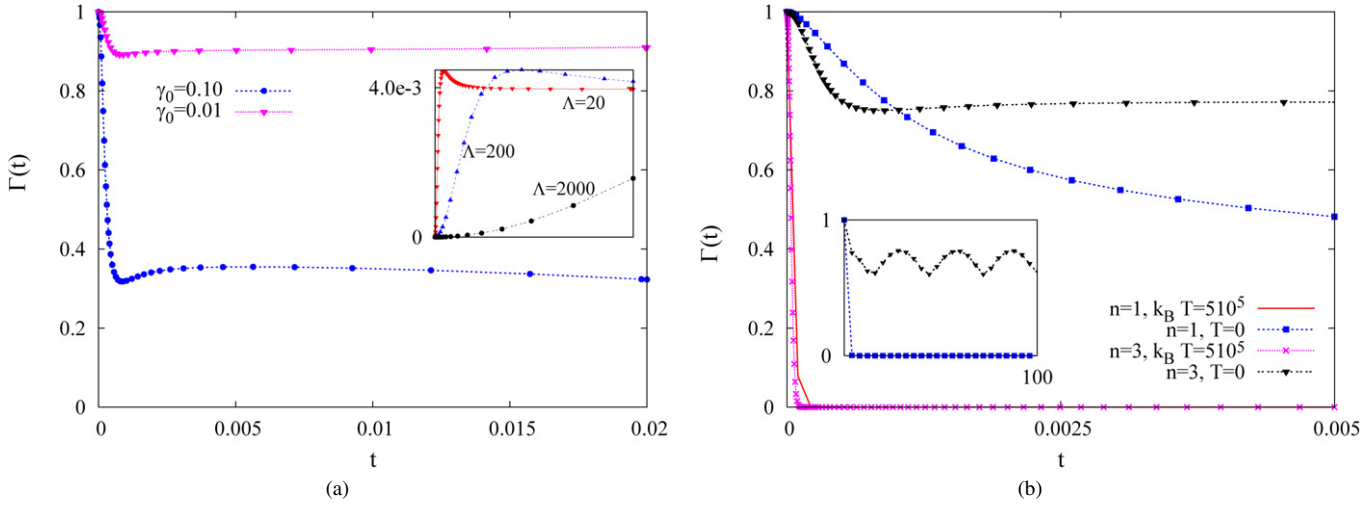

Fig. 1. (a) Decaying factor $\Gamma(t)$ for a supraohmic environment ($n = 3$) at zero temperature. The parameters are measured in units of the bare frequency Ω. We consider the cases $\Lambda = 2000$ ($\Lambda = 200$ when noted), $\Omega = 0.1$, $L_0 = 2$ for different values of γ_0 of the environment. In the inset, we have plotted the time evolution of the linear entropy $SI(t)$ for different values of the cutoff Λ and $\gamma_0 = 0.5$. There is no total decoherence for the underdamped supraohmic environment. (b) A comparison between different decoherence rates for ohmic and supraohmic environments at zero and high temperature is shown in the main plot. Decoherence is as fast in the ohmic case as in the supraohmic for the high temperature limit, while for those times there is no decoherence at zero-T. In the inset, the zero-T behaviour of both environments for longer times. There is not decoherence for the supraohmic environment, while the ohmic takes longer but in the end, it succeeds in destroying the interferences of the main system. Parameters used: $\Lambda = 2000$, $\gamma_0 = 0.1$, $L_0 = 1$ and are measured in units of Ω.

is not a decaying function with time and decoherence shall not be effective in this case.

On the other side, if we ask $\Lambda t \gg 1$ and $\Omega t \geqslant 1$, it can be checked that Eq. (2) reads $\dot{\mathrm{A}}_{\mathrm{int}} \to 0$, since both diffusion coefficients approach to zero so rapidly (it can be shown that $D(t)_{n=3} \sim 2M\gamma_0\Lambda\cos(\Lambda t)/\Lambda t$, and $f(t)_{n=3} \sim \gamma_0$). Thus, the fringe visibility $\mathrm{A}_{\mathrm{int}}$ is a constant and can be approximated by $\mathrm{A}_{\mathrm{int}} \approx 2ML_0^2\gamma_0$ (the value for larger times in order to assure continuity of the coefficient). It is easily deduced that decoherence never occurs for this case, except for unrealistic values of the coupling constant (values outside the perturbative treatment. For more general solutions at larger times see [17]). The decoherence factor shall be a constant value as $\Gamma \sim e^{-2ML_0^2\gamma_0}$ for all times. Since we are considering the underdamped case (small γ_0), the exponent shall not be of order one and $\Gamma(t)$ shall not be much smaller than unity. Decoherence shall be only effective as long as $ML_0^2 \geqslant 1/\gamma_0$, i.e., large macroscopic trajectories. This result is to be contrasted to the one obtained in the high temperature limit, where decoherence occurs in a time estimated as $t_D^{n=3,HT} \sim (\Lambda M k_B T L_0^2\gamma_0)^{-1/2}$ very similar to what occurs in the ohmic environment at the same temperature [15].

In Fig. 1(a), we show the behaviour of the decoherence factor $\Gamma(t)$ for two different values of the coupling constant in the case the supraohmic environment is at zero temperature. As expected, the stronger the coupling with the environment the sooner $\Gamma(t)$ decreases. However, as can be seen in Fig. 1(a) it never reaches zero. This is so because the stronger the environment, the bigger the initial jolt and the more efficient the suppression of the interferences results (the diffusion coefficient is proportional to γ_0). For example, in the case of $\gamma_0 = 0.01$ in Fig. 1(a), we see that $\Gamma(t) \approx 0.9$ after the initial transient and remains steady for all times. Then, interferences are not completely destroyed, just slightly attenuated. Decoherence is definitely not effective at zero-T in the supraohmic case for the set of parameters chosen. Yet more, it is possible to see that the linear entropy has an initial jolt for all values of the frequency cutoff and reaches an asymptotic limit (sooner for smaller values of the frequency cutoff Λ). However, we can also observe that the linear entropy does not significantly increase (reaches an upper limit but far from the maximum possible value for a mixed state) unlike in the ohmic case.

It is important to stress that in the examples shown in the plots, we have used parameters such that decoherence is essentially absent. We have omitted to show examples of macroscopic trajectories (very large L_0) since they are not of much interest from the microscopic point of view. Quantum interference between macroscopically distinguishable trajectories are easily destroyed even for the supraohmic environment at zero temperature.

The non-dissipative character of the supraohmic environment is a consequence of the weakness of the spectral density in the infrared sector, and the dependence with Λ is due to the fact that it is more sensible to the ultraviolet cutoff of the frequency spectrum. The supraohmic QBM model can be viewed as a toy model for a quantum field theory (QFT) scenario. In Ref. [8] has been shown the conditions under which there is decoherence at $T = 0$ for a non-linear interacting field. The supraohmic case is weakly diffusive due to the fact $f(t)$ goes to zero after the initial transient. In this case, decoherence, depends strongly on the coupling constant between system and bath in order to generate remarkable effects. In QFT, diffusive effects come from the particle creation in the environment due to the interaction with the system. When there is a frequency threshold in the environment, only those modes in the system with frequency near the threshold are able to create particles and decohere. This is why the diffusion coefficient is different from zero only for particular values of the parameters. This result is similar to what we have shown so far for the supraohmic environment. In the QBM

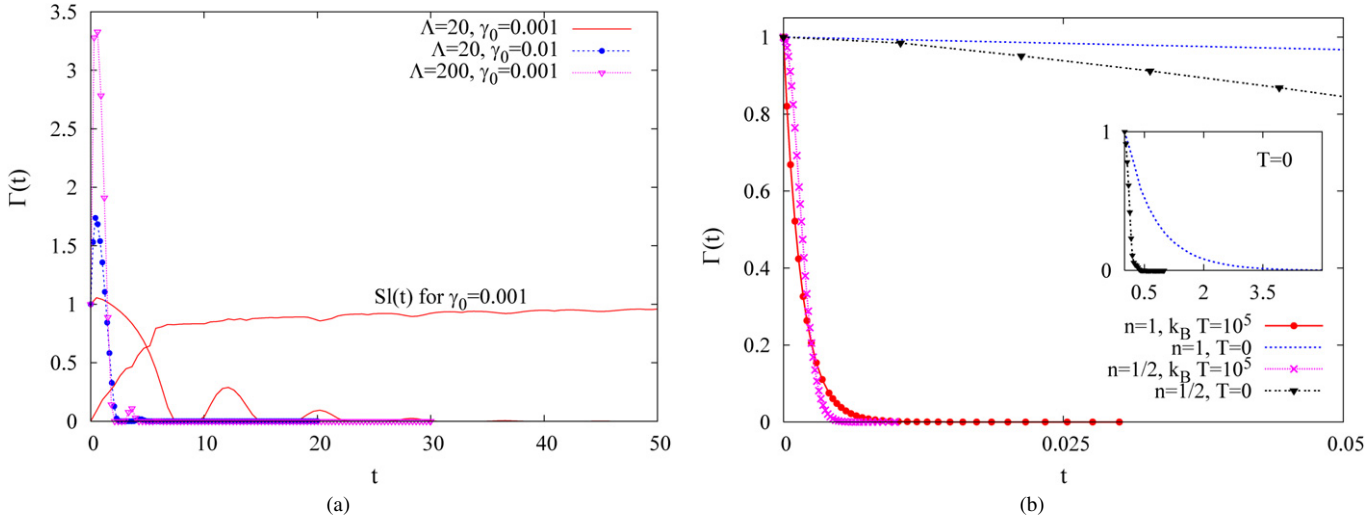

Fig. 2. $\Gamma(t)$ for the case of a system coupled to a subohmic environment ($n = 1/2$) at zero temperature. Parameters are in units of the bare frequency Ω. We consider the case $\Omega = 0.1$, $L_0 = 2$ for different values of γ_0 and the frequency cutoff Λ. It is also shown the linear entropy $Sl(t)$ for the indicated case. (b) A comparison between different decoherence rates for ohmic and subohmic environments at zero and high temperature is shown in the main plot. Decoherence is as fast in the ohmic case as in the subohmic for the high temperature limit, while for those times there is yet no decoherence at zero-T. In the inset, the zero-T behaviour of both environments for longer times. The decoherence timescale is shorter for the subohmic case at strictly zero temperature. Parameters used: $\Lambda = 200$, $\gamma_0 = 0.01$, $L_0 = 0.1$ and are measured in units of Ω.

model we are studying here, the relation between Ω, Λ, and γ_0 is crucial in order to get diffusive effects. Particularly at $T = 0$, when $\Omega \ll \Lambda$ the system is unable to excite the environment in order it "create" particles [8].

In conclusion, whereas supraohmic high-T environments are very efficient inducing decoherence on the main system under certain conditions [12,15], at zero temperature there is a strong condition over the dissipative constant γ_0. As expected, bigger γ_0 implies a stronger coupling to the external environment and therefore, shorter decoherence times.

Finally, in Fig. 1(b), we present a comparison between the decoherence rates for ohmic and supraohmic environments at zero and high temperature. Decoherence is as fast in the ohmic case as in the supraohmic for the high temperature limit. At zero-T there is not decoherence for the supraohmic environment, while the ohmic environment takes longer times but finally succeeds in destroying the quantum interferences, as can be seen in the inset of the figure where the time scale is longer than the one of the main plot.

As for the subohmic environment, we also perform an analytical estimation of the decoherence timescale by computing the fringe visibility factor. In the case that $\Lambda t \gg 1$ and $\Omega \geqslant 1$, Eq. (2) reads

$$\dot{\mathrm{A}}_{\mathrm{int}} \sim \gamma_0 \Lambda \left(2ML_0^2 + \frac{\Gamma_{\mathrm{Euler}}}{\Omega} + \frac{\log(2\Lambda t)}{\Omega} \right). \tag{4}$$

In this case, by integrating in time the above equation, we can obtain the fringe visibility factor $\mathrm{A}_{\mathrm{int}} \sim \gamma_0 \Lambda t/\Omega \log(2\Lambda t)$. It is easy to deduce the decoherence timescale as $t_D \leqslant \Omega/(\gamma_0 \Lambda)$. On the other hand, if we ask $\Omega t \ll 1$, we would obtain a similar decoherence time since the diffusion coefficients presented for the subohmic environment depend slightly on the ratio Ω/Λ. It is important to note that in order to be able to neglect the initial transient, one needs to have product $\Omega/\gamma_0 > 1$.

We can check our estimations with the help of Fig. 2(a), where we present the evolution of $\Gamma(t)$ for a system coupled to a subohmic environment at zero temperature. There is a peculiar feature in this case. The exponential $\Gamma(t)$ initially grows but immediately after, decreases and reaches zero [12]. We can easily note that the dependence with the coupling constant and the frequency cutoff is similar to the ohmic case (the bigger Λ and γ_0, the sooner $\Gamma(t) \to 0$) in the high temperature limit. In Fig. 2(a) we present the time dependence of the linear entropy $S_l(t)$ for just one curve (for the sake of clarity), where it is easy to observe that the saturation of this quantity is reached (the asymptotic limit closer to one when the corresponding solid curve is closer to zero and total decoherence is effective).

Finally, a very distinctive difference between this type of environment and the ohmic one at zero temperature is that the former is much more effective in producing decoherence and does so in a shorter decoherence timescale. In Fig. 2(b), we show a comparison between the ohmic and subohmic environments at zero temperature and in the high temperature limit. Therein, it is easy to see, that the subohmic environment is very efficient in inducing decoherence on the quantum test particle not only at high temperature but at strictly zero temperature as well.

4. Noise induced energy excitation in the high temperature limit

We shall start by studying the thermal activation process in a general environment, either ohmic or non-ohmic. For each case, we shall provide analytical arguments and numerical evidence of the existence of this phenomenon and observe that, those systems which are mostly activated, are those whose interferences have already been suppressed by decoherence in a previous timescale.

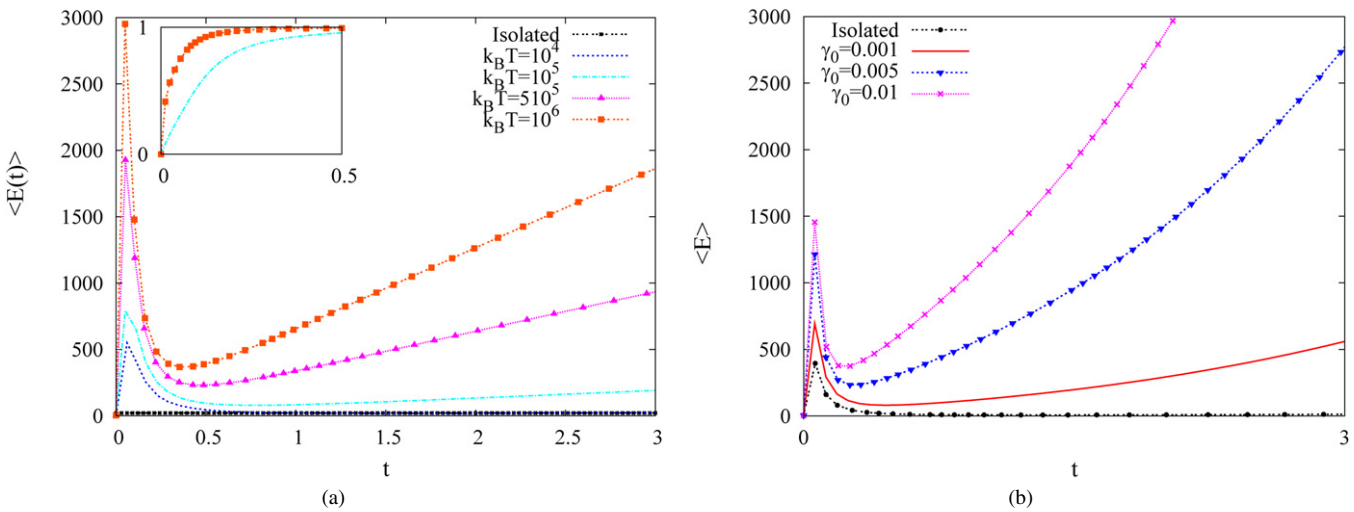

Fig. 3. (a) Evolution in time of the mean energy of the system coupled to an ohmic environment ($n = 1$) in the high temperature limit. We consider the case $\gamma_0 = 0.001$, $\Lambda = 2000$, $\Omega = 0.1$, $L_0 = 2$ for different temperatures of the environment. In the inset, we have plotted the time evolution of the linear entropy for two different environments: $k_B T = 10^5$ and $k_B T = 10^6$ (same colors than in the main picture). Thermal activation is a post-decoherence phenomena. (b) Evolution in time of the mean energy of the system coupled to an ohmic environment ($n = 1$) in the high temperature limit. We consider the case $k_B T = 10^5$, $\Lambda = 2000$, $\Omega = 0.1$, $L_0 = 2$ for different values of the coupling constant γ_0. Big values of γ_0 activate the system earlier. The parameters are measured in units of the bare frequency Ω in all cases.

For all cases, we shall evaluate

$$\langle E(t)\rangle = \frac{1}{2M}\langle p^2\rangle(t) + \frac{M\tilde{\Omega}^2(t)}{2}\langle x^2\rangle(t),$$

where $\langle x^2\rangle = \mathrm{Tr}(\rho_r(t)x^2)$ and $\langle p^2\rangle = \mathrm{Tr}(\rho_r(t)p^2)$ are calculated using the solution of the master equation for a superposition of two Gaussian wave packets, initially located at $x = \pm L_0$. In the following, we shall present the mean energy of the system for different environments at all temperatures so as to show the "noise induced activation" in a quantum Brownian particle.

4.1. Ohmic environment

It is widely accepted that "thermal" activation is a post-decoherence process for a system coupled to an ohmic environment in the high temperature limit. This is so because, after the suppression of the system's interferences due to the presence of the environment, the system and the environment are still in interaction. Therefore, there is still energy exchange between them and consequently, the system increases its energy proportionally to the temperature of the bath for short times. The thermal activation rate for a classical system can be obtained by working with the classical analogue of the master equation for the Wigner function, the Fokker–Planck equation:

$$\dot{W} = \{H_{\mathrm{sys}}, W\}_{\mathrm{PB}} + 2\gamma_0\partial_p(pW) + \mathcal{D}\partial^2_{pp}W. \tag{5}$$

The classical evolution for the average of any physical observable $A(x, p)$ in this regime is then given by:

$$\partial_t\langle A\rangle = -\langle\{H_{\mathrm{sys}}, A\}_{\mathrm{PB}}\rangle + \mathcal{D}\langle\partial^2_p A\rangle - 2\gamma_0\langle p\partial_p A\rangle. \tag{6}$$

If we take $A(x, p)$ to be the Hamiltonian of the main system, we obtain $\partial_t\langle H\rangle = 2\gamma_0(k_B T - \langle p^2\rangle)$ (being $\mathcal{D} = 2\gamma_0 k_B T$ for this case). This expression can be further simplified by assuming T to be much higher than the relevant energy scales in the problem, $\langle p^2\rangle$, during the early stages of the evolution. As a result, the time dependence of the energy of the system is given by:

$$\partial_t\langle H\rangle = 2\gamma_0 k_B T \quad\rightarrow\quad E = 2\gamma_0 k_B T t + E_0, \tag{7}$$

where E_0 is the initial energy of the system. We can then estimate the thermal activation time t_{th} as $t_{\mathrm{th}} = (E - E_0)/(2\gamma_0 k_B T)$. In Fig. 3(a) we have plotted the time evolution of the system's mean energy for the ohmic environment and confirmed its behavior is proportional to the temperature of the bath as indicated in Eq. (7). We can note that, initially in the cutoff timescale, the energy develops a jolt. This is just a transient and does not last long. The energy grows steadily for a while (proportional to $\gamma_0 k_B T$ for short times) and does not depend on the frequency cutoff Λ. It is important to note that we shall always study the dynamics between times $1/\Lambda \ll t \ll t_{\mathrm{sat}} \sim 1/\gamma_0$. As it is already known [12], the decoherence timescale is $t_D \sim 1/(2M\gamma_0 k_B T L_0^2)$ (very early for the environments of Fig. 3(a)). In the inset of that figure, we can see the timescale at which the linear entropy reaches its top value (unity for a mixed state). By this time, interferences have already been destroyed. Besides, the energy of the open system is always bigger than the one of the closed one. Note that not only the decoherence process but also the noise activation of the system depend strongly on the external temperature of the environment. The hotter the environment, the bigger the "activation" (thermal in this case). In Fig. 3(b) we can check its dependence on the value of the coupling constant γ_0 at a fixed temperature and cutoff frequency. As expected, the bigger the coupling constant γ_0, the bigger the increase of energy of the system. In Fig. 3(b) it is shown that the coupling strength sets the time at which the energy starts growing. The long time evolution of the energy in the high temperature case can be found in Ref. [5].

4.2. Nonohmic environments

The previous analysis can be also done for non-ohmic environments. However, expressions are not that easy to deal with since, in this case, neither $f(t)$ is negligible nor $\mathcal{D}$ constant. Thus, we shall restrict ourselves to a numerical analysis based on the thorough analytical study of the decoherence process we have made in a previous section.

We begin our analysis with the supraohmic one. In Fig. 4(a) and Fig. 4(b) we present the evolution in time of the mean energy of the system for different values of the coupling constant and the external temperature respectively. In both cases, it is easy to see that those curves where the initial jolt of the energy is bigger are those cases for which the mean energy will be bigger in the final state (top inset in Fig. 4(a)). In particular, therein, we can see that the mean energy is bigger as the coupling constant grows and in Fig. 4(b), we see the same behavior as the environment gets hotter. It is important to note that the strength of the environment is given by a relation between the three parameters γ_0, k_BT and Λ.

For "strong" supraohmic environments ($2Mk_BTL_0^2\gamma_0 \gg \Lambda$), decoherence happens in timescales of $t_{\mathcal{D}} \sim (\Lambda M\gamma_0 k_BTL_0^2)^{-1}$ since interferences are destroyed very rapidly. Surprisingly, those curves correspond to a considerably bigger value of the final mean energy of the system.

So far, we have shown that when the environment has "succeeded" in the destruction of the interferences, the exchange of energy with the system promotes its "activation". However, for not so strong environments ($Mk_BTL_0^2\gamma_0 < \Lambda$), such as $\gamma_0 = 0.001$ in Fig. 4(a), the interferences are not completely destroyed (the decoherence factor $\Gamma(t)$ is not zero), and then, the exchange of energy is not completely devoted to exciting the system (the environment still tries to suppress the coherences). This case differs qualitatively from the ohmic environment. In particular, it reaches an asymptotic limit in a very short timescale. In the inset of Fig. 4(a) we show the evolution of the mean energy for longer times, when the system enters in the asymptotic regime for the given set of parameters.

Finally, the noise activation is very clear in the case of the subohmic environment. In Fig. 4(c) we have plotted the mean energy of the system for different values of the coupling constant ($\gamma_0 = 0.001$ and $\gamma_0 = 0.005$), different temperatures ($k_BT = 10^5$ and $k_BT = 10^4$) and different cutoffs in the high temperature limit. It is easy to see that the energy grows faster as the coupling constant, the frequency cutoff and the temperature grow. If we recall the behaviour of the decoherence factor $\Gamma(t)$ or the decoherence timescale $t_{\mathcal{D}} \sim (M\gamma_0 L_0^2 k_BT)^{-1}$, we can check once more, that the "noise activation" is a post-decoherence phenomenon since it begins after the system has already lost its interferences.

5. Noise induced energy excitation at strictly zero temperature

As we have already mentioned, there is a widely spread misconception that decoherence tends to zero as a function of the temperature and therefore, there is no decoherence in the

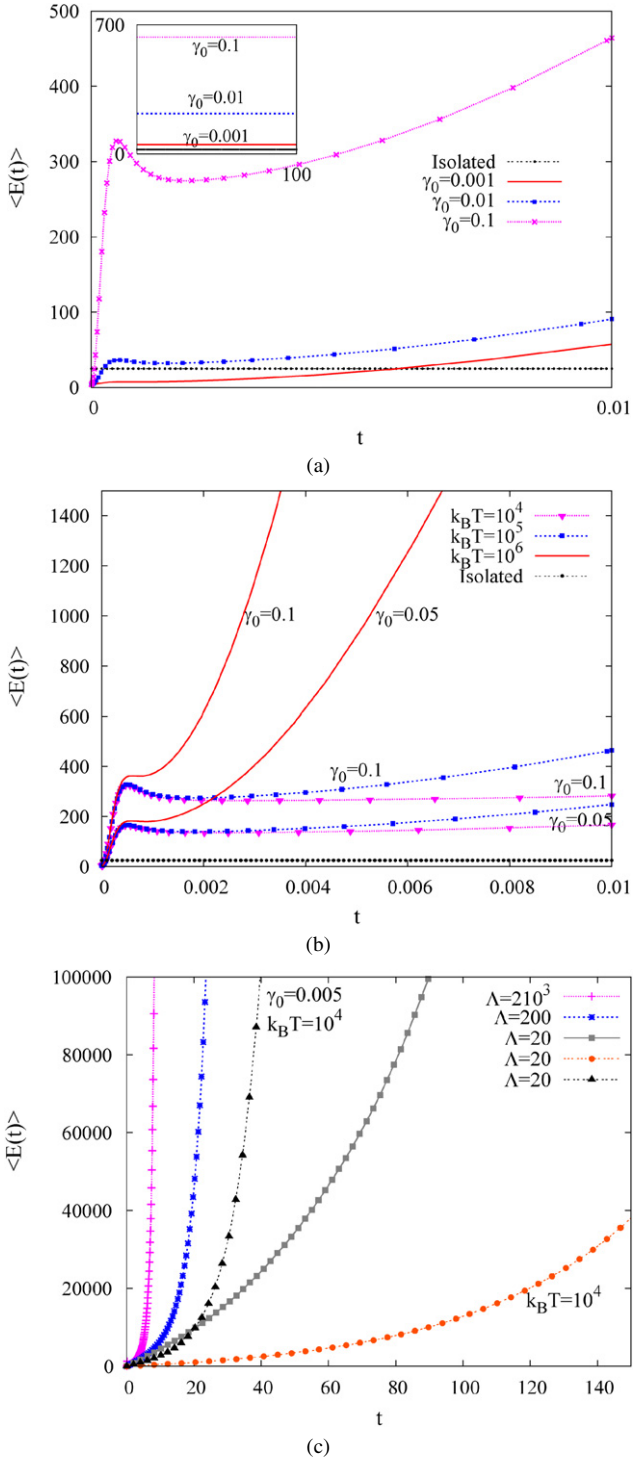

Fig. 4. (a) Evolution in time of the mean energy of the system coupled to an supraohmic environment ($n = 3$) in the high temperature limit. We consider the cases $k_BT = 10^5$, $\Lambda = 2000$, $\Omega = 0.1$, $L_0 = 2$ for different values of γ_0. In the inset, we have plotted the mean energy for longer times. (b) Evolution in time of the mean energy of the system coupled to an supraohmic environment ($n = 3$) in the high temperature limit. We consider the cases $\gamma_0 = 0.001$, $\Lambda = 2000$, $L_0 = 2$ for different temperatures of the environment and coupling constants γ_0. Parameters are measured in units of the bare frequency Ω in all cases. (c) Evolution in time of the mean energy of the system coupled to an subohmic environment ($n = 1/2$) in the high temperature limit $k_BT \gg \hbar\Lambda$. We consider the cases $\Omega = 0.1$, $L_0 = 2$, $\gamma_0 = 0.001$ (and $\gamma_0 = 0.005$ when noted) and $k_BT = 10^5$ ($k_BT = 10^4$ when noted) for different values of the frequency cutoff Λ. Parameters are measured in units of the bare frequency Ω in all cases.

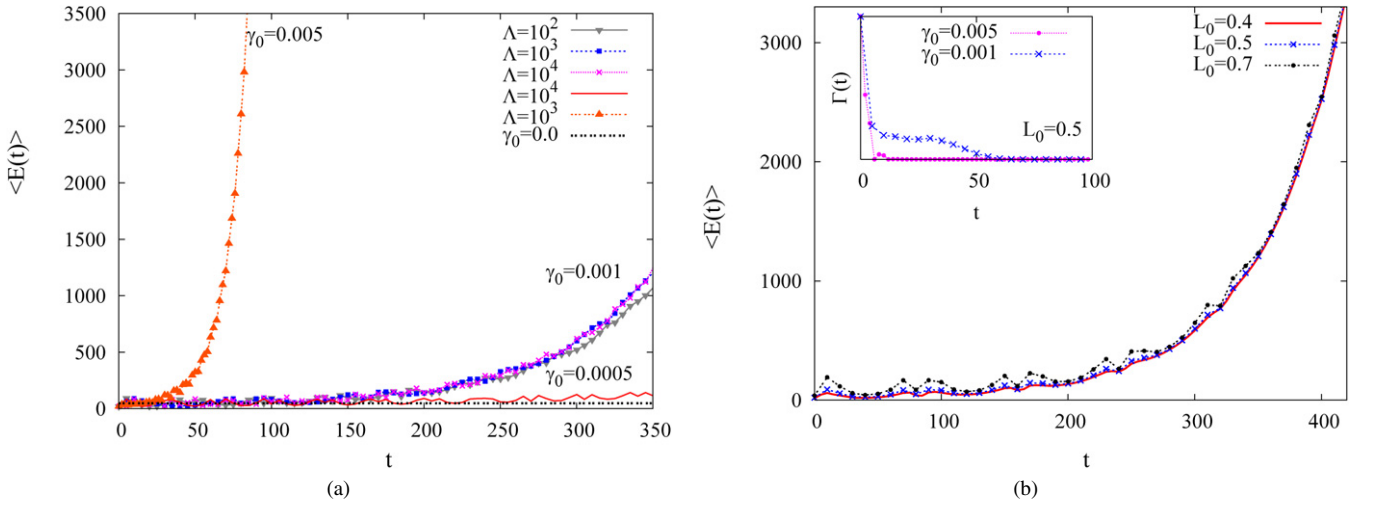

Fig. 5. (a) Evolution in time of the mean energy of the system coupled to an ohmic environment ($n = 1$) at zero temperature. It is clear to see that there is a dependence upon the coupling constant but not on the frequency cutoff. (b) Evolution in time of the mean energy of the system coupled to an ohmic environment ($n = 1$) at zero temperature. The parameters are measured in units of the bare frequency Ω. We consider the case $\gamma_0 = 0.001$, $\Omega = 15$, $\Lambda = 1000$ for different values of L_0. In the inset, we show the time evolution of the decoherence factor for different values of γ_0. Energy excitation actually starts after decoherence becomes effective. The parameters are chosen based on numerical convenience.

limit of zero environmental temperature. If that were the case, physics should be different in the opposite thermal limit. Many questions consequently arise. What do we expect to find if the system is coupled to an ohmic environment at zero temperature? Is it possible to find "activation" in the system? The latter answer is unexpectedly positive. Yet more, we shall see that there is a close connection between decoherence and noise activation since the most "decoherent" environments are the ones with the most visible activation phenomenon. Once more, we shall show that the latter is a post-decoherence process taking place after the interferences have been suppressed.

When trying to interpret the post-decoherence behavior of the open system, several features of its dynamics should be kept in mind. Firstly, one should emphasize that the initial condition is clearly not the ground state of the total action [5]. As soon as the interaction between the main system and the environment is turned on, at $t = 0$, the system will find itself in an excited energy state. The environment will have a non-zero amount of energy in relation to the new initial state. From a purely classical point of view, this energy cannot be responsible for the excitation of the particle to higher values. This argument can be made more quantitative in the following way: the full potential for the system plus environment is

$$V(x, q_n) = V_{\text{sys}}(x) + V_{\text{env}}(q_n) + V_{\text{int}}(x, q_n), \tag{8}$$

with $V_{\text{sys}}(x) = -\frac{1}{2}\Omega^2 x^2$, $V_{\text{env}}(q_n) = \sum_n \frac{1}{2}\omega^2 m_n^2 q_n^2$ and $V_{\text{int}}(x, q_n) = \sum_n C_n x q_n$. Classically, the initial condition is $x = 0$, and, because the environment is at $T = 0$, $q_n = 0$. So, for the full action, the energy terms of the initial condition are given by $V_{\text{sys}} = 0$ (the minimum of V_{sys}), $V_{\text{env}} = 0$, and $V_{\text{int}} = 0$. Consequently, the total initial potential energy of the system plus environment is $V = 0$. Note that classically, the value of the total energy is the same as the energy of the isolated main particle, even when the interaction with the environment is "switched on". This is a consequence of taking zero temperature for the environment. The quantum fluctuations present in the initial state of the environment must play a role in the "activation" [5]. One should note that these fluctuations are not "vacuum fluctuations" of the full system. Nonetheless, the fact that they have such a clear effect on the evolution of the system is quite remarkable. Whereas in the high-T case the quantum and classical oscillators composing the bath had identical distributions, they behave in a markedly different way as $T \to 0$. The quantum nature of the environment, which could be ignored at high-T, leads in this limit to important non-negligible effects.

In terms of the master equation, the quantum fluctuations of the bath oscillators generate non-zero $f(t)$ and $\mathcal{D}(t)$ terms, making diffusive phenomena possible. This is particularly true in the case of the anomalous diffusion coefficient $f(t)$. In the ohmic case it can be shown that it depends logarithmically on the cutoff Λ and thus can be considerably large [3]. Diffusion effects induced by quantum fluctuations are thus responsible for exciting the particle. Though this process is very different from high-T thermal activation, we conjecture that it may still be interpreted in terms of a modified classical setting. The key ingredient is that the classical bath should mimic the properties of the quantum $T = 0$ environment. Considering the classical and quantum versions of the noise kernel $\nu(s)$, it is possible to show that a bath of classical oscillators with a frequency dependent temperature $T(\omega) = \hbar\omega/2$ should reproduce the effects of the initial quantum state. In fact, for this choice of classical environment one obtains $f(t)$ and $\mathcal{D}(t)$ terms identical to those of the $T = 0$ quantum case. Our main point is that after decoherence takes place, a quantum open system at $T = 0$ should behave as a classical open system in contact with a classical bath whose oscillators are excited in a way that reproduces the fluctuations of the corresponding quantum environment [5]. Hereafter, we shall show noise induced activation happening in different environments a strictly zero temperature.

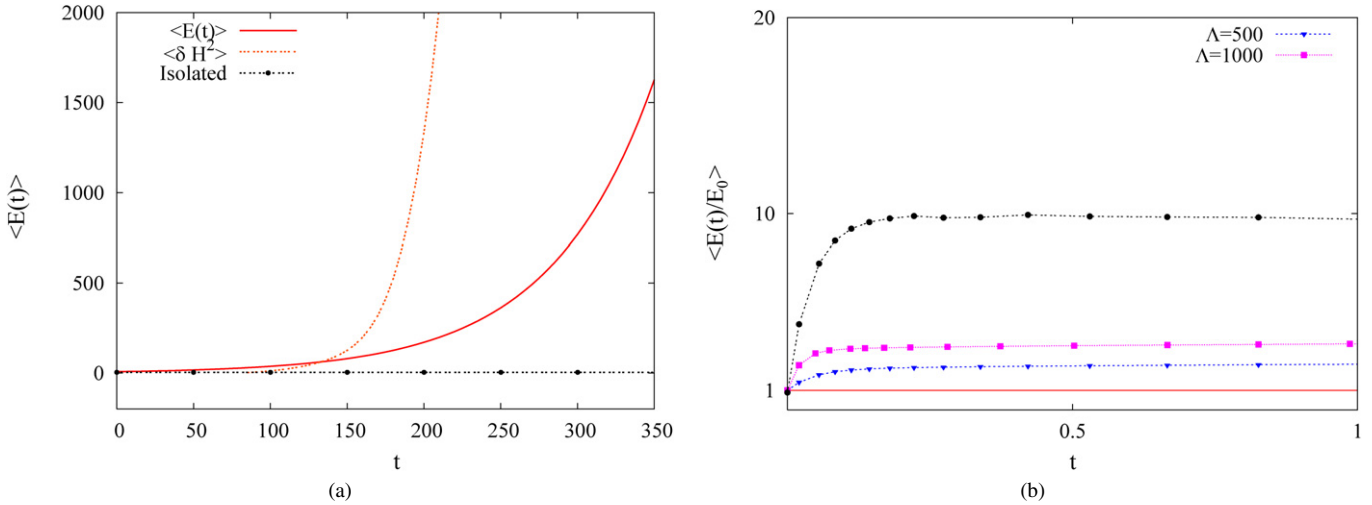

Fig. 6. (a) Evolution in time for the mean energy of the system and the mean dispersion of the energy for an ohmic environment at zero temperature. It is also shown the mean energy of the isolated system. Parameters used: $\Lambda = 10^3$, $\Omega = 15$, $L_0 = 0.5$ and $\gamma_0 = 0.001$. Fluctuations in energy are relevant in the excitation process. The parameters are measured in units of the bare frequency Ω in all cases. (b) Long time behaviour for an ohmic environment at zero temperature for different values of the parameters. In all cases the value of the mean energy of the Brownian particle is bigger than the mean energy of the isolated system (solid line). The parameters used are measured in units of γ_0 so as to include different runs in the same figure. We consider the case $\Omega = 1$ and $\gamma_0 = 0.1$ and different values of L_0: $L_0 = 0$ which means only one Brownian particle (squares and triangles) and $L_0 = 0.5$ (circles). E_0 is the energy of the isolated system. Parameters were deliberate choosen due to the long-time difficulty in the numerical runs.

5.1. Ohmic environments

In Fig. 5(a) we numerically prove our statement for an ohmic environment at zero temperature. Therein, we have plotted the time evolution of the mean energy of the system for different values of the frequency cutoff and the coupling constant γ_0 for an ohmic environment. The bigger the value of γ_0, the sooner the energy of the system grows. Initially, the mean energy of the system is lower than that it would be if the system was isolated (no interaction with the environment). However, after a time $t \geqslant t_{\mathcal{D}}$, timescale for which the interferences have already been suppressed due to decoherence, the mean energy increases considerably because of the interaction with the environment. The system gains energy at the expense of the environment which turns out to be an energy source. Clearly, it is a "post-decoherence" process, very much like the thermal activation at high temperature. In Fig. 5(b), we have plotted the mean energy of the system for different values of the initial distance between the Gaussian wave packets L_0 [16]. As expected, the energy does not depend much on this latter parameter: the bigger the initial distance L_0 is, the sooner the decoherence process takes place and the "noise activation" starts (since decoherence times is in general proportional to L_0^{-2}). But this time difference is subtle. In the inset of Fig. 5(b), we can observe the exponential decay of $\Gamma(t)$ which indicates the timescale at which interferences are completely destroyed. Notably, this timescale coincides with the beginning of the "noise-induced activation" phenomenon.

In Fig. 6(a) we have plotted the mean energy of the open and closed system and the mean dispersion of the energy of the open system for zero temperature. Obviously, the mean energy of the closed system remains steady. In the open case, we can see that, initially, it is lower than the isolated case, but immediately starts growing. The dispersion of the energy shows that the fluctuations are extremely important and are responsible for the increase of the mean energy since it is an uniform growing function of time.

Finally, we would like to show the correct long time behaviour of the mean energy. Consequently, we numerically solved the master equation for times of the order of the saturation time $t_{\text{sat}} \sim 1/\gamma_0$. As we are working in the underdamped limit, achieving these long timescales is a numerically expensive task. Nonetheless, we present the correct asyntotic behaviour by an appropiate selection of the physical parameters. In Fig. 6(b) we show the correct long time behaviour for the ohmic environment at zero temperature for different values of the parameters. In all cases the value of the mean energy of the Brownian particle is bigger than the mean energy of the isolated system (solid line).

5.2. Nonohmic environments

We shall extend the above analysis to the supraohmic environments. For example, in Fig. 7(a), we have plotted the time evolution of the mean energy of the system at zero temperature. Therein, we can see that the energy has the initial jolt but at long time it remains constant (see bottom inset). It reaches a value and stays with that energy for ever since much of what happens in the supraohmic environment is just a consequence of that initial jolt, not a dynamic response. By the way, this type of environment at strictly zero temperature, does not induce effective decoherence on the system. No decoherence implies no energy activation. In Fig. 7(a) we have included a big value of γ_0 ($\gamma_0 = 0.5$) which produces decoherence and therefore energy activation. However, this value is not appropriate because it is not well covered by the underdamped approximation (i.e., weakly coupling with the environment). All the other values of γ_0 included in the figure, correspond to environments not

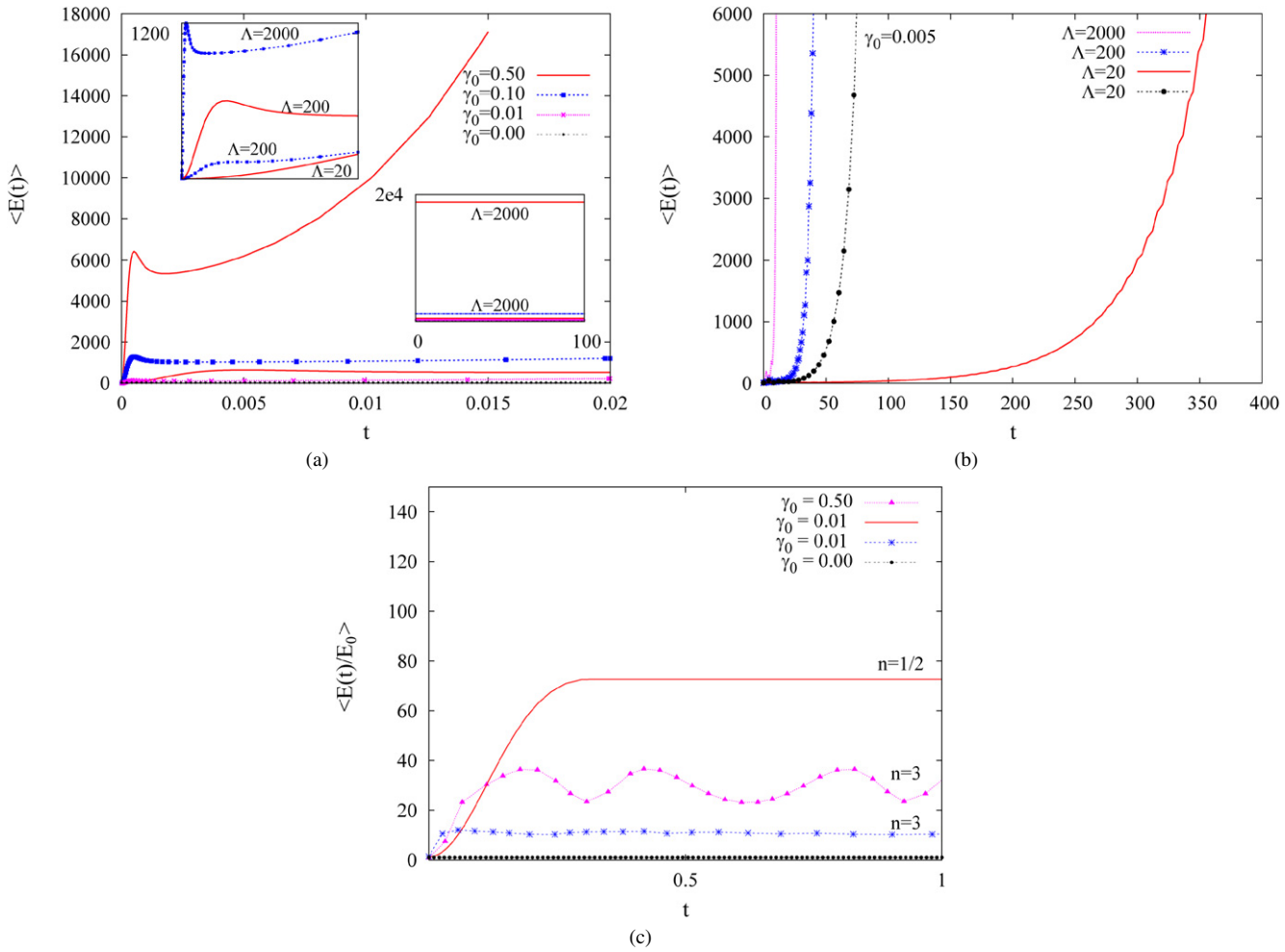

Fig. 7. (a) Evolution in time of the mean energy of the system coupled to an supraohmic environment ($n = 3$) at zero temperature. The parameters are measured in units of the bare frequency Ω. We consider the cases $\Lambda = 2000$, $\Omega = 0.1$, $L_0 = 2$ for different values of the cutoff frequency and the coupling constant γ_0. In the top inset, we have plotted the mean energy in a shorter scale to see the dependence on the frequency cutoff. In the bottom inset, we have plot the mean energy for longer times for some values of the main plot. (b) Evolution in time of the mean energy of the system coupled to an subohmic environment ($n = 1/2$) at zero temperature. The parameters are measured in units of the bare frequency Ω. We consider the cases $\Omega = 0.1$, $L_0 = 2$, $\gamma_0 = 0.001$ for different values of the frequency cutoff Λ. We also plotted the case $\gamma_0 = 0.005$ and $\Lambda = 20$ in order to compare. (c) Long time behaviour for non-ohmic environments at zero temperature. We consider the case of supraohmic $n = 2$ and subohmic $n = 1/2$ environments. The parameters used are measured in units of γ_0 so as to include different runs in the same figure. We consider the case $\Omega = 1$ and $L_0 = 2$ for different values of γ_0 so as to distinguish between "strong" and "weak" supraohmic environments. E_0 is the energy of the isolated system. Parameters were deliberate choosen due to the long-time difficulty in the numerical runs.

strong enough to neither destroy the interferences nor "activate" the system.

As for the subohmic environment at zero temperature, we can observe the same dynamics as in the high temperature limit, but on a longer timescale as it is shown in Fig. 7(b). In this case, it is also possible to check that the activation timescale t_{act} is subsequent to the decoherence timescale $t_{\text{D}} \sim \Omega/(\Lambda\gamma_0)$ with the help of Fig. 7(b).

Finally, in Fig. 7(c) we show the long time evolution of the mean energy for non-ohmic environments for different values of the parameters. In all cases, we can see that an asyntotic behaviour is reached for the very long timescales.

6. Final remarks

We have studied general environments, namely ohmic and non-ohmic and showed that the evolution of a QBM particle can be summarized in terms of two main environmental induced physical phenomena: decoherence and energy activation.

We have presented the different physical features of the non-unitary dynamics in the case of a quantum system coupled to a general environment at zero temperature. In this thermal regime, the quantum fluctuations of the environmental oscillators, absent in a purely classical evolution, lead to non-zero diffusive terms. Their effect is felt primarily through the anomalous diffusion coefficient $f(t)$ that can have a large magnitude in the ohmic and subohmic environments. The supraohmic case is weakly diffusive due to the fact $f(t)$ goes to zero after the initial transient. In this case, both decoherence and energy excitation, depend strongly on the coupling constant between system and bath in order to generate remarkable effects. Yet more, we have confirmed previous results on decoherence in quantum field theory by the use of this model.

We have shown that these non-trivial diffusion effects induced by the quantum environment are large enough to excite the particle to higher energy levels. Particularly it is a post-decoherence process which means that no total decoherence implies no energy activation, as was clearly shown in the case of a supraohmic environment at zero temperature. This is to be contrasted with the case where the environment is classical forbidding any kind of activation phenomena. Though the late time evolution in the presence of a quantum vacuum is in nature very different from high-T thermal activation, we suggest that it could still be interpreted in terms of a purely classical setting, if the environment oscillators are described by a particular non-thermal statistical state. We will pursue this line of enquire in depth in a forthcoming publication.

A system entangled with environment states has a number of properties which distinguishes it from systems for which the ground state factorizes. In order to understand the physics underlying the entanglement process, it is important to consider the evolution of the energy of the main system. The energy is always an observable, and at zero temperature, fluctuations in the energy of the main system are a direct indication of the system-environment interaction. We have shown that the process we have called noise induced energy activation is a post-decoherence process, therefore, the evolution of the linear entropy shows that the state of the system in all of these cases is a mixed state. Thus, energy here can be used as a measure of the degree of entanglement for mixed states, under a non-unitary evolution. What's more, simple systems with well known isolated quantum mechanical properties become "entanglement meters" as nicely explained by Jordan and Büttiker in [4].

Acknowledgements

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