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The Simulation of Situation Models Aids Analogical Transfer Between Algebra Problems

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Several studies on analogical transfer to algebra word problems have demonstrated that adapting solutions learned from worked examples to nonisomorphic problems of the same type is challenging and that most instructional aids do not alleviate this difficulty. At the same time, various authors have suggested that transfer difficulties sometimes originate in students' lack of disposition to relate algebraic formulas to the real-world situations to which they refer. We designed a noninteractive intervention encouraging students to elaborate situation models for base and target problems, and to ground algebraic formalisms in these representations. One experimental group simulated situation models by physical object manipulation, whereas another experimental group performed those simulations mentally. Both conditions outperformed a control group that did not run simulations. This intervention was more effective when the transformations posed by target problems were intrinsically more difficult to assimilate into the learned equation. Implications for the design of instructional interventions are discussed.

Keywords: analogy; transfer; algebra word problems; problem solving; situation model simulation

Solving a problem by analogy implies transferring a solution from a known problem (the *base problem* [BP]) onto a new problem whose solution is unknown (the *target problem* [TP]). In learning environments, the solution to a BP serves as scaffolding for the application of a general method until the student has gained the fluidity required to apply this general method directly (LeFevre & Dixon, 1986; Pirolli & Anderson, 1985). In a series of studies of analogical transfer to algebra word problems, Reed, Dempster, and Ettinger (1985) provided participants with a worked example of a basic task-completion problem (i.e., a step-by-step solution to a concrete problem), and investigated the extent to which college students can adapt the learned procedure to solve problems with structural variations. For example,

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after learning to calculate the time that two painters would require to jointly paint a wall given the amounts of time that each one would need to paint it on his own, a new problem introducing a structural variation could be one in which the problem text stated that, overall, one of the painters worked 1 hr more than the other. When solving this TP by analogy to the worked example, the extra hour painted by the second painter needs to be introduced into the original equation: $r_1 \cdot t_1 + [r_2 \cdot (t_1 + 1 \text{ hr}) = 1$ (with r_1: rate of the first painter; r_2: rate of the second painter; t_1: time taken by the first painter). Reed et al. (Experiments 3 and 4) found that although 70% of the students transferred the base solution to structurally equivalent TPs, only 12% could solve problems introducing structural variations. In subsequent studies, Reed and colleagues complemented worked examples with several instructional aids intended to help students introduce the structural variations into the base equations (e.g., construction of tables relating base and target quantities and variables, Reed & Ettinger, 1987; provision of rules for introducing specific structural variations into the base equations, Reed & Bolstad, 1991; and instruction on unit cancelation, Reed, 2006). At least regarding task completion problems like those used in this study, the earlier mentioned interventions were generally unsuccessful, with success rates not surpassing 35% (see Reed 1999, for a review). According to Reed (2006), a possible limitation of this kind of interventions may have been their almost exclusive focus on algebraic operations.

Several studies have found that when students engage in elaborations such as generating self-explanations or decomposing solution procedures into different subgoals, they improve their ability to transfer solutions learned from worked examples to novel but related problems (see, e.g., Chi, Bassok, Lewis, Reimann, & Glaser, 1989). However, as conventional worked examples rarely elicit the spontaneous production of this kind of elaborations (Renkl, 1997), various successful interventions have been developed to promote them (e.g., Gerjets, Scheiter & Catrambone, 2006; Renkl, Stark, Gruber, & Mandl, 1998).

As advocated by various authors (e.g., Koedinger, Anderson, Hadley, & Mark, 1997; Nathan, 1998; Nathan, Kintsch, & Young, 1992; Reed, 2006), a successful way of aiding the construction of equations for word problems consists in stimulating students to ground algebraic formalisms in the situation models of the problems, that is, in an approximate perceptual-like representation about how the events described by the problem text take place in the real world. As opposed to experts, novices do not tend to ground equations in situation models, thus failing to notice, for example, when a problem's solution—albeit mathematically correct—leads to a semantically absurd situation (Paige & Simon, 1966). Greeno (1989) explains many of these results as instances where abstract and situational representations coexist as disconnected systems, leading problem solvers to perform operations on symbolic expressions that are no longer faithful to the situations to which they are intended to refer. It is by establishing a correspondence of the symbols to the situation that one roots the formalisms into the space of permissible and expected events (Greeno, 1989). Based on evidence that fostering a connection between formalisms and their real-world referents helps participants build equations (e.g., Nathan et al., 1992), our main purpose was to determine whether a stimulation to ground equation structures in the situation models of the problems would also help participants adapt learned solutions to novel algebra problems with structural variations.[1]

Situation model simulations of certain algebra problems can be carried out either physically (e.g., via manipulating physical objects) or mentally. In domains as diverse as text comprehension (Glenberg, Gutierrez, Levin, Japuntich, & Kaschak, 2004), metaphor comprehension (Wilson & Gibbs, 2007), and arithmetic (Glenberg, Willford, Gibson, Goldberg, &

Zhu, 2011), physical and imagined simulations yielded comparable effects. A second objective of this study was to assess whether imagined simulations are as effective as physical simulations in helping participants transfer solutions to TPs with structural variations.

The type of variations imposed by TPs has received little attention within studies of analogical transfer to algebra word problems. In a recent study, Booth and Koedinger (2012) found that the advantage of complementing story problems with diagrams was greater for the more difficult problems. Our third objective was thus to determine whether a stimulation to base the transfer process in situation models of the problems interacts with the difficulty inherent in assimilating a given structural variation into the learned equation (henceforth, *adaptation difficulty*). In what follows, we flesh out a theoretical proposal about the relative adaptation difficulty posed by each of the TPs used in this study.

Figure 1 displays the task-completion problems used as BP and TPs, together with Reed's (1987) representation of the different levels at which the basic equation for task-completion problems can be represented. In Reed's schema, knowledge is represented as a propositional web of nodes (concepts) and predicates (attributes and relations). Concepts appear in rectangles, operations and relations between concepts appear in circles, and numerical values (or the procedures needed to obtain them) appear in ovals. Vertical lines indicate permissible substitutions. While the highest level corresponds to the upper principle $w_1 + w_2 = w_{total}$, the intermediate level corresponds to the variables r and t; when multiplied, their product conforms the left terms of the principle. The lowest level refers to the known and unknown quantities and the procedures needed to obtain them. We added dotted lines to represent the variations introduced by the different TPs used in this study. Based on Reed's idea about the different levels at which the equation of task-completion problems can be represented, it was hypothesized that the adaptation difficulty of a given structural variation is determined by the degree of comprehension of the equation structure that is required to assimilate such variation into the base equation.

In the TP with a speed variation (TP*speed*), calculation of the speed of one of the painters only requires that one knows the amount of time it would take him to finish the task on his own. This value is obtained by doubling the amount of time taken by the other painter to finish the task. Given that knowledge of the equation structure is not involved in this variation, it was postulated that it would not pose adaptation difficulties.

In the TP with a work variation (TP*work*), there are two ways of assimilating the prepainted chunk of wall into the base equation: (1) $r_1 \cdot t + r_2 \cdot t + \mathbf{1/3} = 1$ and (2) $r_1 \cdot t + r_2 \cdot t = 1 - \mathbf{1/3}$. Because both alternatives presuppose comprehension of the upper level of the equation structure (i.e., $w_1 + w_2 = w_{total}$), it was postulated that assimilating this variation into the equation would be more difficult than in TP*speed*.

In the TP with time variation (TP*time*), there are three ways of introducing the extra hour spent by the second painter into the base equation: (1) $r_1 \cdot t + r_2 \cdot t + \boldsymbol{r_2} \cdot \mathbf{1hr} = 1$; (2) $r_1 \cdot t + r_2 \cdot t = 1 - \boldsymbol{r_2} \cdot \mathbf{1hr}$, and (3) $r_1 \cdot t + r_2 \cdot (t\ 1\ \mathbf{1hr}) = 1$. In all of the cases, the problem solver needs to understand that there is a certain amount of time during which the second painter works unaccompanied by the first painter, and that the portion of wall corresponding to that period comes from multiplying his speed (r_2) by such duration (1h), as dictated by the intermediate level of the equation structure. Because all three alternatives require comprehending not only the upper but also the intermediate level of the equation structure, we postulated that assimilating this variation into the equation would be more demanding than either TP*work* or TP*speed*.

(a)

Base problem (BP): Peter can paint a wall in 10 hours, while John can paint that same wall in 15 hours. If they start painting the wall together at 12 o'clock, at what time will it be finished?

Target problem with speed variation (TP*speed*): Ned can paint a wall in 8 hours, while Louis takes twice as long to paint the same wall. If they start painting the wall together at 12 o'clock, at what time will it be finished?

Target problem with work variation (TP*work*): Bob can paint a wall in 20 hours, while Mark can paint that same wall in 12 hours. One third of the wall has been painted by other painters. If Bob and Mark start painting what is remaining of the wall at 12 o'clock, at what time will it be finished?

Target problem with time variation (TP*time*): Fred can paint a wall in 8 hours, while Bob can paint that same wall in 12 hours. They mostly paint it together but, overall, Bob paints one more hour than Fred. If painting started at 12 o'clock, at what time will it be finished?

(b)

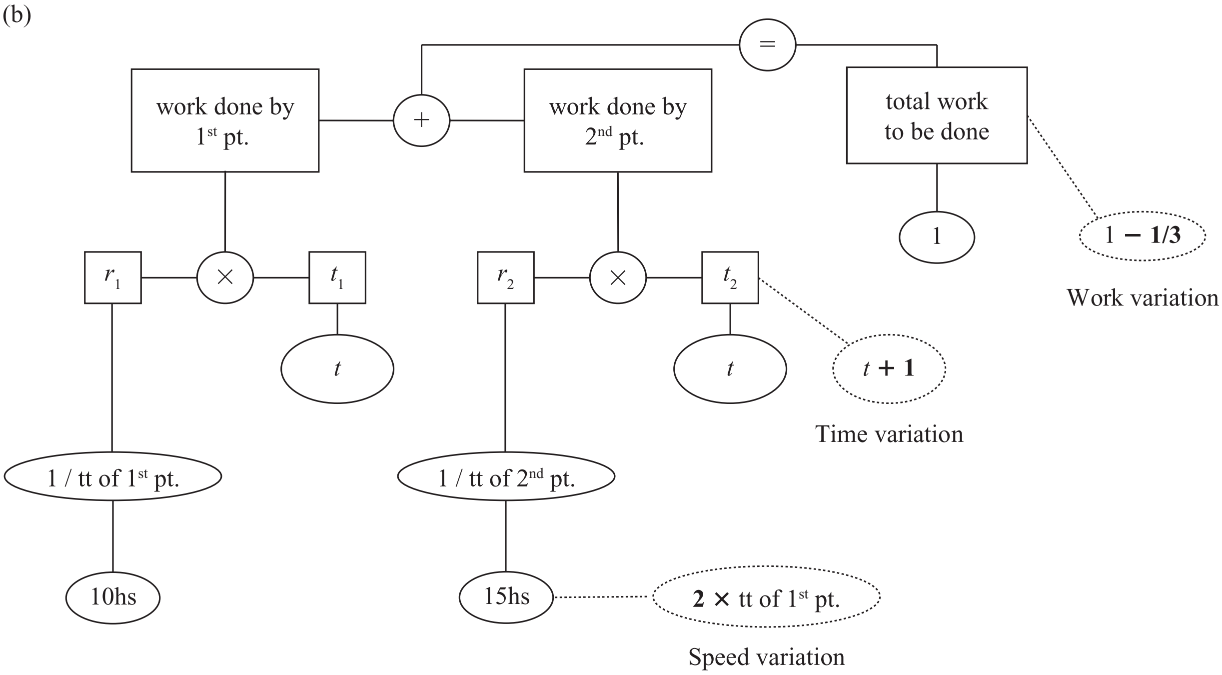

FIGURE 1. Top (a): Base problem and target problems used in the present experiment. Bottom (b): Hierarchical representation of the equation structure of task-completion problems (adapted from Reed, 1987). Dotted lines indicate the adaptation required by each of the target problems. *Note.* tt: total time; pt: painter.

To assess whether transfer was in fact more difficult for TP*time* than for TP*work* and more difficult for TP*work* than for TP*speed*, an independent group of eighteen 12th graders at Estación Limay High School (the same population as in the experiment reported here) received the BP and its solution. Each participant was asked to solve each of the TPs based on the BP and its solution. Transfer performance was 72% for TP*speed*, 56% for TP*work*, and 17% for TP*time*, thus supporting our theory-driven prediction concerning the adaptation difficulties posed by each TP. Our third objective was thus to assess whether the benefit of our intervention, as was the case with Booth and Koedinger's (2012) study, would be greater for TPs for which transfer is intrinsically more demanding.

In the present experiment, all participants studied a worked example of a task-completion algebra problem prior to solving the three TPs presented in Figure 1. Whereas the physical simulation group (PSG) had to simulate situation models of the BP and the TPs via manipulating physical objects, the imagined simulation group (ISG) was asked to simulate those situation models internally. The no simulation group (NSG) was not asked to simulate the situation models during any phase of the experiment.

METHOD

Participants and Design

Sixty 12th year students at Estación Limay High School in Rio Negro, Argentina, volunteered to participate in the experiment. They were randomly assigned to each of the three groups (20 to the PSG, 20 to the ISG, and 20 to the NSG). In this 3×3 design, the type of simulation of the situation models of the problems (physical, imagined, and no simulation) received between-subjects manipulation, whereas the adaptation difficulty of each of the TPs (low, medium, and high) received within-subjects manipulation.

Materials and Procedure

All participants were told that their task would be to carry out a problem-solving activity comprising a pretest, an instruction on how to solve such type of problems, and three test problems. After describing the procedure followed with the PSG, we indicate the differences between the PSG and the other groups.

At pretest, participants of the PSG were given the BP and were allotted 5 mins to solve it by any means possible. However, they were encouraged to construct an appropriate equation. Because no participants gave an appropriate equation at pretest, all the participants were required to carry out the subsequent phases of the experiment.

During the instructional phase, participants of the PSG were first asked to perform a qualitative simulation of the situation depicted by the problem text, in which the objects and their corresponding interactions are represented with approximations rather than exact magnitudes. They were presented with a magnetic rectangle vertically attached to its base (i.e., a toy wall) and two toy painters located at opposing edges of the visible side of the wall. Participants were asked to read the BP aloud at a slow pace. After each sentence was completed, the experimenter asked participants to simulate the content of the sentence with the given materials. In this way, they were asked to simulate each painter painting the wall alone and then to use both painters together to represent how the situation unfolds from beginning to end. During the simulation, participants had to indicate which aspect of the situation the problem was asking them to calculate. After carrying out this qualitative simulation of the BP, participants were given 5 mins to study a worked solution for the BP (Table 1), which remained available during subsequent phases.

Once the allotted study time had elapsed, participants were required to carry out a simulation of the quantitative situation model of the problem and to relate each component of the quantitative simulation to its corresponding expression in the equation. As opposed to the qualitative situation model—in which the unknown magnitudes were represented with approximations—the quantitative situation model requires taking into account the exact magnitudes taken from the problem's solution. To this end, participants were provided with a toy clock set to 12 o'clock and 20 small boxes, each containing several magnetized strips with widths corresponding to fractions of the wall (from 1/2s to 1/20s), and labeled accordingly. Participants received a sheet of paper displaying the base equation with its calculated

TABLE 1. Worked Example of the Task-Completion Problem Used as Base Analog in the Experiment

(A) Base problem: Peter can paint a wall in 10 hours, while John can paint that same wall in 15 hours. If they start painting the wall together at 12 o'clock, at what time will it be finished?

Solution: This problem is a work problem in which two people work together to complete a task. The amount of task completed by each person is found by multiplying his rate of work by the amount of time he works, which is expressed as follows: Rate of work $\times$ Time of work $=$ part of work done

Because Peter takes 10 hr to paint the wall, he finishes 1/10 of the wall in 1 hr. In t hr he finishes $1/10 \cdot t$. John finishes 1/15 of the wall in 1 hr. In t hr he paints $1/15 \cdot t$. The following table summarizes this information:

Worker	Rate of Work (Part of Task/hr)	Time of Work (hr)	Work Done (Part of Task)
Peter	1/10	t	$1/10 \cdot t$
John	1/15	t	$1/15 \cdot t$

If the task is finished, the sum of the fractional part finished by Peter and the fractional part finished by John must equal 1; $(1/10) \cdot t + (1/15) \cdot t = 1$

Solving for t yields the following: $(1/10 + 1/15) \cdot t = 1$

then, $(3/30 + 2/30) \cdot t = 1;$

Finally, $t = 30/5\ h = 6h.$

Answer: if they started at 12, then they finish at 6.

values—that is, $1/10 \cdot 6$ hr $+ 1/15 \cdot 6$ hr $= 1$—and requiring them to perform a series of tasks. First, they had to (a) indicate where in the equation the part completed by the first painter was represented, and (b) represent in the wall how many times the first painter completed his hourly portion of the wall (this could be accomplished by placing the appropriate strips on the wall and advancing the arrows of the clock each time the first painter completed his hourly chunk of wall). This two-step sequence was repeated for the second painter. Finally, participants were asked to (a) mark in the equation the part that represents the sum of the fractions painted by both painters and (b) represent with the strips how both painters worked together on an hour-by-hour basis.

During the transfer phase, participants received the three TPs displayed on the top part of Figure 1, presented in counterbalanced order. After receiving each TP, participants ran a qualitative simulation of its situation model (i.e., only the wall and toy painters were available). After the qualitative simulation of the TP was completed, participants were asked to carry out the qualitative simulation of the BP once again and to identify any differences between them. After comparing each TP to the BP in terms of their qualitative situation models, participants were given 6 mins to find the appropriate equation for the TP via adapting the learned equation. The generic equation for task-completion problems was displayed below the text of each TP (i.e., $r_1 \cdot t_1 + r_2 \cdot t_2 = w_{total}$). A rectangle was provided to introduce the equation for the problem, and blank space was left to write down the necessary calculations. Because participants were not required to solve the equation, the lower part of the page was left for participants to explain how the solution to the equation, if calculated, should yield an answer to the problem. This additional information would allow us to determine whether participants understood the relation between the unknown and the final answer to the problem. For example, in TP*time*, the solution to the most common correct equations yields the time taken by the first painter, who is said to have worked 1 hr less than the second painter. Given that the painting started at noon, calculating at what time the wall will be completed requires that the problem solver adds 1 hour to the eventual solution of the proposed equation. This sequence was repeated for each of the TPs, and all simulations were videotaped.

The procedure followed by the ISG was identical to that of the PSG, except for the fact that all simulations were carried out internally. Prior to running each simulation, participants of the ISG were presented with the materials used by the PSG for their corresponding activities (i.e., the wall and toy painters for the qualitative simulations, and the wall, painters, clock and boxes of strips for the quantitative simulation of the BP), which remained visible throughout the internal simulations. Even though participants in the ISG were encouraged to include the materials in their simulations, they were asked to close their eyes and to perform the simulations mentally.

The procedure followed by the NSG was similar to that of the simulation groups except that each simulation task was replaced by a non-simulation task equivalent to the former in terms of the reconsideration of problem information it elicited (see Table 2 for a detailed comparison between conditions). Neither group received feedback while performing any of the tasks.

RESULTS

Because all participants failed to formulate an equation for the BP at pretest, no participants were excluded from the analysis. Solutions to TPs were scored as correct only when (a) all given and inferable data were correctly incorporated in the equation and (b) the participant correctly stated how the solution of the equation, if calculated, would yield the correct answer. Figure 2 displays the percentages of correct solutions in the three transfer conditions of the study.

Across TPs, transfer to problems with structural variations averaged 68%, which clearly surpasses performance observed by Reed and colleagues in several studies (see Reed, 1999, for a review). Because of ceiling effects in TP*speed* (95% of correct solutions in the NSG), the subsequent analysis of the advantages of the simulation of situation models on analogical transfer will be limited to the performance measures of TP*work* and TP*time*. A 2×3 mixed effects analysis of variance (ANOVA) with repeated measures was accomplished to assess the effects of the adaptation difficulty (TP*work*: medium; TP*time*: high) and the type of situation model simulation (PSG, ISG, and NSG) on analogical transfer. Main effects were observed for both independent variables: adaptation difficulty, $F(1, 60) = 31.933$, $MSE = 0.104$, $p < .001$, and type of situation model simulation, $F(2, 60) = 10.476$, $MSE = 0.222$, $p < .001$. An interaction between both factors was also found to be significant, $F(2, 60) = 3.433$, $MSE = 0.104$, $p < .05$.

TABLE 2. Procedure Followed in the Three Different Transfer Conditions

Physical Simulation Group	Imagined Simulation Group	No Simulation Group
Pretest: Participants attempt to build an equation for the BP.	*Pretest:* Participants attempt to build an equation for the BP.	*Pretest:* Participants attempt to build an equation for the BP.
Instructional phase: (participants are presented with a toy wall and two toy painters)	*Instructional phase:* (participants are presented with a toy wall and two toy painters)	*Instructional phase:* (not applicable)
1. Qualitative simulation of BP with physical objects. Participants are required to: (a) Simulate the time it takes each painter to paint the wall.	1. Qualitative simulation of the BP, carried out internally: Participants are required to: (a) Simulate the time it takes each painter to paint the wall.	1. Equivalent non-simulation tasks. Participants are required to: (a) Reread the problem text.
(b) Simulate them working together.	(b) Simulate them working together.	(b) Write down which painter is fastest, and which one paints a larger portion of the wall.
(c) Indicate what aspect of such situation the problem is asking them to find out.	(c) Indicate what aspect of such situation the problem is asking them to find out.	(c) Write down what the problem is asking them to find out.
2. Participants study a solution to the BP (5 min).	2. Participants study a solution to the BP (5 min).	2. Participants study a solution to the BP (5 min).
3. Quantitative simulation of the BP and relation between the equational structure and the situation model:	3. Quantitative simulation of the BP and relation between the equational structure and the situation model:	3. Equivalent non-simulation tasks:
(a) In the equation, participants indicate the representation of the fraction painted by the first painter.	(a) In the equation, participants indicate the representation of the fraction painted by the first painter.	(a) In the equation, participants indicate the representation of the fraction painted by the first painter.

(Continued)

TABLE 2. Procedure Followed in the Three Different Transfer Conditions

Physical Simulation Group	Imagined Simulation Group	No Simulation Group
(Participants are presented with boxes of strips and a toy clock)	(Participants are presented with boxes of strips and a toy clock)	(not applicable)
(b) Physical simulation of how the first painter paints his part of the wall. This is done by placing the appropriate strips on the wall and advancing the toy clock after the placement of each strip.	(b) Internal simulation of how the first painter completes his part of the task.	(b) Transcribe the hourly fraction of wall painted by the first painter; state how many times he paints such fraction, and indicate at what times he begins and finishes the task.
(c) Repeat steps "a" and "b" with the second painter.	(c) Repeat steps "a" and "b" with the second painter.	(c) Repeat steps "a" and "b" with the second painter.
(d) In the equation, mark the part that represents the sum of the fractions painted by both painters.	(d) In the equation, mark the part that represents the sum of the fractions painted by both painters.	(d) In the equation, mark the part that represents the sum of the fractions painted by both painters.
(e) Simulate with the strips how both painters worked together on an hour-by-hour basis.	(e) Carry out an internal simulation both painters working together on an hour-by-hour basis.	(e) Repeat step "b" for the first and the second painters.
Transfer phase: (identical for the 4 TPs)	*Transfer phase:* (identical for the 4 TPs)	*Transfer phase:* (identical for the 4 TPs)
5. Qualitative simulation of the TP with realistic physical objects (see step 1)	5. Qualitative simulation of the TP carried out internally (see step 1)	5. Equivalent non-simulation tasks for the TP (see step 1).
6. Repeat qualitative simulation of the BP with realistic physical objects, indicating any relevant differences with the TP.	6. Repeat qualitative internal simulation of the BP, indicating any relevant differences with the TP.	6. Equivalent non-simulative tasks for the BP, indicating relevant differences between current TP and the BP.
7. Participants attempt to build an equation for the TP (6 min).	7. Participants attempt to build an equation for the TP (6 min).	7. Participants attempt to build an equation for the TP (6 min).
8. Participants indicate how the equation, when solved, should yield an answer to the problem.	8. Participants indicate how the equation, when solved, should yield an answer to the problem.	8. Participants indicate how the equation, when solved, should yield an answer to the problem.

Note. BP = base problem; TP = target problem.

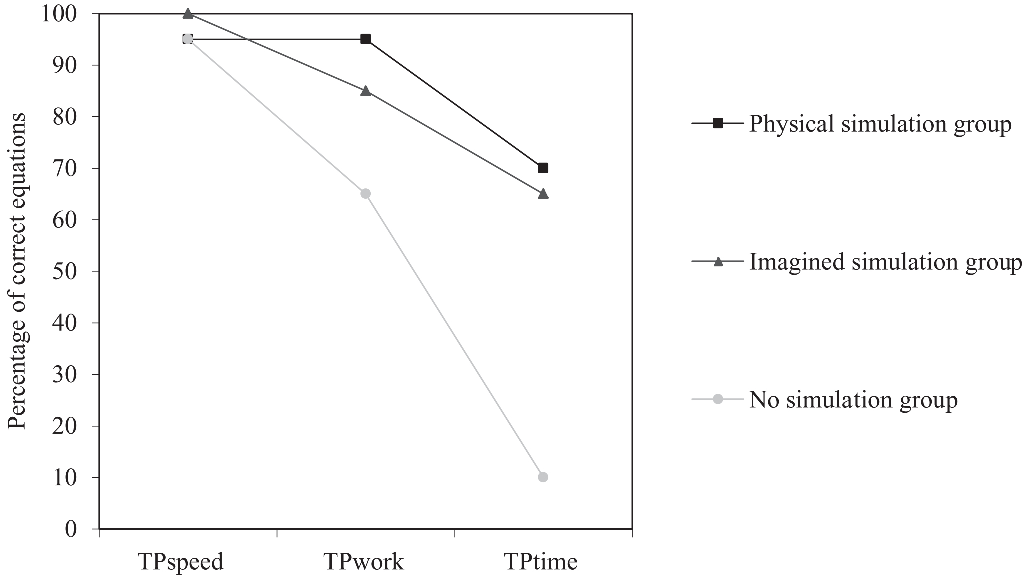

FIGURE 2. Percentages of correct solutions in the three conditions of the experiment.

Paired comparisons revealed that the PSG (83% of correct answers) outperformed the NSG (38%), $p < .001$ (Bonferroni adjustments). The ISG (75% correct) also outperformed the NSG, $p < .001$. These results confirm our hypothesis that promoting situation model simulations aids transfer of a base solution to structurally different TPs. Bonferroni comparisons also revealed that performance in both simulation groups did not differ, $p > .05$.

To determine whether the promotion of transfer advantage found in situation model simulation was higher for structural variations bearing greater adaptation difficulty, two separate 2 × 2 ANOVAs with repeated measures on the factor adaptation difficulty were conducted. One ANOVA compared NSG with ISG, and the other compared NSG with PSG.

The 2 × 2 ANOVA with type of situation model simulation (no simulation and imagined simulation) as a between-subjects factor and adaptation difficulty (medium and high) as a within-subjects factor revealed a significant interaction, $F(1, 40) = 5.712$, $MSE = 0.107$, $p < .05$. Bonferroni comparisons found that the ISG outperformed the NSG in TP*time* (65% vs. 10%), $p < .001$, but not in TP*work* (85% vs. 65%), $p > .05$. Contrasts for paired samples revealed that performance in TP*work* was higher than that in TP*time*, both within the ISG, $t(19) = 2.179$, $p < .05$, and the NSG, $t(19) = 4.819$, $p < .001$. Similarly, a 2 × 2 ANOVA with type of simulation (no simulation and physical simulation) as a between-subjects factor and adaptation difficulty (medium and high) as a within-subjects factor revealed a strong tendency toward interaction between these factors, $F(1,40) = 3.931$, $MSE = 0.114$, $p = .055$. Bonferroni comparisons showed that the PSG reliably outperformed the NSG both in TP*time* (70% vs. 10%), $p < .001$, and TP*work* (95% vs. 65%), $p < .05$. Contrasts for paired samples showed that performance on TP*work* was higher than on TP*time*, both within the PSG, $t(19) = 2.517$, $p < .05$, and the NSG, $t(19) = 4.819$, $p < .001$. The observed interactions suggest that the advantage of simulating situation models is greater for variations posing a greater adaptation difficulty. The simulations carried out by the PSG revealed that participants correctly simulated the qualitative situation model of the TPs as well as their relevant differences with that of the BP in 95% of the trials. The quantitative simulation of the BP and its solution was carried out successfully in 90% of the trials.

Reed et al. (1985) documented the serious difficulties faced by college students when it comes to assimilate a structural variation into a learned equation. Several authors (e.g., Koedinger et al., 1997; Nathan, 1998; Nathan et al., 1992; Reed, 1999) have suggested that stimulating students to ground algebraic formalisms in the situation models of the problems helps them build equations for word problems. In line with this finding, this study set forth to determine whether a stimulation to ground equation structures in the situation models of the problems would also help participants in adapting the learned solutions to nonisomorphic TPs, a task that had proven immune to several prior interventions (see, e.g., Reed, 1999, 2006). To that end, we presented three groups of participants with a worked example of task-completion algebra problems. In addition to studying the BP and its solution, participants in two experimental conditions were asked to simulate the situation model of the BP and to explicitly relate critical aspects of such simulation to their counterparts in the learned equation. Whereas the PSG had to perform such simulations via manipulation of physical materials, the ISG had to carry out those simulations internally. During the transfer phase, participants of all groups received three TPs introducing structural variations, which demanded adapting the base equation to different extents. Whereas for some of the TPs, assimilating the structural variations only required comprehending a few isolated components of the base equation, in other TPs, assimilating the structural variation required a more articulated and complete comprehension of the equation structure. After reading each of the TPs—but before attempting to generate the appropriate equations—participants were again encouraged to simulate the situation model of the TP, and to contrast it against that of the BP to pinpoint their differences. Participants of a control group received the same worked example and the same TPs as the simulation groups but with the difference that for each simulation activity carried out by the experimental groups, the control group had to perform a nonsimulation activity that was equivalent to the simulations in terms of the inferences and the reconsideration of problem information it promoted.

The situation models generated by participants of the PSG demonstrated that students in our study were able to build appropriate situation models for the BP and TPs, and to compare them to pinpoint their differences. This first result suggests that student's tendency not to ground their formalisms in situation representations of the problems is somewhat dispositional and not the result of cognitive limitations (see, e.g., Walkington, Sherman, & Petrosino, 2012). However, the fact that they succeeded in generating such representations should not be taken to prove that building appropriate simulations is attainable by any learner and for any type of problem. As it occurs with the comprehension of the diagrams that complement the information provided by texts, the generation of simulations for increasingly complex problems might rely on several individual skills, such as spatial ability, working memory capacity, or training (see Acevedo Nistal, Van Dooren, Clarebou, Elen, & Verschaffel, 2009, for a review). Research in both mathematics education and cognitive development suggests that students sometimes need effective instruction to correctly understand or generate external representations (Fueyo & Bushell, 1998; Uttal, Scudder, & DeLoache, 1997).

The main target of our study was to assess whether a set of activities promoting a connection between situation models and equation structures would help students adapt the learned solutions to novel problems introducing structural variations. As in the case of diagrams (see, e.g., Beckmann, 2004), we reasoned that simulations of situation models could help participants in deciding what operations to use and in evaluating whether those operations are conceptually sound. As predicted, the groups that were asked to ground equation structures in the situation models of the problems outperformed a control group not required to carry out such simulations. An analysis of the relation between participants' solution strategies and their relative probabilities of leading to correct equations can shed light on the specific ways through which our intervention might have helped participants in adapting the learned solutions to novel problems. In the case of TP*work*, the challenge consisted in introducing into the basic equation the fraction of wall that had already been painted. As was mentioned in a previous section, this fraction can be introduced either into the left term of the basic equation (Strategy 1) or into the right term, albeit with the opposite sign (Strategy 2). An inspection of participants' solutions showed that the probability of placing the appropriate sign to the inserted fraction differed across strategies. Although 97% of participants' attempts to insert the fraction into the right term involved the appropriate sign (the "−"), only 25% of the attempts to insert it into the left term led to the correct sign (the "+"). Now focusing on differences between groups, protocols revealed that although participants of the NSG showed a slight preference for the former strategy, participants in the simulation conditions almost invariably attempted to introduce the prepainted fraction into the right term (i.e., the alternative that had higher chances of leading to the correct sign). In light of this difference, we speculate that our intervention to ground equation structures in situation models might have helped participants in interpreting the right term as the total work to be completed and thus to regard it as the natural place from which to subtract the portion of wall that was already painted.

Regarding TP*time*, the challenge rests in the fact that the second painter works 1 hr more than the first. This variation to the basic problem can be assimilated by setting the working time of the second painter to be $t_1 + 1\text{h}$ (Strategy 3) or by introducing the work that resulted from this extra hour either into the left term (Strategy 1) or into the right term of the equation (Strategy 2), albeit with a different sign. As with TP*work*, participants' solutions showed that these strategies had different probabilities of leading to a successful equation. While attempts to follow Strategies 1 and 2 were infrequent but always successful, the attempts to follow Strategy 3 were successful in only 49% of the cases, with all errors originating in an omission of the parentheses.

The superior performance of the simulation groups stems from a more accurate use of parentheses during attempts to introduce the extra hour directly (Strategy 3) as well as from a more frequent use of Strategy 1 than in the NSG. Thus, we speculate that the grounding of equations in situation models might have helped participants represent the extra hour of the second painter as a separate chunk of painted wall of size $r_2 \cdot 1$ hr, leading either to introduce this expression into the left term of the equation (Strategy 1) or else to acknowledge the anomaly that would arise from omitting the parentheses that are required to insert the extra hour directly (Strategy 3).

The present results extend prior research by Nathan and colleagues (Nathan, 1998; Nathan et al., 1992) in two ways. On the one hand, they demonstrate that the advantage of encouraging students to generate situation models of the base and TPs goes beyond helping them generate proper equations for task-completion problems: It can help them transfer a learned solution to nonisomorphic problems introducing structural variations. On the other hand, the fact that our participants benefitted from such simulations without receiving any kind of feedback suggests that this type of intervention could in principle be applied to standard noninteractive instructional contexts (e.g., classroom activities and homework problem-solving), far easier to implement than computer-based tutoring systems.

As was suggested, one of the main transfer advantages of carrying out the requested simulations might have consisted in fostering an accurate conceptualization of the relevant similarities and differences between the BP and the TPs, thus assuring greater control during the process of assimilating the critical differences into the base equation. It should be taken into account, however, that the transfer performance of our independent group was higher than the performance obtained by Reed et al. (1985) on very similar problems. Just as in Booth and Koedinger's (2012) study on the benefit of diagrams, it is possible that taking advantage of interventions of the type pursued in this study requires some basic mathematical skills. Further studies would be necessary to ascertain whether the benefit of simulating the situation models of the problems may also generalize to students with weaker mathematical abilities.

As opposed to the high amount of surface features shared by the TPs and the BP used in this study, the presence of perceptual and thematic mismatches might complicate both noticing the relevant similarities between the problems and adequately conceptualizing the critical differences that the problems might maintain. For example, consider asking participants to transfer a learned procedure from the BP used in this study to a hypothetical TP that is isomorphic to our TP_{work}, but in which two pipes jointly fill a tank at different rates, with a given fraction of the tank already filled at the onset of the process. Even if participants were invited to simulate both problems with highly realistic manipulatives (e.g., toy painters + wall + paint and toy pipes + tank + water, respectively) and even to tell the differences between them, it is likely that the change in content will add some opacity to the partial isomorphism between the problems. Fyfe, McNeil, Son, and Goldstone (2014) have thoroughly discussed the trade-off between realistic and idealized simulations: While grounding an abstract principle in realistic representations is best for eliciting an adequate comprehension of the principle but suboptimal for later recognizing such principle when embedded in a different content, the opposite is true for idealized representations. As demonstrated in a recent series of studies (e.g., Braithwaite & Goldstone, 2013; McNeil & Fyfe, 2012; see Fyfe et al., 2014, for a review), this trade-off can be circumvented by means of starting off with realistic simulations, and progressively fading away their concreteness in favor of more abstract, or general, representations. Going back to our example, it is conceivable that a progression from rather realistic simulations such as those used in the PSG and the ISG of this study toward more idealized, "stripped away" representations of the BP (e.g., two bars or dots approaching each other at different speeds) could help students acknowledge the relevant similarities between seemingly distinct situations, while at the same time allowing an appropriate conceptualization of eventual structural differences between the problems.

The second objective of our experiment was to investigate whether an internal simulation of situation models would be as effective as a physical simulation in promoting analogical transfer. In line with previous studies comparing the benefits of physical and mental simulations on text comprehension (Glenberg et al., 2004), comprehension of metaphors (Wilson & Gibbs, 2007), and solution to simple arithmetic problems (Glenberg et al., 2011), we found that both types of simulations yielded comparable effects on analogical transfer to nonisomorphic algebra problems. The fact that performance of the ISG was not inferior than performance of the PSG suggests that at least for problems not prone to overloading working memory (e.g., the task-completion problems included in this study), transfer performance can be boosted by rather austere interventions, which do not require investing in expensive manipulative objects. Given the sufficiency of mental simulations for representing the situation models of certain problems, a sensible educational goal could consist in promoting participants' disposition to carry out these activities when appropriate. However, external representations may prove more beneficial than imagined simulations for problems whose situation models impose a greater cognitive load (e.g., a similar TP but with four painters instead of two). Inviting students to decide in what cases physical representations can be safely surrogated by internal simulations seems to be a powerful way of promoting a greater understanding of representations in general, as well as of the relative advantages and disadvantages of different types of representations for problem solving (see e.g., diSessa & Sherin, 2000).

The third objective of our study consisted in assessing whether the effectiveness of an intervention to ground equation structures in the situation models of the problems interacts with the intrinsic transfer difficulty of the TPs. In a recent study using diagrams, Booth and Koedinger (2012) found that the advantage of complementing story problems with such representations was greater for the more difficult problems. Consistent with such results, our intervention to ground equation structures in the situation models of the problems was maximally useful for the problems that had proven to impose a greater transfer difficulty to an independent group of participants. Our interpretation of this interaction is based on the different levels of depth at which the equation of task-completion problems can be represented (Figure 1). Based on this hierarchical representation, we had predicted a gradient of transfer difficulty across TPs, which was confirmed by an independent group of participants. The fact that the relative effectiveness of our instructional intervention across TPs reproduced this exact ordering invites hypothesizing that the importance of grounding equation structures in the situation models of the problems is greater for structural variations whose incorporation into the learned equation demands a deeper knowledge of the equation structure, that is, more levels in Reed's (1987) hierarchical representation of task-completion problems. Future studies should explore whether the predictive power of this kind of hierarchical representations generalizes to word problems governed by different equation structures.

Taking a broader perspective, the observed variation in transfer difficulty across TPs also relates to the theoretical discussion about the relative difficulty of the component subprocesses of analogical reasoning. After analyzing several results from analogical problem solving in algebra and combinatorics, Holyoak, Novick, and Melz (1994) argued that the adaptation subprocess is, in general, a more difficult subprocess than mapping and inference generation. Albeit indirectly, our results also shed light on the intrinsic difficulty of adaptation and the extent to which it can vary. Taking into account that most participants in the NSG were able to map the elements of the TPs onto their counterparts in the BP as well as to detect the critical differences between the problems—two preconditions that should be met when assessing the difficulty of the adaptation process—the observed variability in transfer performance across TPs suggests that adaptation can range from the very difficult to the very easy. As before, we conjecture that at least within the domain of algebra word problems, part of this variability relates to the number of different layers in the representation of the base equation that should be comprehended to assimilate a given structural variation into the equation. Beyond its relevance for the study of analogical transfer, advancing our understanding of the cognitive demands posed by each particular structural variation seems crucial for honing in on the optimal sequence of practice exercises to be presented during instruction, thus fostering a gradual and complete exploration of the variables and relations involved in the structure of the problems.

NOTE

1. The intervention assessed in this study was not limited to the encoding of the base analog. In a way similar to other interventions aimed at alleviating adaptation difficulties (e.g., Reed & Bolstad, 1991; Reed & Ettinger, 1987), our intervention also involved the processing of target analogs.

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